FLEXURAL WAVE LOCALIZATION IN STRUCTURED AND DISORDERED THIN PLATES

A DIRECT VALIDATION OF THE LANDSCAPE THEORY

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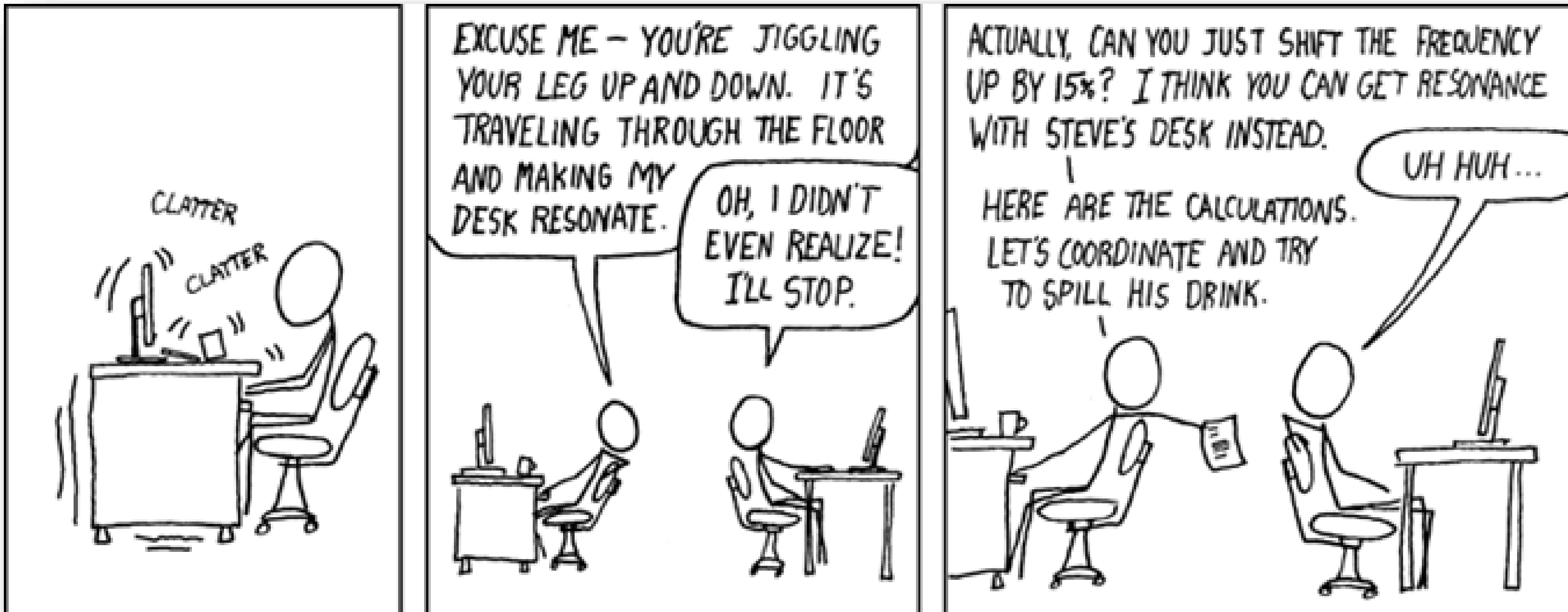
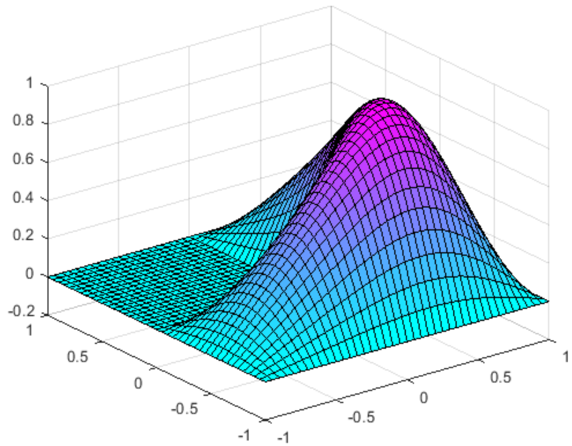
THIS TALK … IN A CARTOON



EIGENMODES AND EIGENVALUES

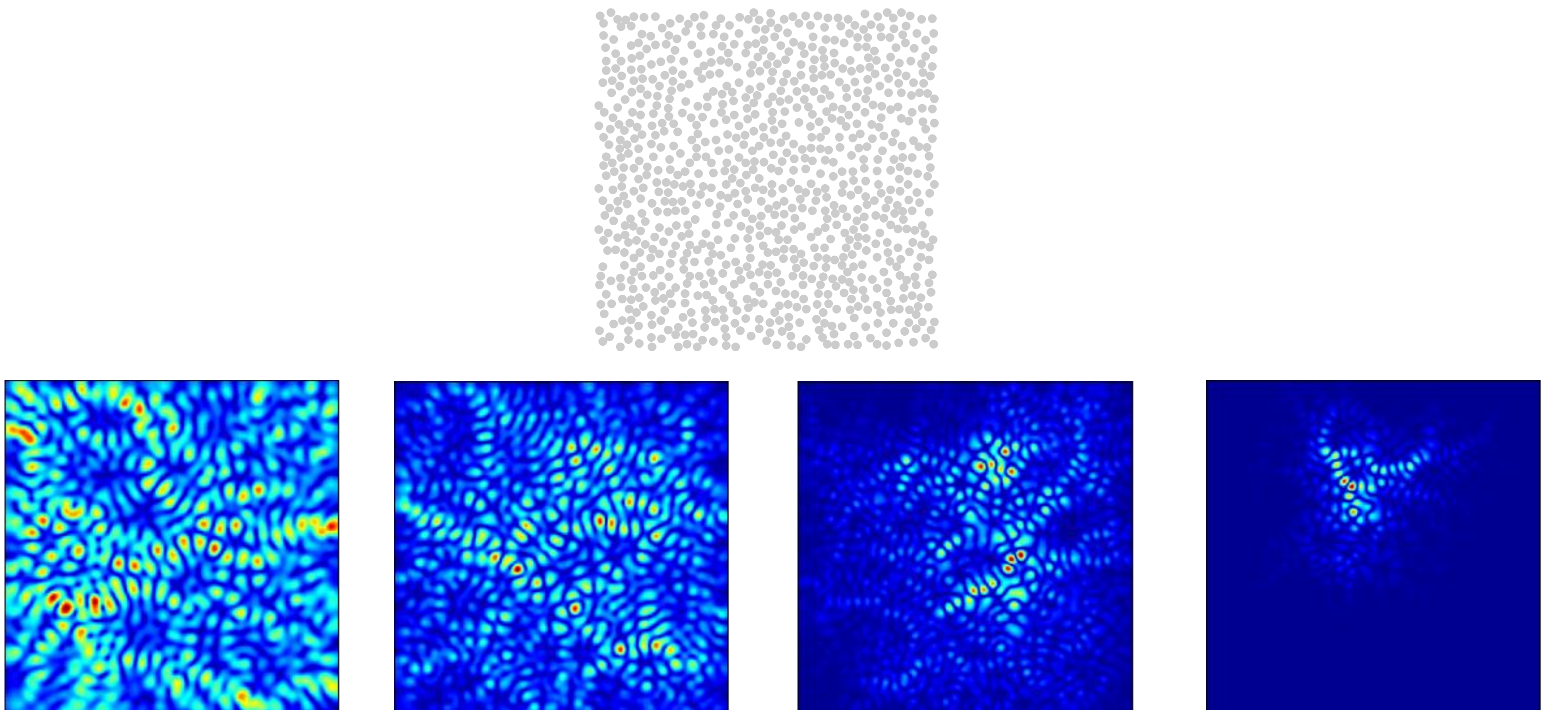
- Key ingredients to understand the physical properties of most vibratory systems



Eigenmode of L-Shaped Membrane

- Yet their prediction remains an open problem in many cases

EXAMPLE 1 : DISORDERED SYSTEMS



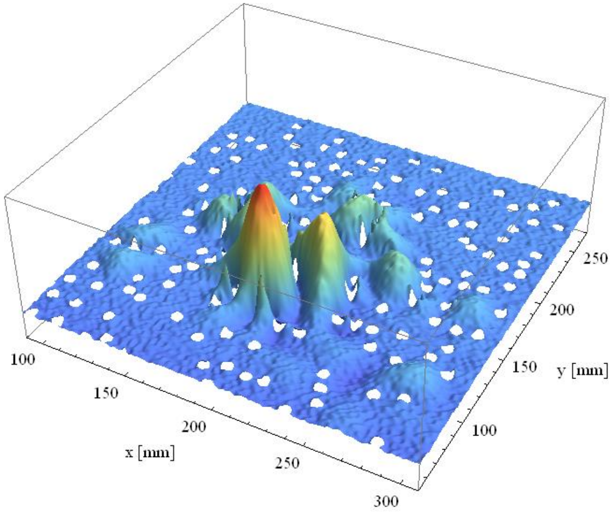
$\Delta n=0.25$ $\Delta n=0.50$ $\Delta n=0.85$ $\Delta n=1.00$

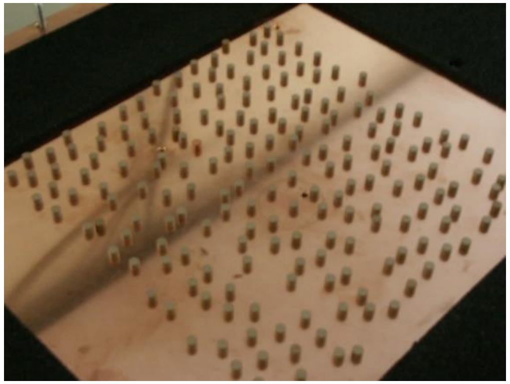
Δn = index contrast between scatterers and matrix

DISORDERED-INDUCED LOCALIZATION

Anderson Localization

"Absence of diffusion
in certain random lattices",
P.W. Anderson, *Phys. Rev 109(1958)*.

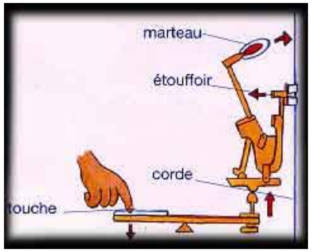


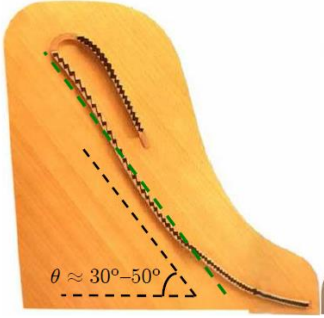


Laurent et al., PRL99, 253902 (2007)

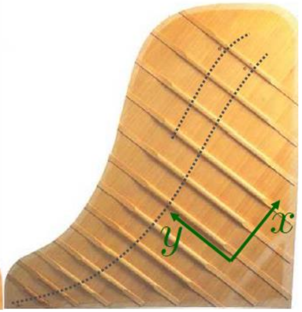
EXAMPLE 2: ARCHITECTURE OF MUSIC INSTRUMENTS

The design of soundboard

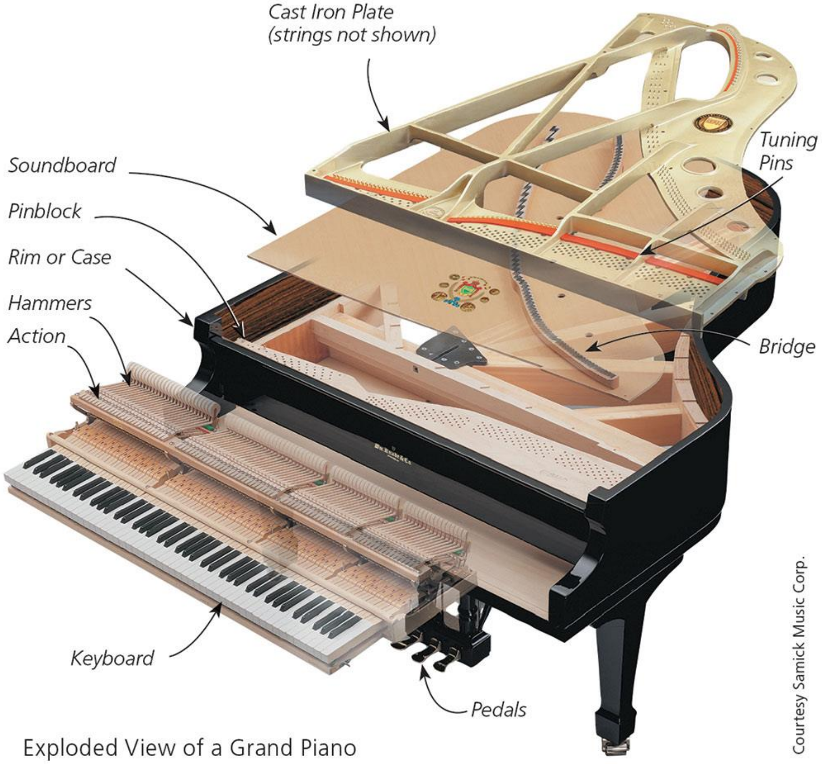




Bridges/chevalet



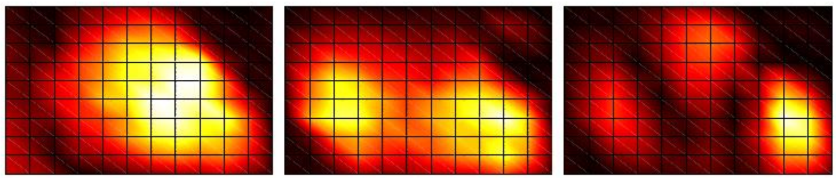
Ribs/traverses



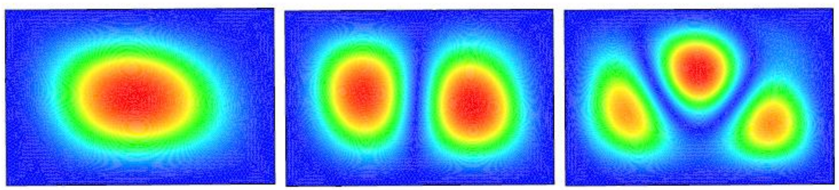
Exploded View of a Grand Piano

Xavier Boutillon (LMS Polytechnique), Vibroacoustics of the piano soundboard

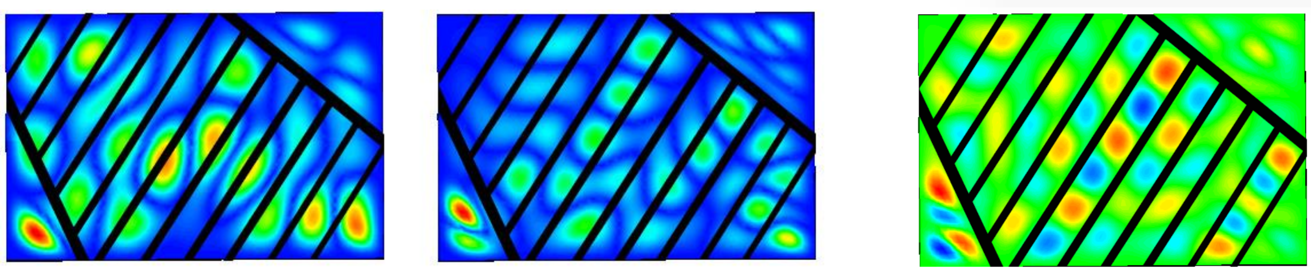
LOW AND HIGH FREQUENCY MODES



(a) (1,1)-mode *(b)* (2,1)-mode *(c)* (3,1)-mode



First 3 modal shapes of the upright piano soundboard : measured and simulated



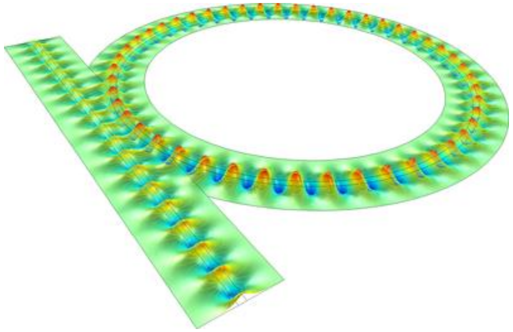
*f*n = 776 *Hz* *f*n = 1089 *Hz* *f*n = 1593 *Hz*

Intensity (and field) distribution of higher modes of the upright piano soundboard

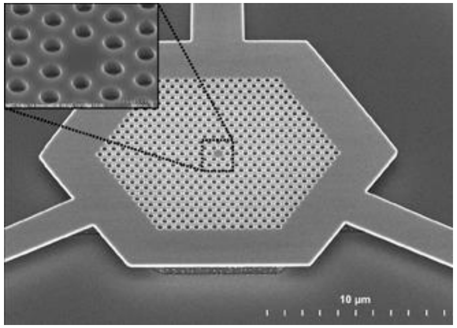
OTHER STRING INSTRUMENTS

OTHER EXAMPLES

- Optical resonators



- Laser cavities
  - microcavity lasers
  - external cavity semiconductor laser
  - dfb laser
  - random lasers
  - …

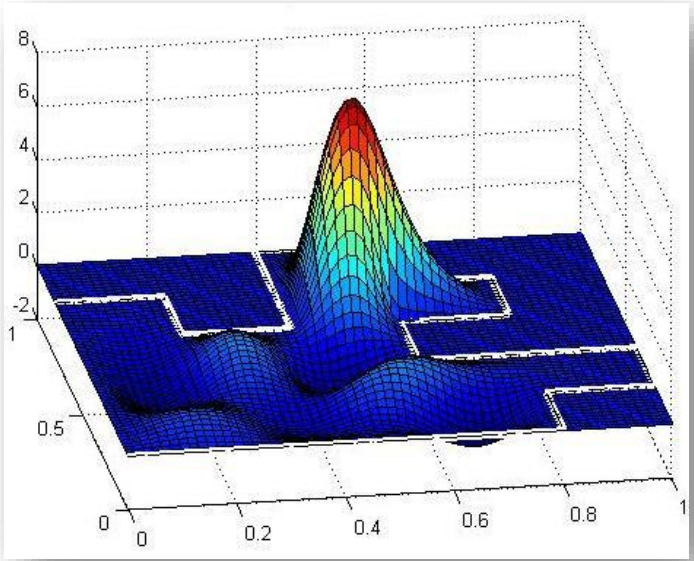


- Microwave resonators

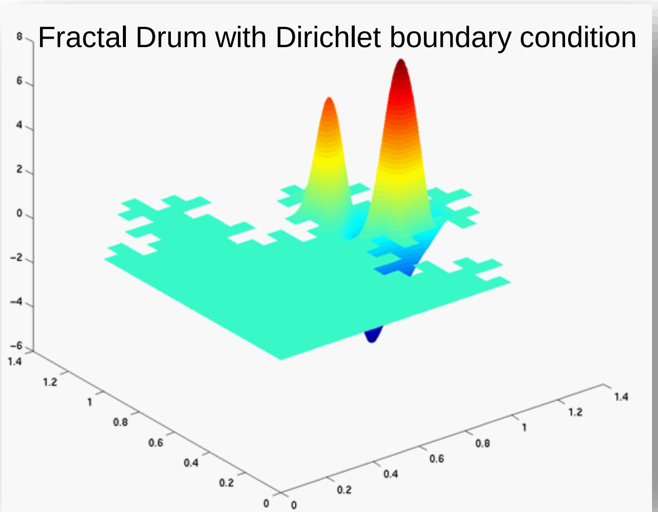
- …

WAVE LOCALIZATION : A BROAD DEFINITION

A state is said to be "localized"
if the vibration amplitude is large
only in a restricted region
of the total domain



WAVE LOCALIZATION IN COMPLEX STRUCTURES



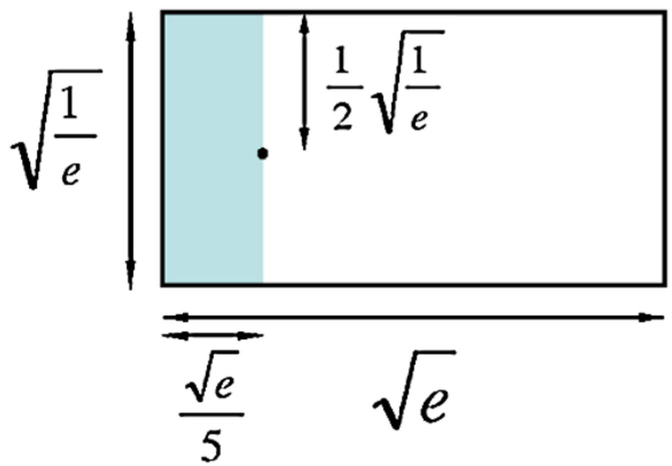
Félix et al., *J Sound Vib*, 2007

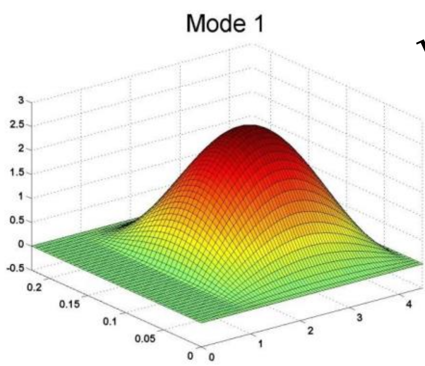
Heilman & Strichartz, *Not Am Math Soc,* 2010

“Localized eigenfunctions : Here you see them, there you don’t”

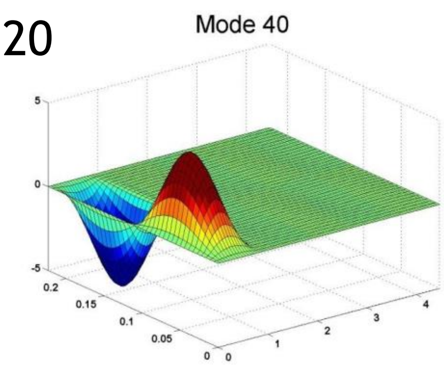
Applications:
- Structural engineering
- Noise abatement walls
- Clamped plate problem
- control of vibrations
- Musical instrument design

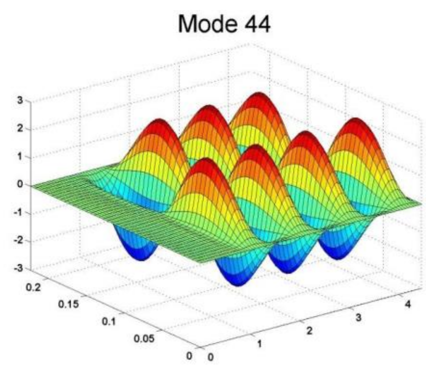
WAVE LOCALIZATION IN A CLAMPED PLATE





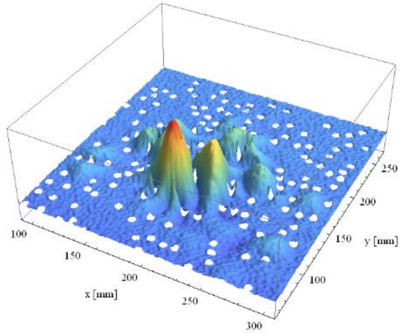
$\sqrt{e}$=20





M.Filoche & Mayboroda, *PRL103*, 254301 (2009)

QUESTIONS



- What is the effective potential that traps the waves ?

  Are there "**invisible confining boundaries**"?

- Need for a **predictive tool** :
  - To retrieve the spatial and spectral properties of the localized modes, without resorting to a full eigenvalue problem
  - To predict the frequency dependence of the mode extension/distribution

- The **inverse problem**: Can we engineer plates with particular vibrational properties ?

THE LOCALIZATION LANDSCAPE

A new theoretical tool to retrieve crucial information on the spatial and frequency properties of these localized waves

M.Filoche and S. Mayboroda, PNAS (2012)

THE LOCALIZATION LANDSCAPE

Elliptic operator $\boldsymbol{L}$

$$\boldsymbol{L} = H = -\Delta + V \rightarrow \mathrm{H}\varphi = -i\hbar\frac{\partial\varphi}{\partial t}$$

$$\boldsymbol{L} = -\Delta \qquad \rightarrow -\Delta\varphi = -\tfrac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

$$\boldsymbol{L} = -\Delta^{\mathbf{2}} \qquad \rightarrow -\Delta^2\varphi = -\tfrac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$$

The properties of the system are entirely contained in the eigenfunctions and eigenvalues of the operator $\boldsymbol{L}$

$$\boxed{\boldsymbol{L\varphi_k = \lambda_k\varphi_k} \qquad \text{with } \varphi_k|_{\partial\Omega} = 0 \qquad \lambda_k > 0}$$

THE LOCALIZATION LANDSCAPE

Proposition :

for every $x \in \Omega$ $\quad\quad |\boldsymbol{\varphi_k(x)}| \leq \boldsymbol{\lambda_k u(x)}$

with normalization $\|\varphi_{k\,max}\| = 1$

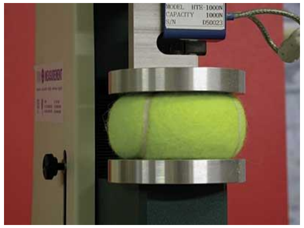
where $\boldsymbol{u}$ is the solution of the Dirichlet boundary problem:

$$\boldsymbol{Lu = 1} \text{ with } u|_{\partial\Omega} = 0$$

$\boldsymbol{u}$ is called the Landscape function the solution of

$$\forall x \in \Omega, \qquad u(x) = \int_{\Omega} |G(x,y)|dy$$

u corresponds to the "static deformation" of the system under uniform load



M.Filoche and S. Mayboroda, PNAS (2012)

THE *LOCALIZATION LANDSCAPE*

- The map of u is a landscape with many valleys and peaks.
- It partitions the domain into sub-regions.
- These sub-regions correspond to the localization regions of the different modes.

