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Assignment 5

Q1. (a) $l(1) = 1$

(b) $l(n) = 2\,l(n-1) + 2^{n-1}$

(c) Homogeneous form is $l(n) = 2\,l(n-1)$, CE: $r^2 = 2r \Rightarrow r = 2, 0$

$l^h(n) = \alpha \cdot 2^n$

Suppose $l^P(n) = n\beta \cdot 2^n$ (because 2 is the root of CE, so we multiply n)

$n\beta \cdot 2^n = 2 \cdot (n-1)\beta \cdot 2^{n-1} + 2^{n-1} \Rightarrow \beta = \frac{1}{2} \Rightarrow l^P(n) = n \cdot 2^{n-1}$

Hence, $l(n) = n \cdot 2^{n-1} + \alpha \cdot 2^n$

$l(1) = 1 \cdot 2^0 + \alpha \cdot 2^1 = 1 \Rightarrow \alpha = 0$

$l(n) = n \cdot 2^{n-1}$

Q2. (a) There're 10 ballons in a row, and we should cut them into 4 pieces, each piece should contains at least one.

$O_\uparrow O_\uparrow O_\uparrow O_\uparrow O_\uparrow O_\uparrow O_\uparrow O_\uparrow O_\uparrow O$

We need to choose 3 place to cut them into 4 pieces, there're 9 places we can choose, so $\binom{9}{3} = 84$ is the answer.

(b) $G(x) = \frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} x^k$

Hence, it is the coefficient of term x^3 of generating function $G(x) = \frac{1}{(1-x)^9}$

Q3. (a) $2^n 3^{\frac{n(n-1)}{2}}$. When (a,a) with $a \in A$, it can be in R or not, 2^n. (a,b) and (b,a), we have $(a,b) \in A$, $(b,a) \notin A$, and $(a,b) \notin A$, $(b,a) \in A$, and $(a,b) \notin A$, $(b,a) \notin A$, $3^{\frac{n(n-1)}{2}}$.

(b) $2^{n(n-1)}$. Only $(a,a) \notin A$, others no restriction.

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(c) $2^{n^2} - 2^{n^2-n+1}$. All the relations number is 2^{n^2}, both reflexive and irreflexive are 2^{n^2-n}.

(d) 2^n. Symmetric and antisymmetric implies $(a,b) \notin R$ $(a \neq b)$. Only (a,a) can $\in R$ or $\notin R$, but this doesn't influence the transitivity.

~~Q4. Prove it. R is reflexive, so $\forall a \in A$, $(a,a) \in R$.~~

Q4. Disprove. For $A = \{1,2,3\}$, $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1)\}$.
R is reflexive and symmetric, but not transitive, because $(2,1) \in R$, $(1,3) \in R$, $(2,3) \notin R$.

Q5. For $\forall (a,b) \in R$, by definition of $R^2 = R \circ R$, $(a,b) \in R$, $(b,b) \in R$ (because R is reflexive), then $(a,b) \in R^2$.
By arbitrariness of (a,b), $R \subseteq R^2$.

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Q6. If $(a,b) \in R_1 \cap R_2$, then $(a,b) \in R_1$, since R_1 symmetric, $(b,a) \in R_1$. Similarly, $(a,b) \in R_2 \Rightarrow (b,a) \in R_2$, then $(b,a) \in R_1 \cap R_2$.
Hence $R_1 \cap R_2$ is symmetric.
If $(a,b) \in R_1 \cup R_2$, then $(a,b) \in R_1$ or $(a,b) \in R_2$. Since both R_1 and R_2 are symmetric, so $(b,a) \in R_1$ or $(b,a) \in R_2$, $(b,a) \in R_1 \cup R_2$.
Hence $R_1 \cup R_2$ is symmetric.

Q7. Suppose (a,b) is in the symmetric closure of the transitive closure of R. Then (a,b), (b,a) has at least one in the transitive closure of R. ~~Their exi~~
There exist at least one path in $a \to b$ and $b \to a$ this 2 paths.
① $(a,b) \in R$, then (a,b) is in transitive closure of symmetric closure of R.
② $(b,a) \in R$, then (a,b) is in symmetric closure of R, then (a,b) is in trasitive closure and symmetric closure of R.
According to the arbitrariness of (a,b), the conclusion holds.

Q8. Connectivity Relation R^* equals the transitive closure of R.

