CSCE 222-200, Discrete Structures for Computing, Honors
Fall 2021

Homework 2

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Instructions:

- The exercises are from the textbook. MAKE SURE YOU HAVE THE CORRECT EDITION! You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Each exercise is worth 5 points.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance on the cover sheet, and write up your solutions on your own.
- *Turn in your pdf file on Canvas by 3:00 PM on Wednesday, Sep 22.*

LaTeX hints: Read this .tex file for some explanations that are in the comments.

Math formulas are enclosed in $ signs, e.g., `$x + y = z$` becomes $x + y = z$.

Logical operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$.

Here is a truth table using the "tabular" environment:

p	$\neg p$
T	F
F	T

**** Delete the instructions and the LaTeX hints in your solution. ****

Exercises for Section 1.6 (pp. 82–84):

12: (Use the result of Exercise 11 without proving it. I found it easier to first use the rules of inference and then use Exercise 11.)

$(p \wedge t) \rightarrow (r \vee s)$	Premise
$q \rightarrow (u \wedge t)$	Premise
$u \rightarrow p$	Premise
$\neg s$	Premise
q	Rremise (result from exercise 11)
$u \wedge t$	Modus Ponens
u	Simplification
t	Simplification
p	Modus Ponens
$p \wedge t$	Conjunction
$r \vee s$	Modus Ponens
r	Disjunctive Syllogism

20(a): It is not a true statement. Even if a^2 is a positive real number the square root can be a negative number.

28: (Hint: Work backwards, assuming that you did end up with nine zeros.)
$\neg R(x) \rightarrow \forall x(P(x) \vee \neg Q(x))$, the contrapositive of the initial premise.
Since $\forall x(P(x) \vee Q(x))$ is a premise, it has to be that $\forall x(\neg R(x) \rightarrow P(x))$. It is implied that the value really depends on $P(x)$ since $Q(x)$ appears in both its original and negated form in the statements.

Exercises for Section 1.8 (pp. 113–117):

6:

a	b	c	$min(a, min(b, c))$	$min(min(a, b), c)$
1	0	-2	-2	-2
-1.4	-3.5	-7.4	-7.4	-7.4
0	0	0	0	0
1.1	1.2	1.3	1.1	1.1

8: If $x = 2$ and $y = 3$, we have $5(2) + 5(3) = 25$. Without loss of generality, we do not have to test a case where x is odd and y is even as it is identical to this case.

28: Proof by contradiction. Assuming we started with 9 0s, then the previous configuration would've had to be all 1's. In order for that configuration to be possible, its previous configuration would have to have alternating 1's and 0's without any consecutive element being the same. However, since there are 9 positions, two of the same numbers are always consecutive. Hence, it is impossible to end up at a configuration of 9 0s.
36: Using a proof by contradiction.
Assume $\sqrt[3]{2} = \frac{a}{b}$ where a and b are relatively prime positive integers.
It follows that $2b^3 = a^3$ by multiplying both sides by b and cubing them.
This equation shows that a^3 is an even integer, implying that $a = 2c$.
The first equation be rewritten with this substitution as $b^3 = 4c^3$. This implies that b is also even.
Hence, since a and b are even integers, we know that they are not relatively prime, so the cube root of 2 is irrational.

Exercises for Section 2.1 (pp. 131–133):

10(b)–(f): Be careful, as $\{2\}$ is NOT the same as 2.
b: 2 is not an element of the set.
c: 2 is an element of the set.
d: 2 is not an element of the set.
e: 2 is not an element of the set.
f: 2 is not an element of the set.

26: Only subdivision d is a powerset of the set a, b.

50: If S is member of S, this leads to a contradiction. S contains the set of sets that do not belong to itself, but in this case S belongs to itself and so shouldn't be in the set. However, if S were not in the set S, then it is a set that doesn't contain itself and should be in the set S, hence it's contradiction either way.

Exercises for Section 2.2 (pp. 144–146):

34:
Using the provided identity, $(A - B) \cap (B - C) \cap (A - C)$ turns to $(A \cap \widetilde{B}) \cap (B \cap \widetilde{C}) \cap (A \cap \widetilde{C})$. Using the commutative law we get $(A \cap \widetilde{A}) \cap (B \cap \widetilde{B}) \cap (C \cap \widetilde{C})$. The intersection of a set and its negation is the empty set. Hence, the statement simplifies to the empty set.

72(e): Only consider the special case where A is a subset of B and B is a subset of C.
If the cardinality of A, B, and C are 1, 2, and 3 respectively, it is easy to construct an inequality.

This also implies that A is a proper subset of B and B is a proper subset of C. The inequaltiy that forms is $\frac{2}{3} \leq \frac{5}{6}$ which is true. In the case where cardinality of all three sets is 1 or 0, then we get $0 \leq 0$.

Exercises for Section 2.3 (pp. 161–164):

34 a: Every value being able to map from A to C implies that for every value c in C, there is a b in B such that f(b) = c. b is derived from a value x in A, such that g(x) = b. Therefore, for every element c in C, there is an x in A, such that f(g(x)) = c implying that f is onto.
34 b: If f(g(x)) is one-to-one this means that f(a) = f(b) implies a = b. For a and b, there are values in A, x and y, such that g(x) = a and g(y) = b. Since a = b, g(x) = g(y) when g is one-to-one. This means x = y, and so if f(g(x)) is one-to-one, g(x) is one-to-one.
34 c: If f(g(x)) is a bijection, then for every c in C, there is a x in A such that f(g(x)) = c and if f(g(a)) = f(g(b)) then g(a) = g(b). If g(x) is onto, this means for every b in B, there is an x in A such that g(x) = b. This means if f(g(x)) is a bijection then f(a) = f(b) implies a = b. a and b are elements in B. There is an x and y in A such that g(x) = a, and g(y) = b. Hence, g(x) = g(y). Therefore, if g is onto then f is one-to-one.

Assuming f is one-to-one, this means we know that f(g(x)) = f(b) implies g(x) = b. Hence, since f(g(x)) is a bijection, then for every element b in B there must be a x in A such that g(x) = a. Hence, the statement that if f(g(x)) is a bijection then f is one-to-one iff g is onto is true.

76(e): This is a false statement. Suppose $x = 0.5$ and $y = 0.5$, then $\lfloor 0.5 \rfloor + \lfloor 0.5 \rfloor + \lfloor 1 \rfloor = \lfloor 2(0.5) \rfloor + \lfloor 2(0.5) \rfloor$. This leads to the statement $1 = 2$ which is false.