Supplier Encroachment with Multiple Retailers

3 Model Setting

s: Supplier

$r_i, i = 1, 2, \cdots, n$: Retailers

n: number of retailers

w: wholesale price

$p = a - Q$: inverse demand function

- $a > 0$: market potential
- Q: total quantity in market
- p: market clearing price

For Q:

- $Q = \sum_{i=1}^n q_{r_i}$: Only retailer
  - q_{r_i}: r_i retailer's quantity
- $Q = \sum_{i=1}^n q_{r_i} + q_s$: Encroachment
  - q_s: selling quantity

c: supplier unit selling cost

- $c \in (0, a)$

0: retailer's unit selling cost

Decision sequence:

1. Supplier decide w
2. retailers determine q_{r_i} simultaneously
3. supplier decide q_s

Analysis

No Supplier Encroachment

Profit function:

$\pi_s = w \sum_{j=1}^n q_{r_j}$
$\pi_{r_i} = \left(a - \sum_{j=1}^n q_{r_j}^n - w\right) q_{r_i}^n$

r_i's order given w:

$$q_{r_i}(w) = \frac{a - w}{(n+1)}, i = 1, 2, \cdots, n$$

Then w:

$$w^n = \frac{a}{2}$$

Then:

$q_{r_i}^n = \frac{a}{2(n+1)}, \pi_{r_i}^n = \frac{a^2}{4(n+1)^2}$ and $\pi_s^n = \frac{na^2}{4(n+1)}$

Then:

Proposition 1

> When the supplier cannot encroach into the retail market, in equilibrium,
>
> 1. the optimal wholesale price w^n is independent of n, the number of downstream retailers;
> 2. each retailer's order quantity $q_{r_i}^n.\ i = 1, 2, \cdots, n$, decreases in n while the total demand of all retailers $\sum_{j=1}^n q_{r_j}^n$ increases in n;
> 3. the supplier's profit π_s^n increases in n while that of a retailer $\pi_{r_i}^n$ decreases in n. The total supply chain profit $\pi_c^n = \pi_s^n + \sum_{j=1}^n \pi_{r_j}^n$ increases in n.

4.2 Potential Supplier Encroachment

Now:

(1)

$\pi_s = w \sum_{j=1}^n q_{r_j} + \left(a - \sum_{j=1}^n q_{r_j} - q_s - c\right) q_s$

Then:

(2)

$q_s\left(q_{r_i}\right) = \left(\frac{a - \sum_{j=1}^n q_{r_j} - c}{2}\right)^+$

Then:

$\pi_{r_i} = \left(a - \sum_{j=1}^n q_{r_j} - q_s\left(q_{r_i}\right) - w\right) q_{r_i}$

Then:

(3)

$$(q_{r_i}(w), q_s(w)) = \begin{cases} \left(\frac{a+c-2w}{(n+1)}, \frac{a-(2n+1)c+2nw}{2(n+1)}\right) & \text{if } w > \frac{(2n+1)c-a}{2n}; \\ \left(\frac{a-c}{n}, 0\right) & \text{if } \frac{(n+1)c-a}{n} < w \leq \frac{(2n+1)c-a}{2n}; \\ \left(\frac{a-w}{(n+1)}, 0\right) & \text{if } w \leq \frac{(n+1)c-a}{n}. \end{cases}$$

Then we get:

$$\pi_s(w) = \begin{cases} nw\frac{a+c-2w}{(n+1)} + \left(\frac{a-(2n+1)c+2nw}{2(n+1)}\right)^2, & \text{if } w > \frac{(2n+1)c-a}{2n}; \\ w(a-c), & \text{if } \frac{(n+1)c-a}{n} < w \leq \frac{(2n+1)c-a}{2n} \\ nw\frac{a-w}{(n+1)}, & \text{if } w \leq \frac{(n+1)c-a}{n}. \end{cases}$$

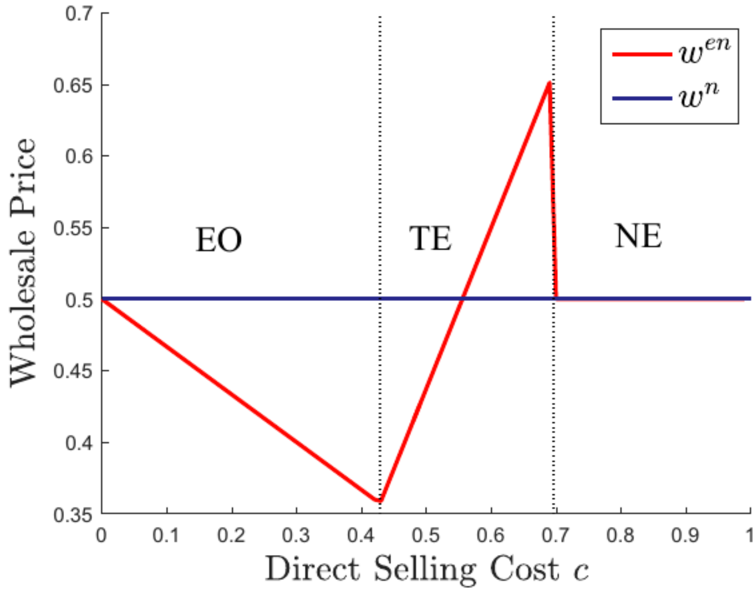
Following Proposition:

Proposition 2

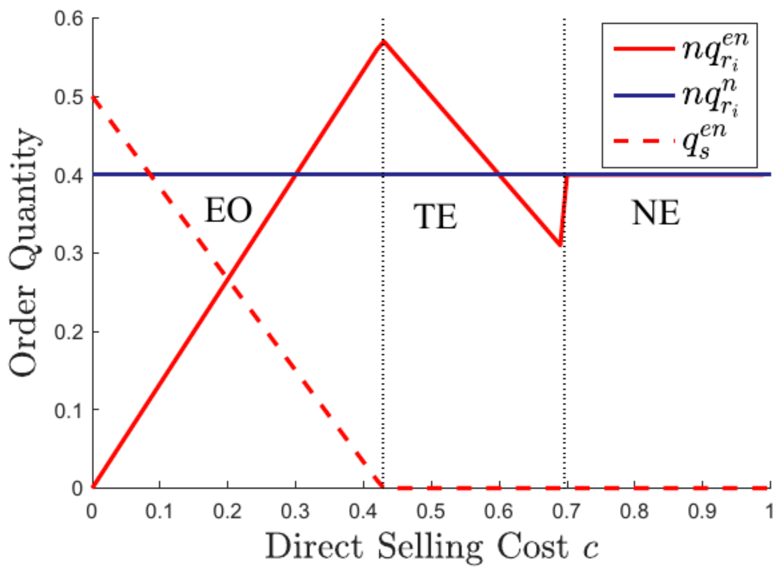
> When the supplier has the option to encroach into the retail market, the supplier encroaches only when his unit direct-selling $\cos tc \leq \underline{c}$ and never encroaches when $c > \bar{c}$, where:
>
> $\bar{c} = \frac{2(n+1)^2 + n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a$ and $\underline{c} = \frac{n+2}{2+3n}a$

Table 1 Equilibrium Wholesale Price, Quantities and Profits: Potential Encroachment

	Encroachment occurrence (EO)	Threat of encroachment (TE)	No encroachment (NE)
	$\boldsymbol{c \le \underline{c}}$	$\boldsymbol{\underline{c} < c \le \bar{c}}$	$\boldsymbol{c > \bar{c}}$
$\boldsymbol{W}^{en}$	$\frac{(n+2)a - nc}{2(n+2)}$	$\frac{(2n+1)c - a}{2n}$	$\frac{a}{2}$
$q_{r_i}^{en}$	$\frac{2c}{(n+2)}$	$\frac{a-c}{n}$	$\frac{a}{2(n+1)}$
q_s^{en}	$\frac{(n+2)a - (2+3n)c}{2(n+2)}$	0	0
$\pi_{r_j}^{en}$	$\frac{2c^2}{(n+2)^2}$	$\frac{(a-c)^2}{2n^2}$	$\frac{a^2}{4(n+1)^2}$
π_s^{en}	$\frac{nc((n+2)a - nc)}{(n+2)^2} + \frac{((n+2)a-(2+3n)c)^2}{4(n+2)^2}$	$\frac{((2n+1)c-a)(a-c)}{2n}$	$\frac{na^2}{4(n+1)}$



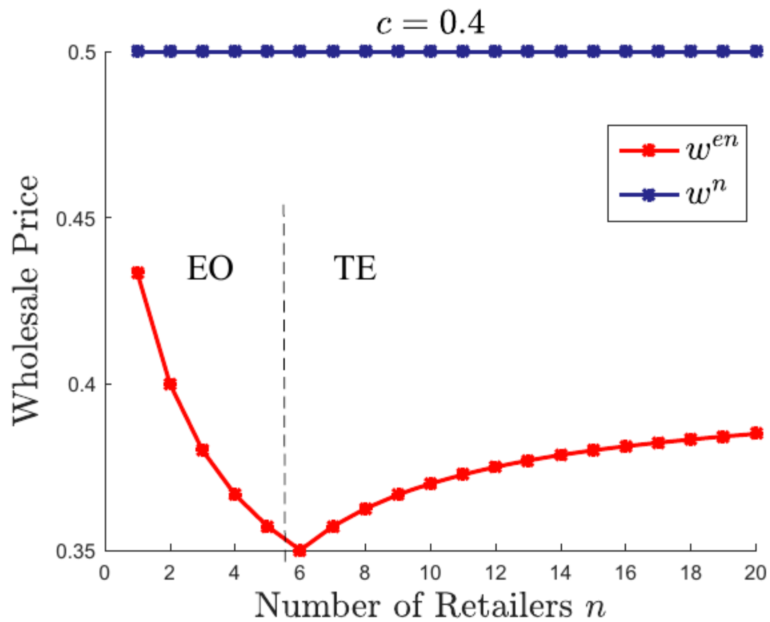
(a)

