现代信号处理

Lecture 04

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基本计算　$x(n)$

1. 移位： $y_1(n) = x(n-k)$

$y_2(n) = x(n+k)$

整个序列移动

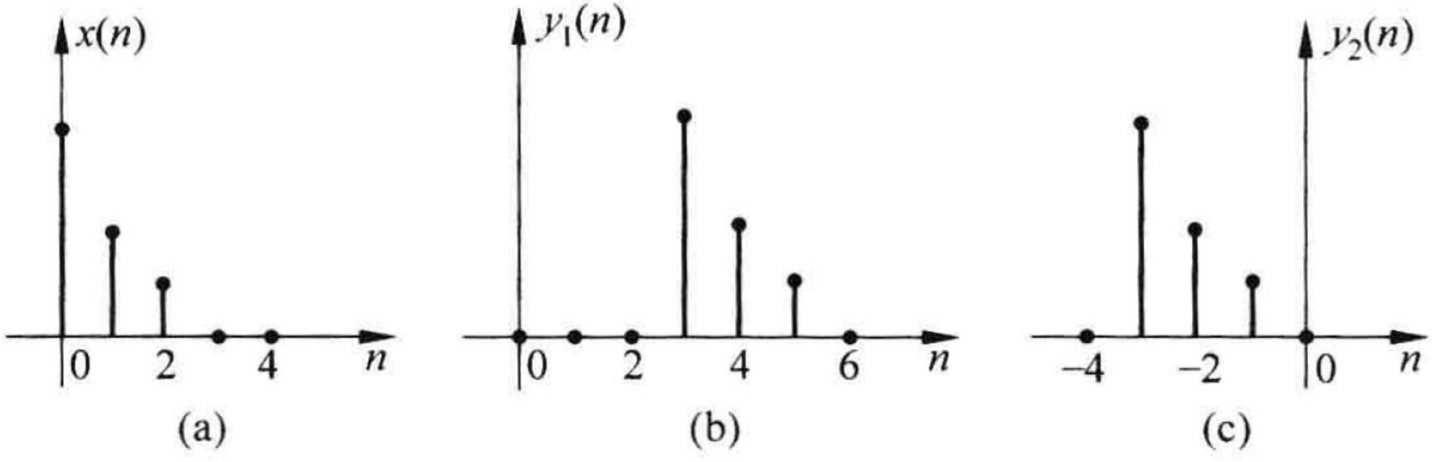


图 1.1.5　序列的移位

$k=3$

n ：当前时刻

$n-k$ ：过去时刻

$n+k$ ：将来时刻

$x(n-1)$

是 $x(n)$ 的单位延迟

以后用z^{-1}表示

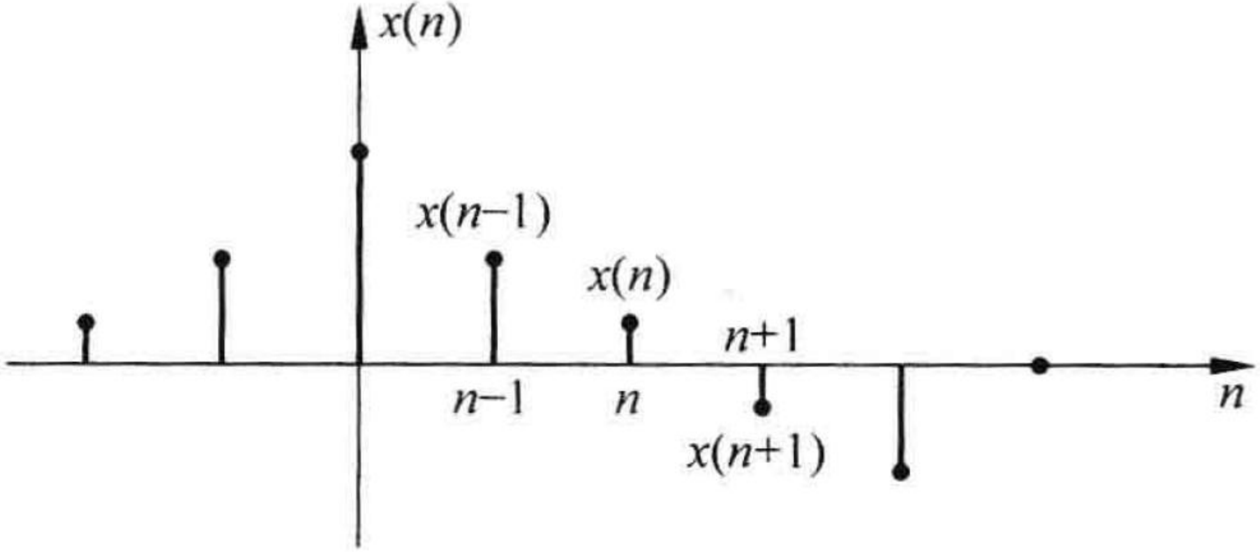


图 1.1.6　$x(n)$中各时间值的含义

$\delta(n)$的“抽取”性质

$x(k)$是$x(n)$在某一时刻k的值：

$$x(k)=x(n)\delta(n-k)$$

