Homework 1

Due Date: September 24, 2025

1. Let D be an arbitrary set in $\mathbb{R}^n$. Define $\bar{D} = \{\mathbf{x} \in \mathbb{R}^n | \forall \epsilon > 0, B(\mathbf{x}, \epsilon) \cap D \neq \emptyset\}$. Show that $\bar{D} = \text{int}(D) \cup \partial D = D \cup D'$.

2. D is called closed if $D = \bar{D}$. Show that (1) D is closed if and only if $\mathbb{R}^n \setminus D$ is open; (2) D is closed if and only if for any sequence $\mathbf{x}_n \to \bar{\mathbf{x}}, \{\mathbf{x}_n\}_{n=1}^{\infty} \subseteq D$, then $\bar{\mathbf{x}} \in D$.

3. Let $\mathbf{A}$ be an $m \times n$ matrix and $\mathbf{b}$ be an m vector. Show that $\{\mathbf{x} | \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ is convex.

4. A company must distribute its product from two warehouse locations to two retail outlets. Warehouse A has a total of 48 units, and Warehouse B has a total of 60 units. Forecasting estimates a demand of at most 36 units for Retail Outlet 1 and 72 units for Retail Outlet 2. The unit shipping costs between each warehouse and retail outlet are given in Table 1. The problem is to determine the minimum-cost shipping schedule. Formulate a linear programming model.

   Table 1

Warehouse	Retail Outlet 1	Retail Outlet 2
A	\$6	\$8
B	\$4	\$3

5. (Scheduling Problem) A post office requires different number of employees on different days of the week. The minimum number of employees required on each day is listed in Table 2.

   Each employee works for 5 consecutive days and then takes 2 days off. Formulate a model to find the minimum number employees that need to be hired.

6. Use AI to assist in formulating the Travelling Salesman Problem (TSP) in an alternative way to that presented in class, and explain why the constraints eliminate subtours that do not include the designated origin node.

Table 2

Day	# of employees
1-Monday	17
2-Tuesday	13
3-Wednesday	15
4-Thursday	19
5-Friday	14
6-Saturday	16
7-Sunday	11