Suppose $(a,b) \in R^*$.

① $(a,b) \in R$. Then ~~(a,b)~~ $(b,a) \in R$ since R is symmetric.

So $(b,a) \in R^*$ since $R \subseteq R^*$.

② $(a,b) \notin R$. But $(a,b) \in R^*$ in the trasitive closure of R.

There exists c, such that $(a,c) \in R$, $(c,b) \in R$.

Since R is symmetric, $(c,a) \in R$, $(b,c) \in R$, so $(b,a) \in R^*$.

Above the two cases, $(a,b) \in R^* \Rightarrow (b,a) \in R^*$, R^* symmetric.

Q9. (a) Yes. (b) Yes. (c) No.

Q10. If $(a,b) \in R$, which means $a-b \in \mathbb{Q}$.

Then $b-a \in \mathbb{Q}$, $(b,a) \in R$, R symmetric.

Each $(a,a) \in R$ since $a-a=0 \in \mathbb{Q}$, R ~~reflexible~~ reflexive.

$(a,b), (b,c) \in R$, $a-b \in \mathbb{Q}$, $b-c \in \mathbb{Q}$, then $a-c \in \mathbb{Q}$, $(a,c) \in R$, R transitive.

So R is equivalence relation.

$[1] = [\frac{1}{2}] = \mathbb{Q}$, $[\pi] = \{x \mid x = \pi + y, y \in \mathbb{Q}\}$

Q11. $R_1 \subseteq R_1 \Rightarrow R_1 \preceq R_1$, ~~trans~~ reflexive.

If $R_1 \subseteq R_2$ and $R_1 \neq R_2$, then $R_2 \subseteq R_1$ isn't hold, so $R(s)$ is antisymmetric.

$R_1 \preceq R_2$, $R_2 \preceq R_3 \Rightarrow R_1 \subseteq R_2$, $R_2 \subseteq R_3 \Rightarrow R_1 \subseteq R_3 \Rightarrow R_1 \preceq R_3$, transitive.

Hence $(R(s), \preceq)$ is poset.

Q12. R and S are equivalence relations, R and S are symmetric, reflexive and transitive.

① $(x,y)\in T \Rightarrow (x,y)\in R,\ (x,y)\in S \Rightarrow (y,x)\in R,\ (y,x)\in S$ (~~reflexive~~ symmetric)

$\Rightarrow (y,x)\in T$, T is ~~reflexive~~ symmetric.

② R, S is reflexive $\Rightarrow \forall x\,(x,x)\in R,\ (x,x)\in S \Rightarrow \forall x\ (x,x)\in T \Rightarrow T$ is reflexive.

③ $(x,y),(y,z)\in T \Rightarrow (x,y)\in R,\ (y,z)\in R,\ (x,y)\in S,\ (y,z)\in S$

$\Rightarrow (x,z)\in R,\ (x,z)\in S$ (transitive) $\Rightarrow (x,z)\in T \Rightarrow T$ is transitive.

From ①②③, T is equivalence relation.

Q13. (a) $\forall x\in\mathbb{R},\ f(x)\le f(x)$, so $f\preceq f$, reflexive.

$f\neq g,\ f\preceq g \Rightarrow \forall x\in\mathbb{R},\ f(x)\le g(x) \Rightarrow \forall x\in\mathbb{R},\ f(x)\ge g(x)$ doesn't hold

$\Rightarrow$ antisymmetric

$f\preceq g\preceq h \Rightarrow \forall x\in\mathbb{R},\ f(x)\le g(x),\ g(x)\le h(x) \Rightarrow \forall x\in\mathbb{R},\ f(x)\le h(x)$

$\Rightarrow$ transitive

(b) $f(x)=0,\ g(x)=x$. When $x=1$, $f(x)\le g(x)$, when $x=-1$, $f(x)\ge g(x)$. $\forall x$, $f(x)\ge g(x)$ and $f(x)\le g(x)$ both not holds. $\preceq$ is not total ordering.

Q14. (a) $R=\{\{0\},\{0,1\},\{0,1,2\},\cdots\}$, R doesn't have maximal element.

(b) $\emptyset$ is the greatest minimum of $\mathcal{P}(N)$, so $R\in\mathcal{P}(N)$ must have minimal element.

(c) $T\in\mathcal{P}(N)$ must have minimal element (from (b)), so T can not has neither maximal nor minimal element.