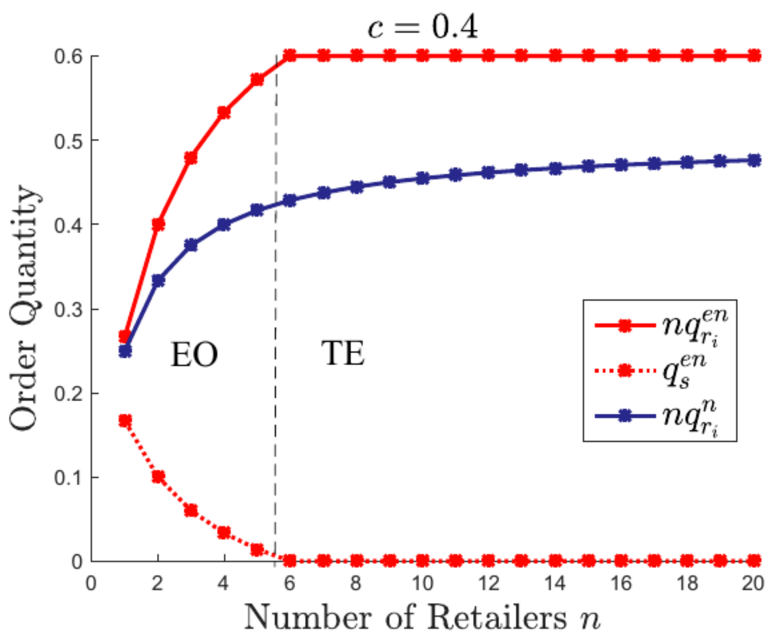


Corollary 1

> When the supplier has the option to encroach into the retail market, in equilibrium
>
> 1. the two direct-selling cost thresholds $\underline{c}$ and $\bar{c}$ both decrease in n;
> 2. the optimal wholesale price w^{en} decreases in n under encroachment occurrence while increases in n under the threat of encroachment.

3. Each retailer's order quantity $q_{r_i}^{en}$ decreases in n. Nonetheless, the total order quantity $\Sigma_{j=1}^{n}\eta_{r_j}^{en}$ remains constant (i.e., $\Sigma_{j=1}^{n}q_{r_j}^{en}=a-c$) under the threat of encroachment, and increases in n under encroachment occurrence.