Indeed all eigenfunction φ satisfies :

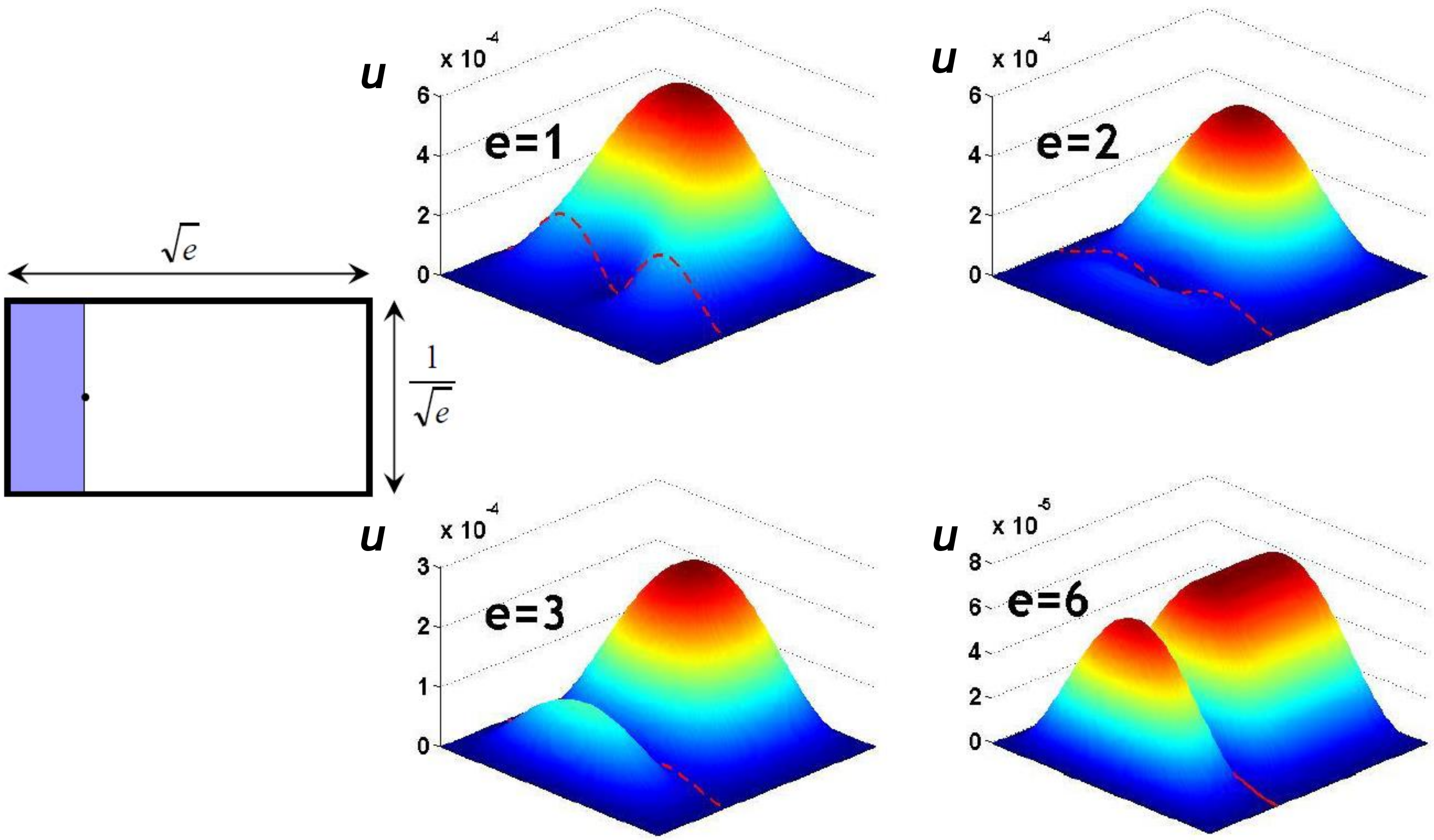
$$\forall x \in \Omega, \quad \boldsymbol{|\varphi_k(x)| \le \lambda_k u(x)}$$

with normalization $\|\varphi_{k\,max}\|=1$

The eigenmodes are "small" when $\boldsymbol{u}$ is "small"

M.Filoche and S. Mayboroda, PNAS (2012)

THE LANDSCAPE IN THE CLAMPED PLATE



THE *LOCALIZATION LANDSCAPE*

- Describes the sub-regions of localization

- Gives, in a single measurement
  - the eigenfrequencies
  - The eigenmode profile
  - The frequency dependence of the mode spatial extension

- This is a static measurement, which does not require a precise knowledge of the microscopic geometry

- It can be used to address the inverse problem and design the system geometry corresponding that a desired spatial distribution of the modes

M.Filoche and S. Mayboroda, PNAS (2012)

OUTLINE

- Part 1: Elastic waves confined in a structured thin plate

  Direct demonstration of the predictive power of the localization landscape in a simple geometry

- Part 2: Disordered-induced localization in thin plates

  Localized modes in

  (a) randomly-pinned plates

  (b) randomly-distributed blind holes in thin plates.

COLLABORATORS

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- **PhD : Olivier Xéridat** (LPMC)**, Marc DUBOIS** (Langevin)
- **Undergraduates : Alexane GONDEL (Ecole des Mines), Florian FEPPON, Aime LABBE, Camille GILLOT, Alix GARELLI, Maxence ERNOULT (Ecole Polytechnique)**

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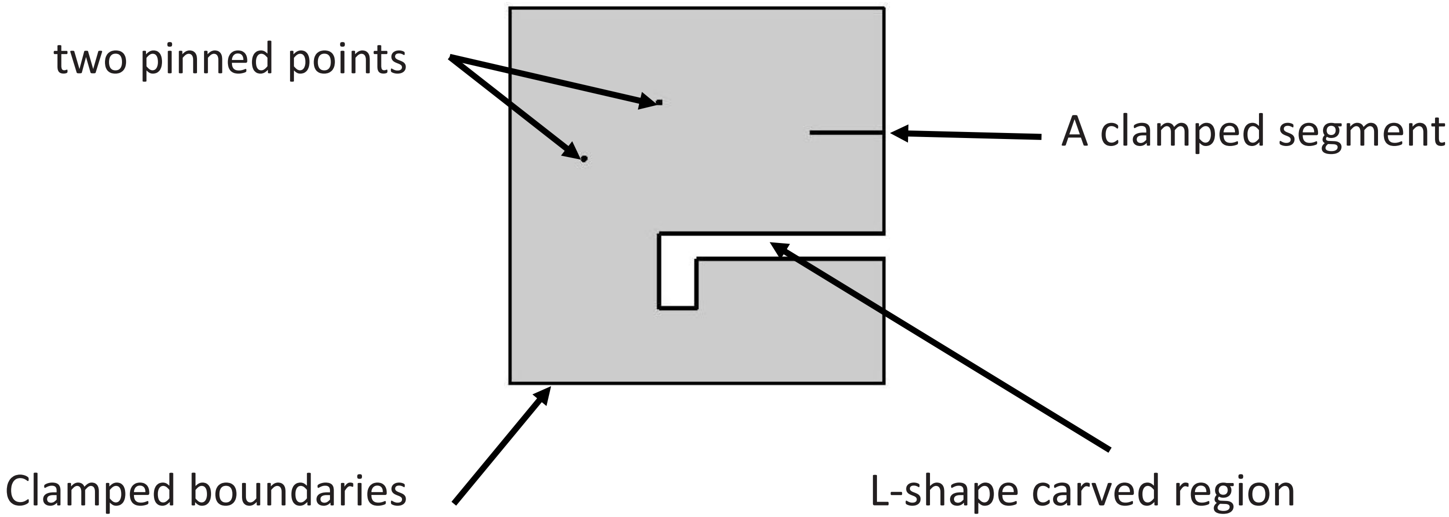
PART 1: ELASTIC WAVES CONFINED IN A STRUCTURED THIN PLATE

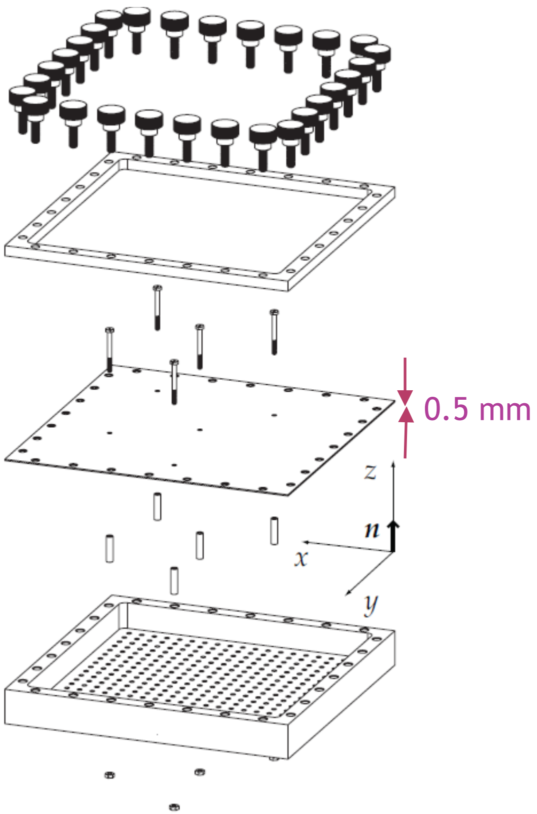
A direct demonstration of the predictive power of the localization landscape theory

G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

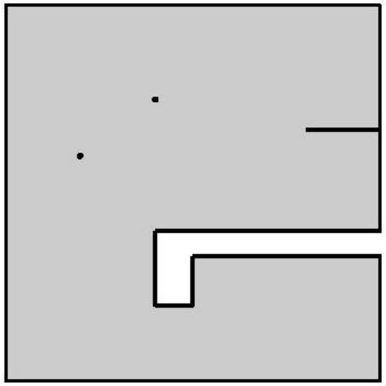
A STRUCTURED SYSTEM

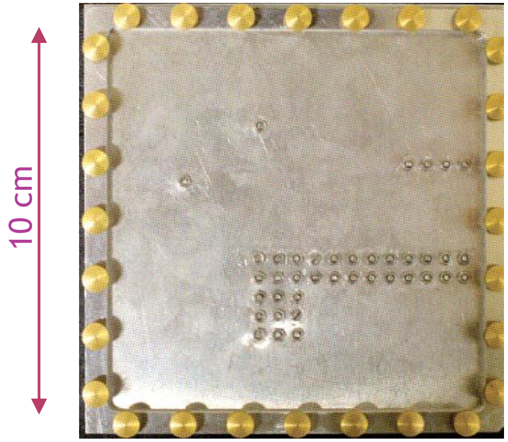


EXPERIMENTAL REALISATION



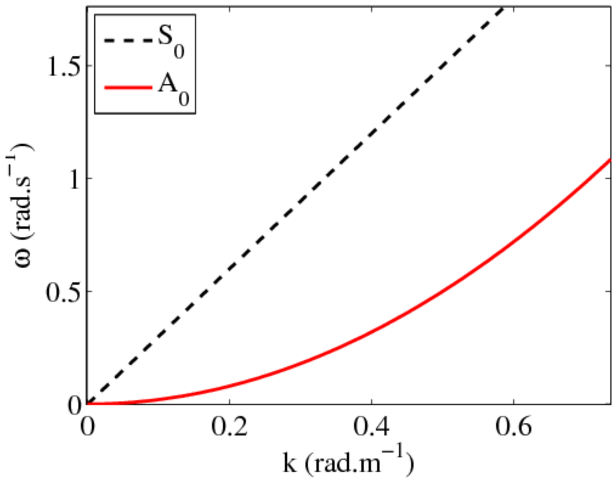
Exploded view

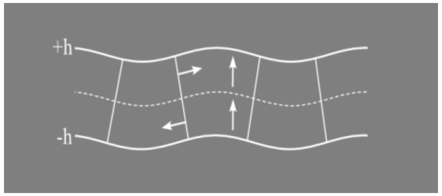




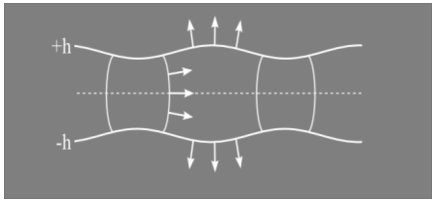
THIN PLATE EQUATION

Lamb waves





A_0 Vertical displacement



S_0

Low frequency approximation: Flexural waves

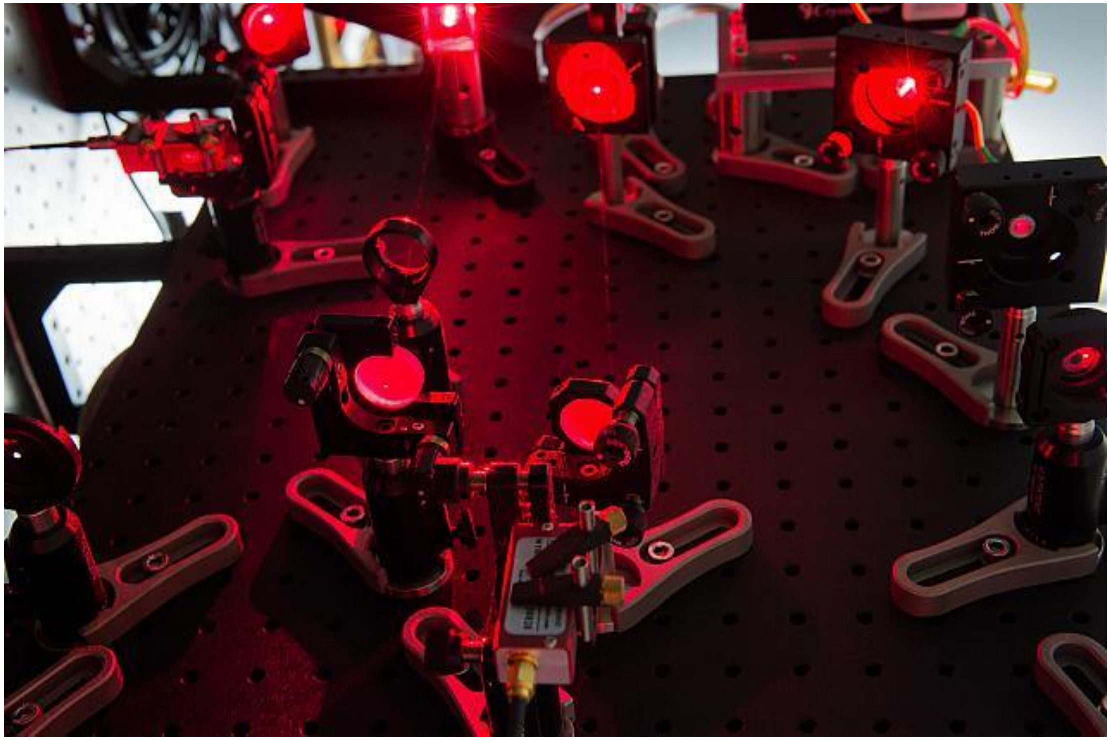
Kirchhoff-Love equation for vertical displacement u:

$$\boldsymbol{\Delta^2 u + \frac{\rho h}{D}\frac{\delta^2 u}{\delta t^2} = 0}, \qquad \omega^2 = \frac{D}{\rho h} k^4$$

with $D = \frac{Eh^3}{12(1-\nu^2)}$

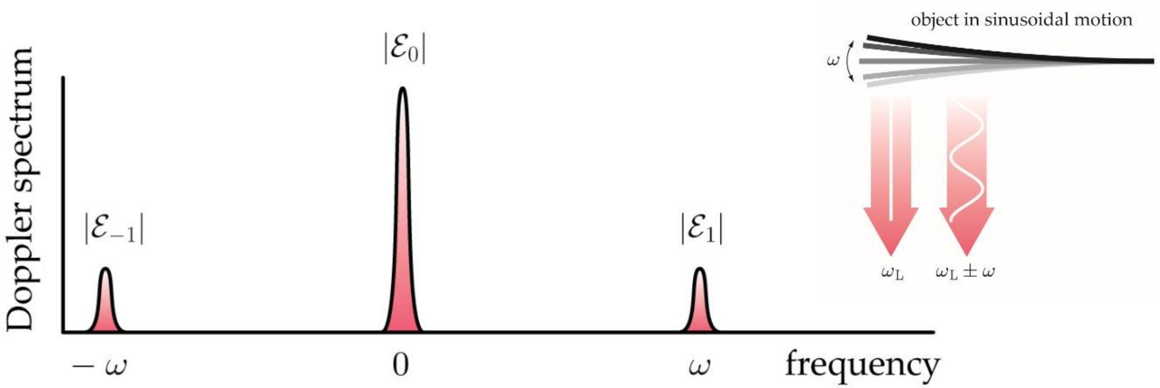
HETERODYNE HOLOGRAPHIC IMAGING

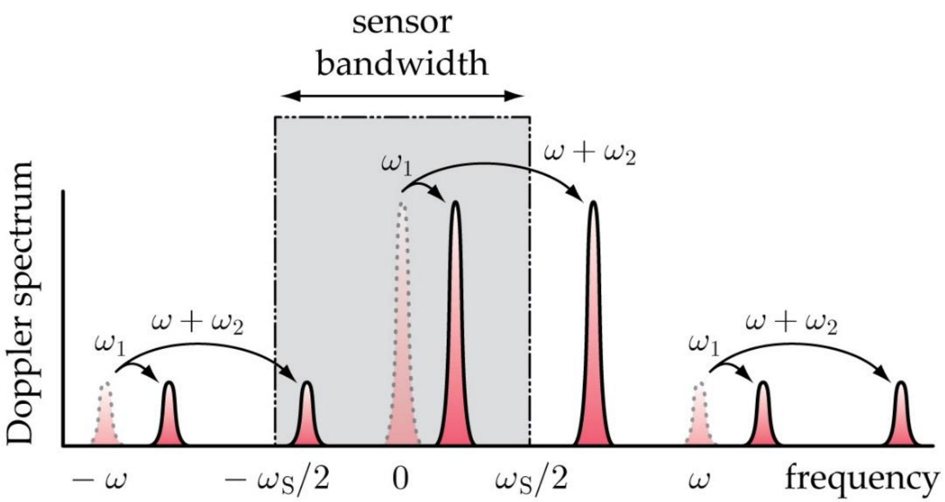
Wide field imaging of out of plane vibration vs. frequency



M. Atlan *et al.* Opt. Lett. 39, 2014

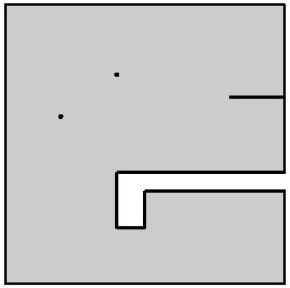
DOUBLE SIDEBAND MODULATION

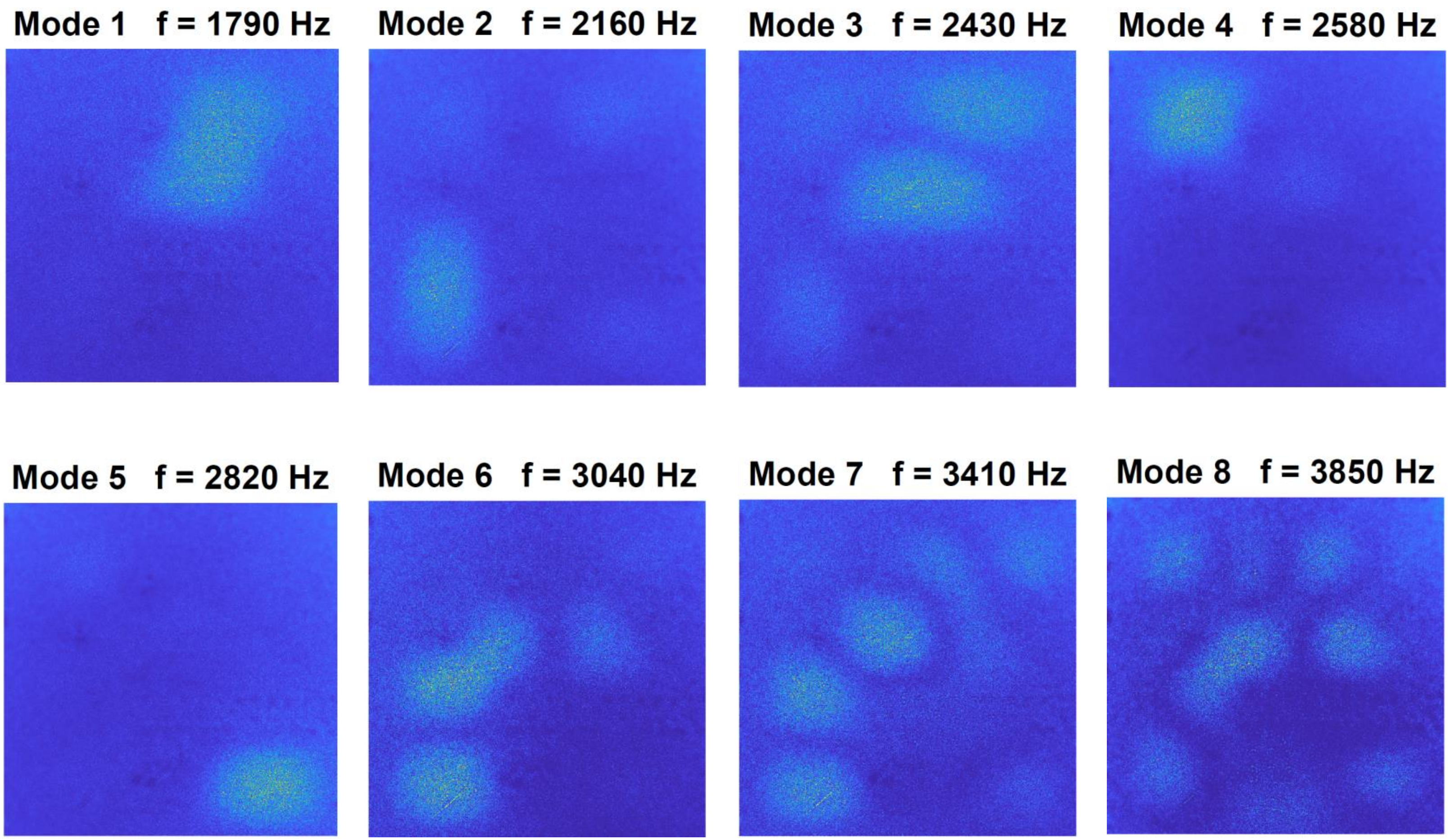




Nicolas Verrier and Michael Atlan. Optics Letters, 38 (2013)

FIRST MODES: INTENSITY



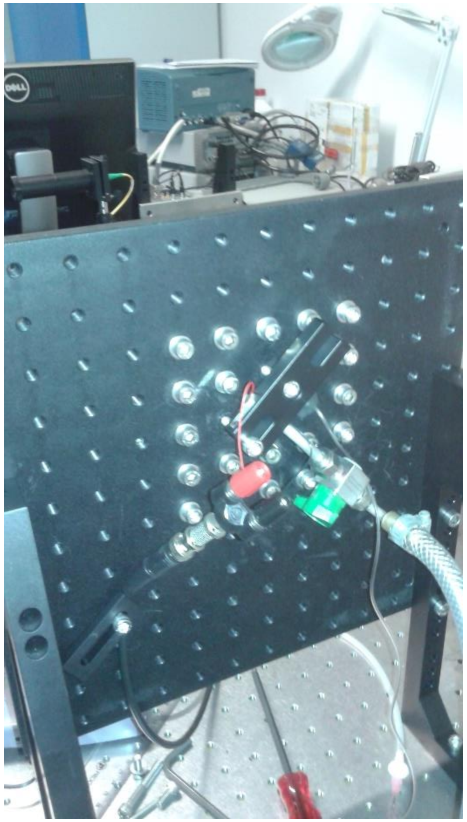


G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

OBJECTIVE

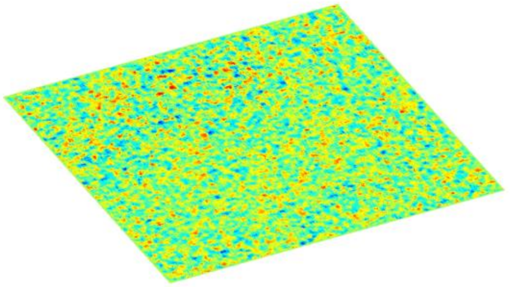
- Can we measure the *localization landscape ?*
- Does the theory apply to a real system ?

