The Bright Side of Supplier Encroachment

Note

Methodology

Backward induction:

- set some variables as given
- solve the optimize problem
- replace the value back to eliminate the variables

Intuitions

Why Everyone benefits from encroachment?

- $\Pi_M^E - \Pi_M^N = \underbrace{\frac{3(a-2c)^2}{24b}}_{\text{Increase in wholesale profit}} + \underbrace{\frac{2c^2}{24b}}_{\text{Supplement from retail profit}} > 0$
- $CS^E - CS^N = \underbrace{\frac{(9a-2c)(3a-2c)}{288b}}_{\text{Price reduction and increased choice}} > 0$
- $c \in \left(\underbrace{\frac{3a}{4\sqrt{2}}}_{\text{Lower bound: avoid excessive squeezing of the retailer}}, \underbrace{\frac{3a}{5}}_{\text{Upper bound: ensure manufacturer's retail competitiveness}} \right)$

Balancing Cost Disadvantages

When c is moderate:

- The manufacturer can credibly compete with the retailer without fully displacing it
- The retailer benefits from a lower w, offsetting partial sales loss to encroachment.

Summary:

Supplier encroachment, when strategically executed, transforms a zero-sum game into a Pareto improvement:

- **Manufacturers** gain from optimized wholesale pricing and direct retail profits.
- **Retailers** benefit if wholesale price cuts offset competitive losses.
- **Consumers** enjoy lower prices and expanded choice.

2 The basic model

In vertical supply chain

- manufacturer(supplier) sell wholesale product to retailer
- retailer sell product to final consumer
- manufacturer may sell product directly to consumers

Some additional retail competition are examined in 3.4

Consumer demand:

- $P = a - bQ$
  - $a, b > 0$
  - P: price
  - Q: quantity

manufacturer produce good at constant unit (marginal) cost: 0

retailer selling cost: 0

- manufacturer's unit cost of selling to consumer: $c \in [0, a)$
- manufacturer unit price: w
- Linear pricing arrangement

Timing:

1. the manufacturer establishes its wholesale price (w).

2. the retailer chooses its profit-maximizing retail output q_R.
3. the manufacturer determines the number of units (q_M) of the homogeneous product it will sell directly to consumers

Backward induction is employed to identify the equilibrium of the game

Key properties are presented in 3

3 Findings

3.1 The No-Encroachment Setting

Benchmark:

- the manufacturer can only reach consumers through its retailer.

the retailer chooses its output q_R to maximize its monopoly profit from retail sales,

- taking the unit wholesale price w as given.

The retailer's problem is:

(1)

$$\underset{q_R}{\text{Maximize}} \left[a - bq_R\right] q_R - wq_R$$

We get:

the retailer's output in the no-encroach-ment setting given unit wholesale price:

(2)

$$q_R^N(w) = \frac{a-w}{2b}$$

Anticipating the retailer's response , the manufacturer chooses w, solving:

(3)

$$\underset{w}{\text{Maximize}} wq_R^N(w) \Leftrightarrow \underset{w}{\text{Maximize}} \frac{w[a-w]}{2b}$$

Then the equilibrium:

(4)

$w^N = \frac{a}{2}$ and $q_R^N = \frac{a}{4b}$

We get profit:

(5)

$\Pi_R^N = \frac{a^2}{16b}$ and $\Pi_M^N = \frac{a^2}{8b}$

Consumer surplus:

(6)

$CS^N = \int_0^{q_R^N} b\left[q_R^N - q\right] dq = \frac{b}{2}\left[q_R^N\right]^2 = \frac{a^2}{32b}$

3.2 The Encroachment Setting

In this setting:

- manufacturer can sell the product directly to consumers
  - after setting the wholesale price
  - and supplying the wholesale product to the (incumbent) retailer.

Given wholesale price w and retailer supply q_R, the manufacturer chooses q_M to:

(7)

$$\underset{q_M}{\text{Maximize}} \left[a - bq_R - bq_M\right] q_M - cq_M + wq_R$$

We get:

(8)

$q_M^E\left(q_R\right) = \frac{a-c-bq_R}{2b}$

Given w, retailer choose q_R:

(9)

$$\underset{q_R}{\text{Maximize}} \left[a - bq_R - bq_M^E\left(q_R\right)\right] q_R - wq_R$$

We get:

(10)

$q_R^E(w) = \frac{a+c-2w}{2b}$

Substitute (8) and (8) into (7) :

(11)

$w^E = \frac{a}{2} - \frac{c}{6}, \quad q_R^E = \frac{2c}{3b}, \quad \text{and} \quad q_M^E = \frac{3a-5c}{6b}$

Then we get:

(12)

$$\Pi_R^E = \frac{2c^2}{9b}, \quad \Pi_M^E = \frac{3a^2-6ac+7c^2}{12b}, \quad \text{and}$$
$$CS^E = \frac{b}{2}\left[q_R^N + q_M^N\right]^2 = \frac{[3a-c]^2}{72b}.$$

3.3 No Encroachment vs Encroachment

Proposition 1 confirm that:

- Encroachment benefits manufacturer and consumer
- manufacturer will encroach $\left(q_M^E > 0\right)$ if and only if its retail cost disadvantage is not too pronounced.

