Problem 1.4

Problem

> 1.4. Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \cdots + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i from producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously. What is the Nash equilibrium? What happens as n approaches infinity?

Solution

Denote π_i as i-th firm's payoff, then:

$\pi_i = (P - c)q_i = q_i(a - Q - c)$

To maximize the payoff, we have:

$a - Q - c - q_i = 0$

That is:

$q_i = a - Q - c$

Then we sum the n firms up:

$\sum_{i=1}^{n} q_i = Q = n(a - Q - c)$

Then we get:

$$Q = \frac{n(a-c)}{n+1}$$

Then:

$$q_i = \frac{a-c}{n+1}$$

Therefore, the Nash Equilibruim is :

$$(\frac{a-c}{n+1}, \ldots, \frac{a-c}{n+1})$$

When n goes to infinity:

$$\lim_{n\to\infty} \frac{a-c}{n+1} = 0$$

Problem 1.5

Problem

> 1.5. Consider the following two finite versions of the Cournot duopoly model. First, suppose each firm must choose either half the monopoly quantity, $q_m/2 = (a-c)/4$, or the Cournot equilibrium quantity, $q_c = (a-c)/3$. No other quantities are feasible. Show that this two-action game is equivalent to the Prisoners' Dilemma: each firm has a strictly dominated strategy, and both are worse off in equilibrium than

> they would be if they cooperated. Second, suppose each firm can choose either $q_m/2$, or q_c, or a third quantity, q'. Find a value for q' such that the game is equivalent to the Cournot model in Section 1.2.A, in the sense that (q_c, q_c) is a unique Nash equilibrium and both firms are worse off in equilibrium than they could be if they cooperated, but neither firm has a strictly dominated strategy.

Solution (a)

The payoff matrix is:

	$q_m/2$	q_c
$q_m/2$	$(\frac{(a-c)^2}{8}, \frac{(a-c)^2}{8})$	$(\frac{5(a-c)^2}{48}, \frac{5(a-c)^2}{36})$
q_c	$(\frac{5(a-c)^2}{36}, \frac{5(a-c)^2}{48})$	$(\frac{(a-c)^2}{9}, \frac{(a-c)^2}{9})$

Then: $q_m/2$ is strictly dominated by q_c, however,

$(q_m/2, q_m/2)$ is worse than (q_c, q_c)

It correspond with Prisoners' Dilemma.

Solution (b)

Assume $q' = \frac{a-c}{k}$, then:

	$q_m/2$	q_c	q'
$q_m/2$	$(\frac{(a-c)^2}{8}, \frac{(a-c)^2}{8})$	$(\frac{5(a-c)^2}{48}, \frac{5(a-c)^2}{36})$	$(\frac{(3k-4)(a-c)^2}{16k}, \frac{(3k-4)(a-c)^2}{4k^2})$
q_c	$(\frac{5(a-c)^2}{36}, \frac{5(a-c)^2}{48})$	$(\frac{(a-c)^2}{9}, \frac{(a-c)^2}{9})$	$(\frac{(2k-3)(a-c)^2}{9k}, \frac{(2k-3)(a-c)^2}{3k^2})$
q'	$(\frac{(3k-4)(a-c)^2}{4k^2}, \frac{(3k-4)(a-c)^2}{16k})$	$(\frac{(2k-3)(a-c)^2}{3k^2}, \frac{(2k-3)(a-c)^2}{9k})$	$(\frac{(k-2)(a-c)^2}{k^2}, \frac{(k-2)(a-c)^2}{k^2})$

To make (q_c, q_c) a Nash Equilibruim, we need:

$$\frac{1}{9} > \frac{2k-3}{3k^2}$$

To make $q_m/2$ not be dominated by q_c, we need:

$$\frac{3k-4}{16k} \geq \frac{2k-3}{9k}$$

To make q' not be dominated, we need:

$$\max(\frac{3k-4}{4k^2} - \frac{5}{36}, \frac{2k-3}{3k^2} - \frac{1}{9}, \frac{k-2}{k^2} - \frac{2k-3}{9k}) \geq 0$$

combined them together, we get:

$$k = \frac{12}{5}$$

that is:

	$q_m/2$	q_c	q'
$q_m/2$	$(\frac{(a-c)^2}{8}, \frac{(a-c)^2}{8})$	$(\frac{5(a-c)^2}{48}, \frac{5(a-c)^2}{36})$	$(\frac{(a-c)^2}{12}, \frac{5(a-c)^2}{36})$
q_c	$(\frac{5(a-c)^2}{36}, \frac{5(a-c)^2}{48})$	$(\frac{(a-c)^2}{9}, \frac{(a-c)^2}{9})$	$(\frac{(a-c)^2}{12}, \frac{5(a-c)^2}{48})$
q'	$(\frac{5(a-c)^2}{36}, \frac{(a-c)^2}{12})$	$(\frac{5(a-c)^2}{48}, \frac{(a-c)^2}{12})$	$(\frac{5(a-c)^2}{72}, \frac{5(a-c)^2}{72})$