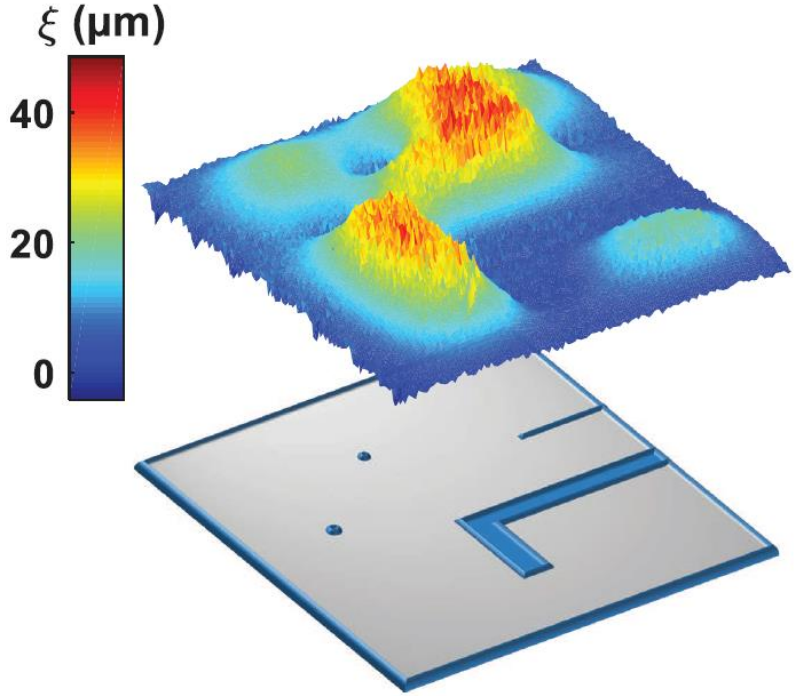
MEASURING THE LANDSCAPE



Measure of the static deformation using holographic interferometry as low pressure is progressively released

MEASURING THE LANDSCAPE

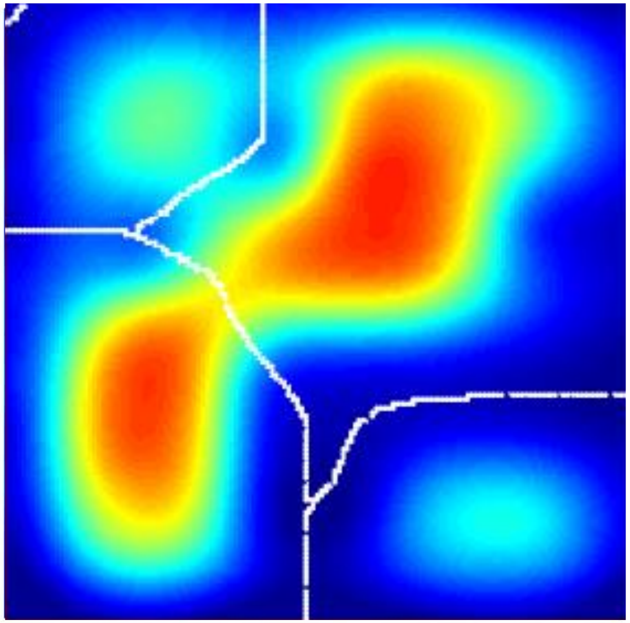




G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

1 - LOCALIZATION REGIONS

The map of u is a landscape with many valleys and peaks

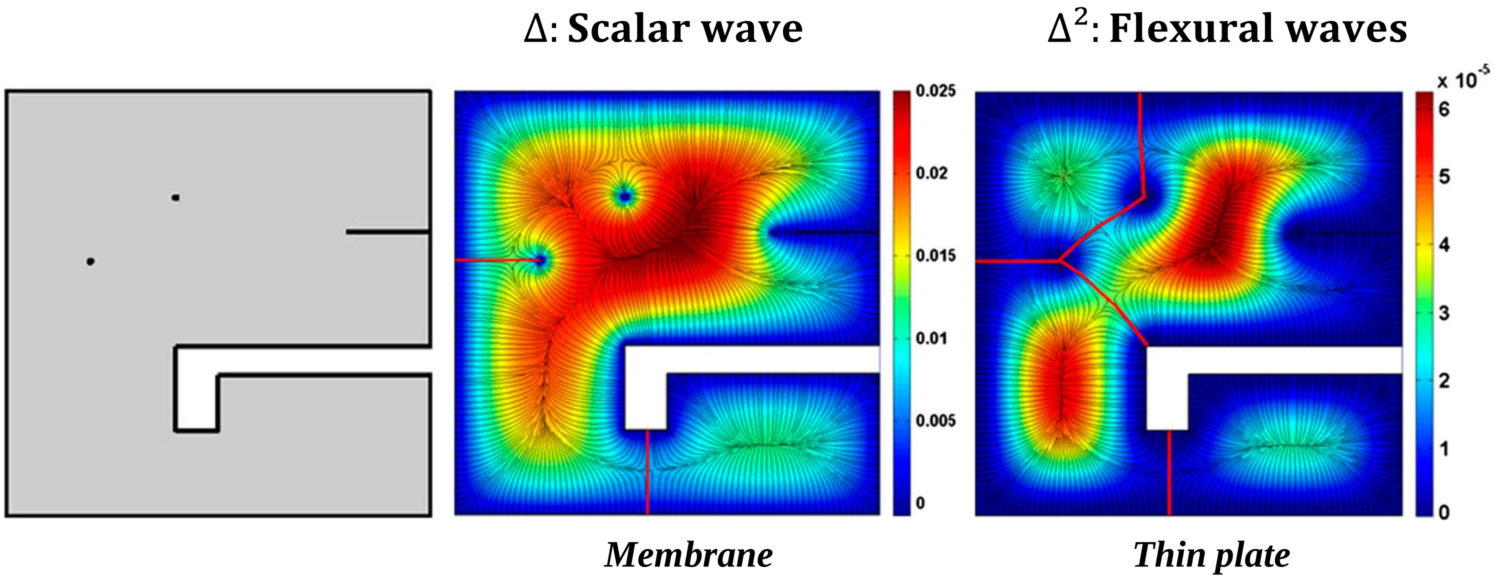


The white lines are the watershed lines of steepest ascent or steepest descent

G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

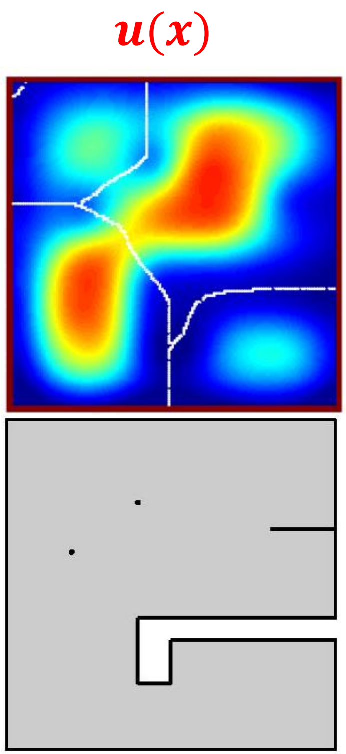
LANDSCAPES FOR DIFFERENT OPERATORS

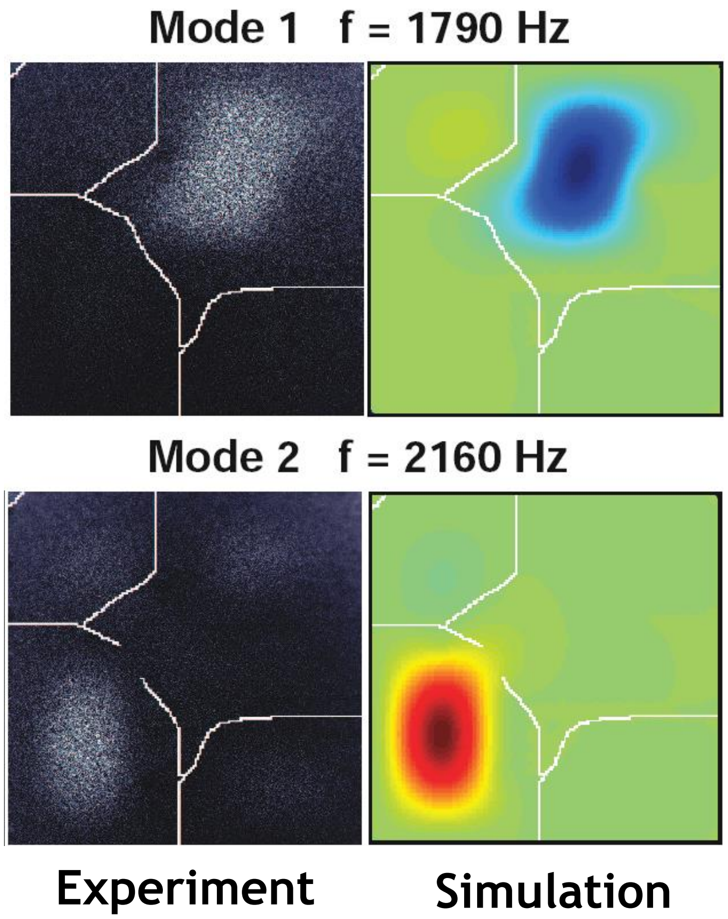
The nature of the operator is embedded in the landscape function



M.Filoche and S. Mayboroda, PNAS (2012)

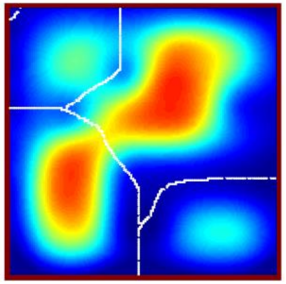
1 - LOCALIZATION REGIONS





1 - LOCALIZATION REGIONS

These sub-regions correspond to the localization regions of the different modes.



All eigenfunctions φ satisfies :

$$\forall x \in \Omega, \quad |\boldsymbol{\varphi_k(x)}| \leq \boldsymbol{\lambda_k u(x)}$$

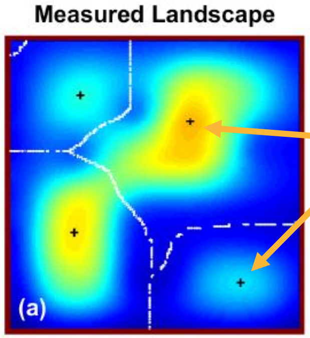
with normalization $\|\varphi_{k\,max}\|$=1

The eigenmodes are “small” where u is “small”

The “valleys” of $\boldsymbol{u(x)}$ are the invisible barriers for waves

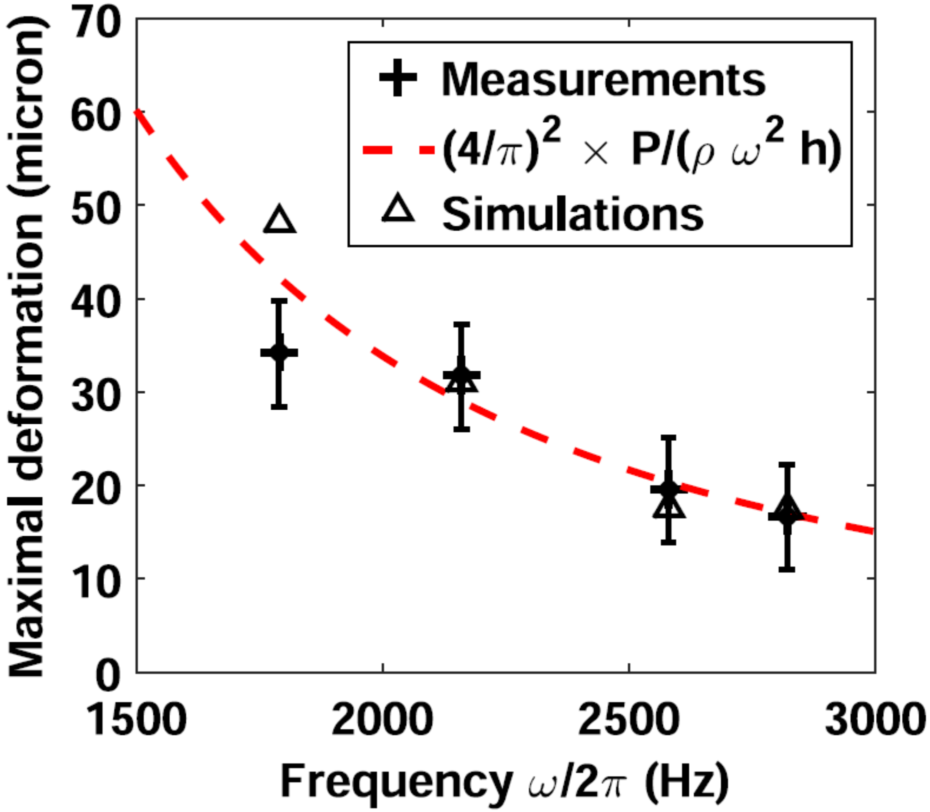
M.Filoche and S. Mayboroda, PNAS (2012)

2 - EIGENVALUE PREDICTION



$\mathbf{max}_{\boldsymbol{i}}(\boldsymbol{u}) = \frac{\mathbf{1.33}^2}{\boldsymbol{\omega}_{\boldsymbol{i}}^{\mathbf{2}}}$ where ω_i is the frequency of the fundamental mode of region i

2 - EIGENVALUE PREDICTION



$$\max_i(u) = \frac{1.33^2}{\omega_i^2}$$

3 - EFFECTIVE VALLEY NETWORK

$$|\varphi_k(x)| \leq \lambda_k u(x)$$

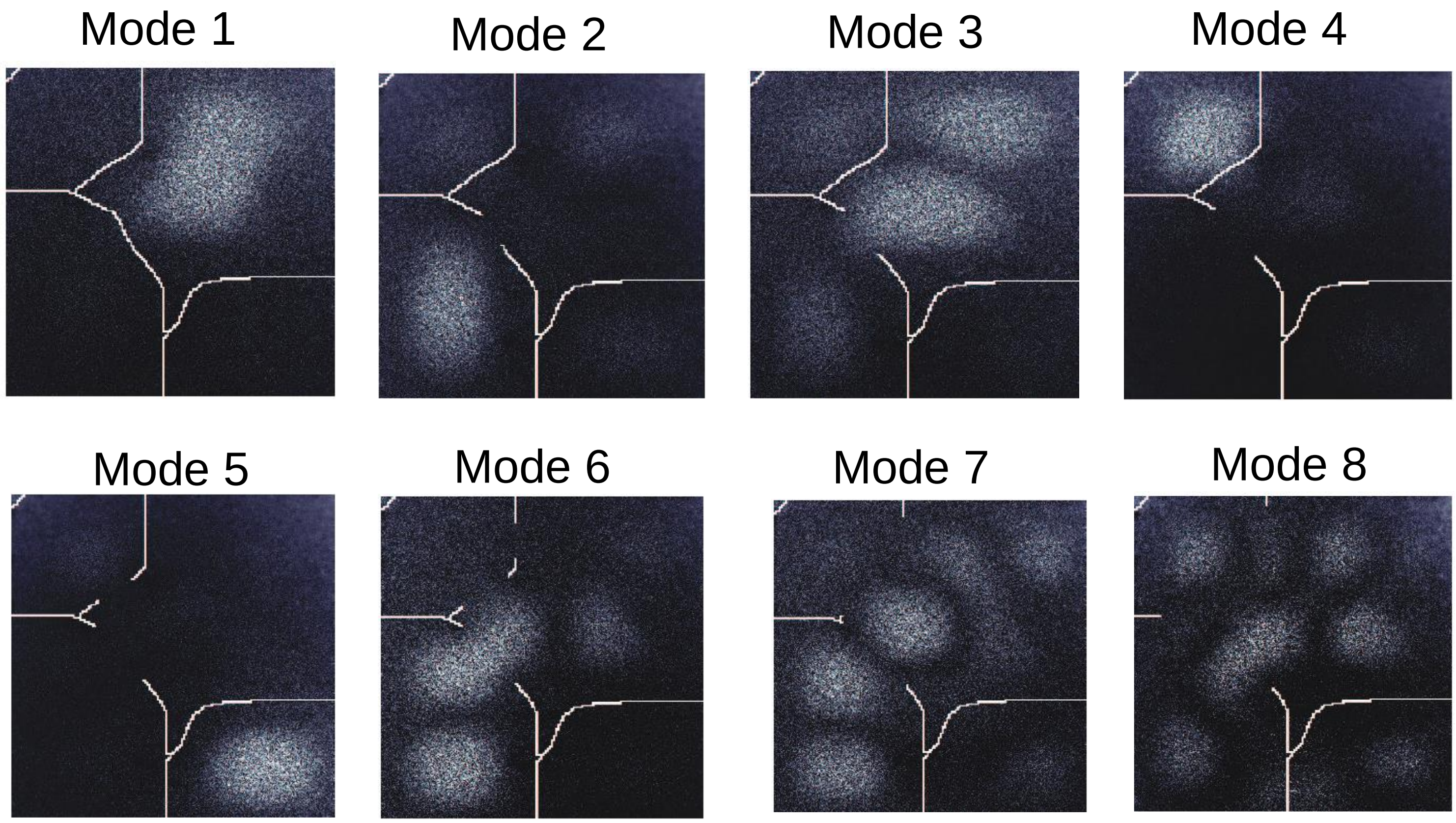
Because φ_k is normalized ($\|\varphi_{k\,max}\|$=1), the constraint exists only if

$$u(x) \leq \lambda_k^{-1}$$

which is referred to as the **effective valley network**

While the landscape *u* is **uniquely defined**,
the valley network **evolves** with the growth of λ.

4 - EXPERIMENTAL TEST OF THE VALLEY NETWORK OPENING

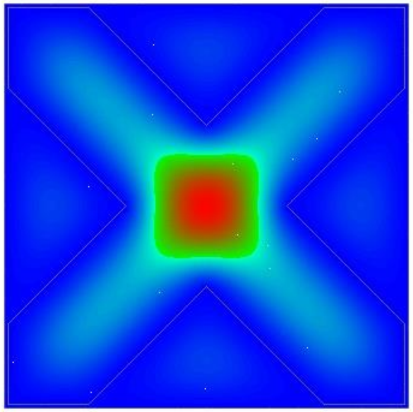


As frequency grows, the valleys “disappear” and the sub-regions begin to merge opening “free” passages for propagation.
At some point, **states are fully delocalized**.

SUMMARY

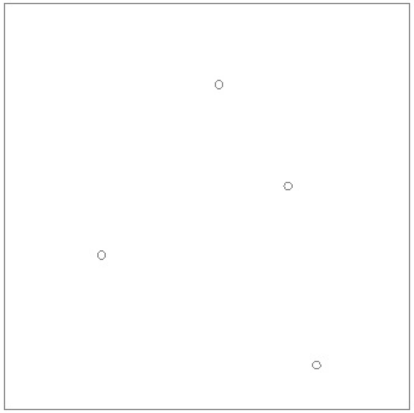
- The *localization landscape* describes the sub-regions of localization
- The local maxima of the *localization landscape* provide with the eigenvalues.
- The effective valley networks gives the frequencies at which gap open and modes extend in the system.

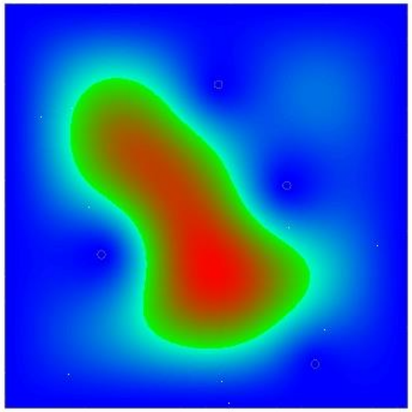
SOLVING THE INVERSE PROBLEM



$\boldsymbol{u}^*$

Define an ideal landscape function $\boldsymbol{u}^*$



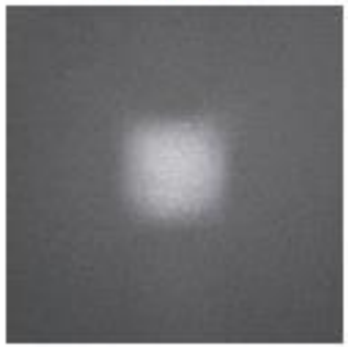


$\boldsymbol{u}$

Minimize $||\boldsymbol{u} - \boldsymbol{u}^*||$

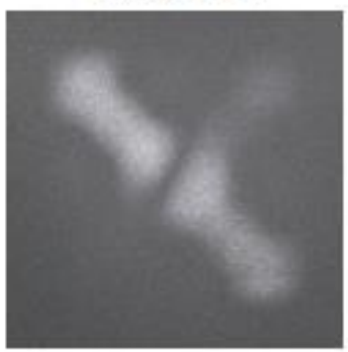
PLATE WITH 16 PINS

1650 Hz



3730 Hz

2950 Hz



5540 Hz

CONCLUSION

- First experimental validation of the theory of the *localization landscape*
- The localization landscape has been directly measured.
- The essential information about localized states is retrieved without resorting to a full eigenvalue problem
  - one simple static measurement provides a rather good picture of the low frequency behavior of the system
  - The opening frequencies of the valleys are accurately predicted

PART 2: DISORDERED-INDUCED LOCALIZATION IN THIN PLATES

I- Localized modes in randomly-pinned plates

II- Localization by resonant scatterers

MOTIVATION

- Exploring Anderson localization in 2D disordered media
  - Observing localized states, mode coupling, necklace states, exceptional points
  - Transition from localized to extended modes
  - Dynamics of Localization

- Trapping elastic waves
  - Energy confinement and control
  - Understanding and controlling the acoustic behavior of complex structures
  - Application to noise reduction and wave control

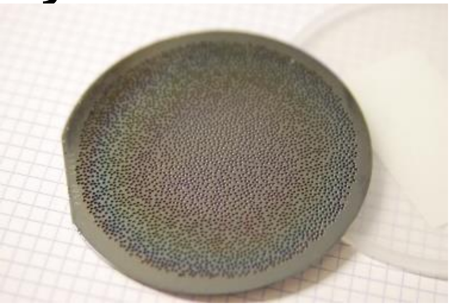
- Predicting localized states of vibration :
  Structural engineering
  - Application to instrument design
  - design of structures with predefined modal characteristics (Optical cavities, lasers)

OBJECTIVES

We explore two different routes to localize elastic waves in disordered thin plates

- 1 - Localization in a randomly pinned plate

- 2 - Localization by resonant scatterers

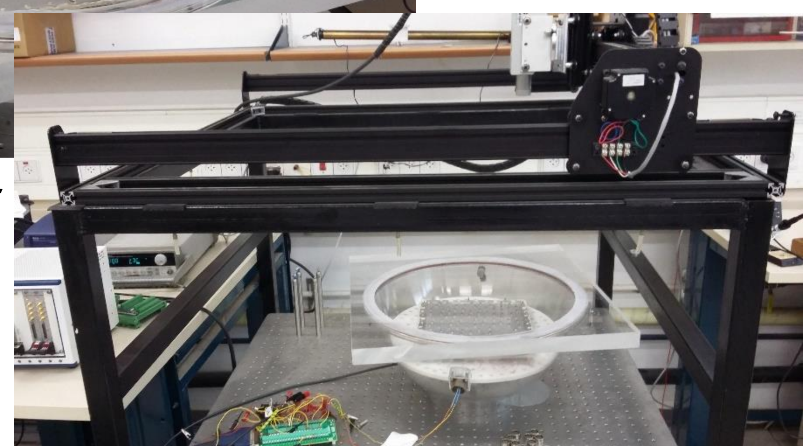


I - Localization in randomly-pinned plate

RANDOMLY PINNED PLATE

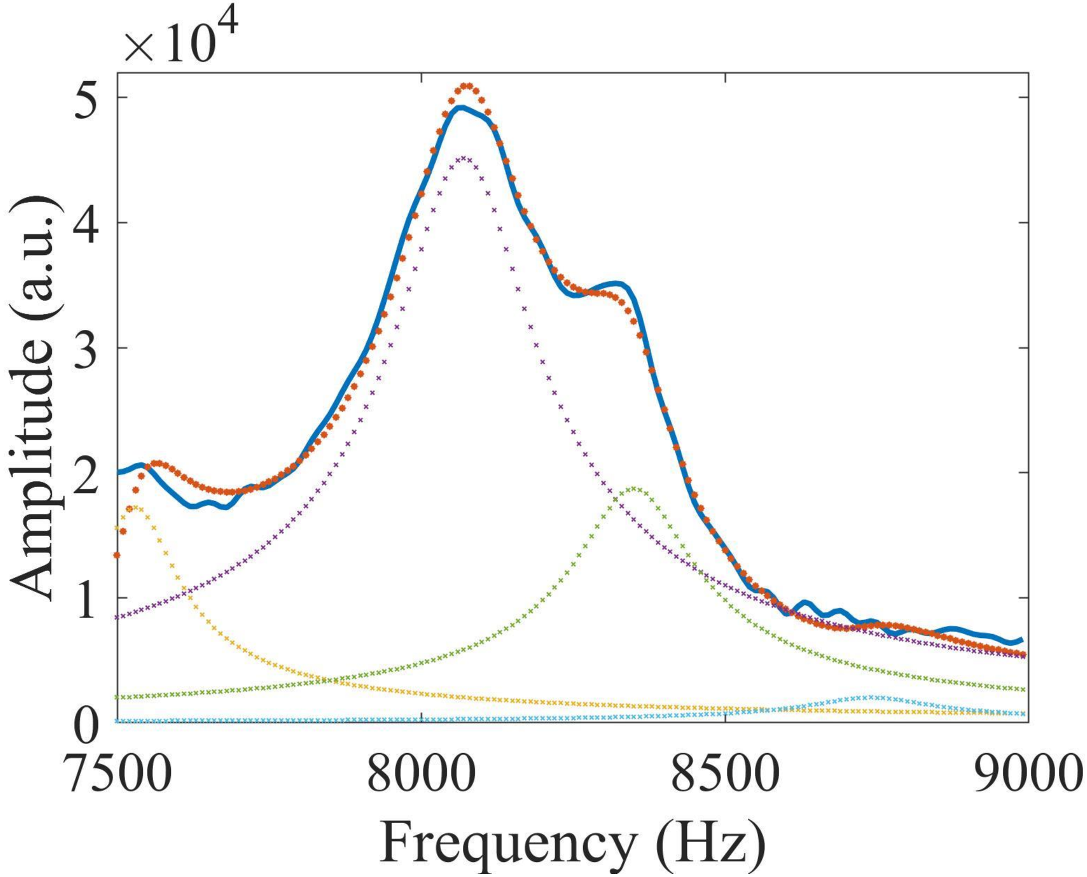
23cmx23cm
100 pins, 3mm-diam
Absorbing boundary conditions