Proposition 1

> The manufacturer encroaches if and only if $c < 3a/5$.
>
> The manufacturer and consumers both benefit from encroachment in this case:
>
> $\Pi_M^E - \Pi_M^N = \left(3[a-2c]^2 + 2c^2\right)/[24b] > 0$
> and $CS^E - CS^N = [9a-2c] \times [3a-2c]/[288b] > 0.$

Key point: encroachment by the manufacturer can alter its preferred wholesale price

Fact: The manufacturer sets a lower wholesale price in the encroachment setting

the retailer may benefit from manufacturer encroachment

Proposition 2

> Encroachment that increases retailer profit arises if and only if $c \in (3a/[4\sqrt{2}], 3a/5)$

We get: systematic reduction in the wholesale price secure Pareto gains

- manufacturer reduces the price of the wholesale product in order to increase the retailer's demand for the input and thereby expand the use of the efficient sales channel
- the substantial wholesale price reduction outweighs the direct reduction in demand due to the manufacturer's retail sales, and the retailer benefits from encroachment

If the wholesale price is a result of bargaining between the parties

Suppose:

Wholesale price is determined by generalized Nash bargaining,

- the weights $\beta \in (0, 1]$ and $1 - \beta$ reflect the bargaining strengths of the manufacturer and the retailer, respectively.

Encroachment that produces Pareto gains will arise in this setting

- if and only if $c \in (3a/[4\sqrt{2}], 3a/[7 - 2\beta])$. The interval is nonempty whenever the manufacturer's bargaining strength is sufficiently pronounced
  - (i.e., $\beta > [7 - 4\sqrt{2}]/2 \approx 0.67$).

Intuitively, substantial manufacturer bargaining strength produces relatively high wholesale prices in the absence of encroachment, which permits the wholesale price reductions that generate Pareto gains under encroachment.

Increase in industry profit:

$$\Pi_R^E + \Pi_M^E - \left[\Pi_R^N + \Pi_M^N\right] = \frac{9a^2 - 72ac + 116c^2}{144b}$$

We get Proposition 3:

Proposition 3

> Encroachment that increases industry profit arises if and only if $c \in [0, 3a/[2[6+\sqrt{7}]])$ or $c \in (3a/2[6-\sqrt{7}], 3a/5)$.

Increase profit when:

- retailer's downstream cost advantage is sufficiently pronounced
- or sufficiently limited

Industry profit can increase by as much as 28% when the retailer's downstream cost advantage is pronounced $(c > 0.45a)$

- primary effect of encroachment is to reduce the wholesale price and thereby limit losses from double marginalization.

Industry profit can increase by as much as 33% when the retailer's cost advantage is limited $(c < 0.17a)$,

- encroachment enables the manufacturer to profit from serving retail customers directly and thereby limit losses from double marginalization by using a direct channel.

Consider **simultaneous encroachment setting**:

(14)

$$w^E = \tfrac{a}{2} - \tfrac{c}{10}, \quad q_R^E = \tfrac{2c}{5b}, \quad \text{and} \quad q_M^E = \tfrac{5a-7c}{10b}$$

And:

(15)

$$\Pi_R^E = \frac{4c^2}{25b}, \quad \Pi_M^E = \frac{5a^2 - 10ac + 9c^2}{20b}, \quad \text{and}$$
$$CS^E = \frac{[5a-3c]^2}{200b}.$$

We get:

Proposition 4

> The retailer, the manufacturer, and consumers all are better off under sequential encroachment than under simultaneous encroachment

3.4 The Effect of Additional Retail Competition

Suppose:

- retailer (denoted R) now faces competition from $n \geq 0$ incumbent rivals
  - (where rival i is denoted R_i').
- For simplicity, each rival is presumed to be a vertically integrated producer of a substitute good who operates with unit cost c.
- The inverse demand function is: $P = a - b\left[q_R + \sum_i q_{R_i'} + q_M\right]$,
  - $q_R, q_{R_i'}$, and q_M denote the retail output of retailer R, established rival R_i', and the manufacturer, respectively.

