CSCI 3104 PS1b

Luna Mcbride

TOTAL POINTS

24 / 44

QUESTION 1

34 pts

1.1 8 / 8

✓ **+ 8 pts** **Correct**

+ 4 pts Correct python code

+ 4 pts Correct Table/Graph

+ 0 pts Incorrect Table

+ 2 pts Code has only a single loop. The flips are counted only for flips that are consecutive.

+ 0 pts Incorrect answer please go through the solutions provided.

+ 0 pts Plagiarism

+ 4 pts The python code is right.

+ 2 pts The graph/Table should have had points untill 2^12

+ 2 pts Code is incorrect only consecutive flips are counted even though two loops are present

+ 0 pts Please submit .py files and not .ipynb files

+ 3 pts Partially correct python code

+ 3 pts Partially correct Table

+ 2 pts Generating numbers in the array only between 1-10. A size n array should have all the numbers from 1-n in shuffled order

+ 2 pts python code loop partially correct

+ 0 pts not attempted

+ 1 pts python code to create the shuffled array

+ 0 pts Code not submitted on canvas but table present

+ 0 pts code has to be submitted in canvas. A table/Graph need to be submitted here. Unable to go through your code because special characters are missing.

Please refer to the solution for a simpler way to do the same.

1.2 3.5 / 4

+ 4 pts Good work, Correct!!!

+ 0 pts Empty or incomplete solution submitted. Please refer to the solution file.

+ 3 pts In the correct direction. But mention a value in terms of n by solving the mentioned formula completely. Please refer to the solution file.

✓ **+ 3.5 pts** **In the correct direction. But evaluate the mentioned formula perfectly. Please refer to the solution file.**

+ 1 pts Incorrect assumption for greatest number of flips.

+ 0 pts Incorrect assumption for greatest number of flips. Please refer to the solution file.

+ 2 pts Correct assumption but wrong or incomplete mathematical evaluation.

+ 2 pts Thinking in the right direction but more specific and detailed answer is expected

+ 2 pts Thinking in the right direction but more detailed analysis needs to be done to arrive at the right answer

+ 3.5 pts Incorrect formula

+ 1 pts Thinking in the right direction but more mathematical answer is expected

1.3 0 / 10

+ 10 pts Correct

+ 3 pts Swap performed correctly

+ 3 pts Correctly iterates through first loop and second loop does not give index out of bounds

+ 2 pts Second loop iterates over all relevant elements

+ 2 pts Indexes second loop by subtracting i and ignoring largest element (Conditional on rubric 4 being correct)

✓ **+ 0 pts Incorrect Solution**

- 10 pts Plagiarism

Does not use nested loops over the array: Incomplete solution.

1.4 1 / 4

+ 4 pts Good work!!!

+ 0 pts Empty or incomplete answer submitted. Please refer the solution file.

+ 2 pts Good work, but your thought is focussing on the entire array's largest element index but the LI for inner loop focusses on making jth element the largest among first j elements.

+ 0 pts index i focusses on outer loop whereas LI is asked to be provided for the inner loop. Please read the question carefully and refer to the solution file.

+ 4 pts Good work but work on your explanation to be more precise. For reference, check the solution file.

+ 3 pts In the correct direction, but your solution is restrained to only two elements at a time. Please check solution file for reference.

+ 1 pts Your answer states that right most j elements are sorted. But here the LI is for inner loop and in that loop the jth element is max of all first j elements. Please check solution file for reference

+ 3 pts Please provide a more clear explanation using a variable like i or j. But your solution is in the correct direction

✓ **+ 0 pts Your explanation is unclear as it is not evident which loop you are talking about (inner loop or outer loop). Please refer solution file for reference.**

+ 0 pts Your solution refers to last n-j elements but at every jth iteration, the A[j] is max of first j elements. Please refer solution file for reference.

+ 0 pts Inner loop isn't about min value in Array[i...n] but instead is about fixing max value at A[j] from A[0...j]

+ 0 pts Explained the meaning of for loop iteration range. But didn't explain LI

+ 0 pts Click here to replace this description.

+ 2 pts Click here to replace this description.

+ 1 pts Click here to replace this description.

+ 3 pts Click here to replace this description.

+ 0 pts Plagiarism

+ 1 Point adjustment

1.5 2 / 8

✓ **- 1 pts "Loop Invariant : The last i elements are sorted" is missing**

- 1 pts "Loop Invariant : the last i elements are not smaller than the first n-i elements" is missing

- 1 pts "Initialization : before the loop it is an empty set, Loop invariant hold trivial" is missing

✓ **- 1 pts Termination requires a more thorough explanation of how the final loop terminates**

- 2 pts The assumption part of the Maintenance is not clear

- 2 pts Maintenance : The loop invariant confirmation of the next iteration is not clear.

