

No.

Date

Assignment 6

Q1. (a) No. The graph contains a circle $\{v_1, v_2, v_4\}$, it can not be a bipartite.

(b) For $S = \{v_5\}$, $\bar{S} = V \setminus S$, we only need to cut edge v_1v_5 and v_5v_6 to separate. So the edge connectivity is 2.

(c) No. The degree of v_1, v_3, v_4, v_7 are not even.

(d) Yes. $V=7$, $E=11$, $F=6$, $V-E+F=2$ holds.

(e) There exist a circle $\{v_1, v_2, v_4\}$, so chromatic number ≥ 2 .

$\{v_1, v_6\}$, $\{v_2, v_5, v_7\}$, $\{v_3, v_4\}$ can make it 3. So chromatic number is 3.

Q2. (a) Prove by contradiction.

Suppose G has no circle, we can find the longest path in G , and v_6 is the end point of this path. $\deg(v_6)=2$, suppose v_1, v_2 connected to v_6 and suppose v_1 is in the longest path.

If v_2 is also in the path, creates a circle, if v_2 doesn't, then contradict to v_6 is the end point.

(b) G is disconnected, so we arbitrarily choose v_6, v_1 from different connected component, edge $\{v_6, v_1\} \notin G$, then $\{v_6, v_1\} \in \bar{G}$.

Then for v_2, v_3 in the same component, choose v_4 in different component, from before, $\{v_2, v_4\} \in \bar{G}$, $\{v_3, v_4\} \in \bar{G}$, so v_2 and v_3 are connected in \bar{G} .

Whether 2 vertices are in same connected component in G , they all will be connected in \bar{G} . Hence, \bar{G} is connected.