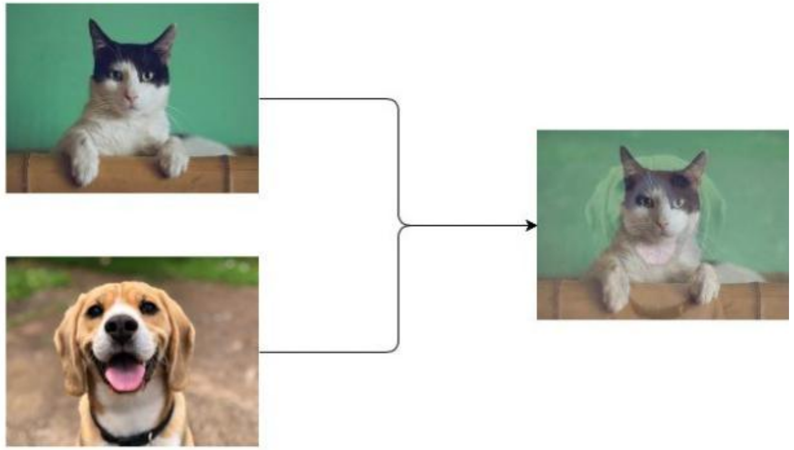
x(n)在的n所有时刻的值为：

$$x(n)=\sum_{k=-\infty}^{\infty}x(k)\delta(n-k)$$

2. 加, 减, 乘: $x_1(n), x_2(n)$

$$y(n) = x_1(n) \pm x_2(n)$$

$$y(n) = x_1(n) \bullet x_2(n)$$

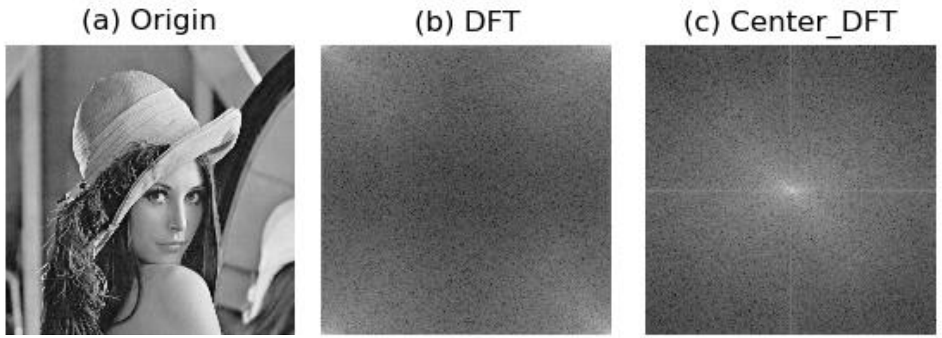


注意：时刻对齐

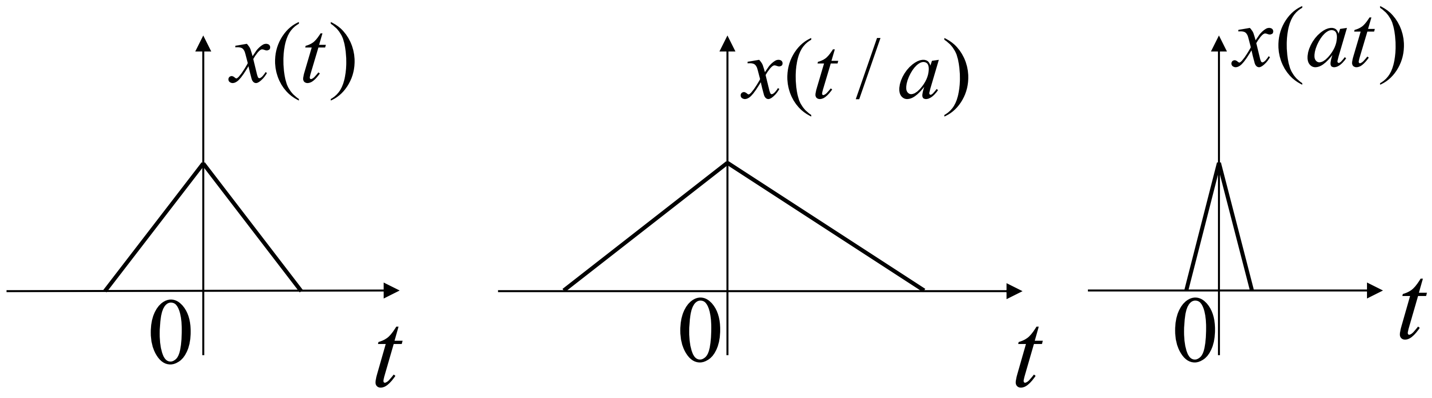
Gaussian Blur

3. 卷积：$y(n) = x_1(n) * x_2(n)$

4. 信号的变换：Z, DFT, DCT



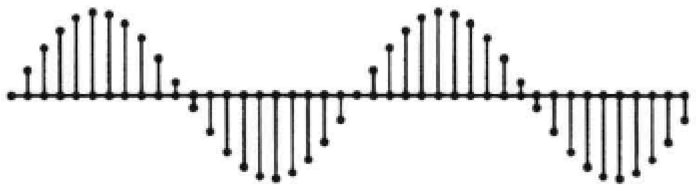
5. 信号时间尺度变化:



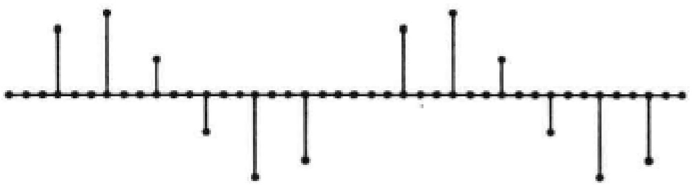
$a>1$

5. 信号时间尺度变化： 离散信号

抽取

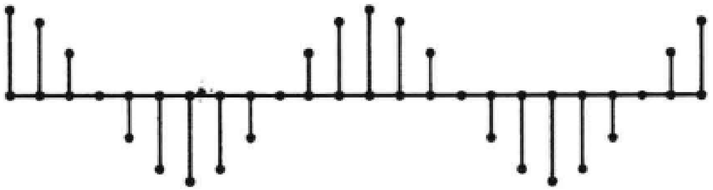


$x(n)$

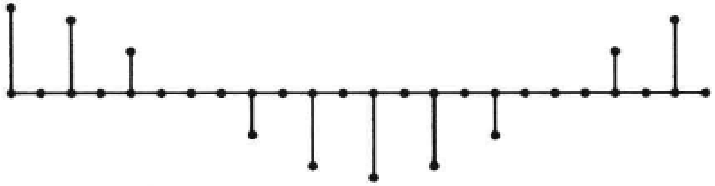


$$x'(n) = x(Mn)$$

插值



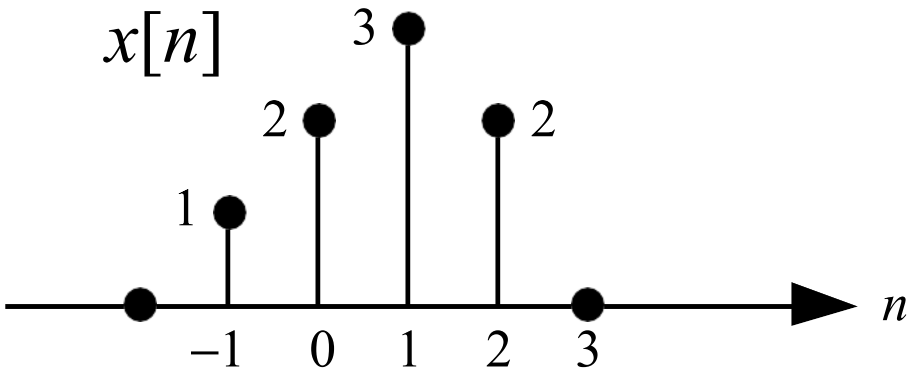
$x(n)$

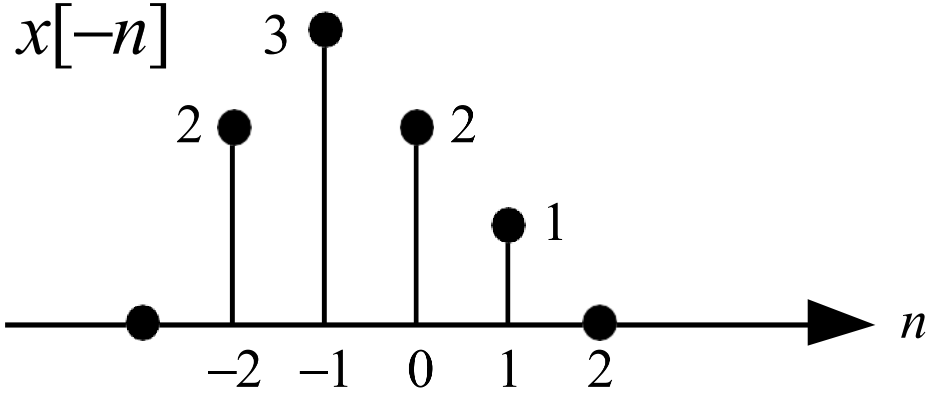


$$x'(n) = \begin{cases} x(n/L) & n = 0, \pm L, \pm 2L, \dots \\ 0 & \text{其他} \end{cases}$$

时间翻转 $x'(n) = x(-n)$