- 2 pts In the Maintenance loop invariant "The last i elements are sorted" is missing

- 2 pts In the Maintenance loop invariant "the last i elements are not smaller than the first n-i elements" is missing

- 0 pts Good Work!

- 6 pts I cannot provide more points when your loop invariant is totally wrong. Please be more careful with indexes next time

- 8 pts Not correct

- 8 pts Potential Copy

- 8 pts Plagiarism

- 4 Point adjustment

I admire your hard work but I cannot give much credit to this. What is "last spot". What do mean by "make more logical sense". Use mathematic expressions.

QUESTION 2

2 6 / 6

+ 6 pts Correct

✓ **+ 6 pts Correct. Even though the question did not**

specify to include the base case, it is good practice in an inductive proof to include them.

+ **4 pts** Missing summation notation and proof is nearly correct

+ **4 pts** Missing some key proof steps

+ **3 pts** Should deal with general case, not for specific value of k/r

+ **2 pts** Wrong understanding of induction. k-case should imply k+1 case.

+ **2 pts** Incorrect/Missing proof for inductive step

+ **1 pts** Missing summation notation and proof is incorrect

+ **0 pts** Not try or totally incorrect

- **6 pts** Plagiarism

QUESTION 3

3 3.5 / 4

+ **0 pts** Good answer! Explained very well.

+ **0 pts** Not attempted or Incorrect answer. Please go through the solution provided.

+ **1 pts** Partially correct. Please read and understand the concept again. Go through the solution provided.

✓ **+ 1 pts Initialization step is correct**

+ **0.5 pts** Initialization step is not explained well or it is not precise

+ **2 pts** Maintenance step is correct.

✓ **+ 1.5 pts Maintenance step is in the right direction but has one mistake. When A[i] != n in maintenance step, ret is not necessarily equal to -1, instead it may store the index of n if n is already found in A[0....i-1].**

+ **0 pts** Maintenance step is incorrect

+ **0.5 pts** Maintenance step is not explained well

✓ **+ 1 pts Termination step is correct**

+ **0.5 pts** Termination step is not explained precisely

+ **2 pts** Please explain the answer properly with all steps

+ **0 pts** Initialization step is not present or incorrrect

+ **0 pts** Termination step is incorrect

+ **1 pts** Maintenance step explanation is in the right direction but not explained clearly

- **4 pts** Plagiarism

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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 1b (44 points) **Fall 2019, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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1. *(34 pts total) Let $A = \langle a_1, a_2, \ldots, a_n \rangle$ be an array of numbers. Let's define a 'flip' as a pair of distinct indices $i, j \in \{1, 2, \ldots, n\}$ such that $i < j$ but $a_i > a_j$. That is, a_i and a_j are out of order.*
   For example - In the array A = [1, 3, 5, 2, 4, 6], (3, 2), (5, 2) and (5, 4) are the only flips i.e. the total number of flips is 3. (Note that in this example the indices are the same as the actual values)

   (a) (8 pts) Write a Python code for an algorithm, which takes as input a positive integer n, **randomly shuffles an array of size n** with elements $[1, \ldots, n]$ and counts the total number of flips in the shuffled array.
   Also, run your code on a bunch of n values from $[2, 2^2, 2^3, ...2^{20}]$ and present your result in a table with one column as the value of n and another as the number of flips. Alternatively, you can present your table in form of a labeled plot with the

2 columns forming the 2 axes.

Note: The .py file should run for you to get points and name the file as `Lastname-Firstname-MMDD-PSXi.pdf`. You need to submit the code via Canvas but the table or plot should be on the main .pdf.

Copy and pasted from the results of my code. This is just one randomized implementation, which is likely different than other runs of the same code:

N	Flips
2	0
4	2
8	12
16	72
32	256
64	1059
128	4330
256	16104
512	66219
1024	269978
2048	1042675
4096	4217964

