CSCE 222-200, Discrete Structures for Computing, Honors
Fall 2021
Homework 5
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Instructions:

- The exercises are from the textbook. MAKE SURE YOU HAVE THE CORRECT EDITION! You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Each exercise is worth 5 points.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance on the cover sheet, and write up your solutions on your own.
- *Turn in your pdf file on Canvas by 3:00 PM, Wednesday, November 10.*

LaTeX hints: Read this .tex file for some explanations that are in the comments.

Math formulas are enclosed in $ signs, e.g., `$x + y = z$` becomes $x + y = z$.

Logical operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$.

Here is a truth table using the "tabular" environment:

p	$\neg p$
T	F
F	T

**** Delete the instructions and the LaTeX hints in your solution. ****

Exercises for Section 6.1 (pp. 416–420):

22 (b), (c), (d), (e), (f):
130, 12, 220, 208, 780

32 (a), (b), (c), (d), (e):
26^8**,** $26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$**,** $26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20$**,** $25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$**.**

Exercises for Section 6.2 (pp. 426–428):

4 (a) and (b):
$5, 13$

12: ** YOUR ANSWER GOES HERE **

Exercises for Section 6.3 (pp. 435–437):

10: $6!$

12a: $C(12, 3)$
12b: $C(12, 0) + C(12, 1) + C(12, 2) + C(12, 3)$
12c:

$$\sum_{n=4}^{12} C(12, n)$$

12d: $C(12, 6)$

34 (a), (b), (c) and (d): For (a), interpret as "at least one a"; for (b), interpret as "at least one a and at least one b".
34a: $C(6, 1) \times 26^5$
34b: $C(6, 1) \times C(5, 1) \times 26^4$
34c: $C(5, 1) \times 24 \times 23 \times 22 \times 21$
34d: $15 \times 24 \times 23 \times 22 \times 21$

Exercises for Section 6.4 (pp. 443–445):

6: 330

14: $C(100, \frac{k}{2})$

18: Each term in the sequence can be written by $a_n = a_{n-1} \times \frac{n-k}{k+1}$ and the value of k increases by 1 for each next term. While $k < \frac{n}{2}$ the value of $\frac{n-k}{k+1}$ is greater than 1, so each next term is increasing and when $k = \frac{1}{2}$, the values of the term and the previous is equal. When $k > \frac{1}{2}$ the value of the multiplicative constant is less than 1 and the sequence starts decreasing.
20a: Using the result from exercise 18, we know that the largest term in the sequence is $C(n, \lfloor n/2 \rfloor)$ and that there are n terms. The sum of all the terms is upper bounded by $sum = C(n, \lfloor n/2 \rfloor) \times n$. We can then say that $\frac{sum}{n} \leq C(n, \lfloor n/2 \rfloor)$.
20b: We can adjust the result from question 18, $\sum_{k=0}^{2n} C(2n, k) = 2^{2n} = 4^n$. The upper bound for the sum of the terms is $sum = C(2n, n) \times n$. Thus, $\frac{sum}{2n} \leq C(2n, n)$.

36:
Attempting to prove the statement for all values of n through regular induction.
Statement: $P(n)$ says $(x + y)^n = \sum_{k=0}^{n} C(n, k)x^{n-k}y^k$
Base case: $(x + y)^0 = C(0, 0)x^{0-0}y^0 = 1$. The statement is true for the base case.
Inductive Step: Assume $P(0)$ to $P(n)$ is true.
Attempting to prove $P(n + 1)$ which is $(x + y)^{n+1} = \sum_{k=0}^{n+1} C(n + 1, k)x^{n+1-k}y^k$
$(x + y)^{n+1}$ can be expressed as $(x + y)^n(x + y)$. $(x + y)^n$ follows the binomial expansion properly from the inductive hypothesis.
Looking at the first few terms in the expansion $C(n, 0)x^n + C(n, 1)x^{n-1}y^1 + C(n, 2)x^{n-2}y^2 + \cdots$ it becomes to easy that in each term after the first, multiplying that term by x and the previous term by y ensures that they are capable of being added. Multiplying $(x + y)^n$ by $x + y$ does exactly this and makes each consecutive term capable of being added together. Using the property of Pascal's Triangle that consecutive elements added create a new row following the pattern of binomial expansion, we know that that is exactly what's happening in this expansion.
Hence, the statement $P(n)$ is true for all n.