将 $x[n]$ 以纵轴为中心作180°翻转





6. 信号的分解

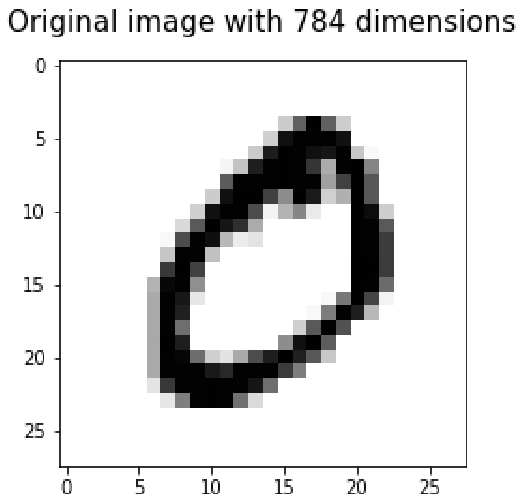
$$x=\sum_{n=1}^{N}\alpha_n\varphi_n$$ 信号的离散表示

$\varphi_1,\varphi_2,\cdots,\varphi_N$ 分解的基向量

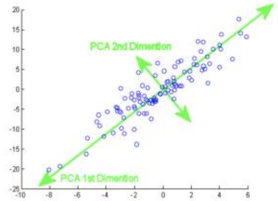
$\alpha_1,\alpha_2,\cdots,\alpha_N$ 分解的系数

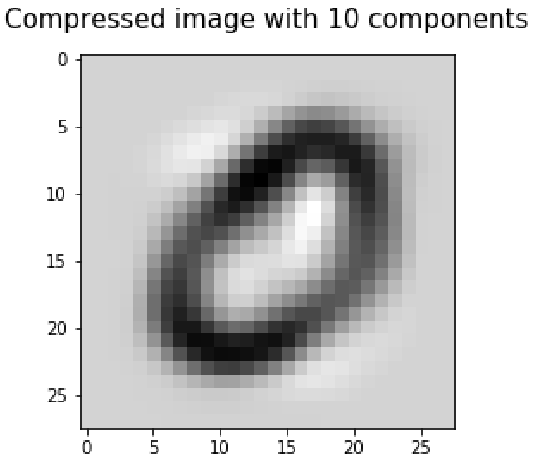
由 $x,\ \varphi_1,\varphi_2,\cdots,\varphi_N$ $\alpha_1,\alpha_2,\cdots,\alpha_N$