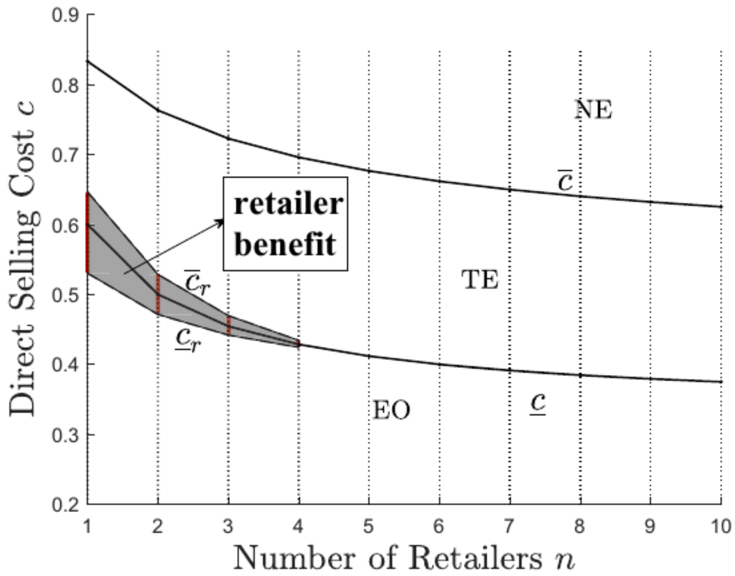


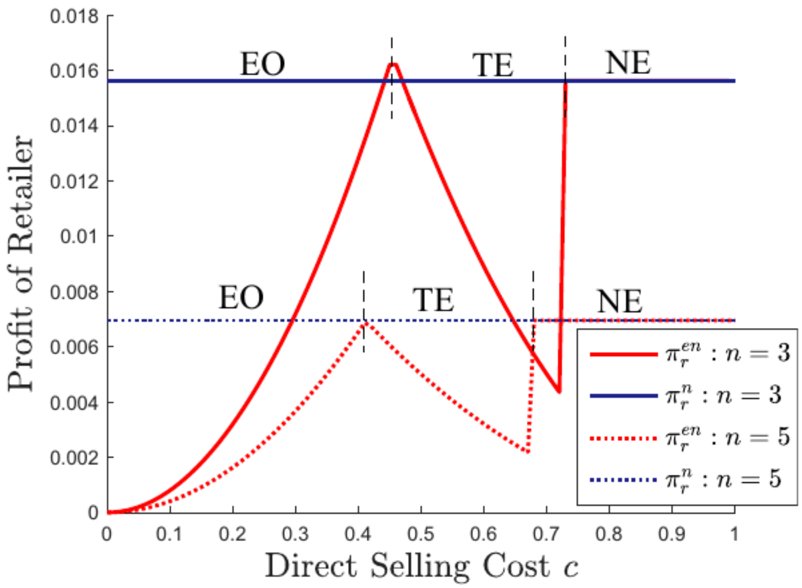
4.3 Impact of Supplier Encroachment

Proposition 3

When the supplier can encroach into the retail market, in equilibrium

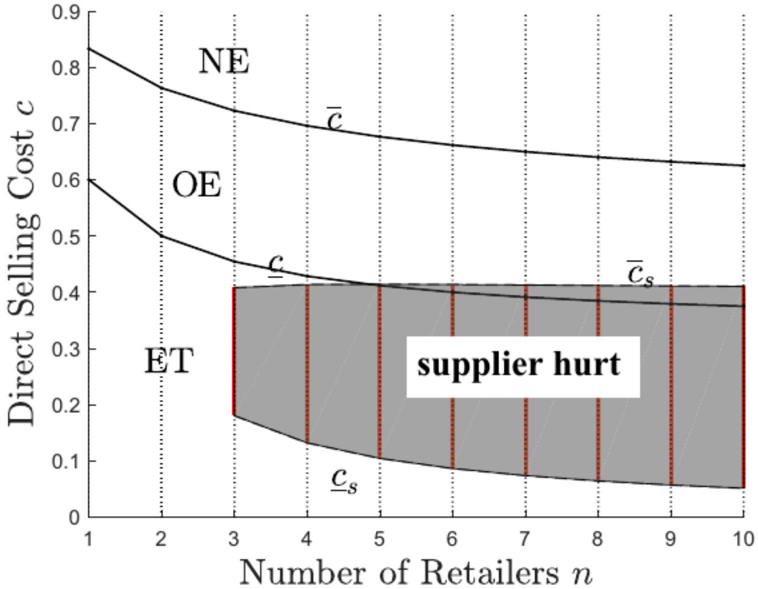
1. if the number of retailers $n > 4$, a retailer is always worse off with supplier encroachment.
2. if $n \leq 4$, a retailer can benefit from supplier encroachment when the direct-selling cost $c \in [\underline{c}_r, \bar{c}_r]$, where $\underline{c}_r = \frac{\sqrt{2}(n+2)a}{4(n+1)}$ and $\bar{c}_r = \frac{2(n+1)-\sqrt{2n}}{2(n+1)}a.$