Vacuum chamber



2D-scan of a laser vibrometer probe

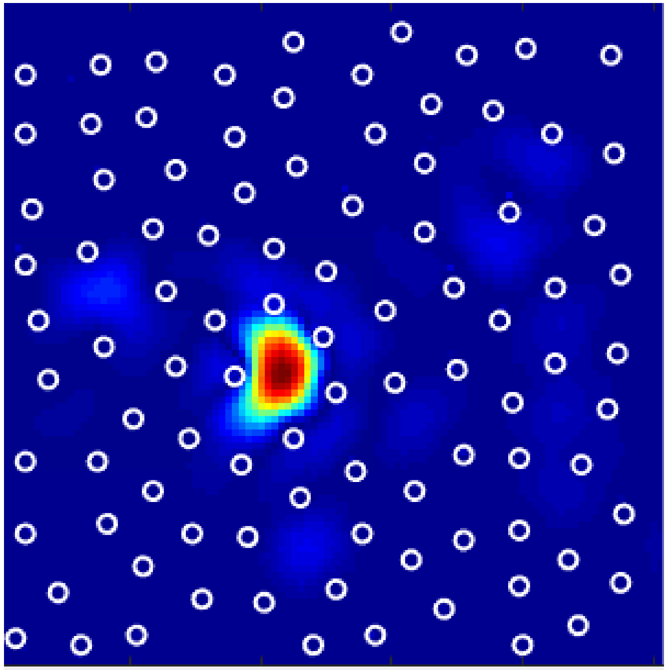
AMPLITUDE SPECTRUM



Spectral decomposition as
a sum of complex Lorentzian functions

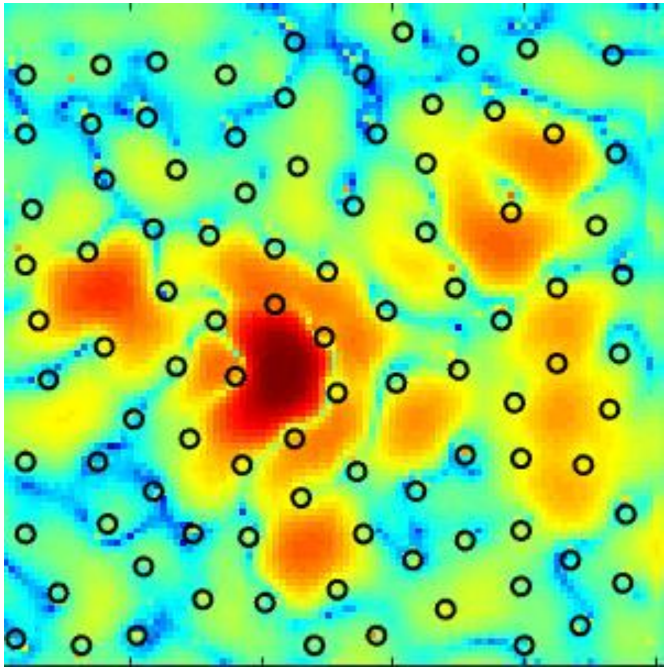
SPATIAL DISTRIBUTION

freq=7940 Hz



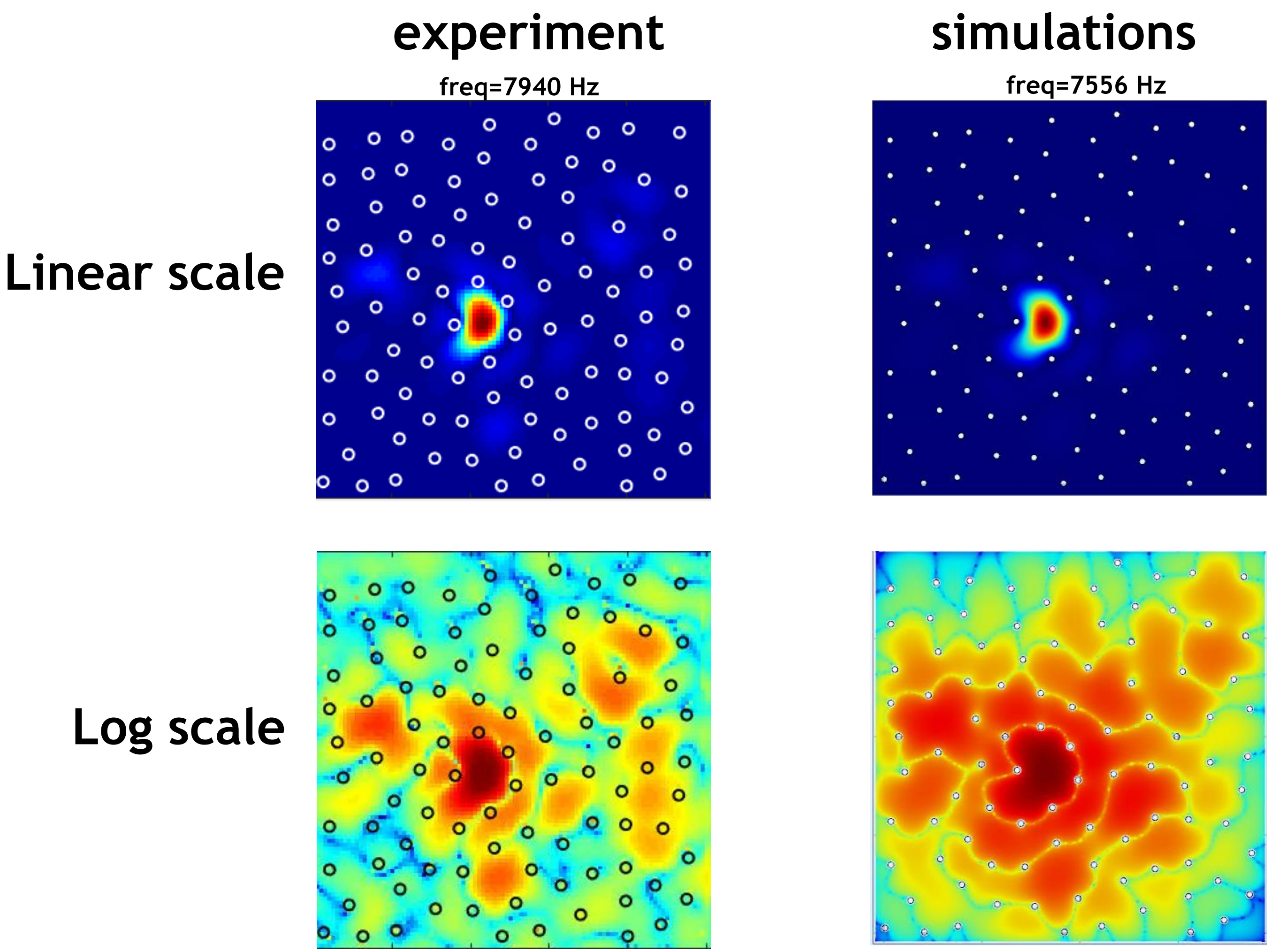
SPATIAL DISTRIBUTION

freq=7940 Hz



Logarithmic scale

SPATIAL DISTRIBUTION



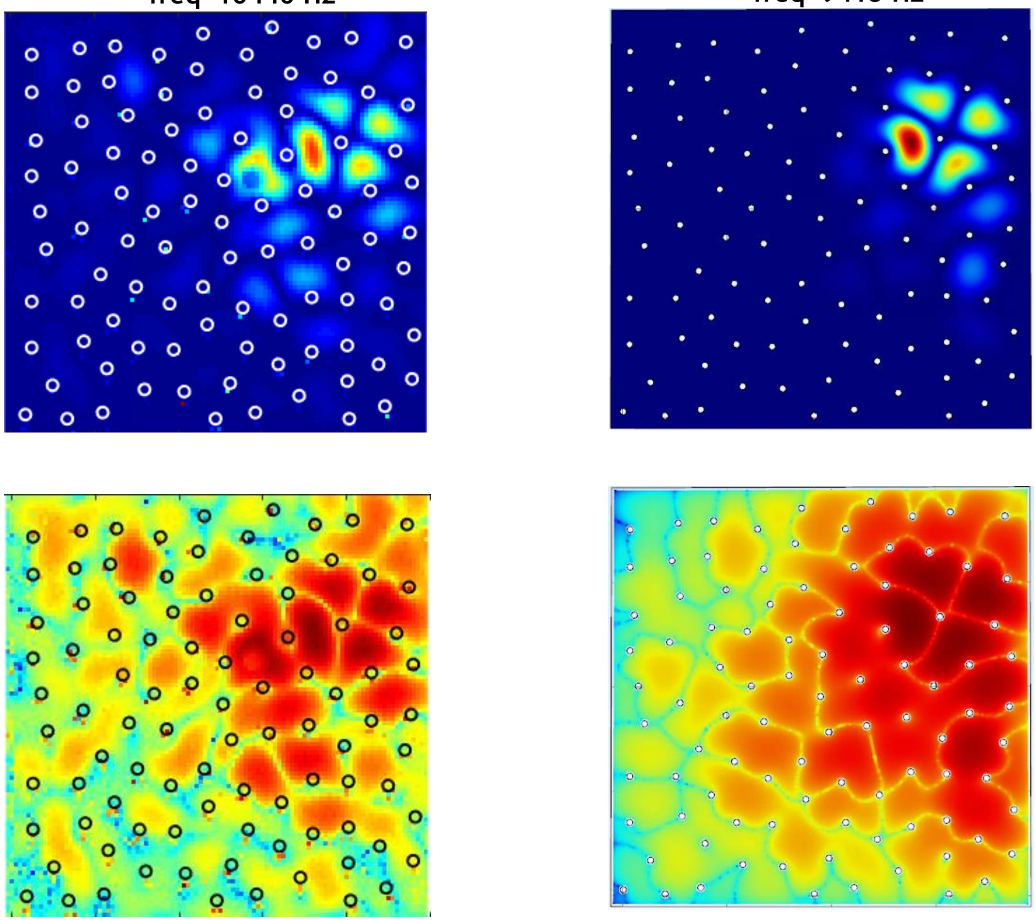
SPATIAL DISTRIBUTION

experiment — freq=10440 Hz

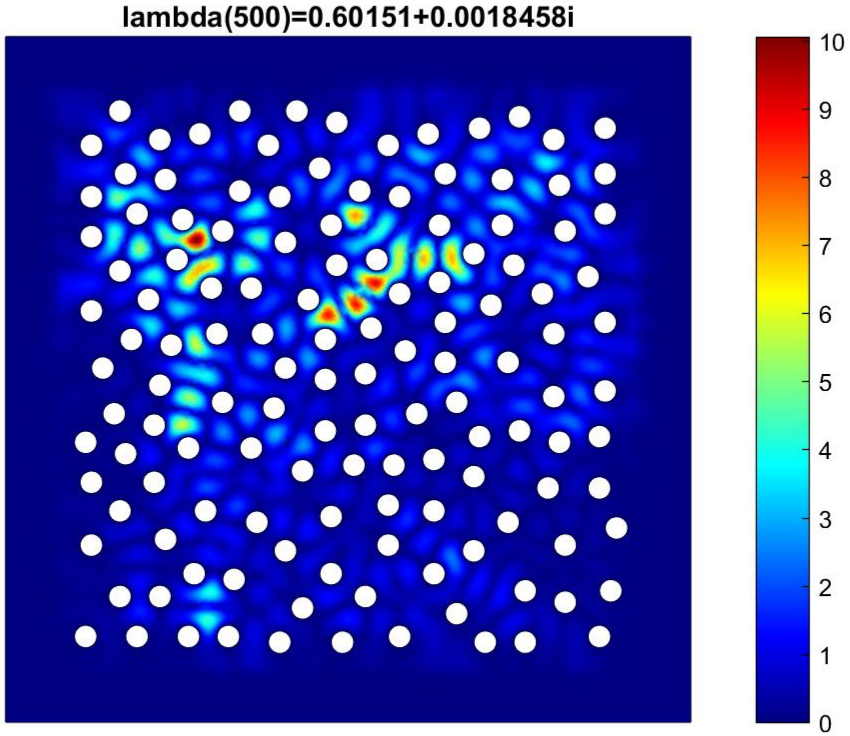
simulations — freq=9415 Hz

Linear scale

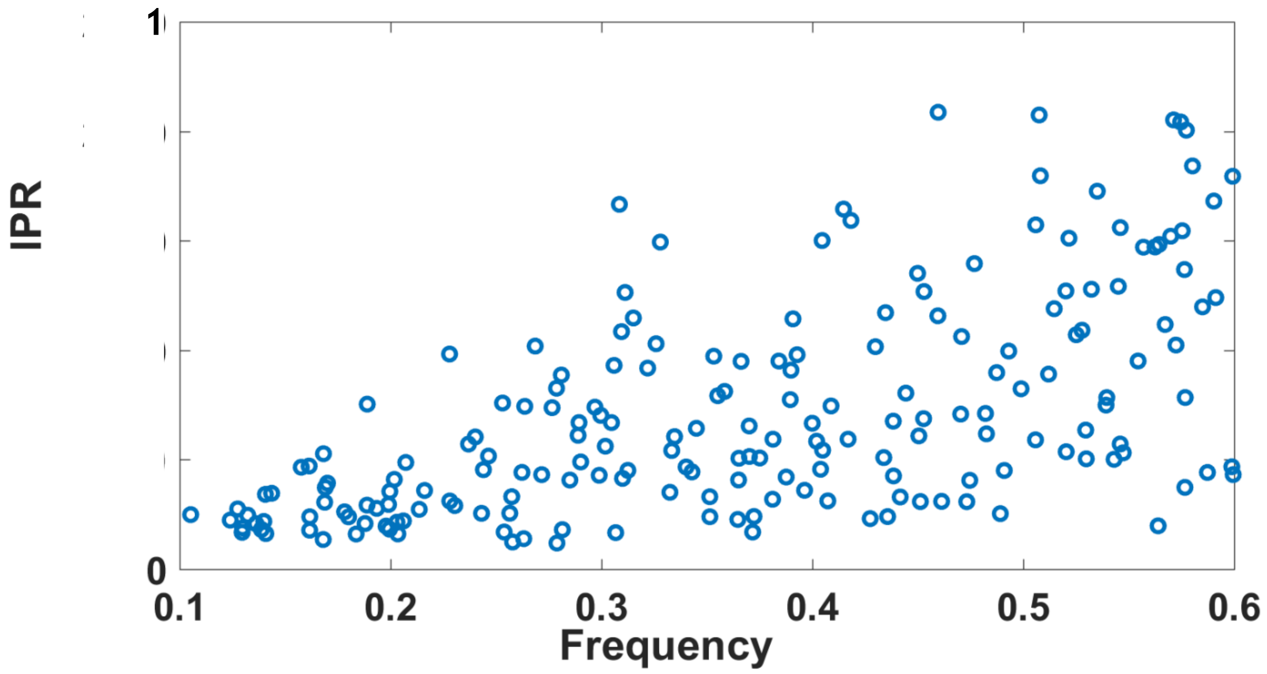
Log scale



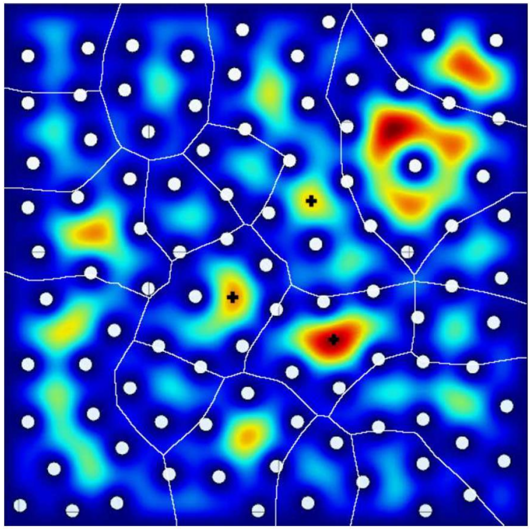
MODE EXTENSION



INVERSE PARTICIPATION RATIO



THE LOCALIZATION LANDSCAPE



Computed
landscape function u

The landscape function u is the solution of

$$\boldsymbol{Lu = 1} \text{ with } u|_{\partial\Omega} = 0$$

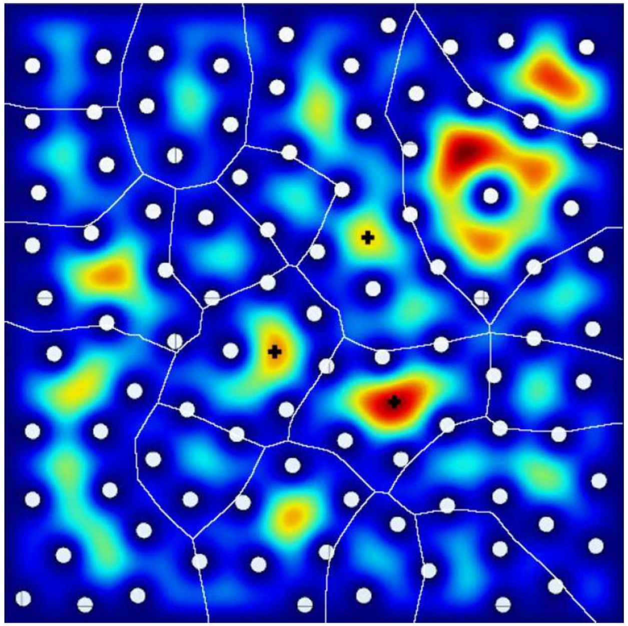
u corresponds to the "static deformation" of the system under uniform load

The regions delimited by the valleys correspond to the localization regions of the different modes.

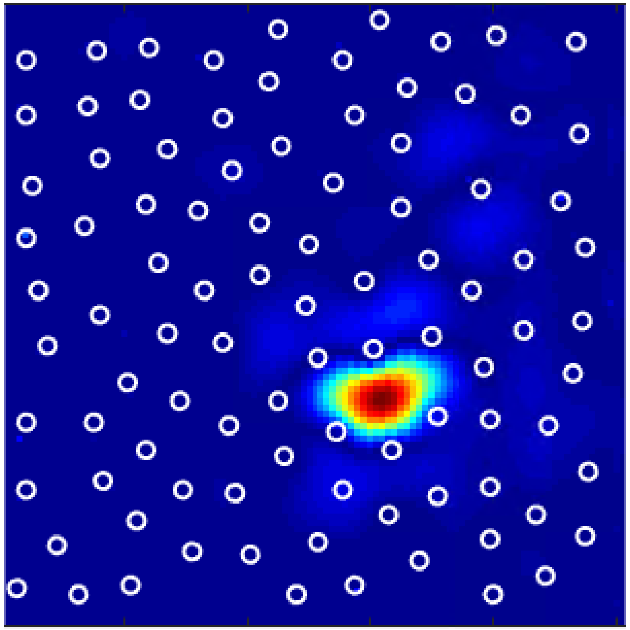
Local maxima give the inverse of the resonant frequencies