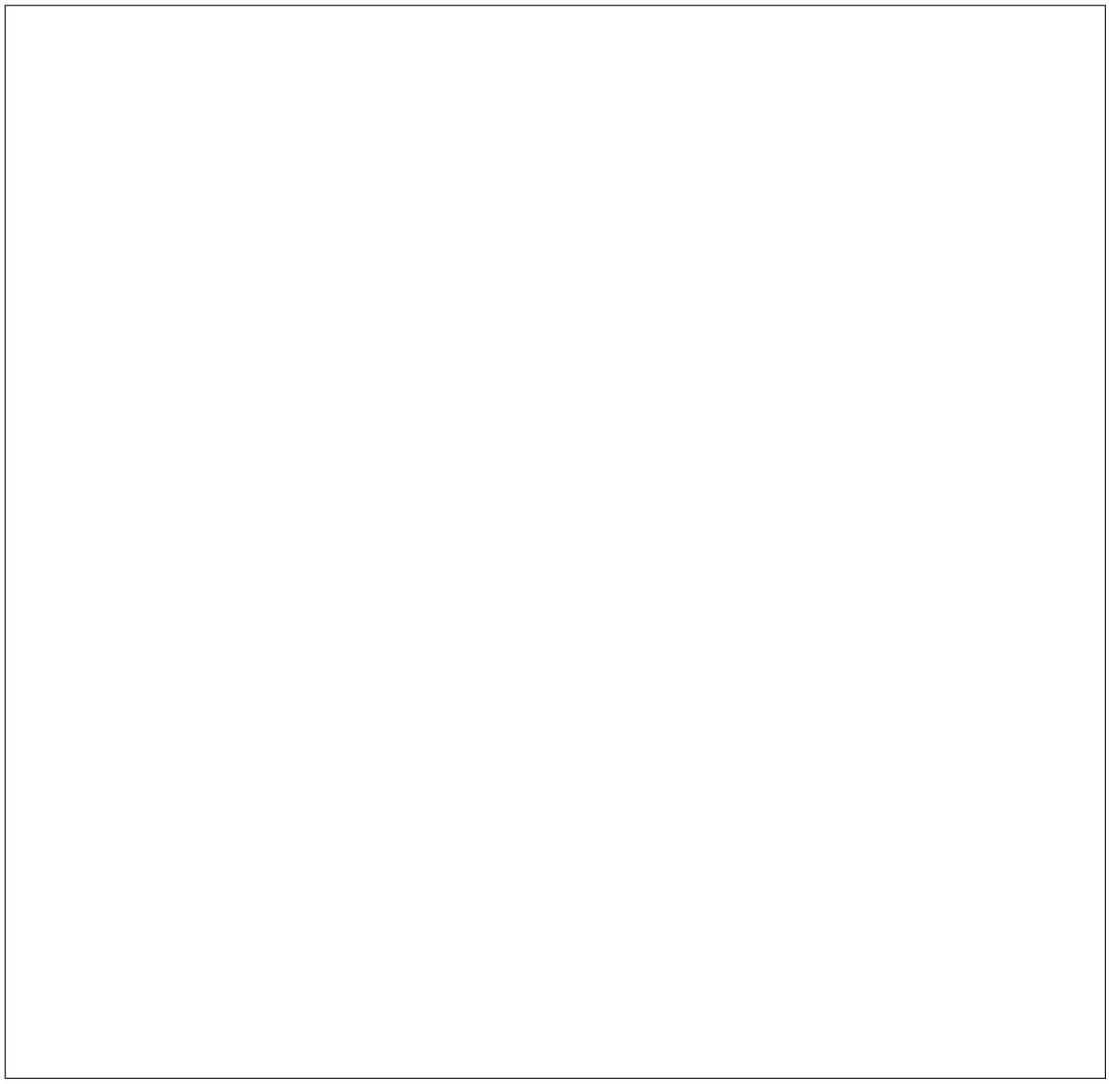
(b) (4 pts) At most, how many flips can A contain in terms of the array size n? Hint: The code you wrote in (a) can help you find this. Explain your answer with a short statement.

$\sum_{i=1}^{n-1} i$

The worst case (or the most flips) occurs when every value is in reverse order. That means every combination downward is another flip. For number, say, 16, that means it pairs with all 15 numbers until one, making 15 combinations from 16 to the end. Repeat that with every number from n to the end, then that is n-1, then n-2... to 1, which fits really well as a sum.

(c) (10 pts) We say that A is sorted if A has no flips. Design a sorting algorithm that, on each pass through A, examines each pair of consecutive elements. If a consecutive pair forms a flip, the algorithm swaps the elements (to fix the out of order pair). So, if your array A was [4,2,7,3,6,9,10], your first pass should swap 4 and 2, then compare (but not swap) 4 and 7, then swap 7 and 3, then swap 7 and 6, etc. Formulate pseudo-code for this algorithm, using nested for loops.
Hint: After the first pass of the outer loop think about where the largest element would be. The second pass can then safely ignore the largest element because it's already in it's desired location. You should keep repeating the process for all elements not in their desired spot.

def sortingalgorithmpseudo (*array*):
—-while not sorted:
———for the length or the array up to last unsorted value
—————check if values are a flip
—————if a flip, then swap the two values
—————if not a flip, leave alone
———end for loop
———check if it is sorted, if not, wrap back
—-once sorted, break while loop



(d) (4 pts) Your algorithm has an inner loop and an outer loop. Provide the 'useful' loop invariant (LI) for the inner loop.You don't need to show the complete LI proof.

