# Lab 10 - Implementing and Understanding Non-dominated Sorting Genetic Algorithm (NSGA II)

CSE, SUSTech

# Outline of This Lab

- Teaching Assistant
- What is Multiobjective Optimization (MOO) Problem?
- What is Non-dominated Solution?
- Non-dominated Sorting Genetic Algorithm (NSGA II)
- Test Problems
- Experimental Setup
- Illustrate the Results!

# Teaching assistant

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# What is Multiobjective Optimization (MOO) Problem?

- More than one objective to be optimised,
- with or without constrains.

$$\min/\max F(x) = (f_1(x), f_2(x), \ldots, f_m(x))$$
$$\text{s.t. } g_j(x) \geq 0,\ j = 1, 2, \ldots, J$$
$$h_k(x) = 0,\ k = 1, 2, \ldots, K$$
$$x^{(L)}_i \leq x_i \leq x^{(U)}_i,\ i = 1, 2, \ldots, I$$

where

- x is a vector of continuous, discrete or mixed variables.
- “s.t.” stands for “subject to”.
- m is the number of objectives.
- $x^{(L)}_i$ and $x^{(U)}_i$ refer to the lower bound and upper bound of $x_i$, respectively.

# What is Non-dominated Solution?
## *-Pareto (帕雷托) Dominance*

- $x_a$ dominates $x_b$ if
  - ✓Solution $x_a$ is no worse than $x_b$ in all objectives.
  - ✓Solution $x_a$ is strictly better than $x_b$ in at least one objective.
  - ✓Denoted as $x_a \preceq x_b$ if minimisation.
- $x_a$ dominates $x_b$ $\Leftrightarrow$ $x_b$ is dominated by $x_a$

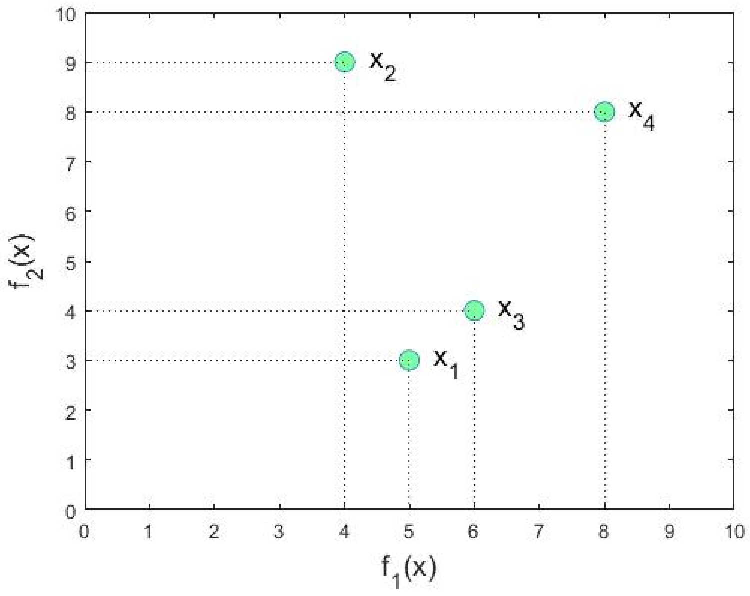


Figure 1: Example: minimise $F(x) = (f_1(x), f_2(x))$.

# What is Non-dominated Solution?
## *-Pareto Front*

- Among a set of solutions P, the non-dominated solution set is a set of solutions that are not dominated by any member of P.
- The non-dominated set of the entire feasible decision space is called the Pareto-optimal set.
- The boundary defined by the set of all points mapped from the Pareto optimal set to objective space is called the Pareto optimal front.

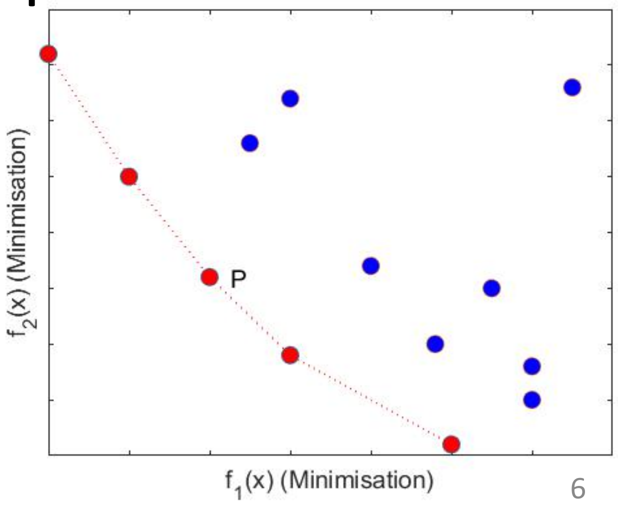



Figure 2: Pareto optimal: red points. Pareto optimal front: dashed red curve.