LANDSCAPE VS. MODES

landscape function

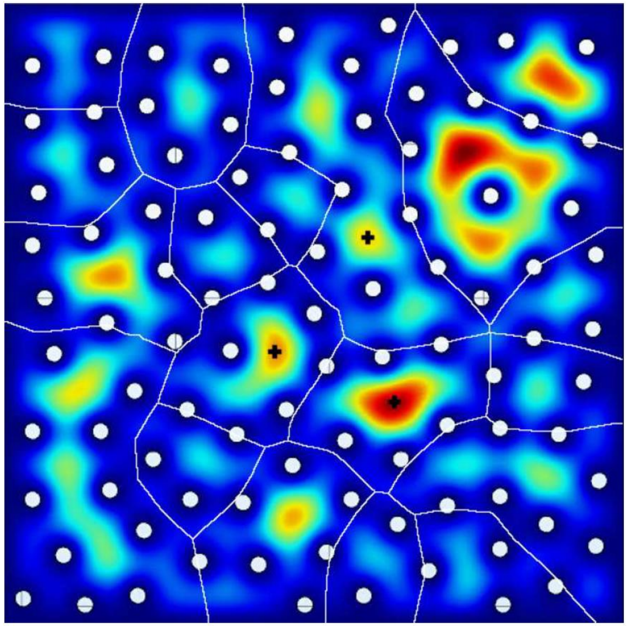


freq=6612 Hz

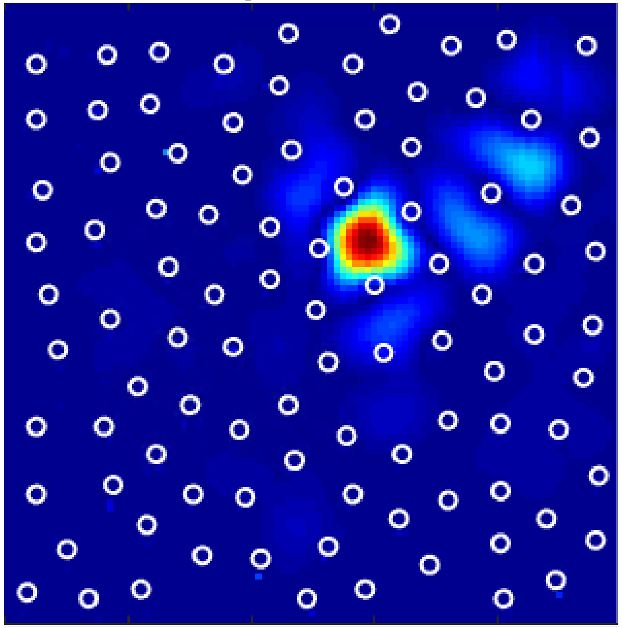


LANDSCAPE VS. MODES

landscape function

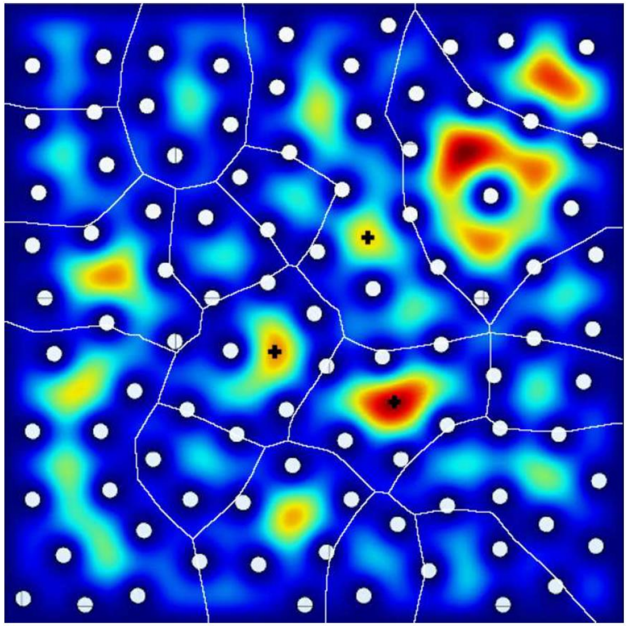


freq=7510 Hz

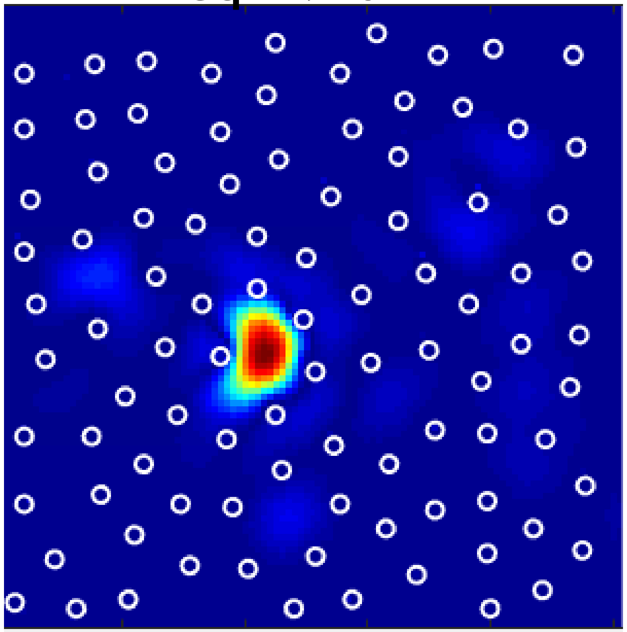


LANDSCAPE VS. MODES

landscape function

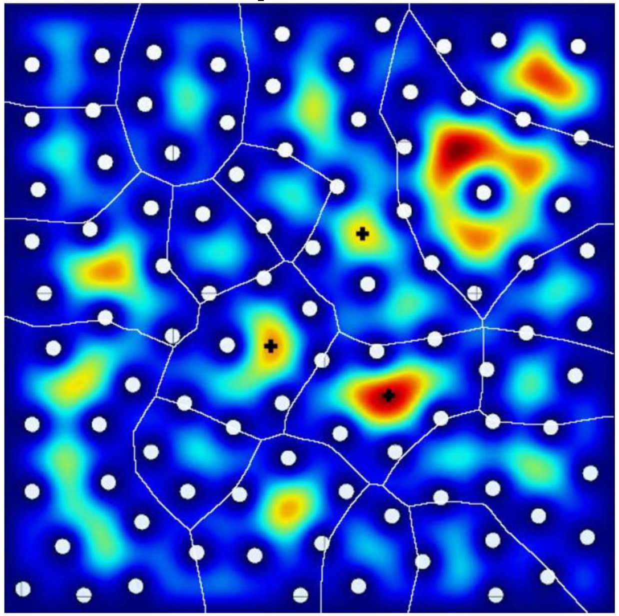


freq=7940 Hz

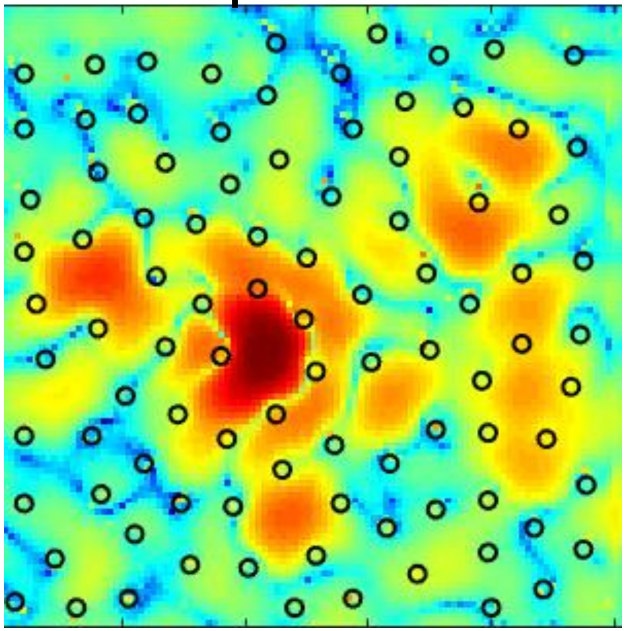


LANDSCAPE VS. MODES

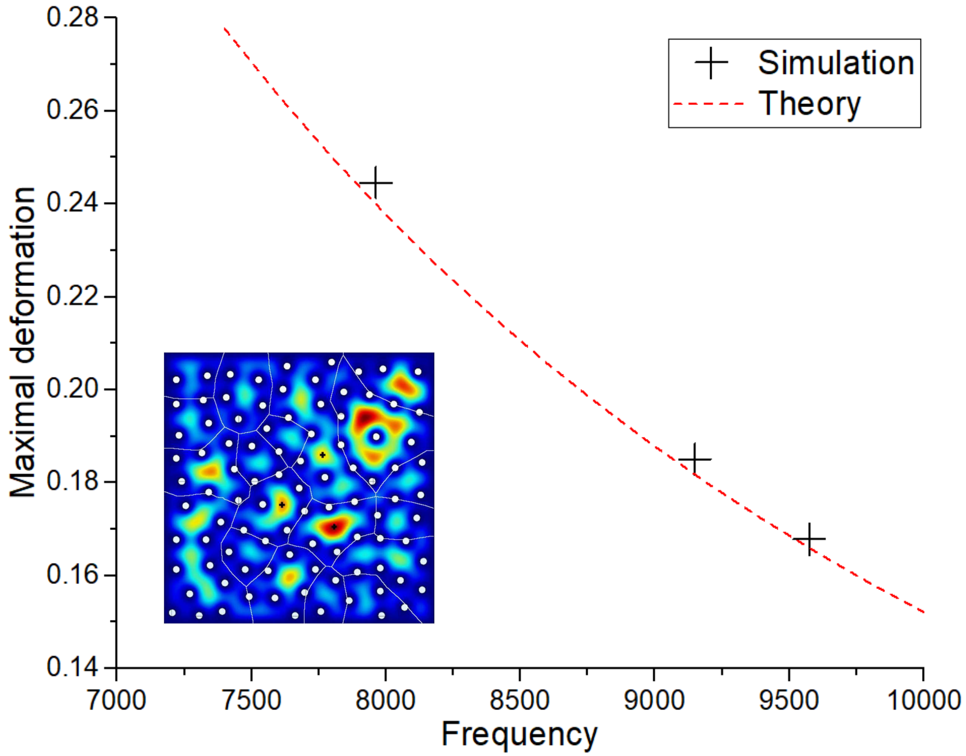
landscape function

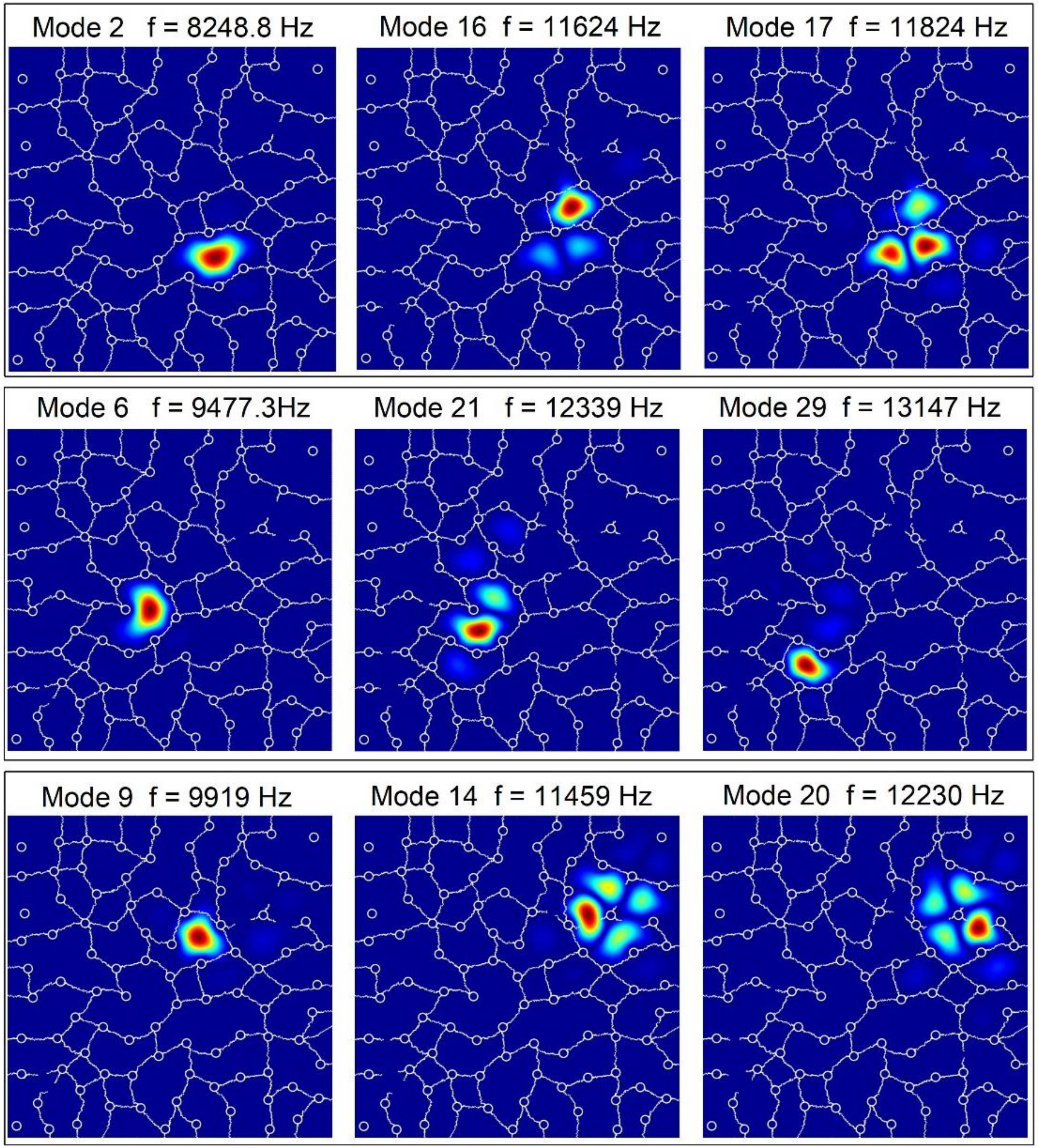


freq=7940 Hz

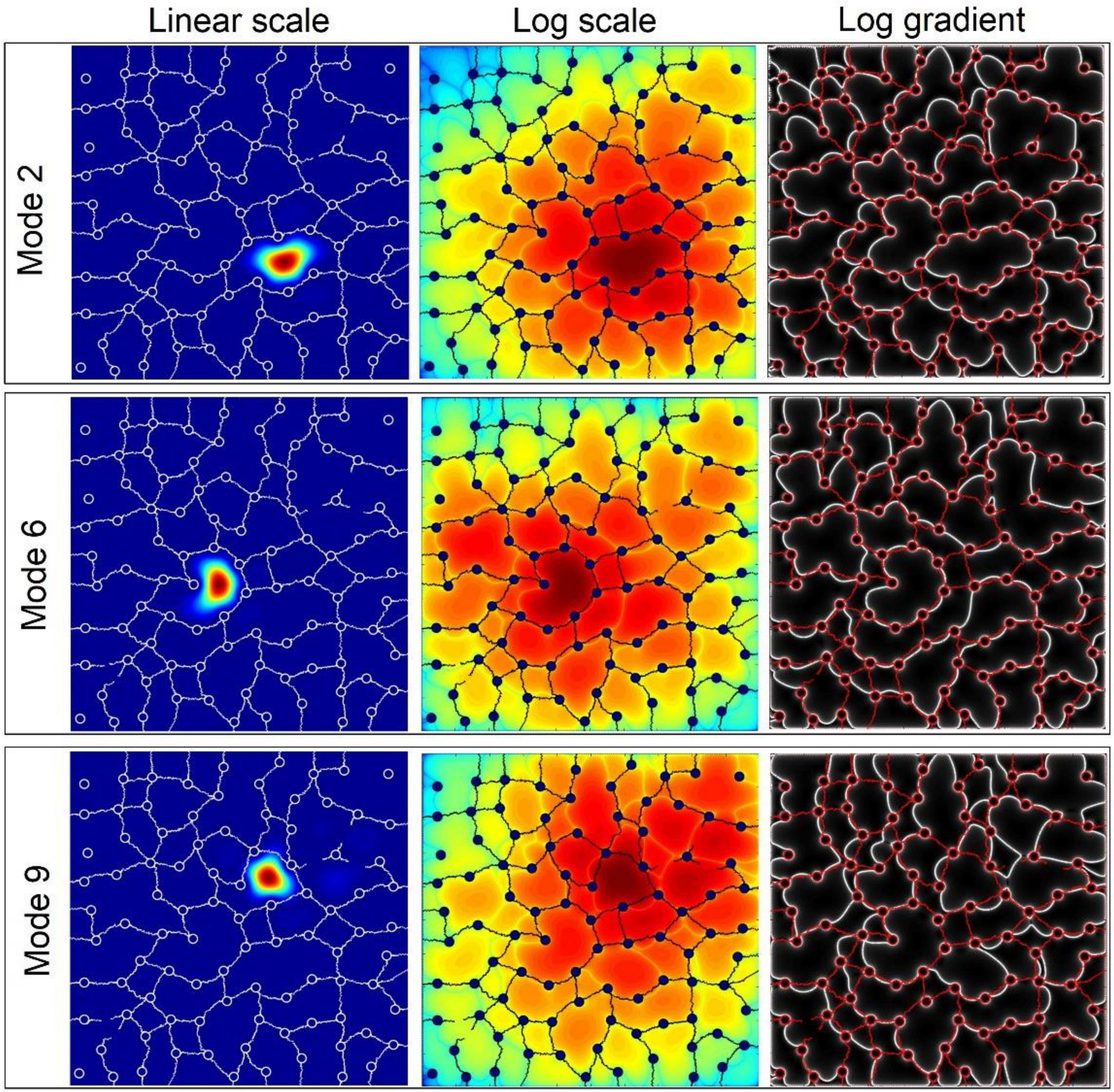


PREDICTION OF THE LOW FREQUENCY SPECTRUM





Mode 2 f = 8248.8 Hz
Mode 16 f = 11624 Hz
Mode 17 f = 11824 Hz
Mode 6 f = 9477.3Hz
Mode 21 f = 12339 Hz
Mode 29 f = 13147 Hz
Mode 9 f = 9919 Hz
Mode 14 f = 11459 Hz
Mode 20 f = 12230 Hz



Linear scale
Log scale
Log gradient
Mode 2
Mode 6
Mode 9

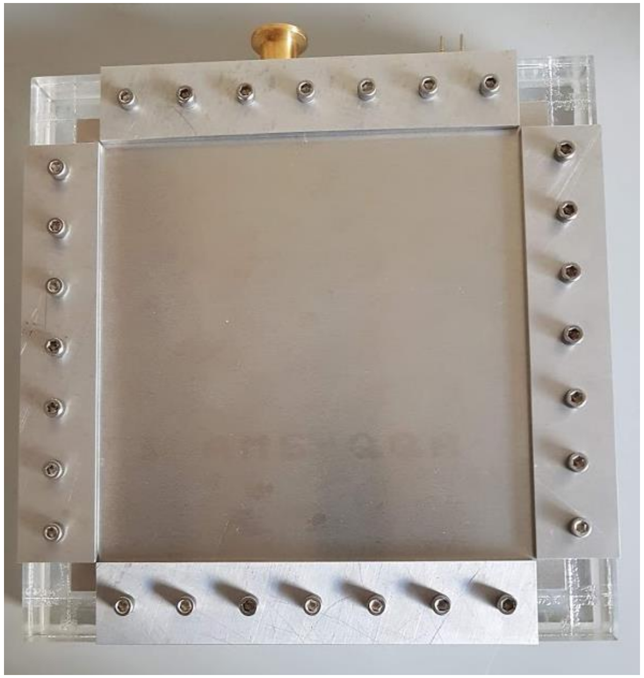
IN SUMMARY

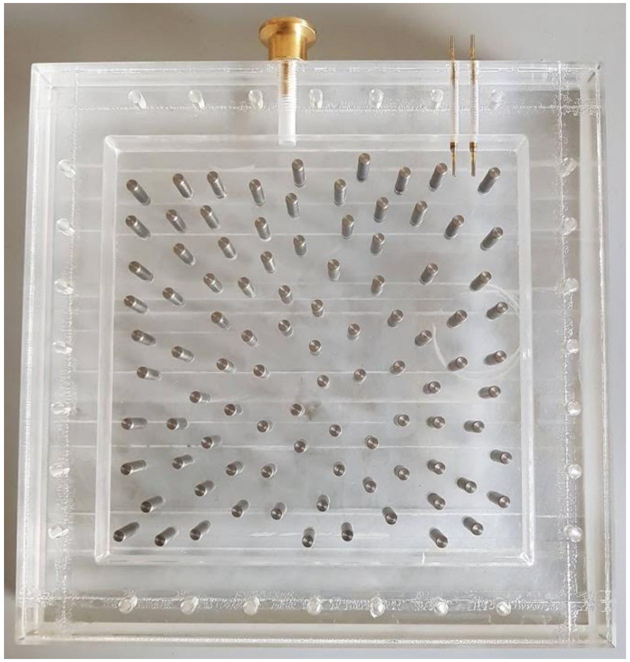
- Disordered-induced localized modes have been measured in a randomly-pinned plate
- The extension of the modes increases with frequency
- The landscape describes the eigenmodes

  Gives their position, described how they decay, gives the low frequency spectrum

COMING NEXT …

- Measuring the landscape in the pinned plate





II - LOCALIZATION BY RESONANT SCATTERERS

MUTIPLE SCATTERING OF ELASTIC WAVES

ELSEVIER

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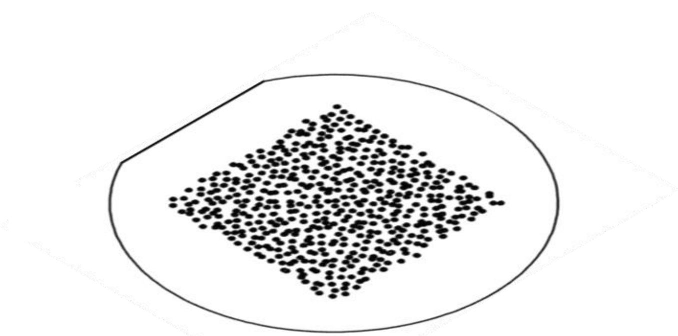
Ultrasonics 42 (2004) 775–779

Ultrasonics

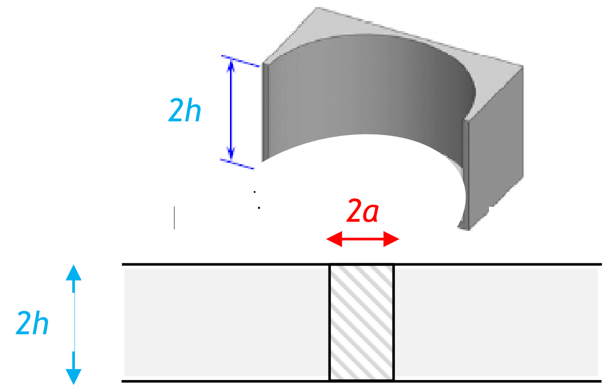
www.elsevier.com/locate/ultras

Time reversed wave propagation experiments in chaotic micro-structured cavities

Rudolf Sprik [a,*], Arnaud Tourin [b]



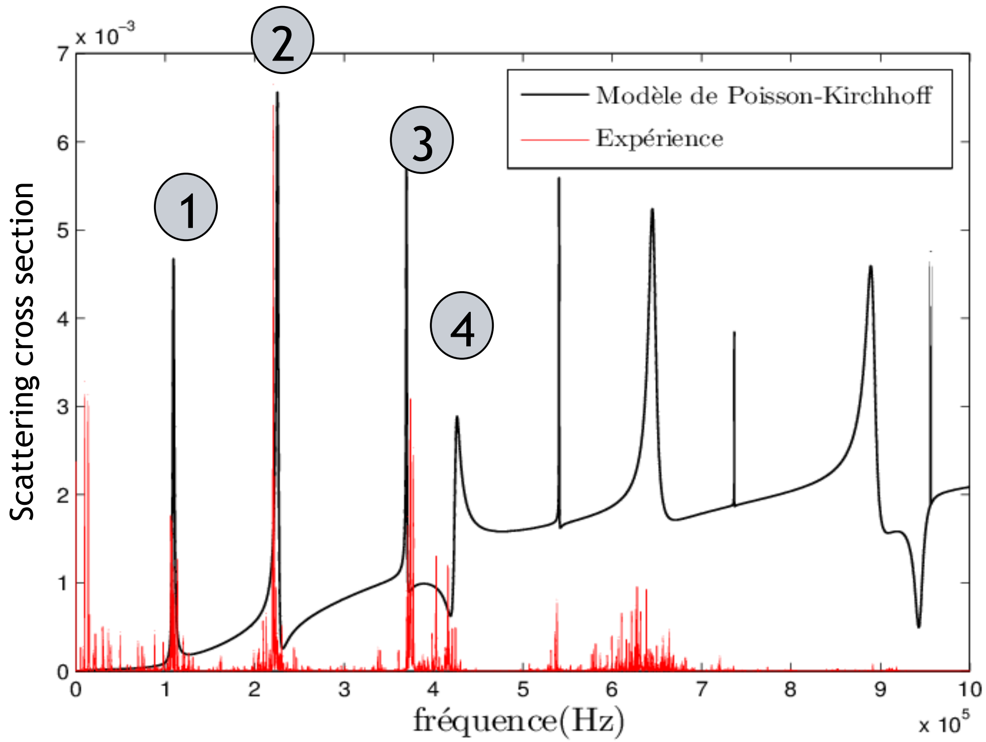
76 mm-diam, 0.35 mm-thick Si-wafer

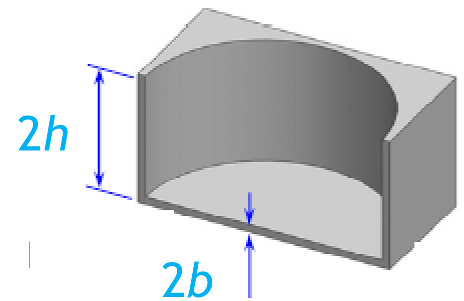


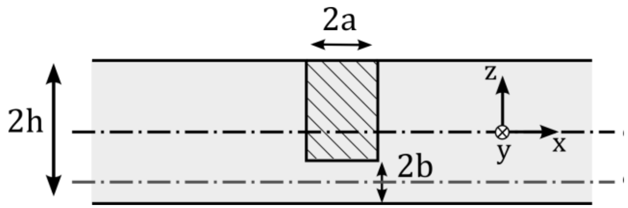
Through hole

Disordered-induced localization was not observed

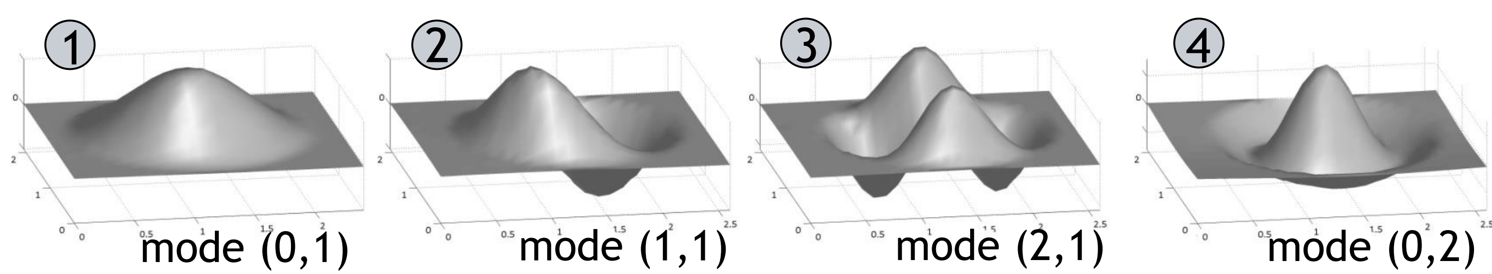
RESONANT SCATTERERS



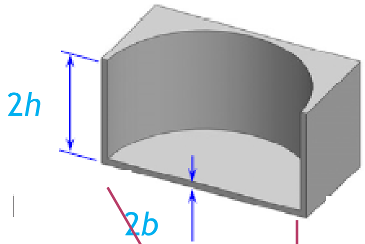


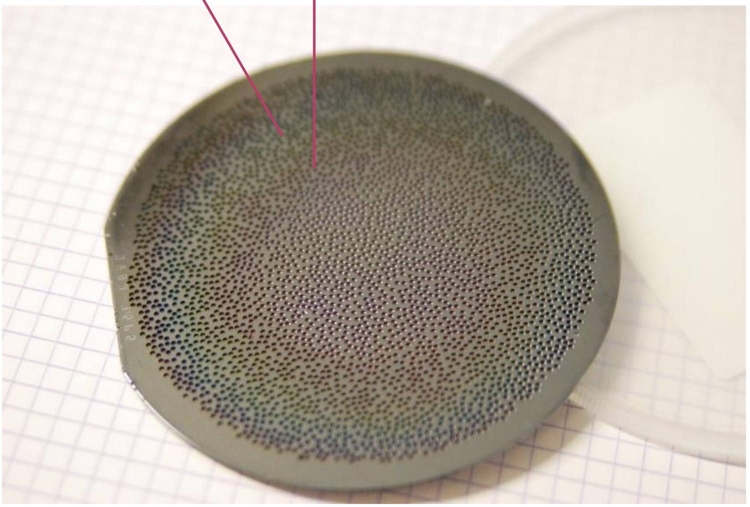


Blind hole



DISORDERED THIN PLATE



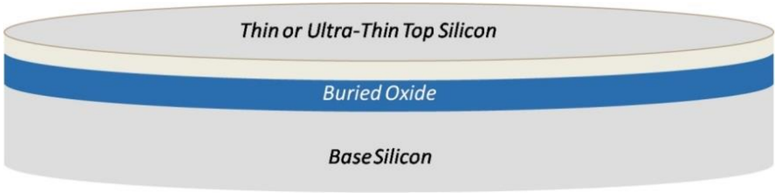


3400 blind holes randomly distributed

SOI wafer : 560 μm-thick

Blind holes : 530 μm-depth, Ø=1 mm
Minimal distance : 100 μm

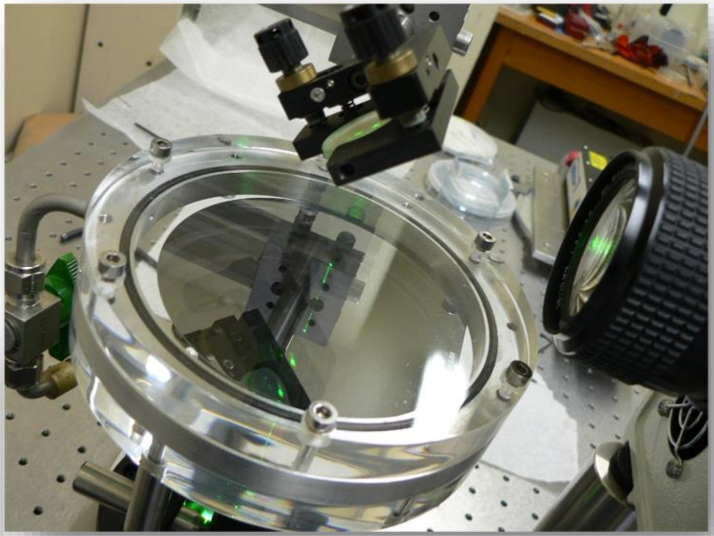
Challenge : reducing blind hole-depth dispersion