Note: i-1 used since the i is incremented. This would be the i before the increment

At the end of each loop, we know that the two values were flipped if not in $x_1 < x_2$ order (or not flipped if in the correct order). There does not technically need the possibility of being equal in this case following the instructions in part A, however, part C makes it unclear whether this is the case, and as such, I will use a less than-equal to sign just in case two indecies have the same value

Therefore:
Loop Invariance: $A[i-2] \leq A[i-1]$

(e) (8 pts) Assume that the inner loop works correctly. Using a loop-invariant proof for the outer loop, formally prove that your pseudo-code correctly sorts the given array. Be sure that your loop invariant and proof cover the initialization, maintenance, and termination conditions.

A useful invariance would be that the end value of the section looked at ((array length-1)-i) (under the "hint" part of part c) and have that as the maximum value. The same less than-equal to discussion from 2d will be applied here. i is also used as a scalar here to get to the end.

Loop Invariance: (A[0]...A[(len(A)-1)-i]) $\leq$ A[(len(A)-1)-(i-1)])
Initialization: The value of the right side overflows past the length of the array. Since the array is not sorted, having the largest value not within the scope makes sense until we can build the array for its size.
Maintenance: With each pass of the while loop, the values keep swapping in the for loop, incedentally putting the absolute biggest in the last spot. We could just use the absolute biggest is always at the end after each pass, however, the biggest not counting the one from the last round would make more logical sense to explain a sorting algorithm.
Termination: The last round comes to having the last value compared to itself, following every other one being put as the biggest of the round. The smallest becomes the smallest by default and the array is henceforth sorted

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2. *(6 pt) If r is a real number not equal to 1, then for every n ≥ 0,*

$$\sum_{i=0}^{n} r^i = \frac{(1 - r^{n+1})}{(1 - r)}.$$

Rewrite the inductive hypothesis from Q3 on PS1a and provide the inductive step to complete the proof by induction. You can refer to Q3 on PS1a to recollect the first 2 steps.

Inductive Hypothesis: For some integer k such that k $\geq$ 0 and n!=1, we can assume $\sum_{i=0}^{k} r^i = \frac{(1-r^{k+1})}{(1-r)}$

Inductive Step: If this relation fits with k, then it should reasonably be said that k+1 should work as well.
$\sum_{i=0}^{k+1} r^i$, Which is the same as $\sum_{i=0}^{k} r^i + r^{k+1}$
Since $\sum_{i=0}^{k} r^i = \frac{(1-r^{k+1})}{(1-r)}$, we can plug in $\frac{(1-r^{k+1})}{(1-r)}$
$\frac{(1-r^{k+1})}{(1-r)} + r^{k+1}$
$\frac{(1-r^{k+1})}{(1-r)} + \frac{(1-r)r^{k+1}}{1-r}$
$\frac{(1-r^{k+1})}{(1-r)} + \frac{(r^{k+1}-r^{k+2}}{1-r}$
$\frac{(1-r^{k+1})+(r^{k+1}-r^{k+2})}{(1-r)}$
$\frac{1-r^{k+1}+r^{k+1}-r^{k+2}}{(1-r)}$
$\frac{1-r^{k+2}}{(1-r)}$
$\frac{1-r^{(k+1)+1}}{(1-r)}$
$\frac{1-r^{k+1}}{(1-r)}$ -> $\frac{1-r^{(k+1)+1}}{(1-r)}$ as k->k+1 (it is just k suited for k+1)

Therefore, via weak induction, the relation ($\sum_{i=0}^{n} r^i = \frac{(1-r^{n+1})}{(1-r)}$) works for a k and a k+1, and thus is a suitable relation for all values n $\geq$ 0

3. *(4 pt) Refer to Q2b on PS1a and finish the LI based proof with all the steps.*

Loop Invariance: if n exists in A[0...i-1], ret=index of n. Else, ret=-1 (following the 1a answer set).
Initialization: No values have been compared, so ret simply takes on its initialized value of -1.
Maintenance: The loop compares the value of A[i] (now A[i-1]) to the needed value we are searching for. If this is the value, we have obtained the index of n and that is what ret becomes. Otherwise, ret remains as -1. Since this is the two stipulations in the invariance, the values fit.
Termination: The invariance terminates with the array. If the value was found, ret has held that index since it was found. If it was not in the array, ret remains as -1. Both defined in the invariance, so even if you do find the end without the value you wanted, the invariance holds.