# What is Non-dominated Solution?
# *-Pareto Optimal Solutions*

- Pareto optimal set in the decision space (决策空间).
- Pareto optimal front in the objective space (目标空间).

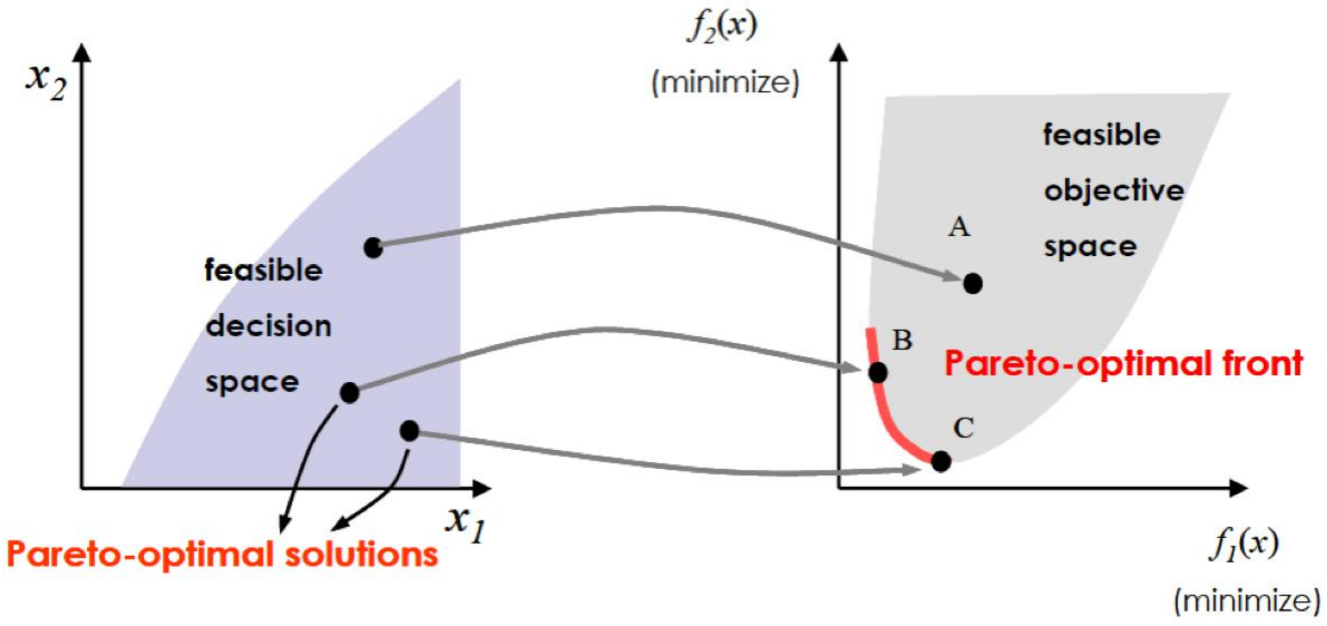
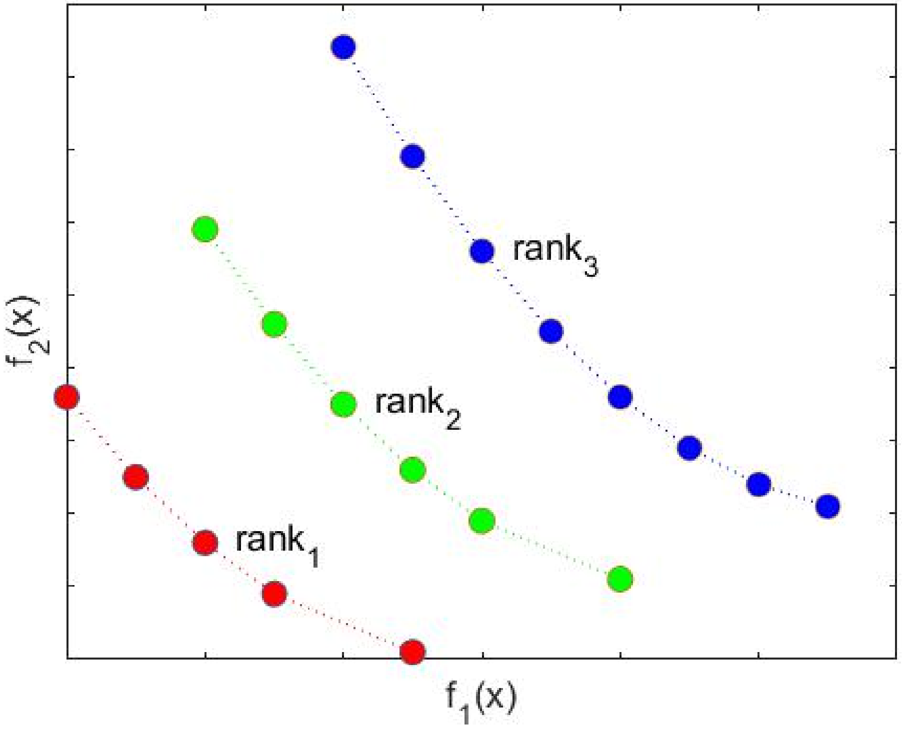



