No.

Date

HW2

Calculate $\dim(\mathbb{F}^n)$, $\dim(\mathbb{F}^{m\times n})$, $\dim(\mathbb{C}^n)$ over $\mathbb{C}$, $\dim(\mathbb{C}^n)$ over $\mathbb{R}$.

Sol. 1) For $\forall \vec{x} \in \mathbb{F}^n$, $\vec{x} = (x_1, x_2, \cdots, x_n)$, $x_i \in \mathbb{F}$, $i = 1, 2, \cdots, n$

Each x_i is independent, so $\dim(\mathbb{F}^n) = n$.

2) For $\forall A \in \mathbb{F}^{m\times n}$, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \vdots \\ a_{m1} & & \cdots & a_{mn} \end{bmatrix}$, $a_{ij} \in \mathbb{F}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$

Each a_{ij} is independent, so $\dim(\mathbb{F}^{m\times n}) = mn$

3) From 1) we know, $\dim(\mathbb{F}^n)$ over $\mathbb{F}$ is n.

Set $\mathbb{F} = \mathbb{C}$, we get $\dim(\mathbb{C}^n)$ over $\mathbb{C}$ is n.

4) For $\forall \vec{z} \in \mathbb{C}^n$, we can write $\vec{z} = \vec{x} + i\vec{y}$, $\vec{x}, \vec{y} \in \mathbb{R}^n$, $\vec{x}$ and $\vec{y}$ are independent.

And we know $\dim(\mathbb{R}^n)$ over $\mathbb{R}$ is n, so $\dim(\mathbb{C}^n) = \dim(\mathbb{R}^n) + \dim(\mathbb{R}^n) = 2n$