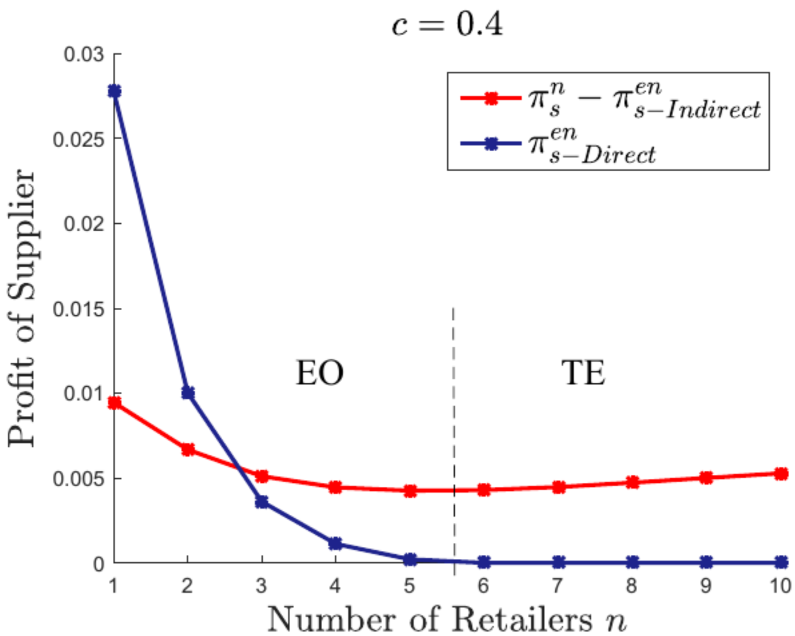




Proposition 4

> When the supplier can encroach into the retail market, in equilibrium
>
> 1. if the number of retailers $n < 3$, the supplier always benefits from having the option to encroach into the retail market.
> 2. if $n \geq 3$, the supplier becomes worse off when being endowed with the option of encroachment if his direct-selling cost $\mathcal{C}_s \leq c \leq \bar{c}_s$, where $\underline{c}_s = \frac{(n+1)(n+2)-\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a$ and $\bar{c}_s = \min\left\{\frac{(n+1)(n+2)+\sqrt{n(n+1)(n-2)(n+2)}}{5n^2+7n+2}a, \frac{2(n+1)^2-n\sqrt{2(n+1)}}{2(n+1)(2n+1)}a\right\}$.



