现代信号处理: Homework 4

Due on Dec.14, 2025

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学号 这里写学号

要求:latex
DDL:2025/12/14 下午 23: 59 分前提交 pdf 电子版
电子版以"homework4-姓名-学号" 形式发送到 12432643@mail.sustech.edu.cn 邮箱

Problem 1

Consider the continuous-time signal $x(t)$ defined as

$$x(t) = \frac{\sin 2\pi t}{\pi t}$$

Prove that $x(t)$ is square integrable, but is not absolutely integrable.

Problem 2

We wish to estimate the amplitudes of exponential signals in noise. The observed data is given by

$$x[n] = \sum_{i=1}^{p} A_i r_i^n + w[n], \quad n = 0, 1, \ldots, N-1$$

where $w[n]$ is white Gaussian noise (WGN) with variance σ^2. Find the MVU estimator of the amplitudes and their covariance. Evaluate your results for the specific case where $p = 2$, $r_1 = 1$, $r_2 = -1$, and N is even.

Problem 3

Consider the observation matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1+\epsilon \end{bmatrix}$$

where ϵ is small. Compute $(\mathbf{H}^T\mathbf{H})^{-1}$, and examine what happens as $\epsilon \to 0$. If the observation vector is $\mathbf{x} = [2 \;\; 2 \;\; 2]^T$, find the MVU estimator. Describe what happens as $\epsilon \to 0$.

Problem 4

In practice, we sometimes encounter the linear model $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $\mathbf{H}$ is composed of random variables. Suppose we ignore this difference and use the usual estimator

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}$$

where we assume the specific realization of $\mathbf{H}$ is known. Prove that if $\mathbf{H}$ and $\mathbf{w}$ are independent, then the mean and covariance of $\hat{\boldsymbol{\theta}}$ are

$$\begin{aligned} E(\hat{\boldsymbol{\theta}}) &= \boldsymbol{\theta} \\ \mathbf{C}_{\hat{\theta}} &= \sigma^2 E_H[(\mathbf{H}^T\mathbf{H})^{-1}] \end{aligned}$$

Problem 5

Suppose we observe a fading signal in noise. We view this fading signal as being derived from another "on" or "off" signal. For example, consider a DC level in WGN, i.e., $x[n] = A + w[n]$, for $n = 0, 1, \ldots, N-1$. When the signal fades, the data model becomes

$$x[n] = \begin{cases} A + w[n] & n = 0, 1, \ldots, M-1 \\ w[n] & n = M, M+1, \ldots, N-1 \end{cases}$$

where the probability of fading is ϵ. Assume we know when the signal has undergone fading. Use the results of Problem 4 to determine the estimator of A and its variance, and compare this result with the case of no fading.

Problem 6

The IID observations $x[n]$ for $n = 0, 1, \ldots, N-1$ have the exponential PDF

$$p(x[n]; \lambda) = \begin{cases} \lambda \exp(-\lambda x[n]) & x[n] > 0 \\ 0 & x[n] < 0 \end{cases}$$

Find a sufficient statistic for λ.

Problem 7

Assume $x[n]$ is the result of a Bernoulli trial (coin toss), with

$$\Pr\{x[n] = 1\} = \theta$$
$$\Pr\{x[n] = 0\} = 1 - \theta$$

and N IID observations are made. Assuming the Neyman-Fisher Factorization Theorem holds for discrete random variables, find a sufficient statistic for θ. Then, assuming completeness, find the MVU estimator of θ.

Problem 8

Consider a sinusoidal signal of known frequency in WGN, i.e.,

$$x[n] = A\cos 2\pi f_0 n + w[n] \quad n = 0, 1, \ldots, N-1$$

where $w[n]$ is WGN with variance σ^2. Find the MVU estimators for the following parameters:

1. Amplitude A, assuming σ^2 is known;
2. Amplitude A and noise variance σ^2.

You may assume the sufficient statistic is complete.