

# CES547T - M1

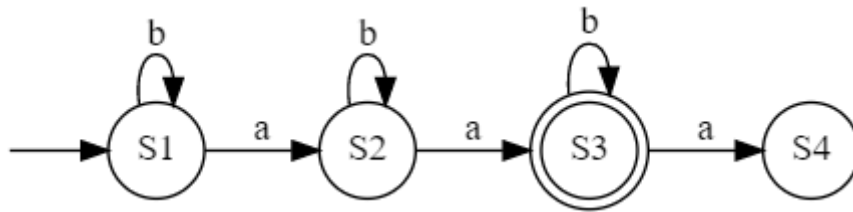
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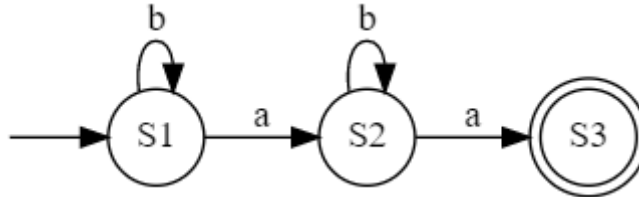
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## 1 Homework

- 2.1(a)** Start at  $S1$ , whenever encounter an  $a$ , move forward. If there is exactly 2  $a$ s in the string, it should stop at  $S3$ .



- 2.1(b)** Similar to **2.1 a**, when encounter the 3rd  $a$ , instead of slipping to  $S4$ , it stops at  $S3$ .



- 4.1(a)**  $(a + b)^*$

1.  $(a + b)^0 = \{\Lambda\} \subseteq L(S)$
2. Assume  $(a + b)^k \subseteq L(S)$ . We have

$$\begin{aligned}
 (a + b)^{k+1} &= (a + b)(a + b)^k \\
 &= (a + b)S \\
 &= S
 \end{aligned}$$

3. Thus by induction. We have  $L(S) = (a + b)^*$ .

- 4.1(b)**  $(a + b)^*a$

1.  $(a + b)^0a = \{a\} \subseteq L(S)$
2. Assume  $(a + b)^ka \subseteq L(S)$ . We have

$$\begin{aligned}
 (a + b)^{k+1}a &= (a + b)(a + b)^ka \\
 &= a(a + b)^ka + b(a + b)^ka \\
 &= aS + bS \\
 &= SS + S \\
 &= S + S \\
 &= S
 \end{aligned}$$

3. Thus by induction. We have  $L(S) = (a + b)^*a$ .

## 2 Quiz

(Only simple notes.)

1. By Cantor's diagonal argument, it's obvious that  $[0, 1]$  is an uncountable set.

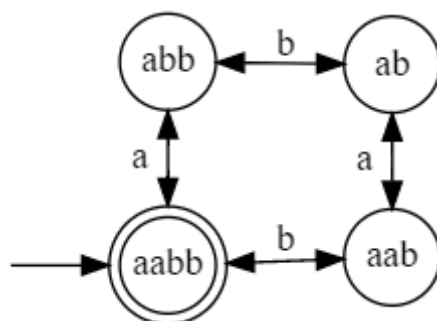
2. Since every positive rational number can be expressed as

$$\frac{p}{q} \text{ where } p, q \in \mathbb{N}$$

Thus  $\mathbb{Q}^+ = \mathbb{N} \times \mathbb{N}$ .

3. Ridiculous. They are both countable sets. Thus they are both equinumerous to  $\mathbb{N}$ .

4. (2.1(g))



5. (4.1(c))  $b(ab)^*$

6. (4.1(h))  $\bigcup_{k \text{ is odd}} (a + b)^k$