1

Problem

> 1. Consider the following game.
>
> | | w | x | y | z |
> |---|---|---|---|---|
> | a | $4,4$ | $1,1$ | $0,2$ | $0,1$ |
> | b | $1,1$ | $1,2$ | $1,0$ | $1,6$ |
> | c | $0,0$ | $2,0$ | $3,2$ | $0,1$ |
> | d | $0,0$ | $0,5$ | $0,2$ | $6,1$ |
>
> (a) Find a mixed strategy for the row player to strictly dominate strategy b.
>
> (b) Compute the set of all Nash equilibria (including mixed strategies). Hint: In any mixed strategy Nash equilibrium, we put zero probability on strictly dominated strategies.

Solution (a)

Notice:

$$\frac{7}{18}a+\frac{7}{18}c+\frac{4}{18}d=(\frac{14}{9},\frac{7}{6},\frac{7}{6},\frac{4}{3})>(1,1,1,1)$$

Solution (b)

After eliminate b:

	w	x	y	z
a	$4,4$	$1,1$	$0,2$	$0,1$
c	$0,0$	$2,0$	$3,2$	$0,1$
d	$0,0$	$0,5$	$0,2$	$6,1$

z is strictly dominated by y:

	w	x	y
a	$4,4$	$1,1$	$0,2$
c	$0,0$	$2,0$	$3,2$
d	$0,0$	$0,5$	$0,2$

d is strictly dominated by a and c:

	w	x	y
a	$4,4$	$1,1$	$0,2$
c	$0,0$	$2,0$	$3,2$

x is strictly dominated by y:

	w	y
a	$4,4$	$0,2$
c	$0,0$	$3,2$

Therefore the mixed nash equilibrium:

$$\left(\left(\frac{1}{2},0,\frac{1}{2},0\right),\left(\frac{3}{7},0,\frac{4}{7},0\right)\right)$$

1.12

Problem

> Find the mixed-strategy Nash equilibrium of the following normal-form game.
>
> | | L | R |
> |---|---|---|
> | T | $2,1$ | $0,2$ |
> | B | $1,2$ | $3,0$ |

Solution

We need:

$$l+2r=2l$$

$$2t+b=3b$$

Then the nash equilibrium:

$$((\frac{1}{2},\frac{1}{2}),(\frac{2}{3},\frac{1}{3}))$$

1.13

Problem

> Each of two firms has one job opening. Suppose that (for reasons not discussed here but relating to the value of filling each opening) the firms offer different wages: firm i offers the wage w_i, where $(1/2)w_1 < w_2 < 2w_1$. Imagine that there are two workers, each of whom can apply to only one firm. The workers simultaneously decide whether to apply to firm 1 or to firm 2 . If only one worker applies to a given firm, that worker gets the job; if both workers apply to one firm, the firm hires one worker at random and the other worker is unemployed (which has a payoff of zero). Solve for the Nash equilibria of the workers' normal-form game. (For more on the wages the firms will choose, see Montgomery [1991].)

Solution

Pure strategy equilibrium:

$$(T, R), (B, L)$$

Mixed strategy equilibrium:

we have:

$$\frac{1}{2}w_1 l + w_1 r = w_2 l + \frac{1}{2}w_2 r$$
$$\frac{1}{2}w_1 u + w_1 d = w_2 u + \frac{1}{2}w_2 d$$

Then we get:

$$\left(\left(\frac{2w_1 - w_2}{w_1 + w_2}, \frac{2w_2 - w_1}{w_1 + w_2}\right), \left(\frac{2w_1 - w_2}{w_1 + w_2}, \frac{2w_2 - w_1}{w_1 + w_2}\right)\right)$$