Figure 3: Image source: “Multi-Objective Optimization” by K. Deb.

# Non-dominated Sorting Genetic Algorithm (NSGA II)
# -*Non-dominated Sorting*

Classify the solutions into a number of mutually exclusive non-dominated sets.

$$F = \bigcup_{i=1}^{3} rank_i$$




[2] Kalyanmoy Deb et al. “A fast and elitist multiobjective genetic algorithm: NSGA-II”. In: IEEE transactions on evolutionary computation 6.2 (2002), pp. 182–197

# Non-dominated Sorting Genetic Algorithm (NSGA II)
# -*Crowding Distance*

- Determine crowding distance.
- Denotes half of the perimeter of the enclosing cuboid with the nearest neighbouring solutions in the same front.
- Estimation of the largest cuboid enclosing a particular solution (density estimation).

normalized *m*th side distance:

$$\mathrm{d}_i^m = \frac{d_{(i+1)(i-1)}^m}{f_{\max}^m - f_{\min}^m}$$

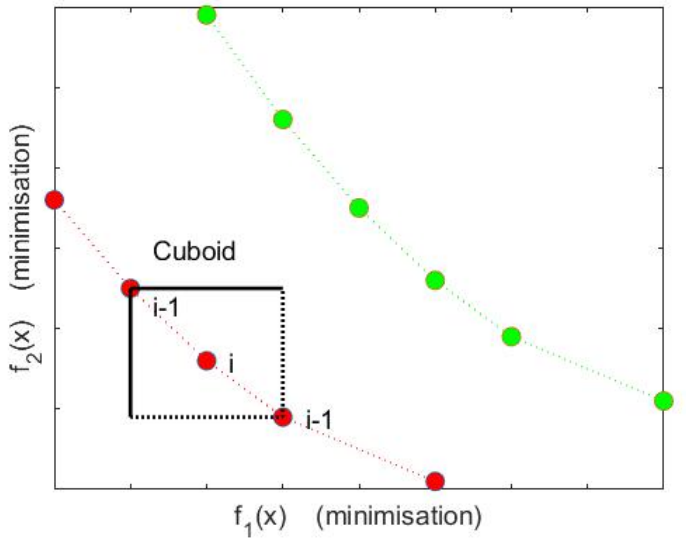



Figure 6: The crowding distance of the ith solution in its front (red) is the average side-length of the cuboid (box) (or half of the perimeter of the enclosing cuboid with the nearest neighbouring solutions in the same front).

# Non-dominated Sorting Genetic Algorithm (NSGA II)
## -*Comparing Solutions*

- Crowding tournament selection
  - ✓Assume that every solution has a non-domination rank and a local crowding distance.
  - ✓A solution $x_a$ wins a tournament against another solution $x_b$:
    - ➢ If the solution $x_a$ has a better rank.
    - ➢ If they have the same rank but solution $x_a$ has a larger crowing distance than solution $x_b$.

