1

Problem

Consider the following game involving N matchsticks positioned on a table. Two players participate in this game, with Player 1 making the initial move, followed by Player 2. The players then continue to take turns until the game concludes. The game is governed by two rules:
(i) A player in his or her turn can pick either 1 or 2 matchsticks.
(ii) The player who picks the last matchstick wins the game.

Answer the following questions:
(a) For $N = 3$, construct the game tree and determine the subgame perfect Nash equilibrium.
(b) For $N = 4$, construct the game tree and determine the subgame perfect Nash equilibrium.
(c) For $N = 5$, construct the game tree and determine the subgame perfect Nash equilibrium.
(d) If $N = 9,999$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.
(e) If $N = 10,000$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.
(f) If $N = 10,001$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.

Solution (a)

Game tree:

- Player 1 picks 2 (A)
  - Player 2 picks 1 (M)
- Player 1 picks 1 (B)
  - Player 2 picks 2 (P)
  - Player 2 picks 1 (Q)
    - Player 1 picks 1 (C)

Backward induction:

Player 2 will pick 2 when Player 1 picks 1, then Player 2 always win.

Subgame perfect Nash equilibrium:

$(AC, MP), (BC, MP)$

Solution (b)

We denote the strategy in Solution a for player 1 as α, for player 2 as β

Game tree:

- Player 1 picks 2 (A)
  - Player 2 picks 2 (M)
  - Player 2 picks 1 (N)
    - Player 1 picks 1 (C)
- Player 1 picks 1 (B)
  - Player 2 act as α, player 1 as β

Backward induction:

If Player picks 2, Player 2 will picks 2.
Because β makes player 1 win, player 1 will pick 1

Player 1 always win

Subgame perfect Nash equilibrium:

$(BC\beta, M\alpha)$

Solution (c)

We denote the strategy in Solution b for player 1 as γ, for player 2 as δ

Game tree:

- Player 1 picks 2 (A)
  - Player 2 act as α, player 1 as β
- Player 1 picks 1 (B)
  - Player 2 act as γ, player 1 as δ

Backward induction:

β always win, so player 1 will choose A

subgame perfect Nash equilibrium:

$(A\beta\delta, \alpha\gamma)$

Solution (d)

Player 2 wins

He will pick the number that makes N proportional to 3

Solution (e)

Player 1 wins

He will pick 1, then pick the number that makes N proportional to 3

Solution (f)

Player 1 wins

He will pick 2, then pick the number that makes N proportional to 3

2

Problem

Consider the following game involving N matchsticks positioned on a table. Two players participate in this game, with Player 1 making the initial move, followed by Player 2. The players then continue to take turns until the game concludes. The game is governed by two rules:
(i) A player in his or her turn can pick either 1 or 3 matchsticks.
(ii) The player who picks the last matchstick loses the game.

Answer the following questions:
(a) For $N = 3$, construct the game tree and determine the subgame perfect Nash equilibrium.

(b) For $N = 4$, construct the game tree and determine the subgame perfect Nash equilibrium.
(c) For $N = 5$, construct the game tree and determine the subgame perfect Nash equilibrium.
(d) For $N = 6$, construct the game tree and determine the subgame perfect Nash equilibrium.
(e) For $N = 7$, construct the game tree and determine the subgame perfect Nash equilibrium.

(f) If $N = 100$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.
(g) If $N = 101$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.
(h) If $N = 102$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.
(i) If $N = 103$, would you prefer to be Player 1 or Player 2? Specify the strategy that could guarantee a victory in this scenario.

Solution (a)

Game tree:

- Player 1 picks 3 (A)
- Player 1 picks 2 (B)
  - Player 2 picks 1 (L)
- Player 1 picks 1 (C)
  - Player 2 picks 2 (O)
  - Player 2 picks 1 (P)
    - Player 1 picks 1 (D)

Backward induction:

in O, P, player 2 picks O

in A, B, C, plauer 1 picks A

subgame perfect Nash equilibrium:

$$(AD, LO)$$

Solution (b)

Denote player 1, 2's strategy in Solution (a) as α, β

game tree:

- Player 1 picks 3 (A)
  - Player 2 picks 1 (L)
- Player 1 picks 2 (B)

    - Player 2 picks 2 (O)
    - Player 2 picks 1 (P)
        - Player 1 picks 1 (D)
- Player 1 picks 1 (C)
    - Player 2 use α, player 1 use β

Backward induction:

in O, P, player 2 choose O

Player 1 always lose

subgame perfect Nash equilibrium

$(AD\beta, LO\alpha), (BD\beta, LO\alpha), (CD\beta, LO\alpha)$

Solution (c)

Denote player 1, 2's strategy in Solution (b) as γ, δ

game tree:

- Player 1 picks 3 (A)
    - Player 2 picks 2 (L)
    - Player 2 picks 1 (M)
        - Player 1 picks 1 (D)
- Player 1 picks 2 (B)
    - Player 2 use α, player 1 use β
- Player 1 picks 1 (C)
    - Player 2 use γ, player 1 use δ

Backward induction:

in L, M, player 2 chooses L

because δ always win, player 1 chooses C

subgame perfect Nash equilibrium:

$(CD\beta\delta, L\alpha\gamma)$

Solution (d)

Denote player 1, 2's strategy in Solution (c) as ϵ, ζ

game tree:

- Player 1 picks 3 (A)
  - Player 2 use α, player 1 use β
- Player 1 picks 2 (B)
  - Player 2 use γ, player 1 use δ
- Player 1 picks 1 (C)
  - Player 2 use ϵ, player 1 use ζ

Backward induction:

δ always win, so Player 1 chooses B

subgame perfect Nash equilibrium :

$(B\beta\delta\zeta, \alpha\gamma\epsilon)$

Solution (e)

Denote player 1, 2's strategy in Solution (d) as η, θ

game tree:

- Player 1 picks 3 (A)$
  - Player 2 use γ, player 1 use δ
- Player 1 picks 2 (B)
  - Player 2 use ϵ, player 1 use ζ
- Player 1 picks 1 (C)
  - Player 2 use η, player 1 use θ

Backward induction:

δ always win, so Player 1 chooses A

subgame perfect Nash equilibrium :

$(A\delta\zeta\theta, \gamma\epsilon\eta)$

Solution (f)

Player 2 wins.

He will pick the number that makes N proportional to 4

Solution (g)

Player 1 wins.

He will first pick 1, then pick the number that makes N proportional to 4

Solution (h)

Player 1 wins.

He will first pick 2, then pick the number that makes N proportional to 4

Solution (h)

Player 1 wins.

He will first pick 3, then pick the number that makes N proportional to 4