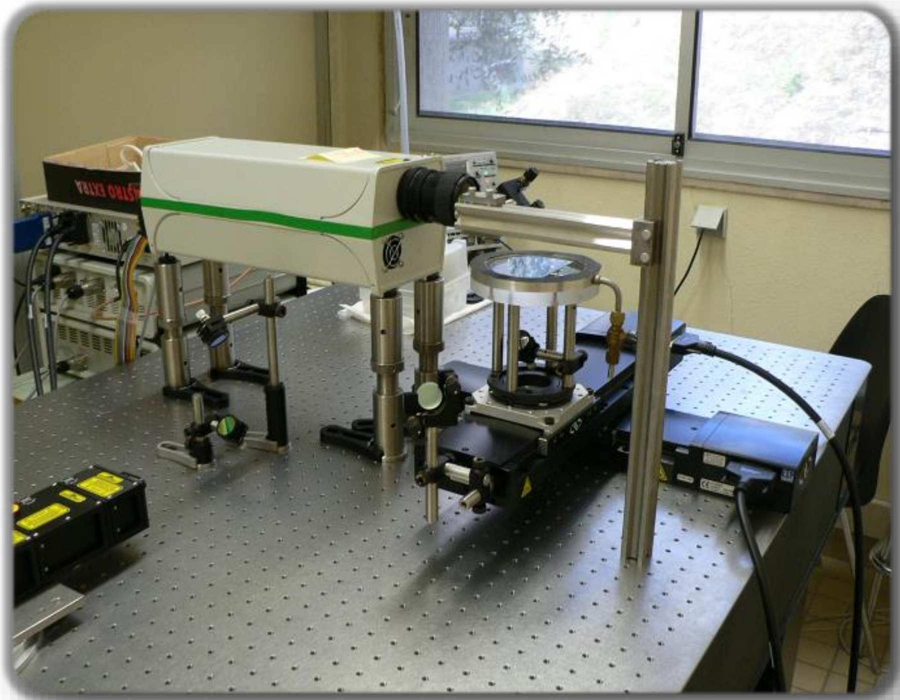


- SOI (Silicon On Insulator)
- Photolithography
- Ion-beam etching

Fabrication by Etienne Herth, FemtoST, Besançon

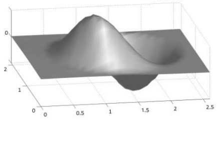
EXPERIMENTAL SETUP

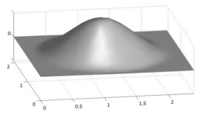


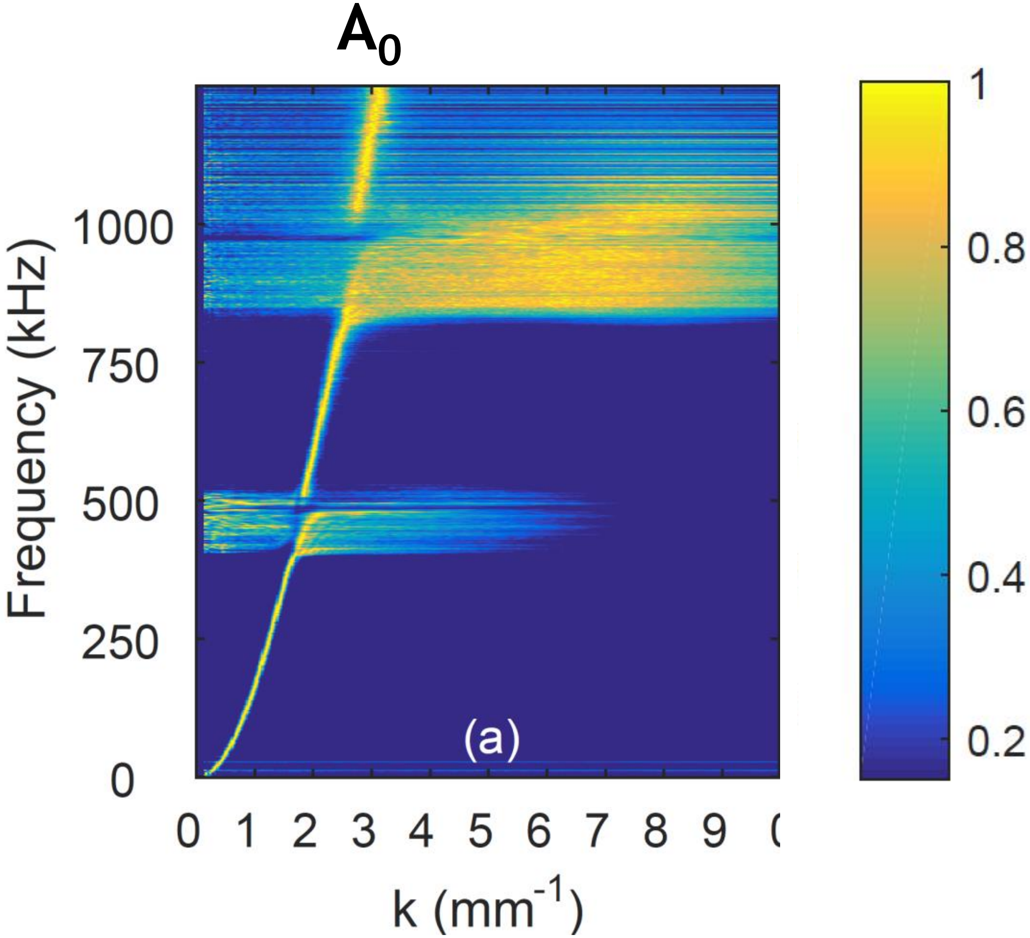


Heterodyne laser interferometer

DISPERSION DIAGRAM

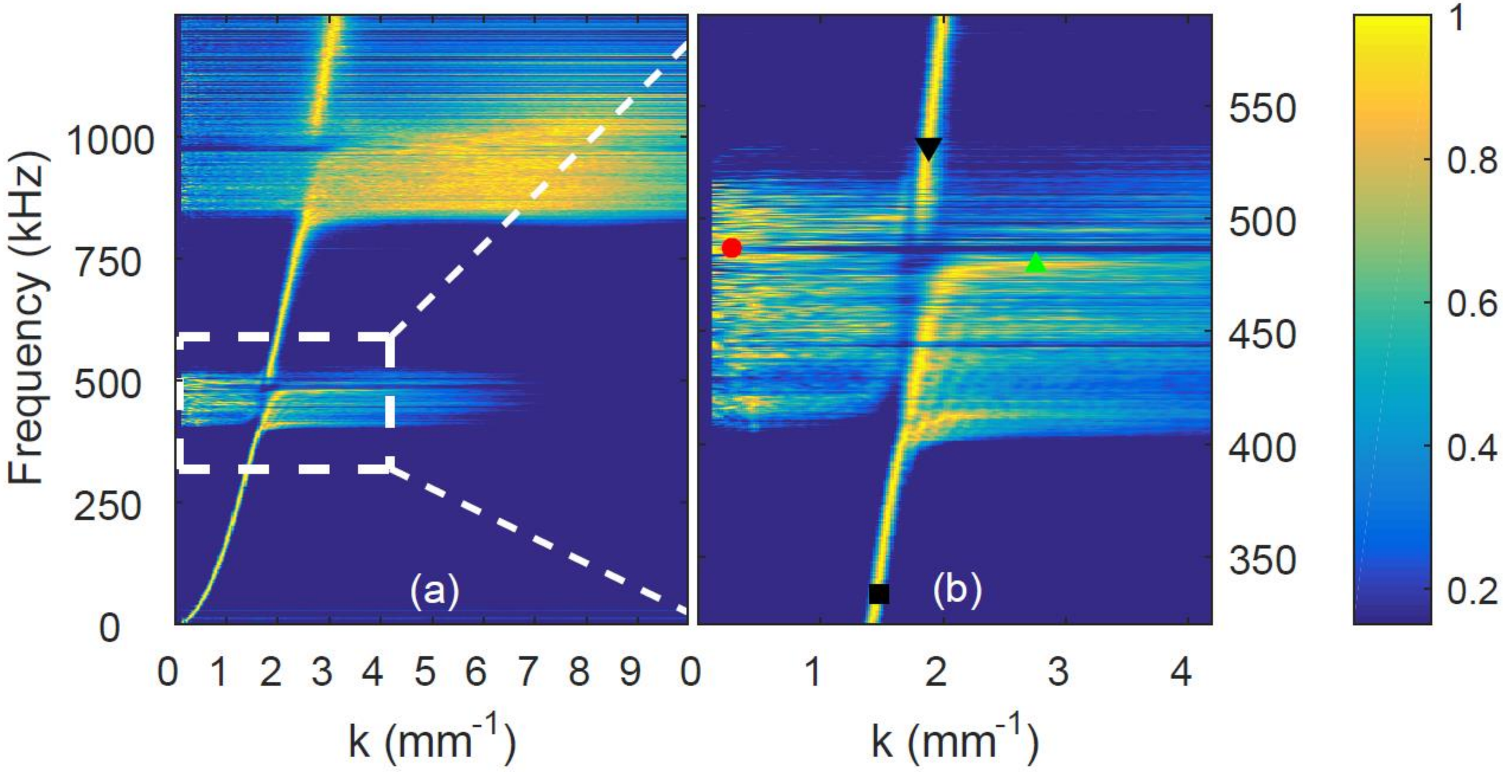




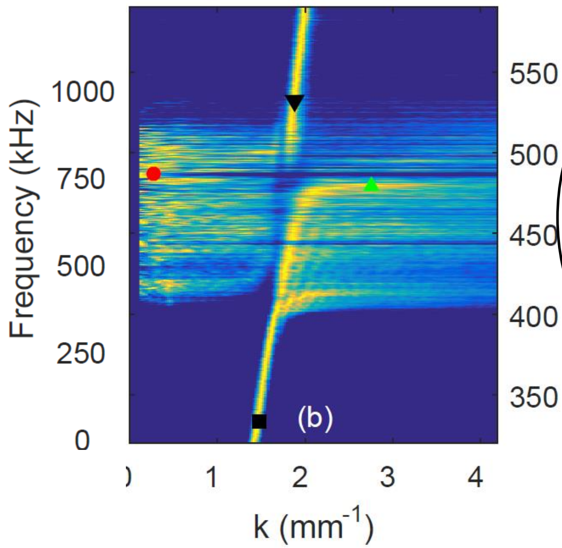


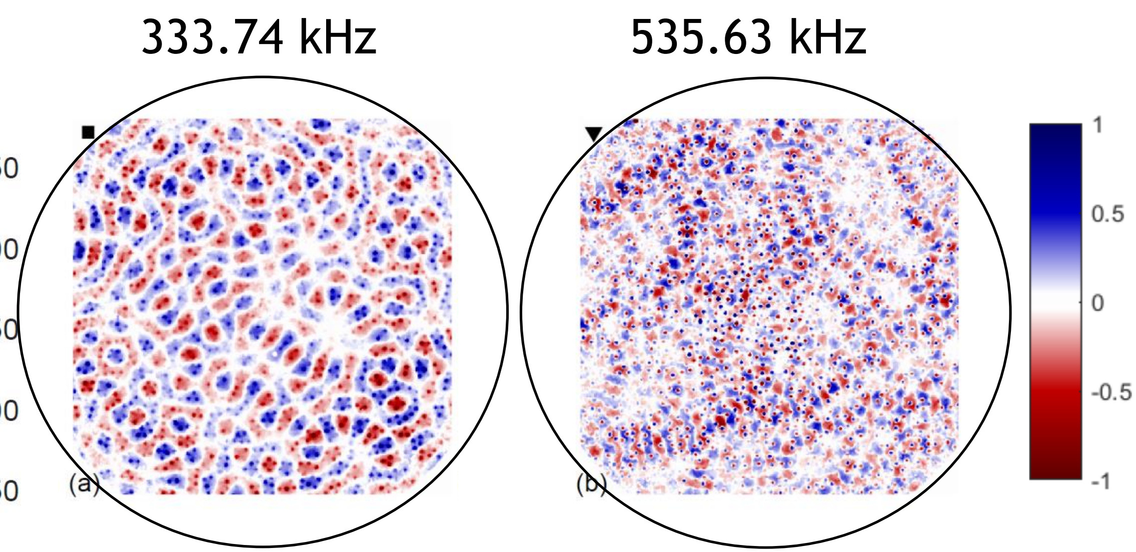
Hybridization gap

DISPERSION DIAGRAM

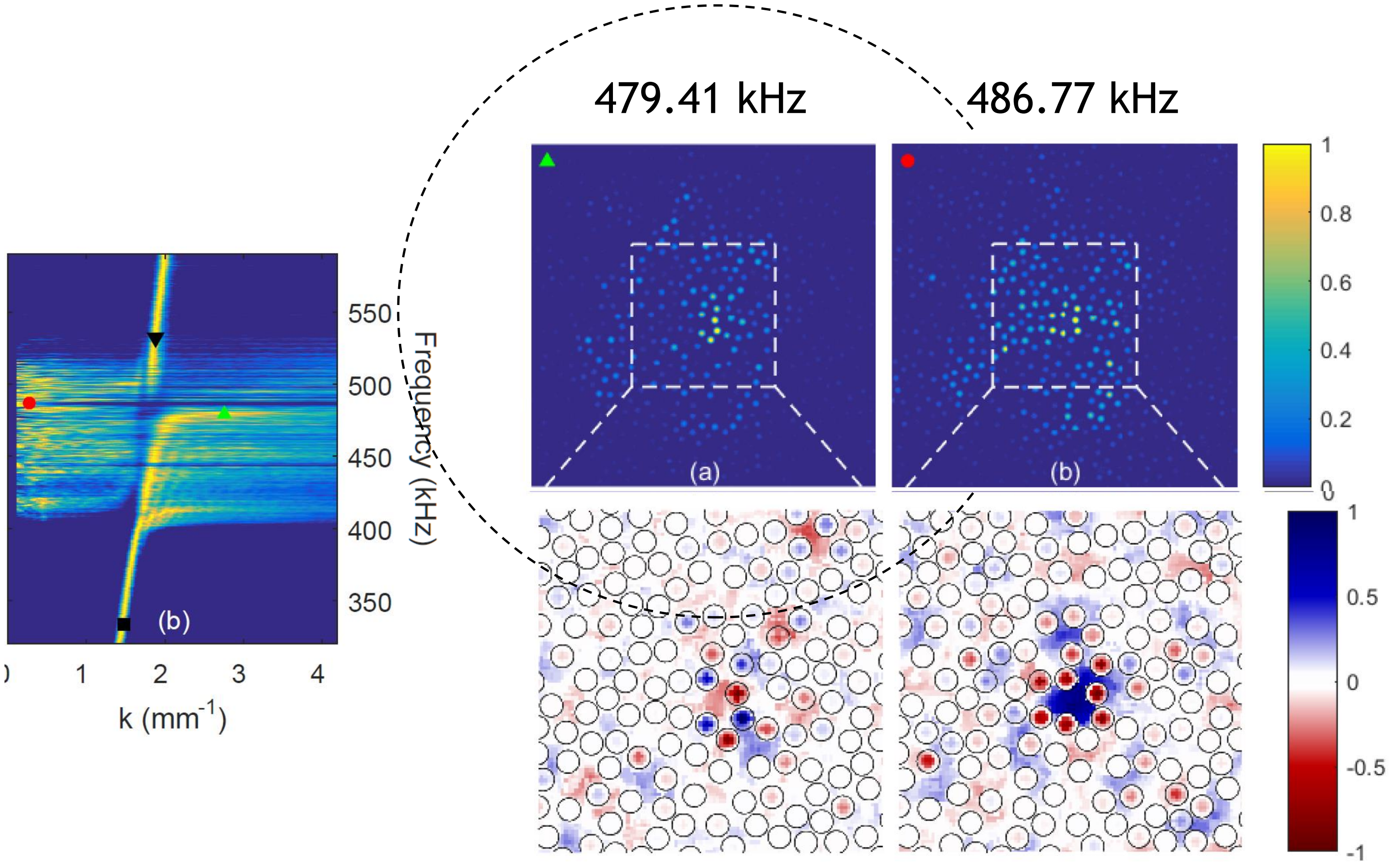


EXTENDED STATED

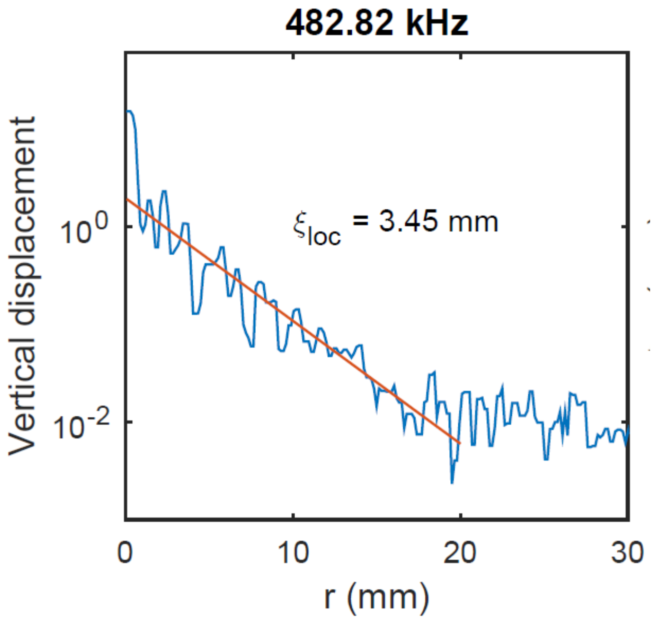




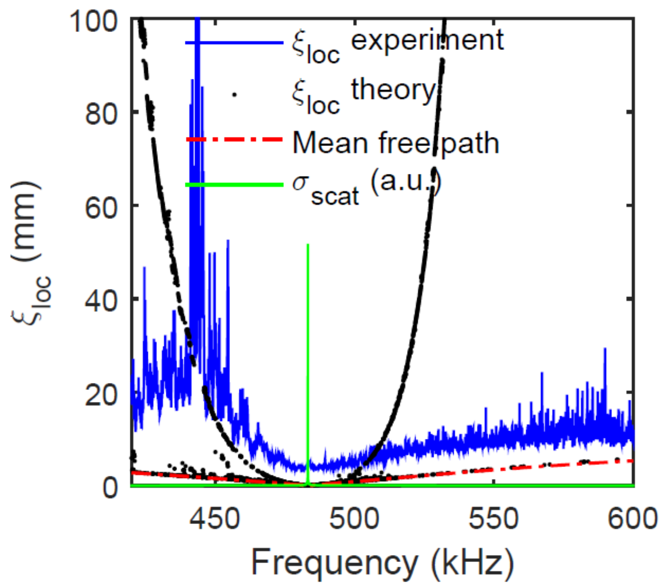
LOCALIZED STATED



LOCALIZATION LENGTH



Mode profile @ 482.82 kHz (semilog)



Localization length vs. frequency

SUMMARY

- Resonant scattering is an interesting route toward localization
- It is demonstrated here for flexural waves in a thin plate with blind holes
- The localization length is measured and shows a minimum near the resonance of the blind hole.
- Two types of mode are identified on both sides of the hybridization gap.

CONCLUSION

We have demonstrated modal confinement and localization in 2 particular situations:

- 1 - Localization by randomly pinned plate
- 2 - Localization by resonant scatterers

They correspond to 2 different situations :

- 1 - Many microcavities possessing their own resonant frequency. At higher frequencies, they coupled to one another.
- 2 - Off-resonance: weak scatering. Near resonance: hybridisation gap and strong confinement of the wavefunction.

The Landscape theory of localization works in the pinned pate.

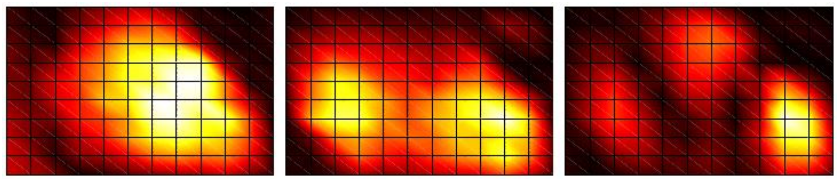
No theory exists yet in the second case.

G. Lefebvre *et al.*, **One single static measurement predicts wave localization in complex structures,** Phys. Rev. Lett. 117, 074301 (2016).

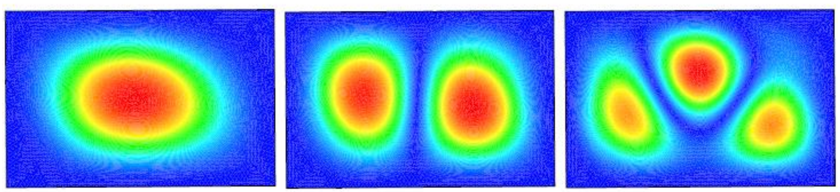
G. Lefebvre *et al.*, **Localization induced by resonant scatterers in thin plates**, in preparation.

THANK YOU FOR YOUR ATTENTION

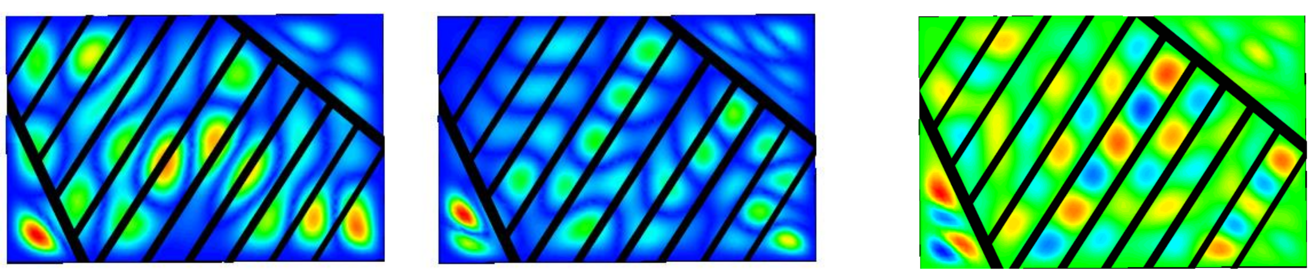
LOW AND HIGH FREQUENCY MODES



(a) (1,1)-mode *(b)* (2,1)-mode *(c)* (3,1)-mode



First three modal shapes of the upright piano soundboard



*f*n = 776 *Hz* *f*n = 1089 *Hz* *f*n = 1593 *Hz*

Intensity (and field) distribution of higher modes of the upright piano soundboard

Xavier Boutillon (LMS Polytechnique), Vibroacoustics of the piano soundboard

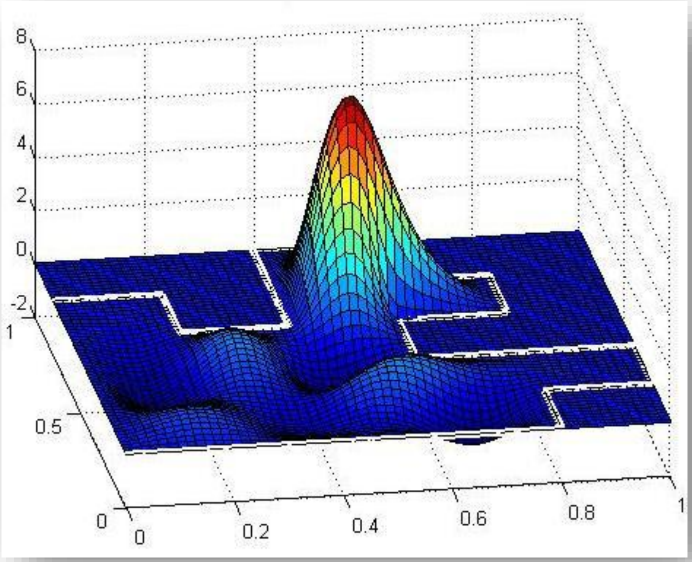
OTHER STRING INSTRUMENTS

MOTIVATION

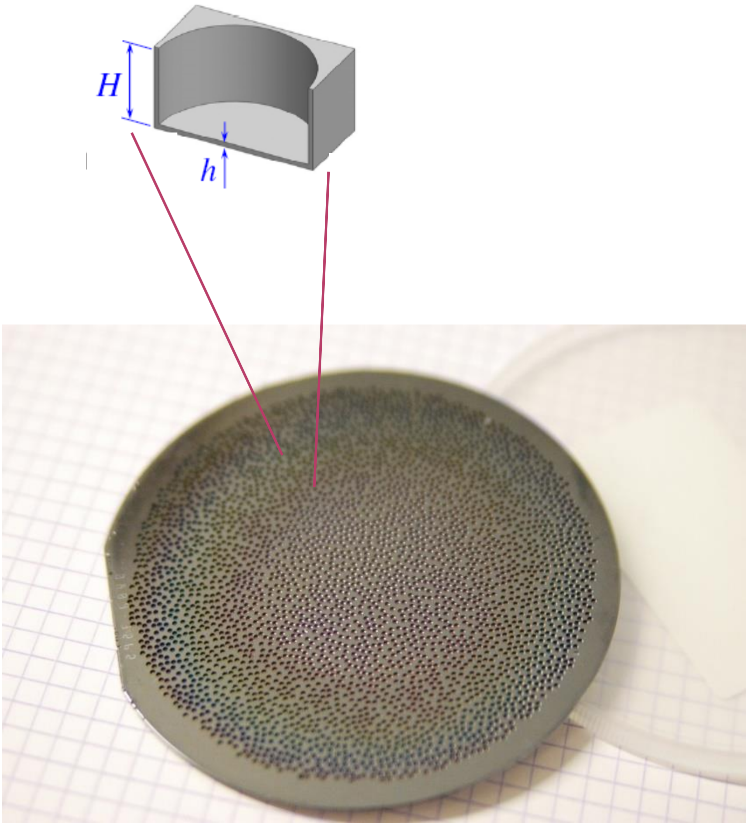
- Eigenmodes and eigenvalues : Key ingredients to understand the physical properties of most vibratory systems
- Yet, the prediction of the spatial and spectral distributions of the stationary waves remains an open problem in many cases.
- And **the design of the structure with predefined modal characteristics** remains a challenge

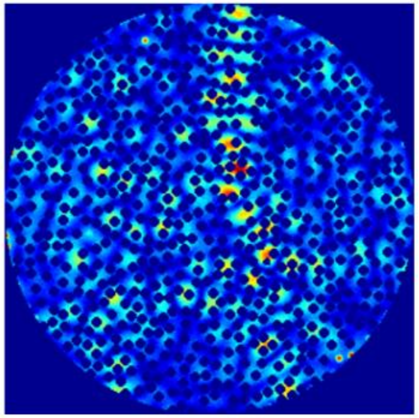
WAVE LOCALIZATION : A BROAD DEFINITION

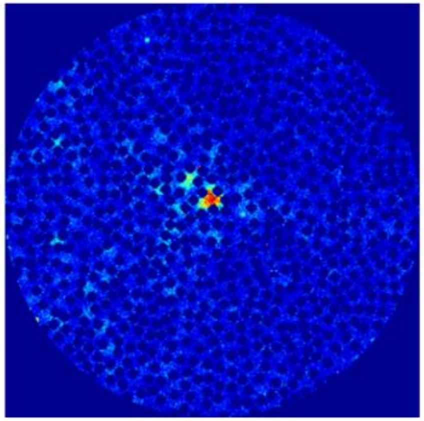
A state is said to be "localized"
if the vibration amplitude is large
only in a restricted region
of the total domain



MODE CONFINEMENT IN A DISORDERED THIN PLATE

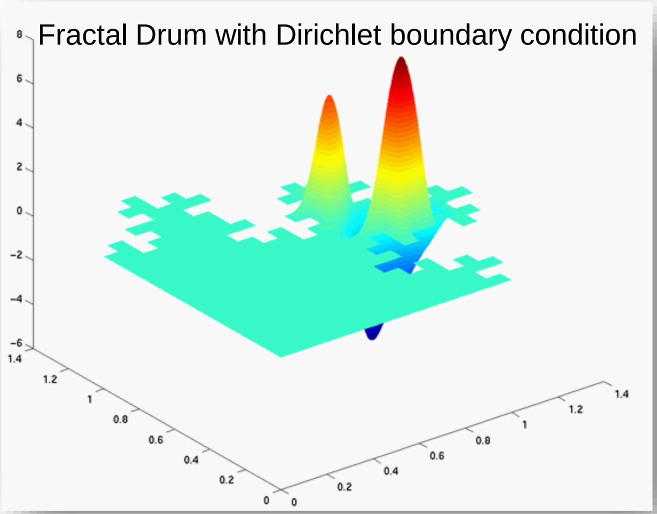



Extended mode


Localized mode

G. Lefebvre et al., to be published

WAVE LOCALIZATION IN COMPLEX STRUCTURES

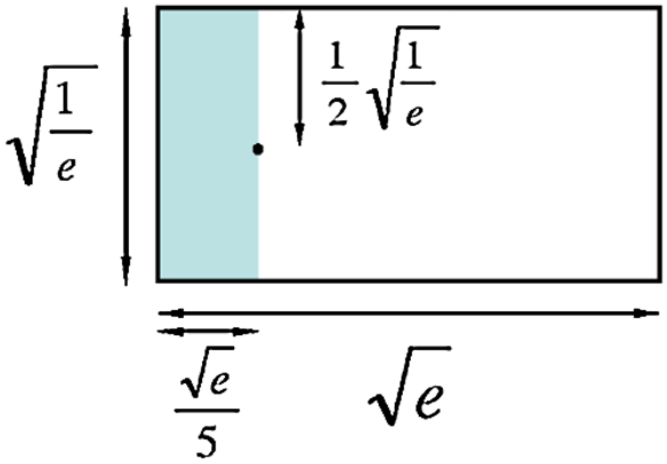


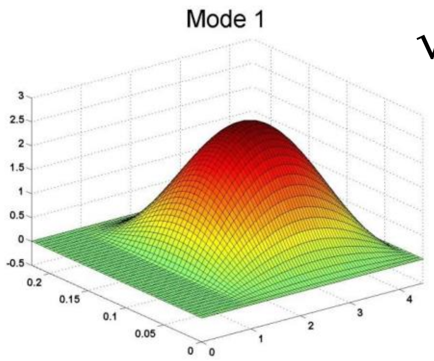
Félix et al., *J Sound Vib*, 2007

Heilman & Strichartz, *Not Am Math Soc,* 2010

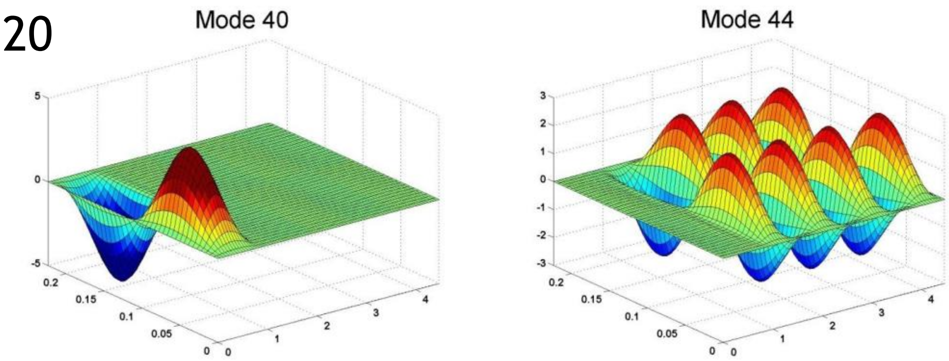
"Localized eigenfunctions: here you see them, there you don't"