信号的分解，或信号的变换



PCA降维





7. 现代信号的处理方式

■模型驱动方式

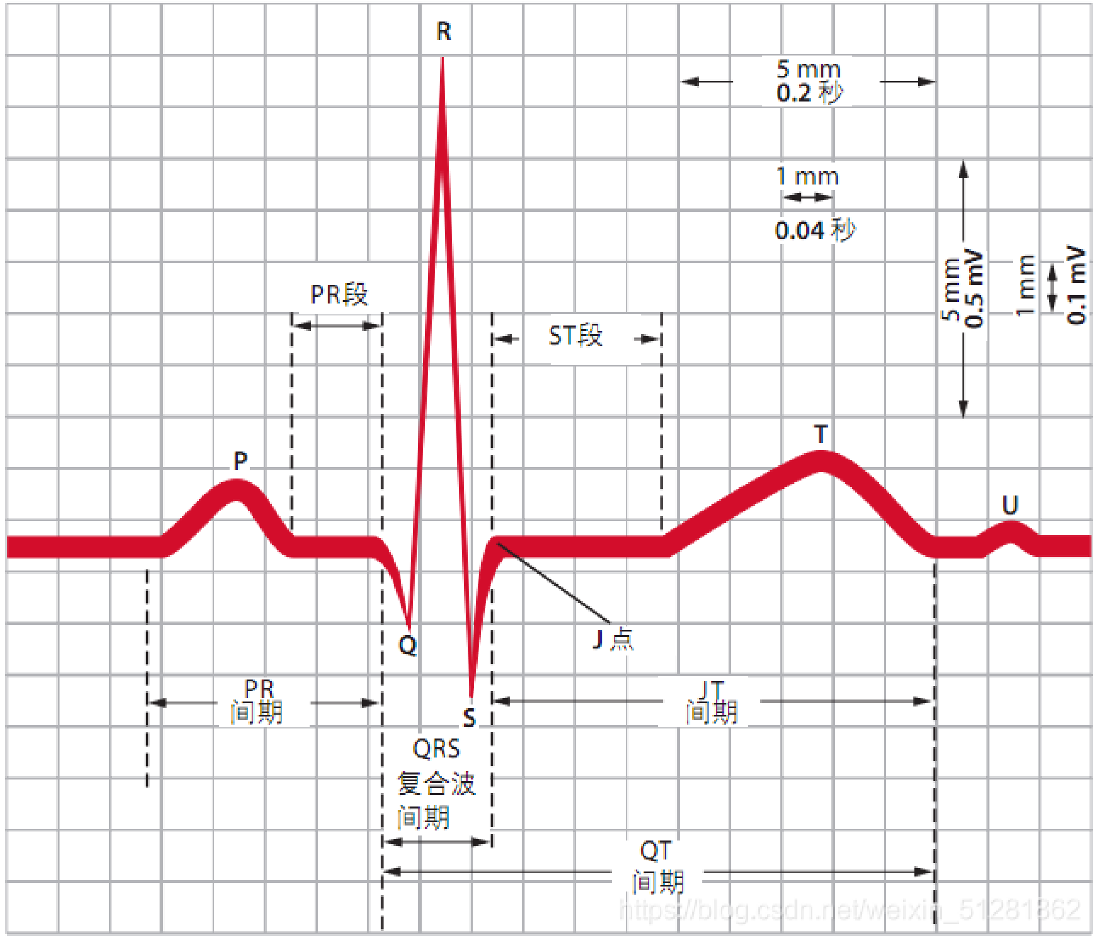
Linear Regression、SVM……

■数据驱动方式

CNN、Transformer……

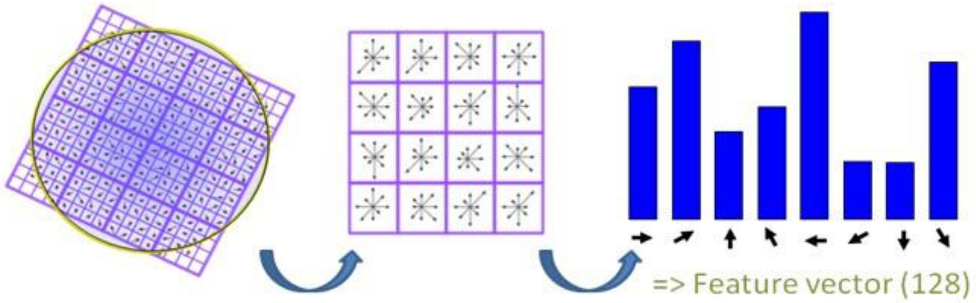
模型驱动的特征提取算法

x：基于特征提取的数据
F：人工设计的显式的函数
θ：需要优化的参数



$$z^{(i)} = F(x^{(i)}; \theta)$$

