Problem

> 2.6. Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$ and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The firms choose their quantities as follows: (1) firm 1 chooses $q_1 \geq 0$; (2) firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3, respectively. What is the subgame-perfect outcome?

Solution

For firm 2:

$$\pi_2 = (P - c)q_2 = (a - (q_1 + q_2 + q_3) - c)\, q_2$$

By maximizing the profit, we can get:

$$q_2 = \frac{a-c-q_1-q_3}{2}$$

Similarly:

$$q_3 = \frac{a-c-q_1-q_2}{2}$$

With $q_2 = q_3$, we get:

$$q_2 + q_3 = \frac{2(a-c-q_1)}{3}$$

Then, for firm 1:

$$Q = q_1 + q_2 + q_3 = q_1 + \frac{2(a-c-q_1)}{3}$$

Then firm 1's profit is:

$$\begin{aligned} \pi_1 &= (P - c)q_1 = \left(\frac{a-q_1+2c}{3} - c\right) q_1 \\ &= \left(\frac{a-q_1+2c-3c}{3}\right) q_1 = \frac{a-q_1-c}{3} q_1 \end{aligned}$$

By maximizing the profit, we can get:

$$q_1 = \frac{a-c}{2}$$

Also we can get:

$q_2 = q_3 = \frac{a-c}{6}$

Then:

$$Q = \frac{5(a-c)}{6}$$

$$P = \frac{a+5c}{6}$$