HW2

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PROBLEM 1: Let C be convex and let $\mathrm{int}(C) \neq \phi$. Then (i) $\overline{\mathrm{int}(C)} = \bar{C}$ and (ii) $\mathrm{int}(C)=\mathrm{int}(\bar{C})$ and then $\partial C = \partial \bar{C}$. **Hint:** $\mathbf{int}(C) \cup \partial C = \bar{C} = \bar{\bar{C}} = \mathbf{int}(\bar{C}) \cup \partial \bar{C}$ **and** $\mathbf{int}(C) \cap \partial C = \phi$.

◀. WRITE YOUR ANSWER HERE ▶

PROBLEM 2: Show that the following functions are convex:

1. $e^{x_1+x_2} + (x_1 - x_2)^2 + x_1^4$;
2. $e^{x_1} + e^{x_2} + (x_1 - 4x_2)^4 - 5$.

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PROBLEM 3: Consider the minimal-objective function of $\mathbf{b}$ for fixed A and $\mathbf{c}$:

$$\begin{aligned} z(\mathbf{b}) = \min\ & \mathbf{c}^{\mathbf{t}}\mathbf{x} \\ \text{s.t. } & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

Show that $z(\mathbf{b})$ as a function of $\mathbf{b}$ is a convex function in $\mathbf{b}$ for all feasible $\mathbf{b}$.

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PROBLEM 4: Let X be a nonempty compact set and let $f : \mathbb{R}^n \to \mathbb{R}$, and $\mathbf{g} : \mathbb{R}^n \to \mathbb{R}^m$. Denote $\theta(\boldsymbol{\lambda}) = \inf\{f(\mathbf{x}) + \langle \boldsymbol{\lambda}, \mathbf{g}(\mathbf{x})\rangle : \mathbf{x} \in X\}$. Prove that $\theta(\boldsymbol{\lambda})$ is concave over $\mathbb{R}^m$.

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PROBLEM 5: Prove that every local solution of the following problem is a global solution as well:

$$\begin{aligned} \min_{x_1,x_2,x_3\in\mathbb{R}} \quad & e^{x_1-2x_2+x_3} + (x_1 - 5x_2 + 6x_3)^2 + (-x_1 + 2x_2 + 3x_3)^6 \\ \text{s.t.} \quad & x_1 + x_2 - 7x_3 = 1 \\ & x_1^2 + x_2^2 + e^{x_1-2x_2-x_3} \leq 2 \\ & x_1 \geq 0 \\ & x_3 \geq 0 \end{aligned}$$

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PROBLEM 6: Consider the optimization problem

$$\begin{aligned} \min_{x \in \mathbb{R}} \quad & 2x^2 - x^3 \\ \text{s.t.} \quad & x \in \{-2, -1, 0, 1, 2\} \end{aligned}$$

1. Convert the above problem to an optimization problem with a linear objective.
2. Draw the feasible set of the reformulated problem.
3. Convexify the reformulated problem and draw the feasible set of the resulting convex problem.

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PROBLEM 7: Employ AI to evaluate the benefits of studying convex programming and to provide a concrete example illustrating how convex programming can be applied to solve practical problems.

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