Wave localization
in a clamped plate



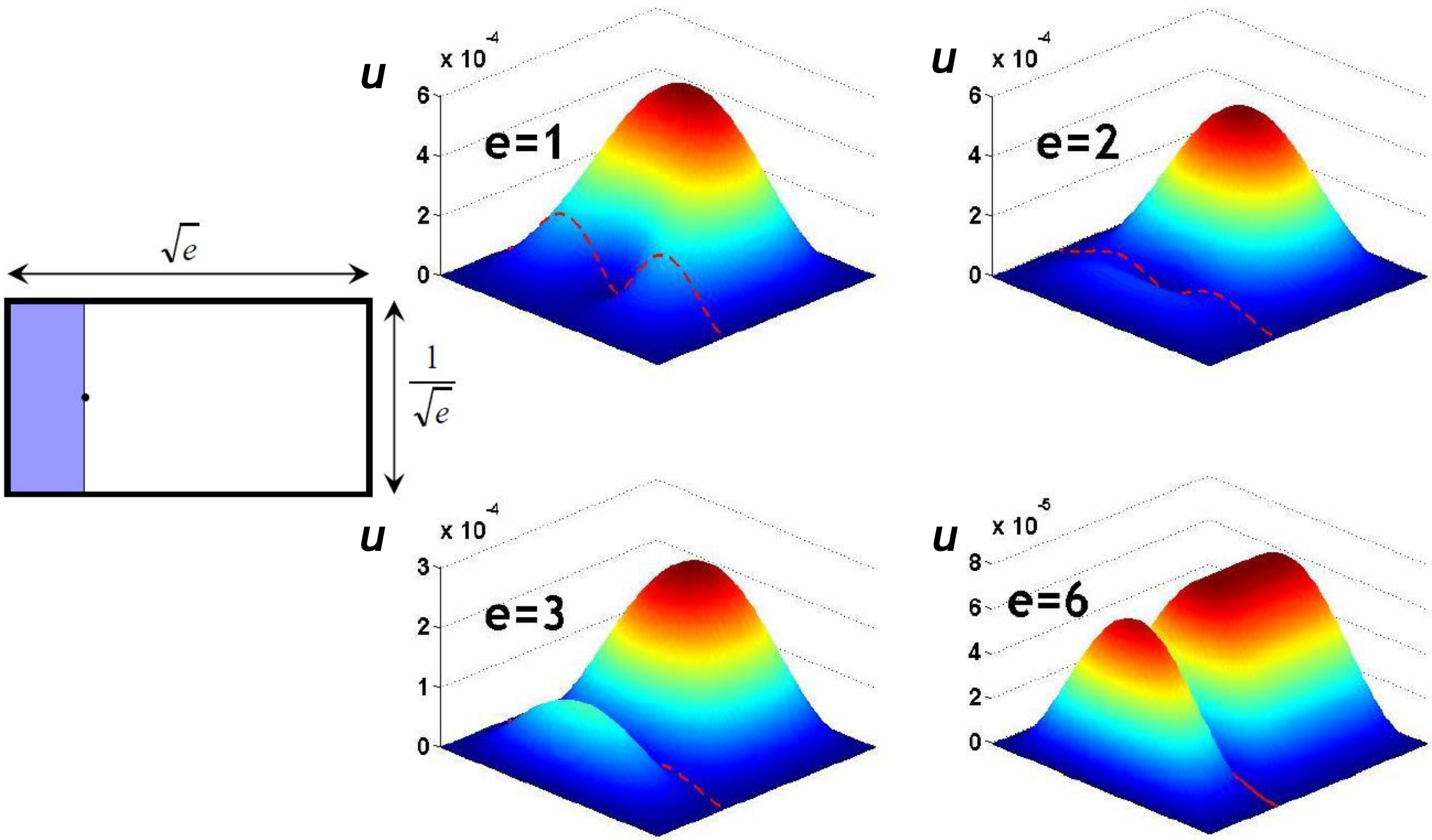


$\sqrt{e}$=20



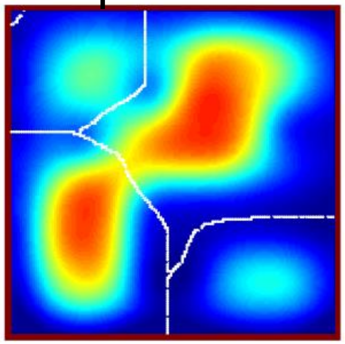
M.Filoche & Mayboroda, *PRL103*, 254301 (2009)

THE LANDSCAPE IN THE CLAMPED PLATE

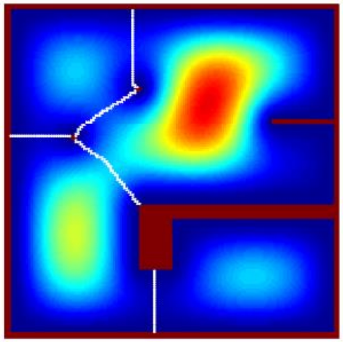


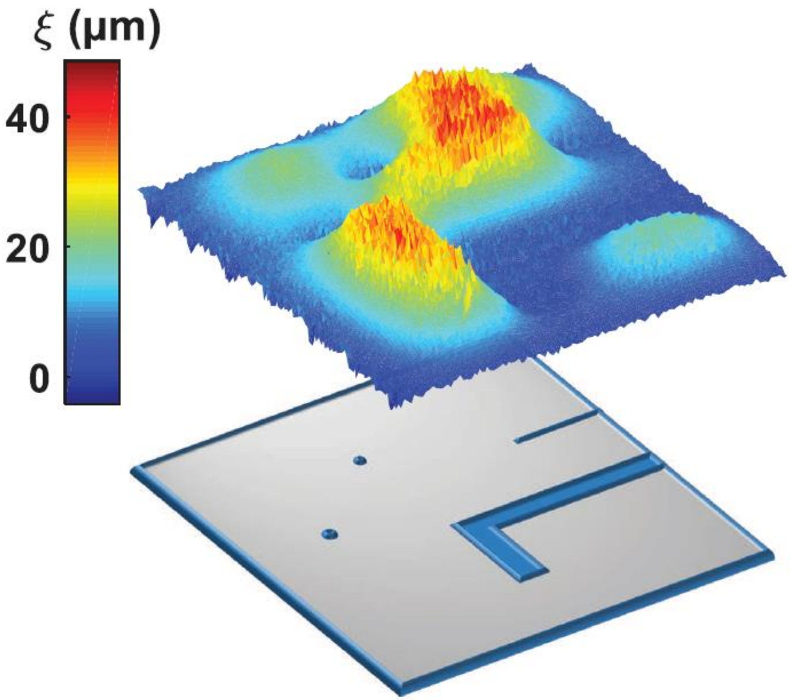
MEASURING THE LANDSCAPE

Experiment

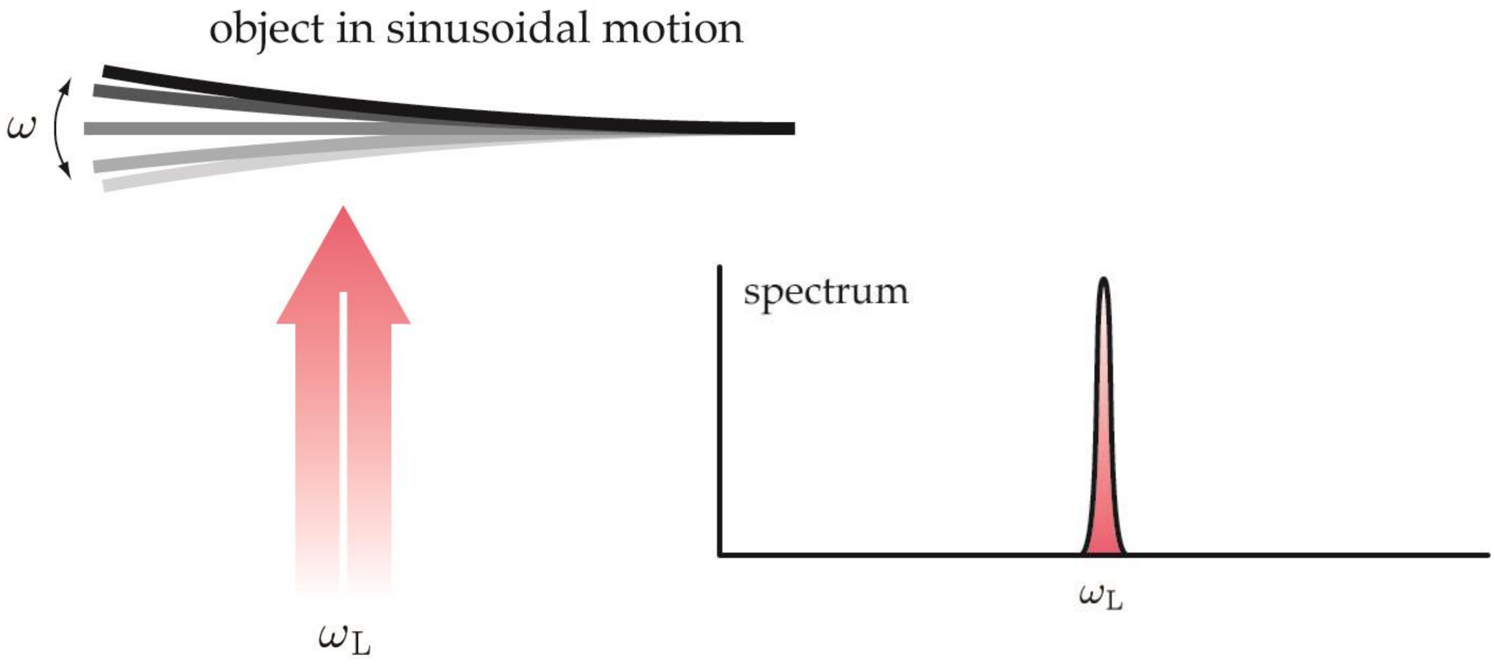


Simulation

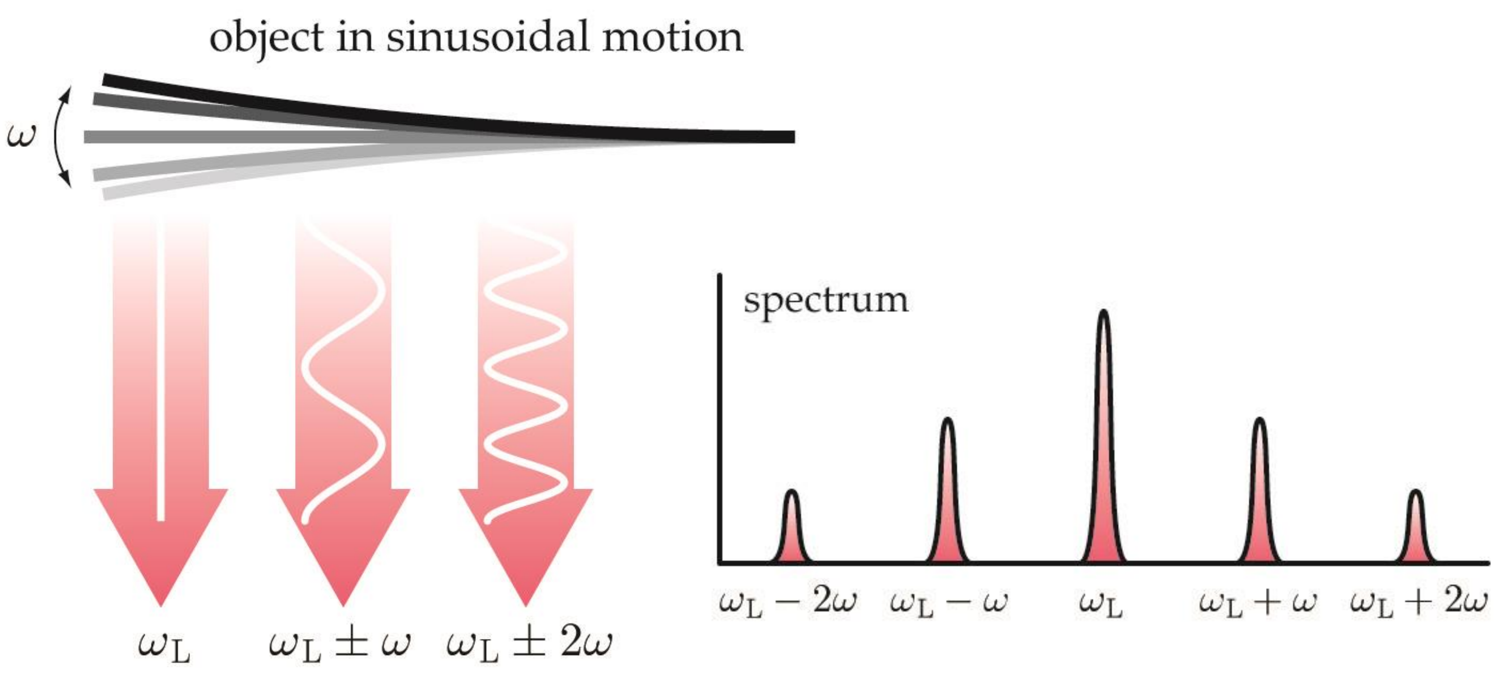




Light scattered by an object in sinusoidal motion

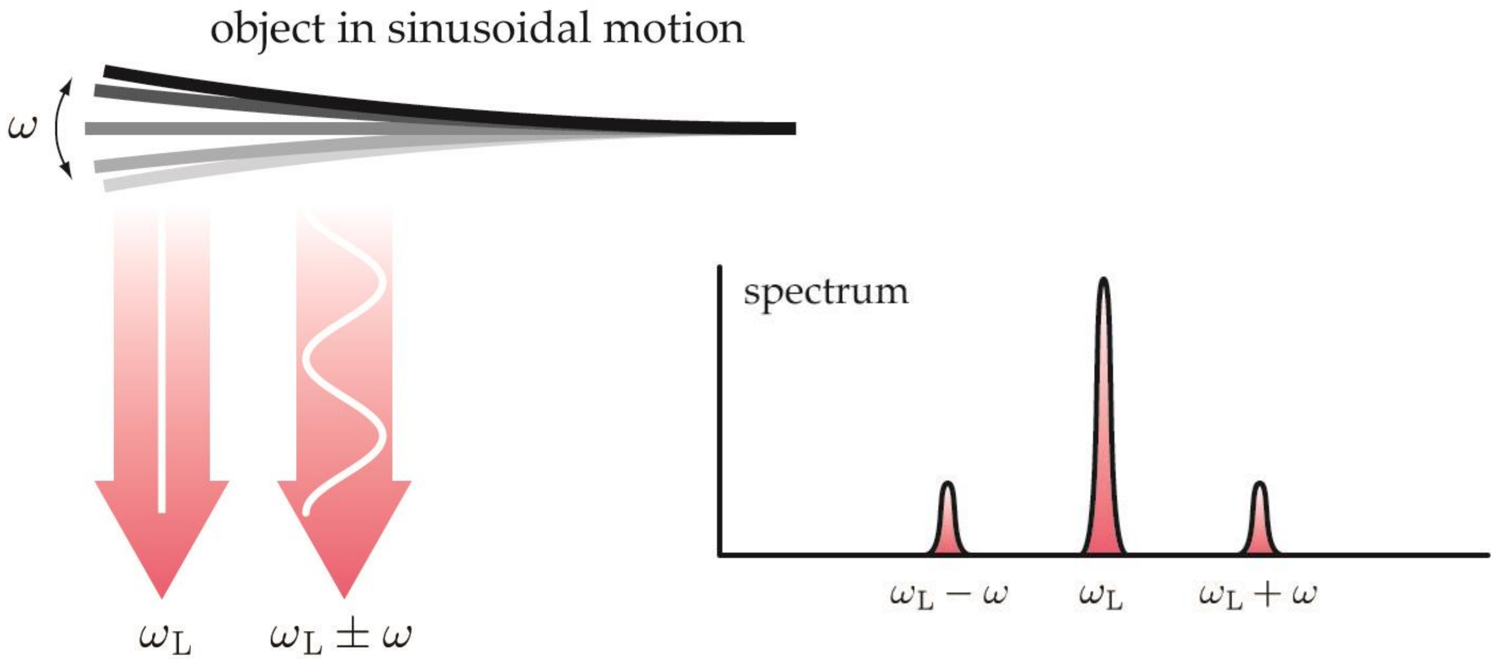


LIGHT SCATTERED BY AN OBJECT IN SINUSOIDAL MOTION



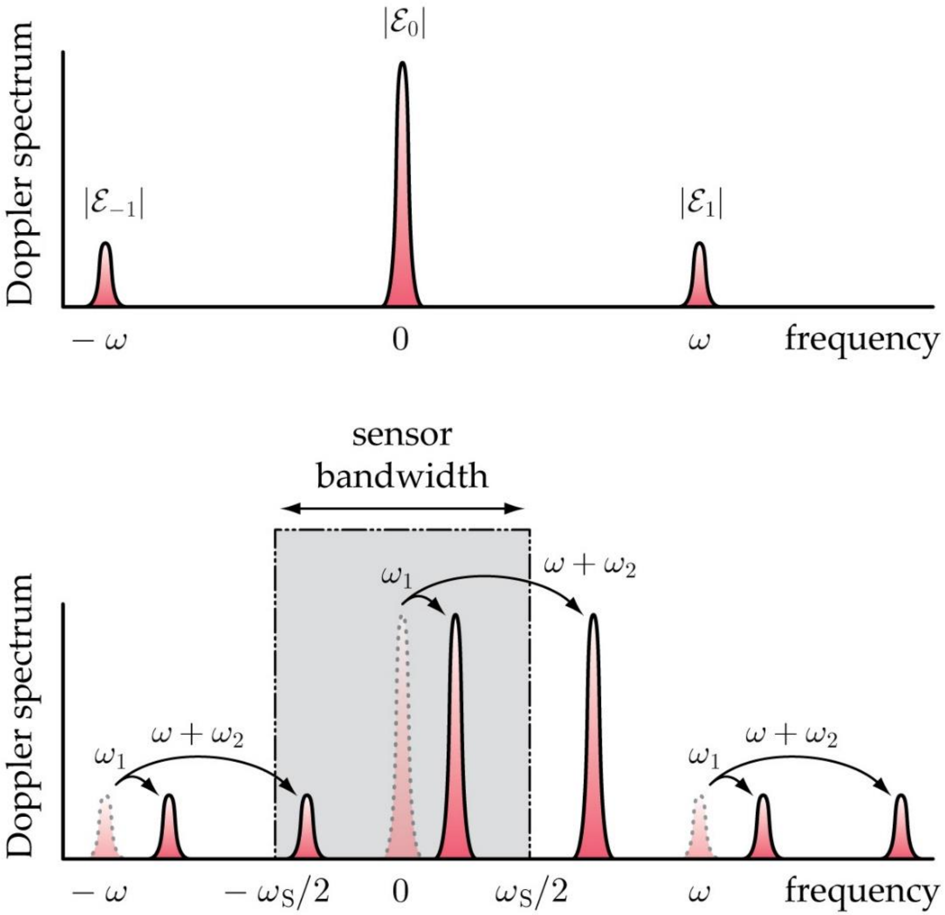
Heterodyne holographic interferometry (M. Atlan *et al.* Opt. Lett. 39, 2014)

SMALL VIBRATIONS



Heterodyne holographic interferometry (M. Atlan *et al.* Opt. Lett. 39, 2014)

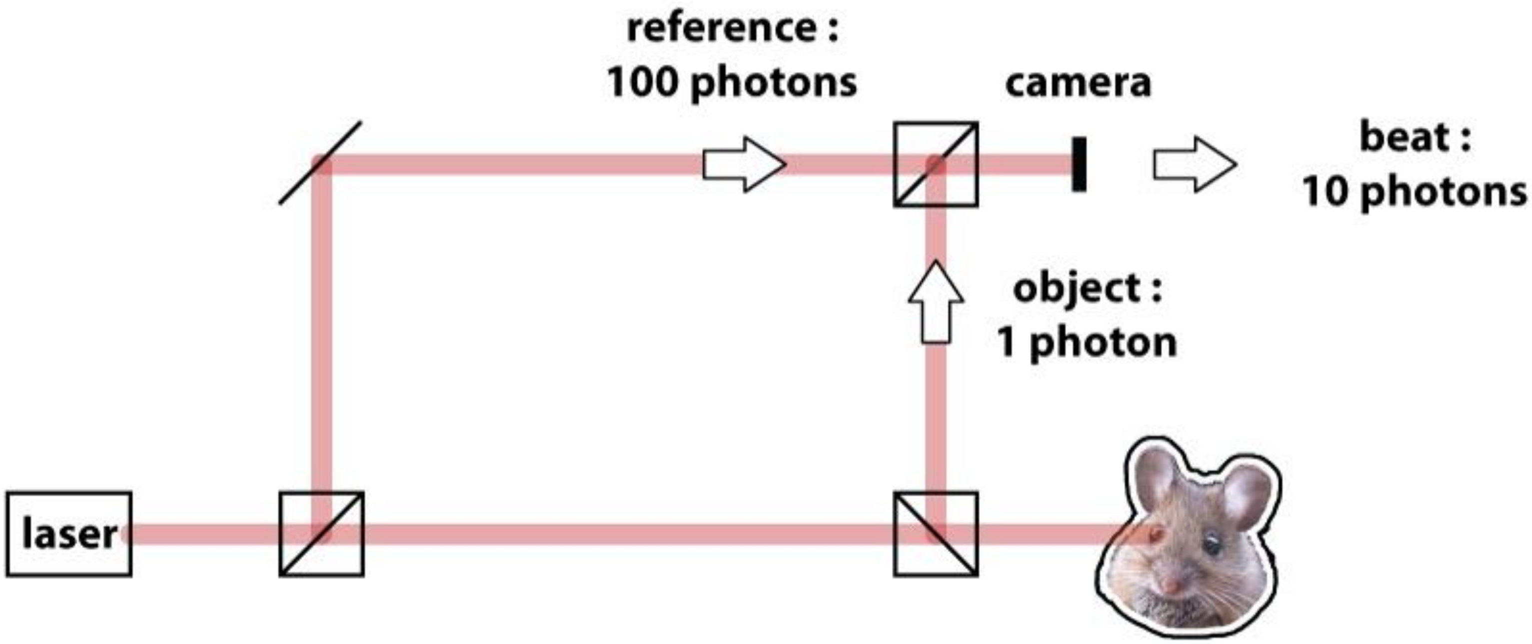
DOUBLE SIDEBAND MODULATION



Nicolas Verrier and Michael Atlan. Optics Letters, 38 (2013)

PRE-DETECTION OPTICAL SIGNAL ENHANCEMENT
HIGH HETERODYNE GAIN REGIME

The weak object wave beats against the reference wave.
Signal modulation depth is enhanced



EXPERIMENTAL VALIDATION OF THE LANDSCAPE THEORY

- Show that the landscape function controls the amplitude of the localized waves = *effective potential*

- Show that the landscape function predicts the eigenvalues of the vibratory modes

- Show that the landscape function gives the evolution of the mode extension vs. frequency = *transition from localized to extended*

G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

$\text{MAX}_I(U)$

$$u = \sum_i \alpha_i \varphi_i \quad \text{with} \quad \alpha_i = (u, \varphi_i) = \int u \phi_i \; dx$$

$$(Lu, \varphi_i) = (1, \varphi_i) = \int \varphi_i \; dx$$

$$(Lu, \varphi_i) = (u, L\varphi_i) = \lambda_i (u, \varphi_i) = \lambda_i \alpha_i$$

$$\alpha_i = \frac{1}{\lambda_i} \int \varphi_i \; dx \; .$$

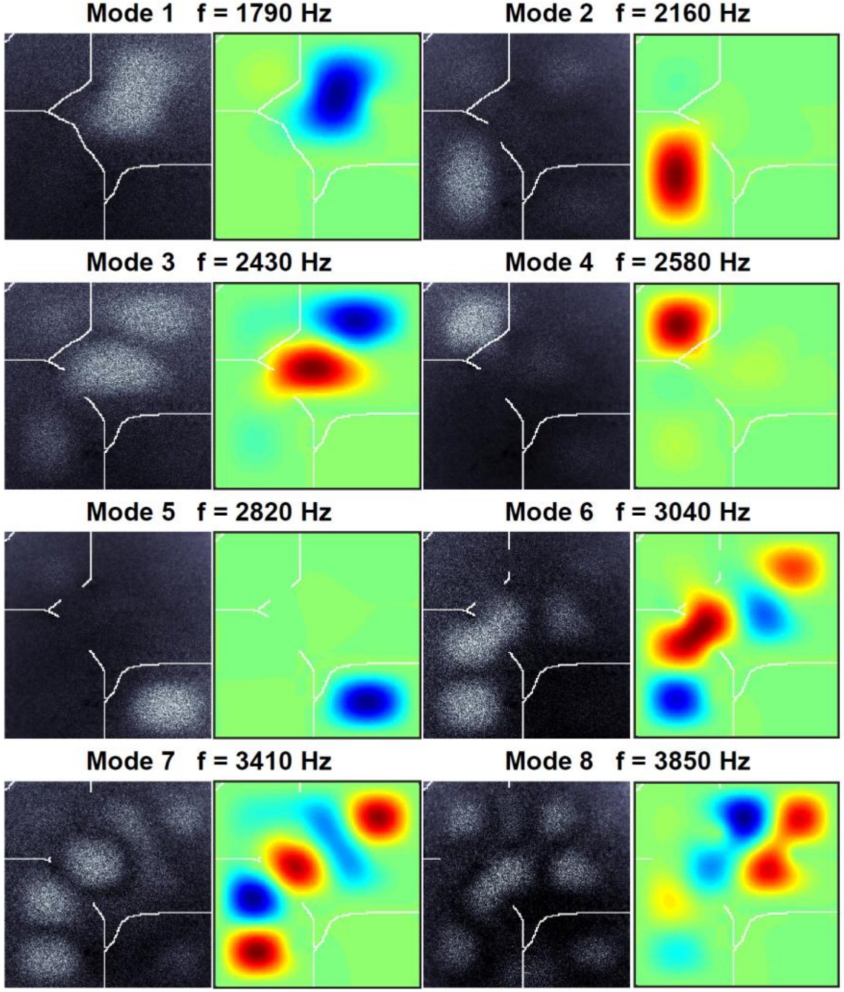
For the fundamental mode of a localization subregion

$$\lambda_0 = \frac{\int \varphi_0 \; dx}{\alpha_0} = \frac{\int \varphi_0 \; dx}{(u, \varphi_0)}$$

u is essentially the fundamental mode

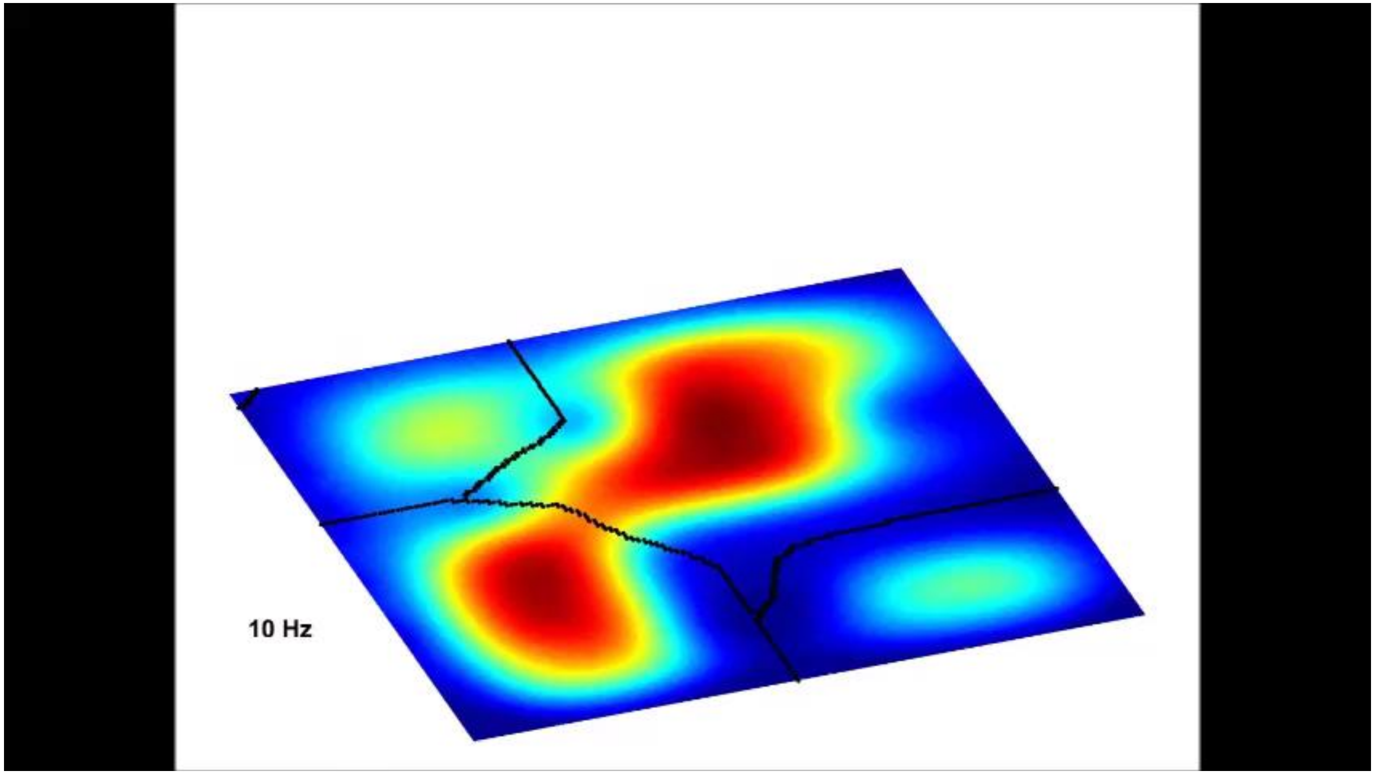
$$\lambda_0 \approx \frac{\int \frac{u}{\|u\|} \; dx}{\left(u, \frac{u}{\|u\|}\right)} = \frac{\int u \; dx}{\int u^2 \; dx}$$