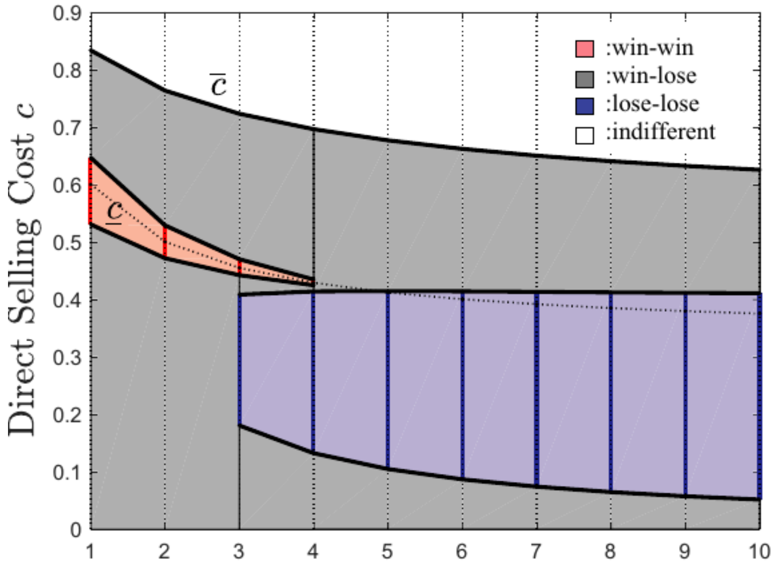


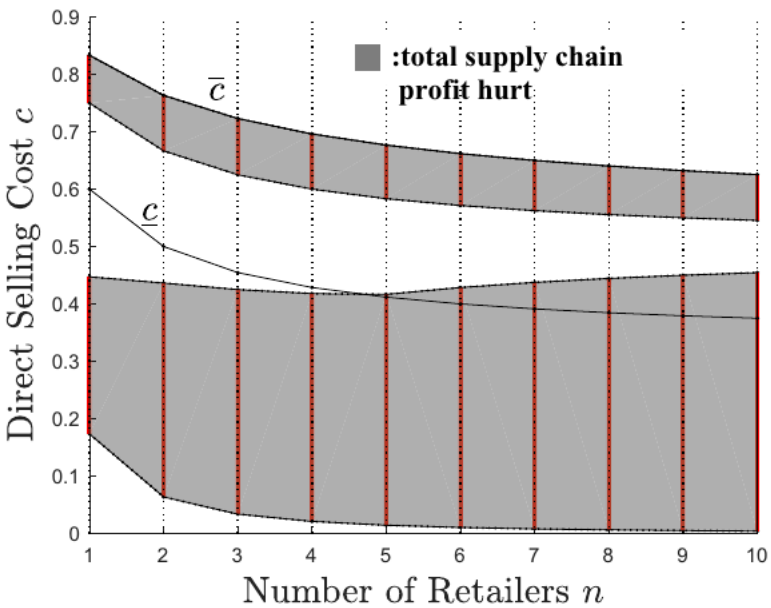
Discussion

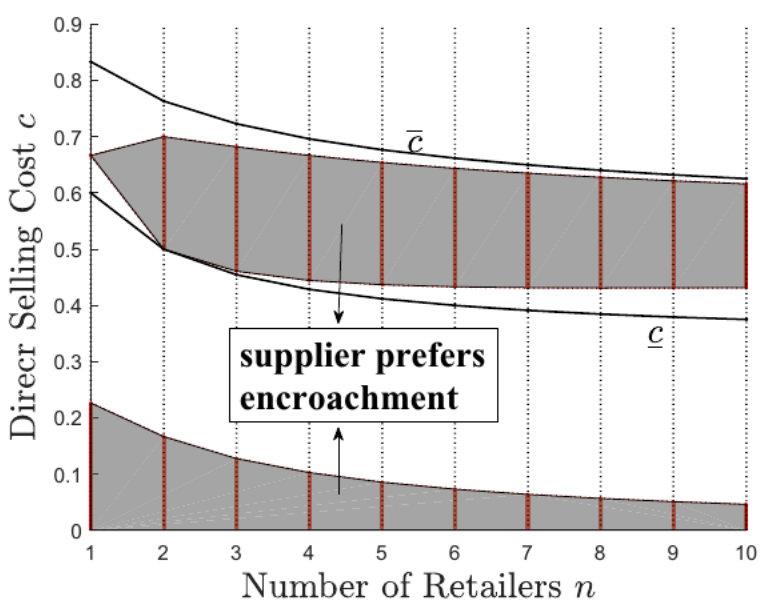
5.1 Market Penetration via Channel Expansion

Proposition 5

> When there are n incumbent retailers in his indirect distribution channel, the supplier should encroach into the retail market and direct sell when his direct-selling cost
>
> $$c \in \left[0, \frac{n+2-\sqrt{n^2-n+2}}{5n+2}a\right] \cup \left[\frac{2(n+1)(n+2)-\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a, \frac{2(n+1)(n+2)+\sqrt{2n(n-1)(n+2)}}{2(2n+1)(n+2)}a\right].$$
>
> Otherwise, the supplier should enroll a new retailer into his indirect channel.


Direct Selling Cost c
:win-win
:win-lose
:lose-lose
:indifferent
$\overline{c}$
$\underline{c}$


Direct Selling Cost c
:total supply chain profit hurt
$\overline{c}$
$\underline{c}$
Number of Retailers n


Direcr Selling Cost c
$\overline{c}$
$\underline{c}$
supplier prefers encroachment
Number of Retailers n

5.2 Imperfect Product Substitution

Then:

$p_{r_i} = a - \sum_{j=1}^{n} q_{r_j} - \theta q_{s'}, i = 1, 2, \cdots, n$ and $p_s = a - q_s - \theta \sum_{j=1}^{n} q_{r_j}$

- $0 < \theta \leq 1$

$$\begin{aligned}\pi_s &= w \sum_{j=1}^{n} q_{r_j} + \left(a - \theta \sum_{j=1}^{n} q_{r_j} - q_s - c\right) q_{s'} \\ \pi_{r_i} &= \left(a - \sum_{j=1}^{n} q_{r_j} - \theta q_s - w\right) q_{r_i}, i = 1, 2, \cdots, n\end{aligned}$$

Proposition 6

> When the channels are imperfect substitutes, the equilibrium wholesale price, quantities and profits are as shown in Table 2. Moreover,
>
> 1. if $n \leq \left[\frac{2\left(2-\theta^2\right)+2\sqrt{2(2-\theta^2)}}{\theta^2}\right]$, the retailer can benefit from supplier encroachment when $c \in \left[\underline{c}_{r-\theta}, \bar{c}_{r-\theta}\right]$, where
> $$\text{where} \quad \bar{c}_{r-\theta} = \frac{\sqrt{2(2-\theta^2)(n+1)} - n\theta}{\sqrt{2(2-\theta^2)(n+1)}} a$$
> $$\underline{c}_{r-\theta} = \frac{4(n+1)-(3n+2)\theta^2-2(1-\theta)\sqrt{2(2-\theta^2)(n+1)}}{2\theta\sqrt{2(2-\theta^2)(n+1)}} a$$
> 2. if $n \geq 3$, the supplier becomes worse off when being endowed with the option of encroachment when $c \in \left[\underline{c}_{s-\theta}, \overline{C}_{s-\theta}\right]$, where
>
> $\underline{C}_{s-\theta} = \frac{4n^2\theta^2-8n^2\theta+4n^2+n\theta^2-8n\theta+8n-2\theta^2+4}{(2-\theta)^2n^2+(8-4\theta-\theta^2)n+4-2\theta^2+\theta\sqrt{n(-n^2+n+2)(2\theta^2-4n+3n\theta^2-4)}} a$, and
>
> $\bar{c}_{s-\theta} = \min\left(\frac{\left((4-2\theta)n+4-2\theta^2\right)(n+1)-\theta^2 n\sqrt{2(n+1)}}{4n^2-2n\theta^2+8n-2\theta^2+4} a, \frac{4n^2\theta^2-8n^2\theta+4n^2+n\theta^2-8n\theta+8n-2\theta^2+4}{(2-\theta)^2n^2+(8-4\theta-\theta^2)n+4-2\theta^2-\theta\sqrt{n(-n^2+n+2)(2\theta^2-4n+3n\theta^2-4)}} a\right)$. where
>
> $\bar{c}_\theta = \frac{\left((4-2\theta)n+4-2\theta^2\right)(n+1)+\theta^2 n\sqrt{2(n+1)}}{(4n+4-2\theta^2)(n+1)} a$ and $c_\theta = \frac{4-2\theta^2+\left(4-2\theta-\theta^2\right)n}{4-2\theta^2+(4-\theta^2)n} a$.

