CES547T - M1

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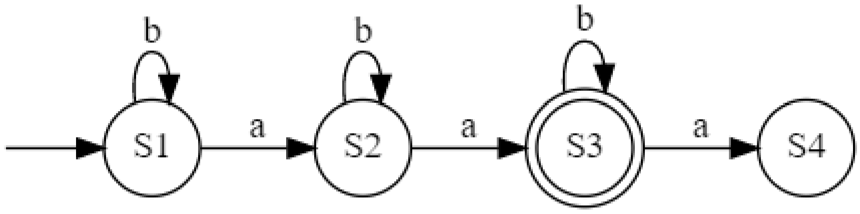
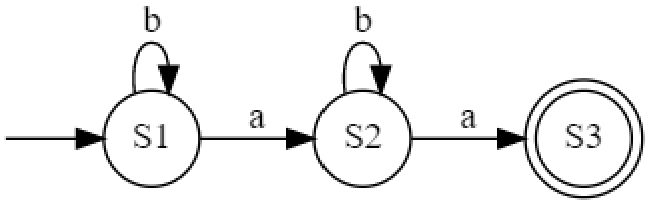
1 Homework

2.1(a) Start at $S1$, whenever encounter an a, move forward. If there is exactly 2 as in the string, it should stop at $S3$.



2.1(b) Similar to **2.1 a**, when incounter the 3rd a, instead of slipping to $S4$, it stops at $S3$.



4.1(a) $(a+b)^*$

1. $(a+b)^0 = \{\Lambda\} \subseteq L(S)$

2. Assume $(a+b)^k \subseteq L(S)$. We have

$$\begin{aligned}(a+b)^{k+1} &= (a+b)(a+b)^k \\ &= (a+b)S \\ &= S\end{aligned}$$

3. Thus by induction. We have $L(S) = (a+b)^*$.

4.1(b) $(a+b)^* a$

1. $(a+b)^0 a = \{a\} \subseteq L(S)$

2. Assume $(a+b)^k a \subseteq L(S)$. We have

$$\begin{aligned}(a+b)^{k+1} a &= (a+b)(a+b)^k a \\ &= a(a+b)^k a + b(a+b)^k a \\ &= aS + bS \\ &= SS + S \\ &= S + S \\ &= S\end{aligned}$$

3. Thus by induction. We have $L(S) = (a+b)^* a$.

2 Quiz

(Only simple notes.)

1. By Cantor's diagonal argument, it's obvious that $[0,1]$ is an uncountable set.

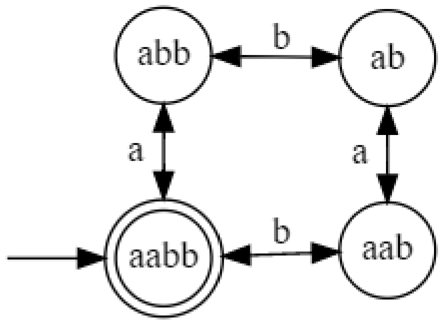
2. Since every positive rational number can be expressed as

$$\frac{p}{q} \text{ where } p, q \in \mathbb{N}$$

Thus $\mathbb{Q}^+ = \mathbb{N} \times \mathbb{N}$.

3. Ridiculous. They are both countable sets. Thus they are both equinumerous to $\mathbb{N}$.

4. **(2.1(g))**



5. **(4.1(c))** $b(ab)*$

6. **(4.1(h))** $\bigcup_{k \text{ is odd}} (a+b)^k$