

CSCE 222-200: Discrete Structures for Computing, Honors Assignment Cover Page Fall 2021

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| Assignment: | Homework 1 |
| Grade (filled in by grader): | |

Please list below all sources (people, books, webpages, etc) consulted regarding this assignment (use the back if necessary):

| CSCE 222 Students | Other People | Printed Material | Web Material (give URL) | Other Sources |
|--------------------------|---------------------|-------------------------|--------------------------------|----------------------|
| 1. | 1. | 1. | 1. | 1. |
| 2. | 2. | 2. | 2. | 2. |
| 3. | 3. | 3. | 3. | 3. |
| 4. | 4. | 4. | 4. | 4. |
| 5. | 5. | 5. | 5. | 5. |

Recall that TAMU Student Rules define academic misconduct to include acquiring answers from any unauthorized source, working with another person when not specifically permitted, observing the work of other students during any exam, providing answers when not specifically authorized to do so, informing any person of the contents of an exam prior to the exam, and failing to credit sources used. *Disciplinary actions range from grade penalty to expulsion.*

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work. In particular, I certify that I have listed above all the sources that I consulted regarding this assignment, and that I have not received or given any assistance that is contrary to the letter or the spirit of the collaboration guidelines for this assignment."

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| Signature: | Aakash Haran |
| Date: | 09-08-2022 |

CSCE 222-200, Discrete Structures for Computing, Honors

Fall 2021

Homework 1

Aakash Haran

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Email: ash3498@tamu.edu

Assignment: Homework 1

Resources used: Discrete Mathematics and its Applications, Eight Edition, Kenneth Rosen

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Signature: Aakash Haran **Date:** 09-03-2021

Instructions:

- The exercises are from the textbook. **MAKE SURE YOU HAVE THE CORRECT EDITION!** You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Each exercise is worth 5 points.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance on the cover sheet, and write up your solutions on your own.
- *Turn in your pdf file on Canvas by 3:00 PM on Wednesday, Sep 8.*

Exercises for Section 1.1 (pp. 13–17):

34(f):

| p | q | $p \leftrightarrow q$ | $p \leftrightarrow \neg q$ | $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$ |
|-----|-----|-----------------------|----------------------------|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

44: *Hint:* If you need to gain intuition for what the formula is stating, consider the case when $n = 3$ (when $i = 1$, j can take on the values 2 and 3, and when $i = 2$, j can take on the value 3) and the case when $n = 4$.

If p_1 to p_n were all false, the expression would still evaluate to true as each of the OR statements would evaluate to true and in turn the AND statements using these results would be true. If at most one expression p_i were true it would still be a tautology as every OR statement would be still be true as it takes the maximum of the two input truth values. However, if two statements p_i and p_j are true, then $p_i \vee p_j$ would evaluate to false and hence the AND statement with that result would result in false, hence changing the result.

50: We can assume happiness is given a truth value of 1, True and sadness was given 0, False though the inverse could be true as well. Then the result would be True for "Fred and John are happy" and False for "Neither Fred nor John or happy".

Exercises for Section 1.2 (pp. 23–26):

18(c): If exactly two inscriptions are true, one inscription is false, which is easier to think about. If Trunk 1 is false, there must be treasure in Trunk 1 according to itself, Trunk 2 says there is treasure in Trunk 1 which must be a true statement and agrees with Trunk 1, and Trunk 3 says there is treasure in Trunk 2, which doesn't contradict any of the propositions. If Trunk 1 is the false statement, the treasure is in Trunk 1 and 2.

If Trunk 2 is false, then there is no treasure in Trunk 1. This agrees with Trunk 1, which says there is no treasure in Trunk 1. Trunk 3 says there is treasure in Trunk 2, which doesn't disagree with any statement. Hence, if Trunk 2 is false, then the treasure must be in Trunk 3 and Trunk 2.

If Trunk 3 is false, then there must not be treasure in Trunk 2. However, Trunk 2 and 1 must be true statements. If Trunk 2 says there is treasure in Trunk 1 and Trunk 1 says there is not treasure in Trunk 1, there is a contradiction. Hence, this case is impossible.

Exercises for Section 1.3 (pp. 38–40):

6:

| p | q | $\neg(p \wedge q)$ | $\neg p \vee \neg q$ |
|-----|-----|--------------------|----------------------|
| T | T | F | F |
| T | F | T | T |
| F | T | T | T |
| F | F | T | T |

The third and fourth columns are identical, proving De Morgan's Law.

12(d):

| p | q | r | $p \vee q$ | $p \rightarrow r$ | $q \rightarrow r$ | $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ |
|-----|-----|-----|------------|-------------------|-------------------|--|
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | T |
| T | F | T | T | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | T | T | T | T |
| F | T | F | T | T | F | T |
| F | F | T | F | T | T | T |
| F | F | F | F | T | T | T |

16(d):

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

$$\text{Using identity } (p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$$

$$[(p \vee q) \wedge [(p \vee q) \rightarrow r]] \rightarrow r$$

$$\text{Using identity } p \rightarrow q \equiv \neg q \vee p$$

$$\neg[(p \vee q) \wedge [\neg(p \vee q) \vee r]] \vee r$$

Using the Associative Law

$$\neg[(p \vee q) \wedge \neg(p \vee q)] \vee r \vee r$$

Simplifying and applying the Identity Law

$$\neg(F \vee r) \vee r$$

$$\neg r \vee r \equiv True$$

Exercises for Section 1.4 (pp. 56–60):

10(c): $\exists x(C(x) \wedge F(x) \wedge \neg D(x))$

20(c): For every x in the domain, when x is not equal to 1, the proposition $P(x)$ is evaluated to true.

46: The two statements are not logically equivalent.

The logical statement $\forall x(P(x) \leftrightarrow Q(x))$ can be rearranged to $\forall x(P(x) \rightarrow Q(x)) \wedge \forall x(Q(x) \rightarrow P(x))$. The statement $\forall xP(x) \leftrightarrow \forall xQ(x)$ can be rearranged to $(\forall xP(x) \rightarrow \forall xQ(x)) \wedge (\forall xQ(x) \rightarrow \forall xP(x))$. Since distributing the universal quantifier over the implies connector changes the logical statement, it is not possible to get the second simplification from the first. $\forall x(P(x) \rightarrow Q(x)) \not\equiv (\forall xP(x) \rightarrow \forall xQ(x))$.

Exercises for Section 1.5 (pp. 68-72):

20(c): $\forall x \exists y [((x < 0) \wedge (y < 0)) \rightarrow (x - y > 0)]$

20(d): $\forall x \forall y (|x + y| \leq |x| + |y|)$

30(e):

The original proposition: $\neg \exists y (\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

The new proposition: $\exists y (\forall x \exists z \neg T(x, y, z) \wedge \exists x \forall z \neg U(x, y, z))$

52: $\exists x(P(x) \wedge \forall y((y \neq x) \rightarrow \neg P(y)))$