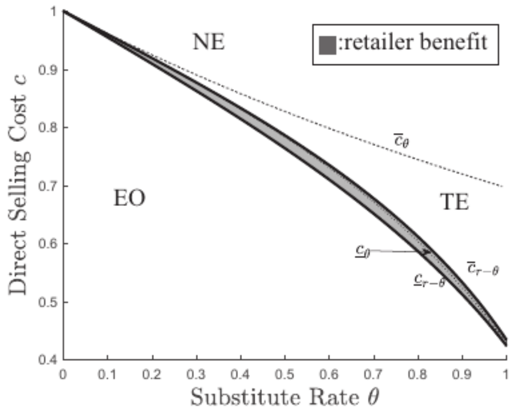
Corollary 2

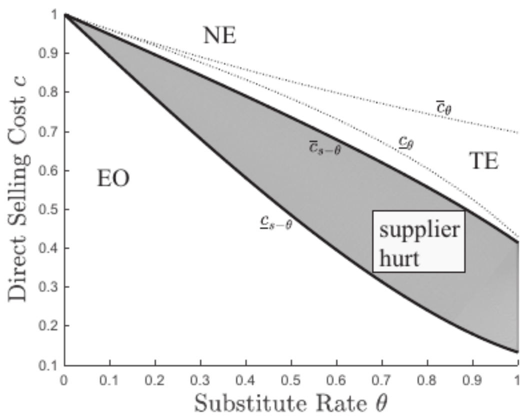
> When the direct and indirect channels are imperfect substitutes, the thresholds stated in Proposition $6, \bar{c}_{r-\theta}, \underline{c}_{r-\theta}, \bar{c}_{s-\theta}, \underline{c}_{s-\theta}$ all decrease in θ

	Encroachment occurrence$\boldsymbol{C} < \underline{\boldsymbol{C}}_g$	Threat of encroachment$\underline{\boldsymbol{c}}_\theta < \boldsymbol{c} \leq \overline{\boldsymbol{c}}_\theta$	No encroachn
W^{en}	$\frac{\left(4-2\theta^2+n\theta^5+4n\left(1-\theta^2\right)\right)a-n\theta^3 c}{8(1+n)-2(3n+2)\theta^2}$	$\frac{\left(2\pi+2-\theta^2\right)c-\left(2\pi(1-\theta)+\left(2-\theta^2\right)\right)a}{2\pi\theta}$	$\frac{a}{2}$
$q_{r_i}^{en}$	$\frac{2(1-\theta)a+2c\theta}{4(1+n)-(3n+2)\theta^2}$	$\frac{a-C}{nG}$	$\frac{a}{2(n+}$
q_s^{en}	$\frac{\left(4-2\theta^2+\left(4-2\theta-\theta^2\right)n\right)a-\left(4-2\theta^2+\left(4-\theta^2\right)\pi\right)c}{8(1+\pi)-2(3n+2)\theta^2}$	0	0
$\pi_{r_j}^{en}$	$\frac{2\left(2-\theta^2\right)\left(a(1-\theta)+c\theta\right)^2}{\left(4(n+1)-(3n+2)b^2\right)^2}$	$\frac{\left(2-\xi^2\right)\left(\bar{a}-c\right)^2}{2\pi^2\theta^2}$	$\frac{a^2}{4(n+}$
π_s^{en}	$\frac{\left(4-2\theta^2+m\theta^2\right)(a-c)^2+8n(1-\theta)(a-c)a+4nc^2}{16(n+1)-4(3n+2)\theta^2}$	$\frac{\left(\left(2\pi+2-\theta^2\right)c-\left(2n(1-\theta)+2-\theta^2\right)a\right)(a-c)}{2\pi\theta^2}$	$\frac{na^2}{4(n+}$

