HW4

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PROBLEM 1: Consider the following problem.

$$\begin{aligned} \text{Minimize} \quad & x_1 + x_2 \\ \text{subject to} \quad & x_1^2 + x_2^2 = 4 \\ & -2x_1 - x_2 \leq 4 \end{aligned}$$

Formulate the Lagrangian dual problem by incorporating both constraints into the objective function via the Lagrangian multipliers u_1 and u_2.

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PROBLEM 2: Using weak duality, prove that the set $\{x = (x_1, x_2, x_3)^T \in \mathbb{R}^3 \mid x_1^2 + x_2^2 + x_3^2 = 1,\ x_1 + 2x_2 + 3x_3 \geq 5\}$ is empty.

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PROBLEM 3: Consider the problem to minimize $x_1^2 + x_2^2$ subject to $x_1 + x_2 - 4 \geq 0$, and $x_1, x_2 \geq 0$.

1. Verify that the optimal solution is $\bar{x} = (2, 2)^t$ with $f(\bar{x}) = 8$.
2. Letting $X = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\}$, write the Lagrangian dual problem. Show that the dual function is $\theta(u) = -\dfrac{u^2}{2} + 4u$. Verify that there is no duality gap for this problem.

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PROBLEM 4: Minimize $f(\mathbf{x}) = e^{-(x_1+x_2)}$ subject to $x_1 + 2x_2 \leq 20$, $x_1 \geq 0$, $x_2 \geq 0$.

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PROBLEM 5: Let A be a positive-definite $n \times n$ symmetric matrix and let $\mathbf{y}$ be a fixed vector in $\mathbb{R}^n$. Show that the maximum value of $f(\mathbf{x}) = \langle \mathbf{y}, \mathbf{x} \rangle$ subject to the constraint $\langle \mathbf{x}, A\mathbf{x} \rangle \leq 1$ is $\langle \mathbf{y}, A^{-1}\mathbf{y} \rangle^{1/2}$. Use this result to establish the generalization of the Cauchy–Schwarz inequality

$$|\langle \mathbf{x}, \mathbf{y} \rangle|^2 \leq \langle \mathbf{x}, A\mathbf{x} \rangle \langle \mathbf{y}, A^{-1}\mathbf{y} \rangle. \tag{1}$$

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PROBLEM 6: Employ AI to evaluate the benefits of studying Lagrangian Dual problems.

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