MEASUREMENTS VS. SIMULATIONS



G. Lefebvre *et al.*, Phys. Rev. Lett. (2017)

4 - EFFECTIVE VALLEY NETWORK



THE LOCALIZATION LANDSCAPE

- Describes the sub-regions of localization

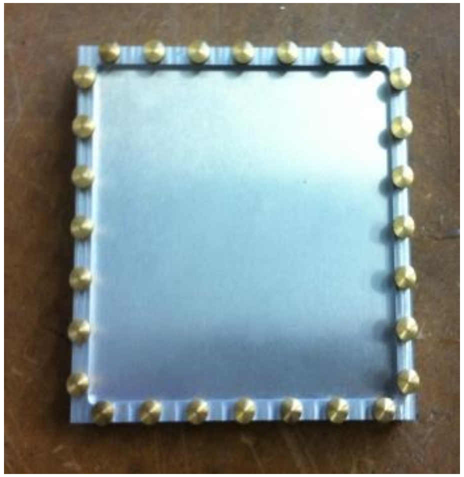
- Gives, in a single measurement
  - the eigenfrequencies
  - The eigenmode profile
  - The frequency dependence of the mode spatial extension

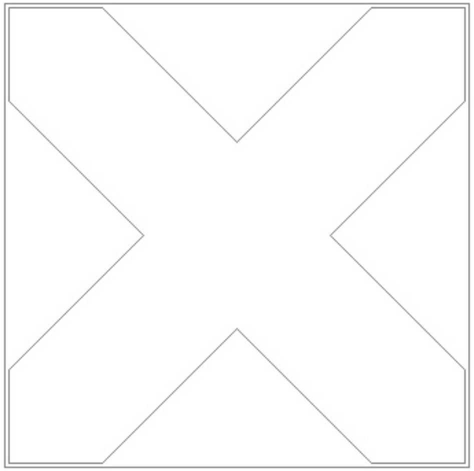
- This is a static measurement, which does not require a precise knowledge of the microscopic geometry

- It can be used to address the inverse problem and design the system geometry corresponding that a desired spatial distribution of the modes

M. Filoche and S. Mayboroda, PNAS (2012)

HOW TO MAKE AN X VIBRATE ?





PROBLEM FORMULATION

Kirchhoff-Love equation
of flexural waves

$$\frac{\partial^2 w}{\partial t^2} + \frac{Eh^2}{12\rho\left(1-\nu^2\right)}\nabla^4 w = 0$$

$\Downarrow$

Steady-state
equation of the eigenmodes

$$\begin{cases} LW_m = \omega_m^2 W_m \quad \text{on} \quad \Omega \\ W_m|_{\partial\Omega} = 0 \\ \mathbf{n}\cdot\nabla W_m|_{\partial\Omega} = 0 \end{cases}$$

$$L = \frac{1}{\alpha^2}\nabla^4.$$

$\Rightarrow$

Derivation of the
Ideal landscape

$$\begin{cases} Lu^* = 1 \quad \text{on} \quad \Omega \\ u^*|_{\partial\Omega\cup\partial\Omega_1} = 0 \\ \mathbf{n}\cdot\nabla u^*|_{\partial\Omega\cup\partial\Omega_1} = 0 \end{cases}$$

$\Uparrow$

Derivation of the
experimental landscape with
N clamped points

$$\begin{cases} Lu = 1 \quad \text{on} \quad \Omega \\ u|_{\partial\Omega\cup A_1\cup\cdots\cup A_N} = 0 \\ \mathbf{n}\cdot\nabla u|_{\partial\Omega\cup A_1\cup\cdots\cup A_N} = 0 \end{cases}$$

$\Uparrow$

Minimization of the distance

$$||u - u^*||$$

FREQUENCY RESPONSE OF A PLATE CLAMPED WITH 16 PINS

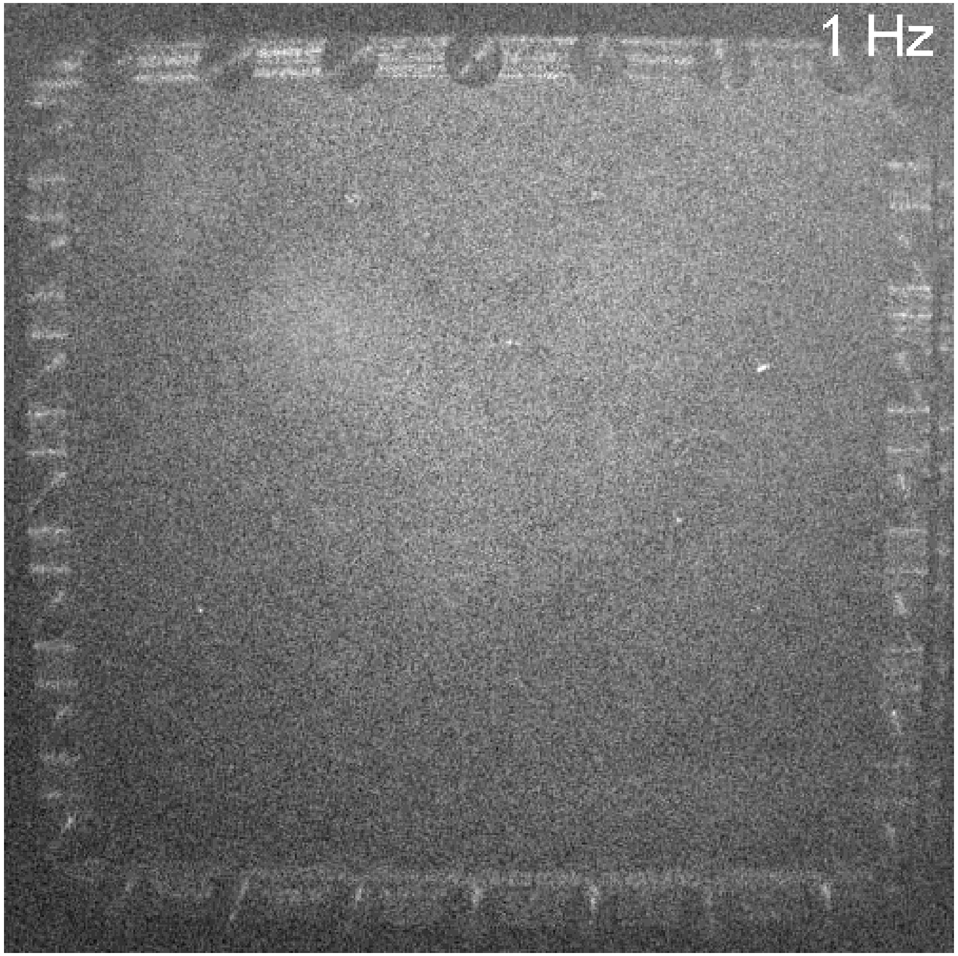
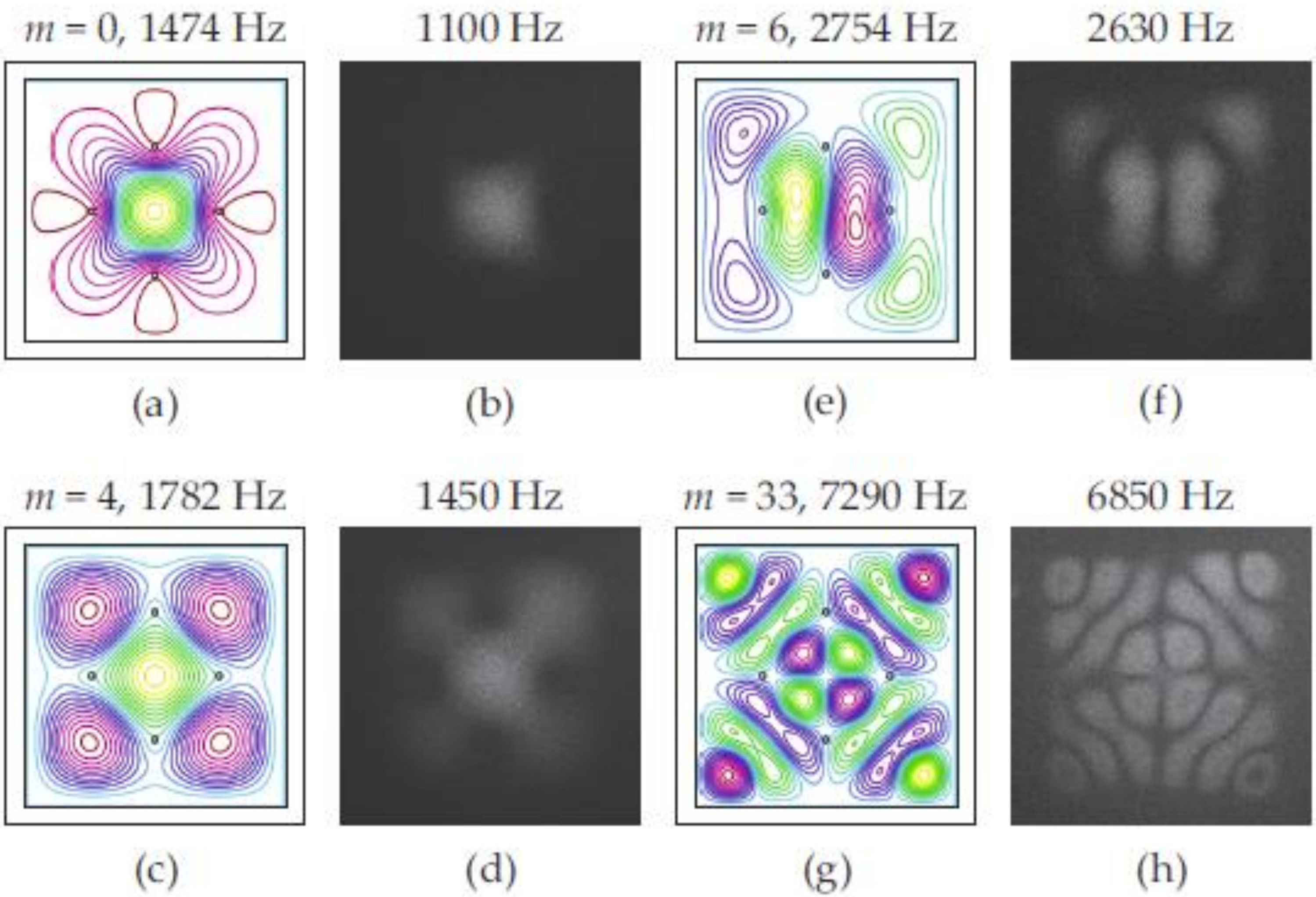
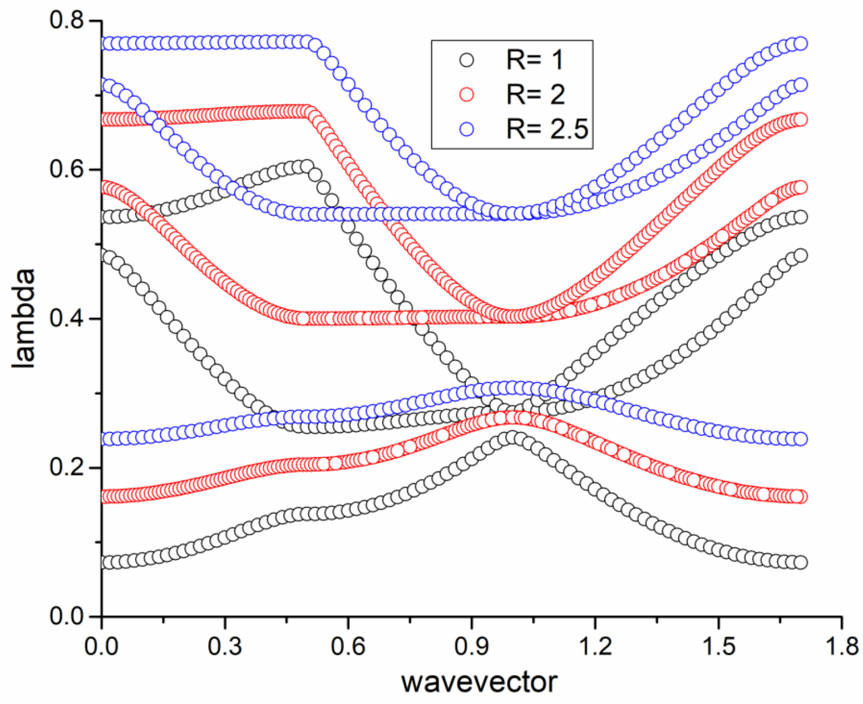


PLATE WITH 4 PINS

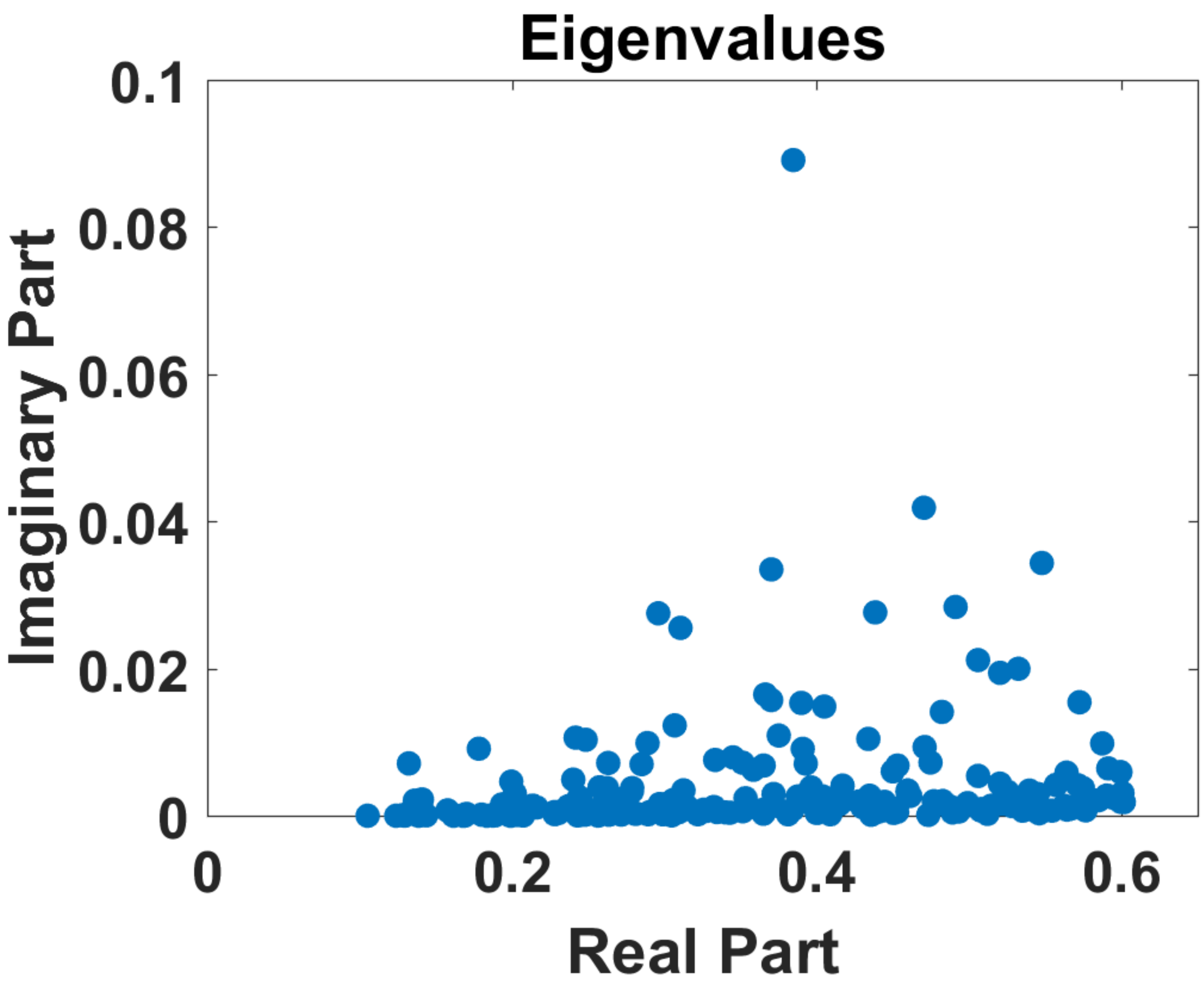


BAND DIAGRAM

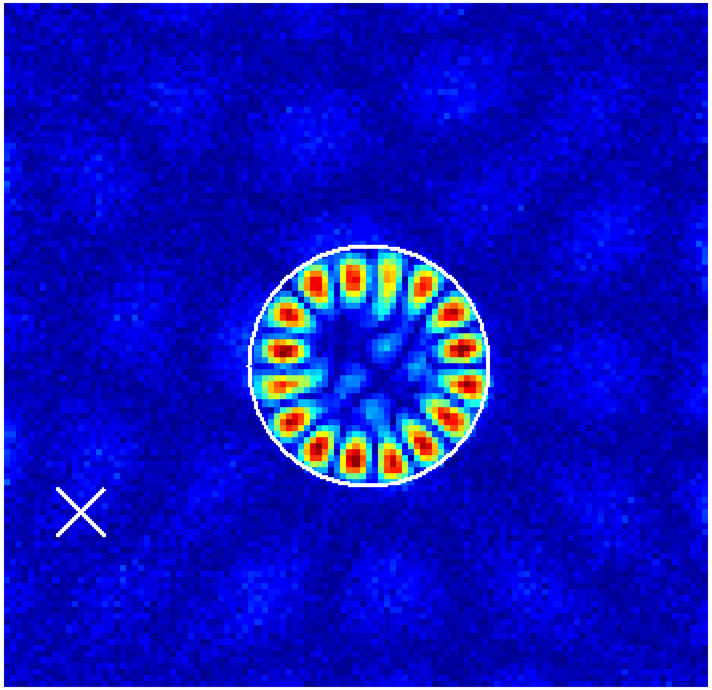


Low frequency gap

DISTRIBUTION OF RESONANCES



BLIND HOLE RESONANCES



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  - Kun TANG (Bar Ilan University, Israel)
- **Fabrication :**
  - Etienne HERTH (FemtoST, Besançon, France)

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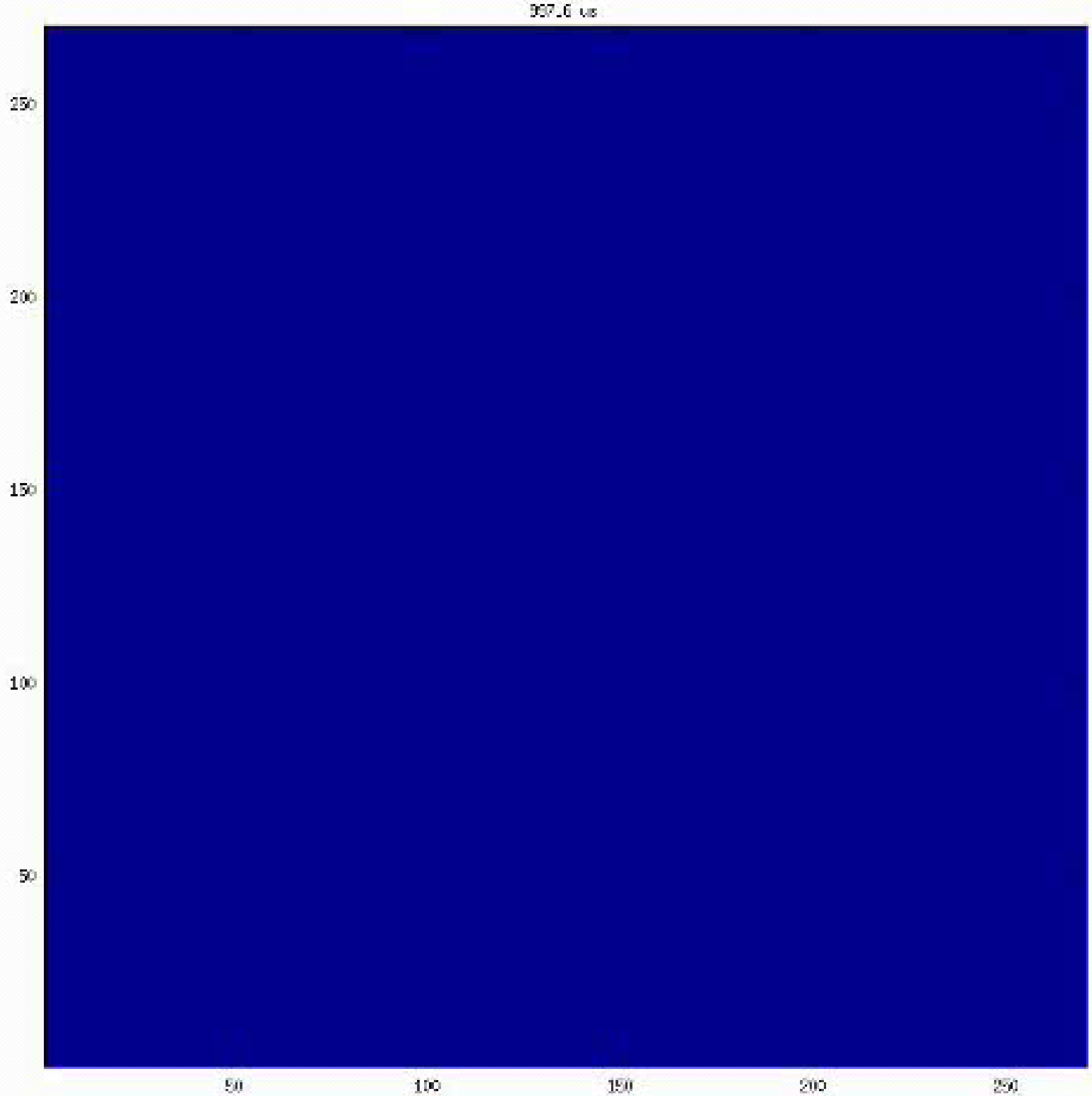
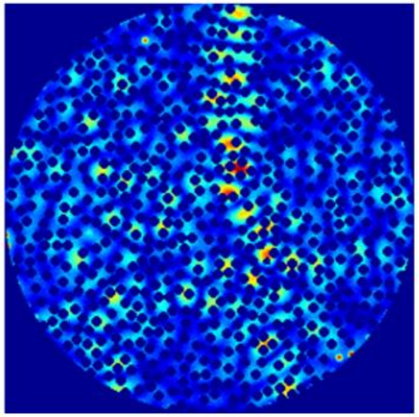
Papers :

G. Lefebvre *et al.*, **One single static measurement predicts wave localization in complex structures,** Phys. Rev. Lett. 117, 074301 (2016).

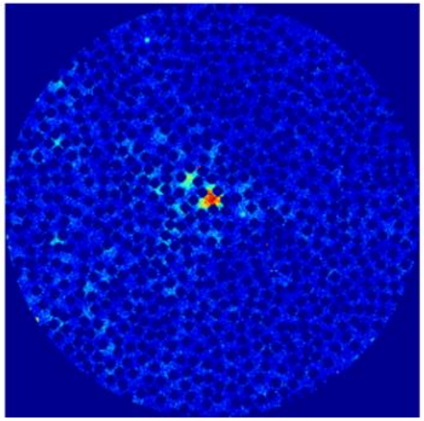
G. Lefebvre *et al.*, **Localization induced by resonant scatterers in thin plates**, in preparation.


250
200
150
100
50
50
100
150
200
250

LOCALIZATION OF FLEXURAL WAVES



Extended mode



Localized mode

PERSPECTIVES

- Can be generalized to any complex system, where the structural or microscopic information is not accessible
- Towards the prediction of the full dynamics from a static measurement
- Extension to other systems, other wave equations
- Inverse problem: towards engineering of modes