# Non-dominated Sorting Genetic Algorithm (NSGA II)

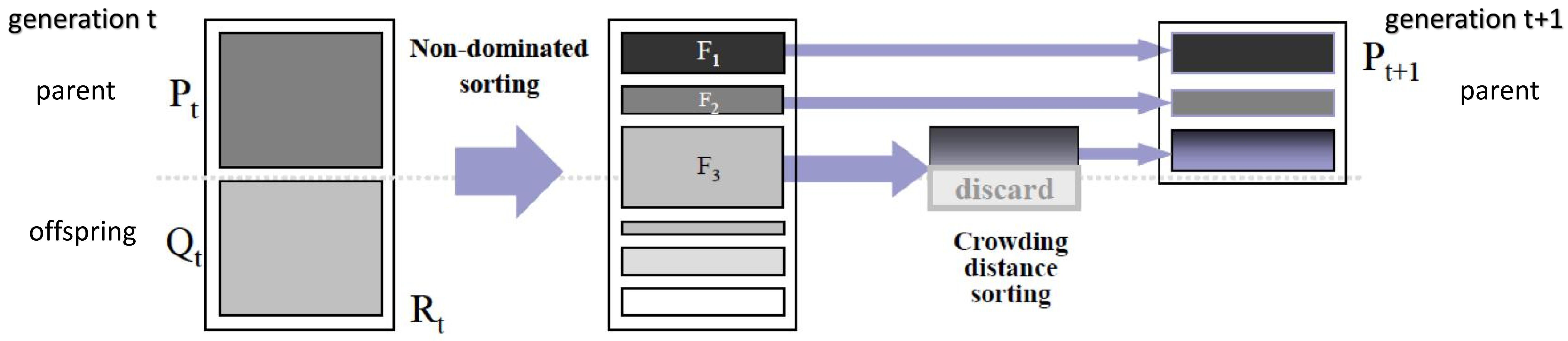



Figure 5: Image source: “Multi-Objective Optimization” by K. Deb.

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# Elitist Non-Dominated Sorting GA (NSGA II)
## *-main loop*

- Step 1
  - ✓Create offspring population $Q_t$ from $P_t$ by using the crowded tournament selection, crossover and mutation operators
- Step 2
  - ✓Combine parent $P_t$ and offspring $Q_t$ populations $R_t = P_t \cup Q_t$
  - ✓Perform a non-dominated sorting to $R_t$ and find different fronts $F_i$
- Step 3
  - ✓Set new population $P_{t+1} = \varnothing$ and set i = 1
  - ✓Until $|P_{t+1}| + |F_i| < N$, perform $P_{t+1} = P_{t+1} \cup F_i$ and increase i
- Step 4
  - ✓Include the most widely spread solutions ($N-|P_{t+1}|$) of $F_i$ in $P_{t+1}$ using the crowding distance values

# Test Problems

| Problem | $n$ | Variable bounds | Objective functions | Optimal solutions | Comments |
|---|---|---|---|---|---|
| ZDT1 | 30 | $[0,1]$ | $f_1(\mathbf{x}) = x_1$<br>$f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})}\right]$<br>$g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$ | $x_1 \in [0,1]$<br>$x_i = 0,$<br>$i = 2, \ldots, n$ | convex |
| ZDT2 | 30 | $[0,1]$ | $f_1(\mathbf{x}) = x_1$<br>$f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - (x_1/g(\mathbf{x}))^2\right]$<br>$g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$ | $x_1 \in [0,1]$<br>$x_i = 0,$<br>$i = 2, \ldots, n$ | nonconvex |
| ZDT3 | 30 | $[0,1]$ | $f_1(\mathbf{x}) = x_1$<br>$f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})} - \frac{x_1}{g(\mathbf{x})}\sin(10\pi x_1)\right]$<br>$g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$ | $x_1 \in [0,1]$<br>$x_i = 0,$<br>$i = 2, \ldots, n$ | convex, disconnected |
| ZDT4 | 10 | $x_1 \in [0,1]$<br>$x_i \in [-5,5],$<br>$i = 2, \ldots, n$ | $f_1(\mathbf{x}) = x_1$<br>$f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})}\right]$<br>$g(\mathbf{x}) = 1 + 10(n-1) + \sum_{i=2}^{n}\left[x_i^2 - 10\cos(4\pi x_i)\right]$ | $x_1 \in [0,1]$<br>$x_i = 0,$<br>$i = 2, \ldots, n$ | nonconvex |

# Experimental Setup

Implement the Non-dominated Sorting Genetic Algorithm (NSGA II) and run it on the four test MOO problems with a maximum of 25 000 function evaluations for each run.

# Illustrate The Results!

Plot 4 figures to display the found nondominated solutions for the four test problems, respectively .

- ✓x-axis: $f_1$
- ✓y-axis: $f_2$