Exercises for Section 6.5 (pp. 454–457):

10 a: 6^{12}
10 b: 6^{36}
10 c:

18: $20! \div 2! \div 4! \div 3! \div 2! \div 3! \div 2! \div 3!$

24: $C(17, 12)$

26: $C(15, 5) \times C(10, 4) \times C(6, 3) \times C(3, 2)$

48: $7! \times 2!$

Exercises for Section 9.1 (pp. 608–610):

6d: Vacuously antisymmetric.
6e: Symmetric, reflexive, transitive
6f: Symmetric, transitive
6h Symmetric, transitive

30a: $\{$(1, 2), (2, 3), (3, 4), (1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)$\}$
30b: $\{$(1, 2), (2, 3), (3, 4)$\}$

32: $\{$(1, 1), (1, 2), (2, 1), (2, 2)$\}$

52b: Both relations should have (a, a) for every a in A. Therefore they are included in the intersection.
52e: If (a, a) is in R then (a, a) is in S and hence the composite will contain (a, a).

Exercises for Section 9.2 (pp. 619–621):

8a: ISBN
8b: If the title and publication date pair for every book is unique.
8c: If the title and number of pages pair for every book is unique.
12 ** YOUR ANSWER GOES HERE **

16 ** YOUR ANSWER GOES HERE **

Exercises for Section 9.3 (pp. 629–627):

4 (a) $\{$(1, 1), (3, 3), (4, 4), (1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (4, 3), (3, 4) $\}$

14a:

0	**1**	**0**
1	**1**	**1**
1	**1**	**1**

14b

0	**1**	**0**
0	**1**	**1**
1	**0**	**0**

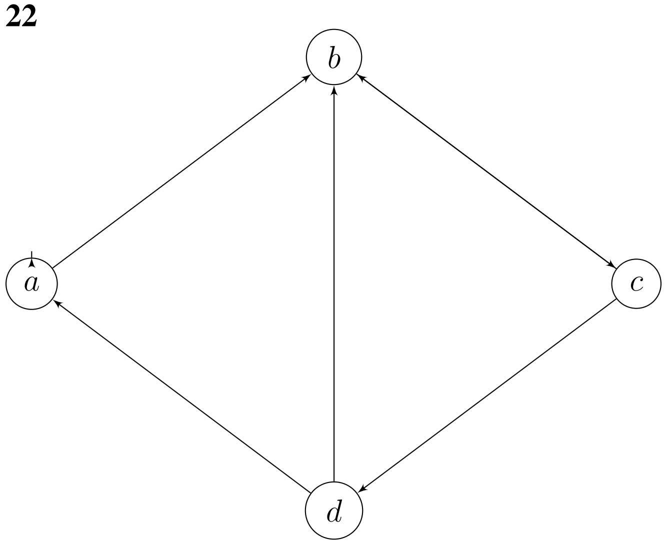
14c

0	**1**	**1**
1	**1**	**1**
0	**1**	**0**

14d

1	**1**	**1**
1	**1**	**1**
0	**1**	**0**

22

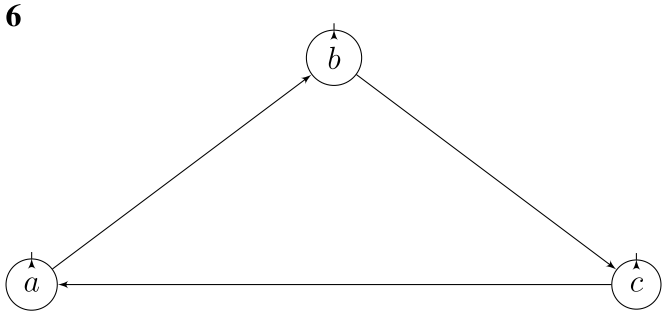


26 $\{$(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, a), (c, d)$\}$

32 (only for the graph in 26 and only for reflexive, symmetric, antisymmetric, and transitive)
The graph represents reflexive relation.

Exercises for Section 9.4 (pp. 637–638):

6



22 ** YOUR ANSWER GOES HERE **

26 (c) ** YOUR ANSWER GOES HERE **