模型驱动的特征提取算法



x：基于特征提取的数据

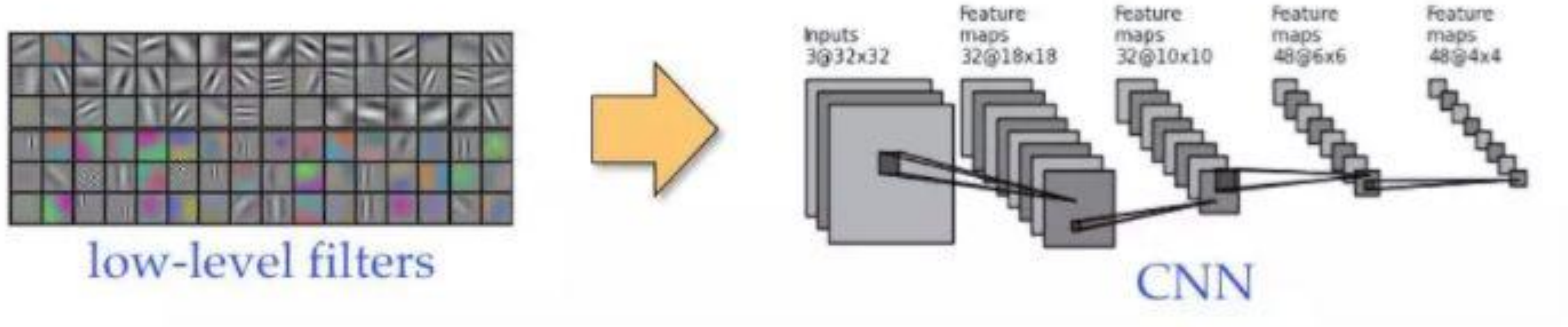
F: 人工设计的显式的函数

θ: 需要优化的参数

$$z^{(i)} = F(x^{(i)}; \theta)$$

数据驱动的特征提取算法

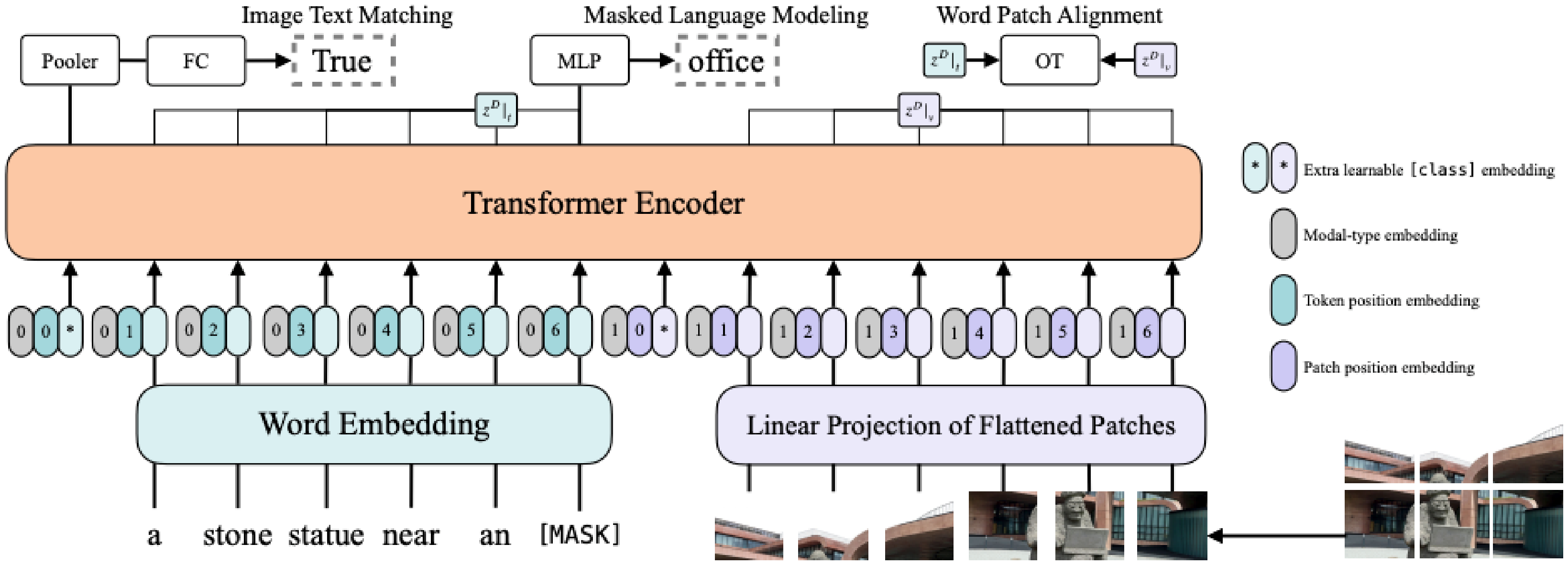
Hand-crafted features have inspired DNN structures capable of learning data-driven representations:

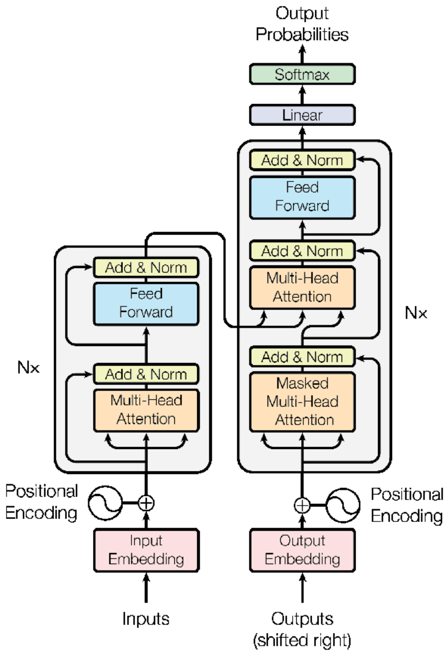


x：不需要提取特征的原始数据
F: 隐式函数，由数据驱动优化
θ: 需要优化的参数

$$z^{(i)} = F(x^{(i)}; \theta)$$

数据驱动的特征提取算法





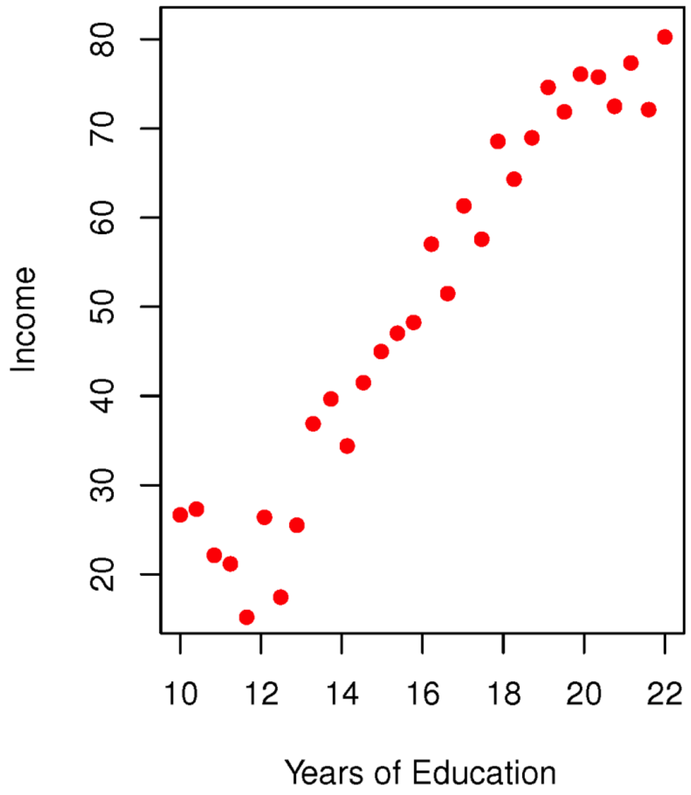
x：不需要提取特征的原始数据

F：隐式函数，由数据驱动优化

θ：需要优化的参数

$$z^{(i)} = F(x^{(i)};\theta)$$

