HW5

Write the eigenvalue, multiplicity, eigenspace, generalized eigenspace.

a) $A_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\det(\lambda I - A_1) = (\lambda - 2)^3 = 0 \qquad \lambda_1 = \lambda_2 = \lambda_3 = 2$

Eigenvalue is 2, multiplicity is 3.

$\mathcal{E} = \mathcal{N}(\lambda I - A_1) = \mathcal{N}(0) = \mathbb{C}^3 \qquad \widetilde{\mathcal{E}} = \mathcal{N}(\lambda I - A_1)^3 = \mathcal{N}(0^3) = \mathcal{N}(0) = \mathbb{C}^3$

b) $A_2 = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\det(\lambda I - A_2) = (\lambda - 2)^3 = 0 \qquad \lambda_1 = \lambda_2 = \lambda_3 = 2$

Eigenvalue is 2, multiplicity is 3.

$\mathcal{E} = \mathcal{N}(\lambda I - A_2) = \mathcal{N}\left(\begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$

$\begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Rightarrow y = 0, \; x, z \in \mathbb{C}$

So $\mathcal{N}\left(\begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = \operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \mathcal{E}$

$\widetilde{\mathcal{E}} = \mathcal{N}(\lambda I - A_2)^3 = \mathcal{N}\left(\begin{bmatrix} 0 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^3\right) = \mathcal{N}(0) = \mathbb{C}^3$