5.3 Costly Channel Expansion

f: fixed cost to enroll a retailer





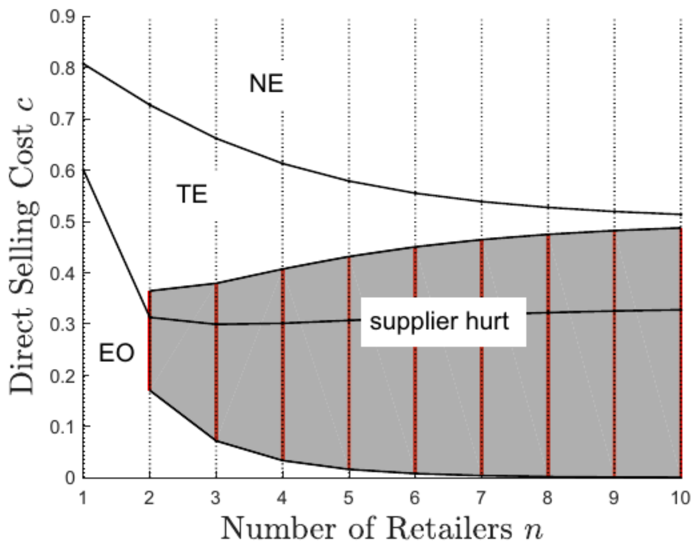
Proposition 7

> When the supplier can proactively choose the number of retailers to enroll into his indirect channel, the equilibrium encroachment strategy, the optimal number of enrolled retailers and equilibrium profits are presented in Table 3, where n^{enf} is a real number, n^{enf*} is an integer,
>
> $\bar{c}_f = \frac{a+\sqrt{2f}+\sqrt{2(2-\sqrt{2})a\sqrt{f}-2f}}{2}$ and $_{c_f} = \frac{2\sqrt{2f}+a}{3}$

	Encroachment occurrence(EO) $c \leq \underline{c}_f$	Threat of encroachment(TE) $\underline{\boldsymbol{c}}_f < \boldsymbol{c} \leq \bar{\boldsymbol{c}}_f$	No encroachment (NE)$\boldsymbol{c} > \bar{\boldsymbol{c}}_f$
n^{ent}	$c\sqrt{\frac{2}{7}} - 2$	$\frac{a-c}{\sqrt{2f}}$	$\frac{a}{2\sqrt{f}} - 1$
$n^{\text{enf*}}$	$\arg\max_{n\in\{\lfloor n^{ent}\rfloor, \lfloor n^{ent}\rfloor+1, \lfloor n^{nt}\rfloor, \lfloor n^{nt}\rfloor+1\}} \pi_s^{enf}(n)$		
$\pi_{r_j}^{\text{enf}}\left(n^{\text{enf}}\right)$	f	f	f
$\pi_s^{\text{enf}}\left(n^{\text{anf}}\right)$	$(c-\sqrt{2f})^2 + \frac{(a-c)^2}{4}$	$(c-\sqrt{2f})(a-c)$	$\frac{(a-2\sqrt{f})^2}{4}$

Proposition 8

> When retailers make their ordering decisions sequentially
>
> 1. if $n \leq 6$, retailer r_1 can benefit from supplier encroachment when $c \in \left[\underline{c}_{r-S}, \bar{c}_{r-S}\right]$, where
>
> $\underline{c}_{r-S} = \min\left\{\frac{2^n+1}{2^{n+3/2}}a, \underline{c}_S\right\},$
> $\bar{c}_{r-S} = \left(1 - \frac{2^{2n}-2^{n+1}+2}{\sqrt{2^{n+2}\left(2^{3n-1}+2^{2n}-2-2^n\sqrt{2^{n-1}(2^n-2)}\right)}}\right)a$ and
> $\underline{c}_S = \frac{(2^n+1)\left(3\times 2^{2n} - 2^{n+1} + 2 - \sqrt{2^{n+3}\left(2^n-2\right)} - 2\sqrt{2}\sqrt{(2^n+1)^{-1}(2^{2n}-2^{n+1}+2)(2^n-1)\sqrt{2^{n+1}(2^n-2)}-2^n(2^n-2)}\right)}{9\times 2^{3n} - 13\times 2^{2^n} + 2^{n+4} - 2 - (2^{n+1}+2)\sqrt{2^{n+1}(2^n-2)a}}$
>
> 2. Retailer $r_i, i = 2, 3, \cdots, n$, is always worse off with supplier encroachment.
> 3. if $n \geq 2$, the supplier becomes worse off when being endowed with the option to encroach if $c \in \left[\underline{c}_{s-S}, \bar{c}_{s-S}\right]$, where $\underline{c}_{s-S} = \frac{2^n(2^n+1)-\sqrt{2^n(2^n-1)(2^n-3)(2^n+1)}}{5\times 2^{2n}-3\times 2^n}a$ and $\bar{c}_{s-S} = \frac{2^{3n}-2^n\sqrt{2^{n+1}(2^n-2)}-\sqrt{2^n(2^{2n-1}-2^n+1)\left((2^n-1)\sqrt{2^{n+1}(2^n-2)}+2\right)}}{2^n\left(2^{2n+1}-2^{n+1}+2-\sqrt{2^{n+1}(2^n-2)}\right)}a.$



NE
TE
EO
supplier hurt
Direct Selling Cost c
Number of Retailers n