# Lab 5 - Implement and Understand PSO algorithm

CSE, SUSTech

# Outline of This Lab

- Teaching Assistant
- How Does The PSO with Inertia Weight Work
- Does The Inertia Weight Matter?
- Test Functions
- Illustrate The Results!

# Teaching assistant

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# How Does The PSO with Inertia Weight Work
# -*Part I*

The PSO Algorithm (Main Loop):

1. Initialize a swarm of particles with random positions and velocities.
2. Evaluate each particle's fitness.
3. Update each particle's pbest and the swarm's gbest.
4. For each particle:
   a. Calculate its new velocity.
   b. Update its position.
5. Repeat from Step 2 until a stopping criterion is met (e.g., max iterations, solution found).

# How Does The PSO with Inertia Weight Work -*Part II*

add inertia weight

$$v_{id} = v_{id} + c_1 rand()\left(p_{id} - x_{id}\right) + c_2 Rand()\left(p_{gd} - x_{id}\right)$$

$$x_{id} = x_{id} + v_{id}$$

$$v_{id} = wv_{id} + c_1 rand()\left(p_{id} - x_{id}\right) + c_2 Rand()\left(p_{gd} - x_{id}\right)$$

$$x_{id} = x_{id} + v_{id}$$

# Does The Inertia Weight Matter?

- Implement the above algorithm with the real-valued representstion with different inertia weight setting

# Test Functions
## *-7 unimodal benchmark functions*

- Unimodal functions: $f_1$-$f_5$
- $f_6$ is the step function (one minimum, discontinuous).
- $f_7$ is a noisy quartic function, where random[0, 1) is a uniformly distributed random variable in [0, 1).

| Test function | $n$ | $S$ | $f_{min}$ |
|---|---|---|---|
| $f_1(\mathbf{x}) = \sum_{i=1}^{n} x_i^2$ | 30 | $[-100, 100]^n$ | 0 |
| $f_2(\mathbf{x}) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$ | 30 | $[-100, 100]^n$ | 0 |
| $f_3(\mathbf{x}) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$ | 30 | $[-10, 10]^n$ | 0 |
| $f_4(\mathbf{x}) = \max_i\{\lvert x_i \rvert, 1 \leq i \leq n\}$ | 30 | $[-100, 100]^n$ | 0 |
| $f_5(\mathbf{x}) = \sum_{i=1}^{n} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$ | 30 | $[-100, 100]^n$ | 0 |
| $f_6(\mathbf{x}) = \sum_{i=1}^{n} (x_i + 0.5)^2$ | 30 | $[-30, 30]^n$ | 0 |
| $f_7(\mathbf{x}) = \sum_{i=1}^{n} i x_i^4 + random[0, 1)$ | 30 | $[-1.28, 1.28]^n$ | 0 |

# Test Functions
## *-8 multimodal benchmark functions*

| Test function | $n$ | $S$ | $f_{min}$ |
|---|---|---|---|
| $f_8(\mathbf{x}) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$ | **30** | $[-500, 500]^n$ | **-12569.5** |
| $f_9(\mathbf{x}) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10)]$ | **30** | $[-5.12, 5.12]^n$ | **0** |
| $f_{10}(\mathbf{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right) +20 + e$ | **30** | $[-32, 32]^n$ | **0** |
| $f_{11}(\mathbf{x}) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ | **30** | $[-600, 600]^n$ | **0** |
| $f_{12}(\mathbf{x}) = \frac{\pi}{n}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] +(y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4),$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a, \\ 0, & -a \le x_i \le a, \\ k(-x_i - a)^m, & x_i < -a. \end{cases}$ | **30** | $[-50, 50]^n$ | **0** |
| $f_{13}(\mathbf{x}) = 0.1\left\{\sin^2(3\pi x_1) + \sum_{i=1}^{n-1}(x_i - 1)^2[1 + sin^2(3\pi x_{i+1})] +(x_n - 1)^2[1 + \sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$ | **30** | $[-50, 50]^n$ | **0** |
| $f_{14}(\mathbf{x}) = \left[\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i - a_{ij})^6}\right]^{-1}$ | **2** | $[-65.536, 65.536]^n$ | **1** |
| $f_{15}(\mathbf{x}) = \sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$ | **4** | $[-5, 5]^n$ | **0.0003075** |

# Experimental Setup

- 15 Test Minimization Functions (7 unimodal + 8 multimodal benchmark functions).
- Population size 50.
- Maximum function evaluation 500, 000.
- Set inertia weight as 0.5, 0.0, and 1.2, respectively
- 30 independent runs for each function and each different inertia weight setting.

# Illustrate The Results!

Plot 45 figures with one for each test function and each inertia weight setting

- ✓x-axis: current generation number.
- ✓y-axis: average fitness value of the best individual of current population over 30 runs.