Name: Luna McBride
ID: 107607144

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 6a (10 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

Name: Luna McBride
ID: 107607144

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 6a (10 points) **Fall 2019, CU-Boulder**

1. (1 pt) What do the edge weights of a graph G in a maximum-flow network represent?

   Solution.
   The edge weights are the capacity; ie the edge weights determine how much of something (like liters of water, gallons of chocolate milk, etcetera) can flow through this edge

2. (2 pts) What are the two conditions that must be met for network flow?

   Solution.
   The two conditions are:
   Capacity: The weight of an edge is not exceeded.
   Conservation: "What goes in must come out", or that you must drain every vertex (besides the source and the sink) completely.

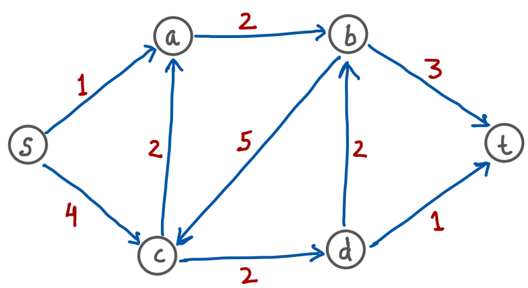
3. (2 pts) What do the edge weights in the residual graph G_f represent? Include both forward and backward edges.

   Solution.
   There are two things an edge can represent in a residual graph:
   Flow away from the source: This is a forward edge. This is the capacity of the edge that could still possibly be used, but currently goes unused.
   Flow toward the source: This is the backward edge. This is the capacity already used in flowing toward the sink.

Name: Luna McBride
ID: 107607144

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 6a (10 points) **Fall 2019, CU-Boulder**

4. (5 pts) Based on the following network and the given edge capacities answer the following.



(a) (1 pts) Can the max flow be 5 (capacity(e_{sa}) + capacity(e_{sc}))? Justify your answer in one sentence.

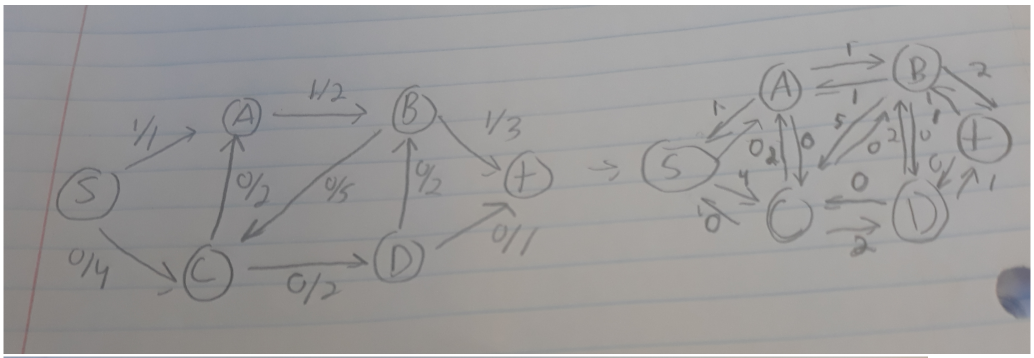
Solution.
The flow cannot possibly be 5. The sink has pipes adding to 4, which means the 5th must be in one of the vertices, breaking the conservaton condition, or trying to overfill one of the edges to the sink, breaking the capacity condition (not to mention issues before the point of the sink, but the sink is most telling).

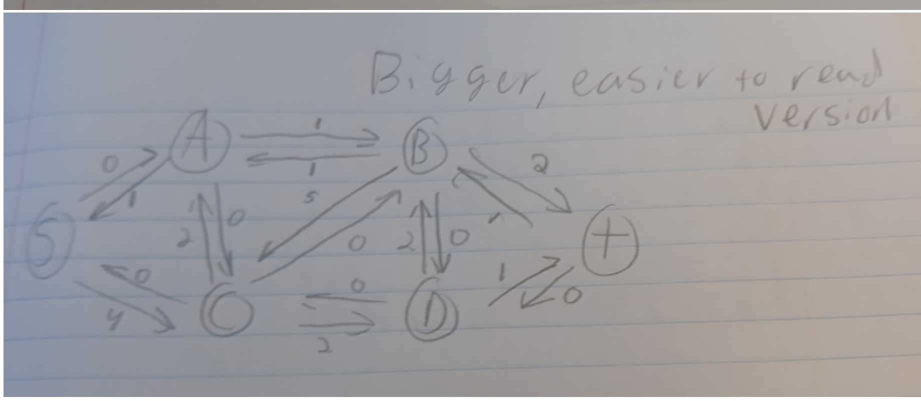
(b) (2 pts) For the graph, identify one simple $s-t$ path and the bottleneck edge value on that path. Also report the maximum allowed flow on this $s-t$ path.

Solution.
Path: S-A-B-T. Bottlenecking value=1 (edge S-A). This means the maximum allowed flow is 1, since we are not counting any brought in from the other vertices.

(c) (2 pts) Assuming all $f(e)$ are initially 0 where f represents flow, what are the residual capacities on the forward and backward edges of G_f after one iteration of the Ford-Fulkerson algorithm. Use the simple path you identified in Part b.

Solution.





Most important, Structured (Forward Edge, Backward Edge):
(S-A: 0, A-S:1)
(A-B:1, B-A: 1)
(B-t:2, t-B:1)