Using the method in 3.1, when no encroachment:

(16)

$$w^N(n) = \frac{a+nc}{2[1+n]}, \quad q_R^N(n) = \frac{a+nc}{2b[2+n]}, \quad \text{and}$$
$$q_{R_i'}^N(n) = \frac{a[3+2n]-c[4+3n]}{2b[2+3n+n^2]}.$$

Using the method in 3.2, When encroachment:

(17)

$$w^E(n) = \frac{a[3+n]+c\left[-1+5n+2n^2\right]}{2[3+6n+2n^2]},$$
$$q_R^E(n) = \frac{na+c\left[2+3n+2n^2\right]}{b[3+6n+2n^2]},$$
$$q_{R_i'}^E(n) = \frac{a[3+2n]-c[5+4n]}{b[3+6n+2n^2]}, \quad \text{and}$$
$$q_M^E(n) = \frac{a[3+2n]-c[5+4n]}{2b[3+6n+2n^2]}.$$

When encroachment arise:

- the manufacturer and consumers gain for all n

Profit:

(18)

$$\Pi_R^N(n) = \frac{[a+nc]^2}{4b[2+n]^2} \text{ and}$$
$$\Pi_R^E(n) = \frac{\left[na+c\left(2+3n+2n^2\right)\right]^2}{2b[3+6n+2n^2]^2}$$

Proposition 5

> For all $n \geq 0$
>
> (i) Encroachment arises if and only if $c < [3+2n]$ a/ $[\,5+4n]$, in which case both the manufacturer and consumers benefit from encroachment; and
>
> (ii) Encroachment that increases retailer profit arises if and only if
>
> $$c \in \left(f(n)a, \frac{[3+2n]a}{[5+4n]}\right), \text{ where}$$
> $$f(n) = \frac{9+36n+40n^2+16n^3+2n^4}{7n+4n^2-4n^3-2n^4+\sqrt{2}[2+n]^2[3+9n+8n^2+2n^3]}.$$

Corollary

> The range of c values for which the retailer benefits from encroachment increases as n increases, i.e., $[3+2n]a/[5+4n] - f(n)a$ is increasing in n

- As n increases, retailer R is weakened by the larger number of retail rivals it faces.

retailer R's demand becomes more sensitive to the established wholesale price as n increases.

- (Retailer R's demand decreases as w increases at the rate $2[1+n]/[2+n]$, which is an increasing function of n.)

In response to this increased sensitivity, the manufacturer lowers the input price.

This benefit of encroachment accrues exclusively to retailer R.

In contrast, the burden of the revenue reduction caused by encroachment is shared by all incumbent retailers.

Consequently, the range of c/a realizations in which retailer R gains from encroachment increases as the number of incumbent retailers (n) increases.

Importantly, the range in which encroachment secures retailer gains in the absence of incumbent retail competition (i.e., when $n = 0$, as specified in Proposition 2 and as illustrated by the region between

the vertical intercepts of the two curves in Figure 1): is smaller than the corresponding range in the presence of incumbent retail competition.

4 Extensions

4.1 Imperfect Substitutes

- $n = 0$
- not perfect substitute

Let the (inverse) demand curve for the retail product of firm i be $P_i = a - q_i - kq_j$,

- P_i is the price of firm i' s product,
- q_i and q_j are the retail outputs of firms i and j, respectively (for $i, j = R, M$).
- The parameter $k \in (0, 1)$ represents the degree of product substitution.

The demands for the two retail products become independent as k approaches 0 .

The retail products become perfect substitutes (as in 3) as k approaches 1 .

We get:

(19)

$$
\begin{aligned}
w^E(k) &= \frac{a}{2} - \frac{k^2[a(1-k)+ck]}{2[8-5k^2]} \\
q_R^E(k) &= \frac{2[a(1-k)+ck]}{8-5k^2}, \\
q_M^E(k) &= \frac{[a-c]\left[8-3k^2\right]-2ak}{2[8-5k^2]}
\end{aligned}
$$

We can see: when the manufacturer encroaches, consumers and the manufacturer both benefit for all values of k

Profit:

(20)

$$\begin{aligned}\Pi_R^E(k) &= \left[a - q_R^E(k) - kq_M^E(k)\right] q_R^E(k) - wq_R^E(k) \\ &= \frac{\left[4-2k^2\right]\left[a(1-k)+ck\right]^2}{\left[8-5k^2\right]^2}.\end{aligned}$$

Compare (20) and (5) we get:

Proposition 6

> With imperfect substitutes:
>
> (i) Encroachment arises if and only if $c <$ ([8-2k $3k^2$] a) / $\left[8-3k^2\right]$, in which case both the manufacturer and consumers benefit from encroachment; and
>
> (ii) Encroachment that increases retailer profit arises if and only if:
>
> $$c \in \left(g_1(k)a, \frac{\left[8-2k-3k^2\right]a}{\left[8-3k^2\right]}\right), \text{ where}$$
> $$g_1(k) = \frac{128-112k-64k^2+57k^3}{4\left[(8-5k^2)\sqrt{4-2k^2}+8(2-2k-k^2+k^3)\right]}$$

- manufacturer always encroaches
- does not affect retailer's profit

Retailer

- benefits from more homogeneity (k increase) for lower wholesale price
- harmed by more intense competition

4.2 Price Competition

Suppose set price rather than quantities:

(21)

$$\begin{aligned}\widetilde{w}^E(k) &= \tfrac{a}{2} - \tfrac{k^2[a(1-k)+ck]}{2[8-5k^2+k^4]}, \\ \tilde{q}_R^E(k) &= \tfrac{\left[2-k^2\right][a(1-k)+ck]}{[1-k^2][8-5k^2+k^4]}, \quad \text{and} \\ \tilde{q}_M^E(k) &= \tfrac{[a-c]\left[8-7k^2+k^4\right]-ak\left[6-5k^2+k^4\right]}{2[1-k^2][8-5k^2+k^4]}\end{aligned}$$

The retailer's profit:

(22)

$$\begin{aligned}\widetilde{\Pi}_R^E(k) &= \left[a - \tilde{q}_R^E(k) - k\tilde{q}_M^E(k)\right]\tilde{q}_R^E(k) - w\tilde{q}_R^E(k) \\ &= \frac{\left[4-2k^2\right]\left[a(1-k)+ck\right]^2}{\left[1-k^2\right]\left[8-5k^2+k^4\right]^2}.\end{aligned}$$

We get:

Proposition 7

> Under retail price competition:
>
> $$\begin{aligned}\widetilde{\Pi}_R^E(k) &= \left[a - \tilde{q}_R^E(k) - k\tilde{q}_M^E(k)\right]\tilde{q}_R^E(k) - w\tilde{q}_R^E(k) \\ &= \frac{\left[4-2k^2\right]\left[a(1-k)+ck\right]^2}{\left[1-k^2\right]\left[8-5k^2+k^4\right]^2}.\end{aligned}$$
>
> (i) Encroachment arises if and only if $c < \left[8-6k- 7k^2+5k^3+k^4-k^5\right]a/\left[8-7k^2+k^4\right]$, in which case both the manufacturer and consumers benefit from encroachment; and
>
> (ii) Encroachment that increases retailer profit arises if and only if $c \in \left(g_2(k)a, \frac{\left[8-6k-7k^2+5k^3+k^4-k^5\right]a}{\left[8-7k^2+k^4\right]}\right), \quad \text{where}$
> $g_2(k) = \frac{128-176k-64k^2+153k^3-51k^5+11k^7-k^9}{4\left[(8-5k^2+k^4)\sqrt{4-6k^2+2k^4}+8(2-2k-k^2+k^3)\right]}$

4.3 Nonlinear Costs

Suppose manufacturer's cost:

$cq_M + c_2[q_M]^2$

We have:

$$\begin{aligned}w^E(c_2) =& \frac{a}{2} - \frac{c+ac_2}{6+22c_2+16c_2^2}, \quad q_R^E(c_2) = \frac{2\left[c+ac_2\right]}{3+8c_2} \\ & q_M^E(c_2) = \frac{3a\left[1+2c_2\right]-c\left[5+8c_2\right]}{6+22c_2+16c_2^2}\end{aligned}$$

Also, the retailer's profit:

$\Pi_R^E(c_2) = \frac{2[1+2c_2][c+ac_2]^2}{[1+c_2][3+8c_2]^2}$

Then we have:

Proposition 8

> In nonlinear costs:
>
> (i) Encroachment arises if and only if $c < 3[1+2c_2]$ a/ [$5+8c_2$], in which case both the manufacturer and consumers benefit from encroachment; and
>
> (ii) Encroachment that increases retailer profit arises if and only if
>
> $$c \in \left(h(c_2)\, a, \frac{3[1+2c_2]a}{[5+8c_2]}\right), \text{ where}$$
> $$h(c_2) = \frac{9+57c_2+80c_2^2}{4\left[8c_2(1+2c_2)+(3+8c_2)\sqrt{2+6c_2+4c_2^2}\right]}$$