拟合问题



$$z^{(i)} = F(x^{(i)}; \theta)$$

x： Year of education

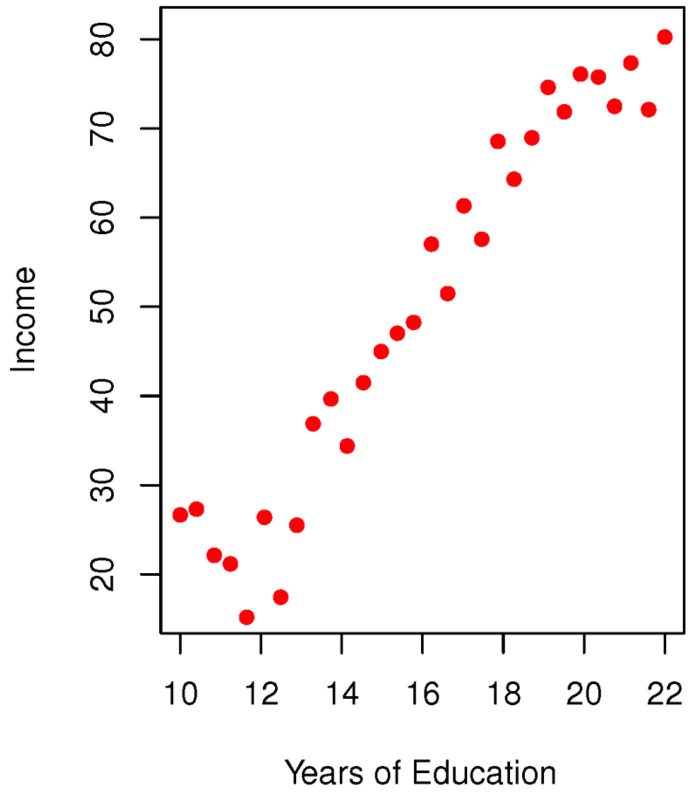
z： Income

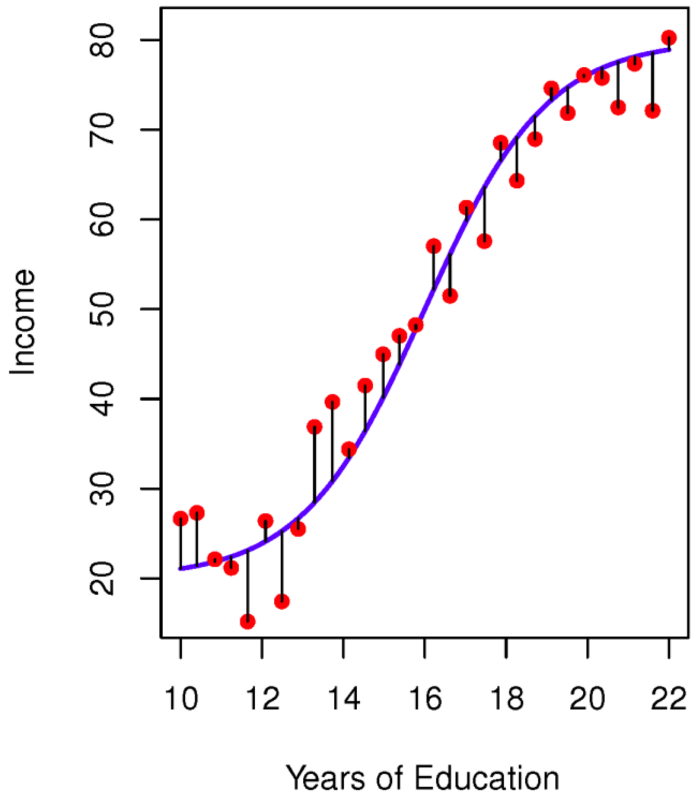
F: 待拟合的函数

θ: 函数中的参数

Can we predict *Income* using *Years of Education*?

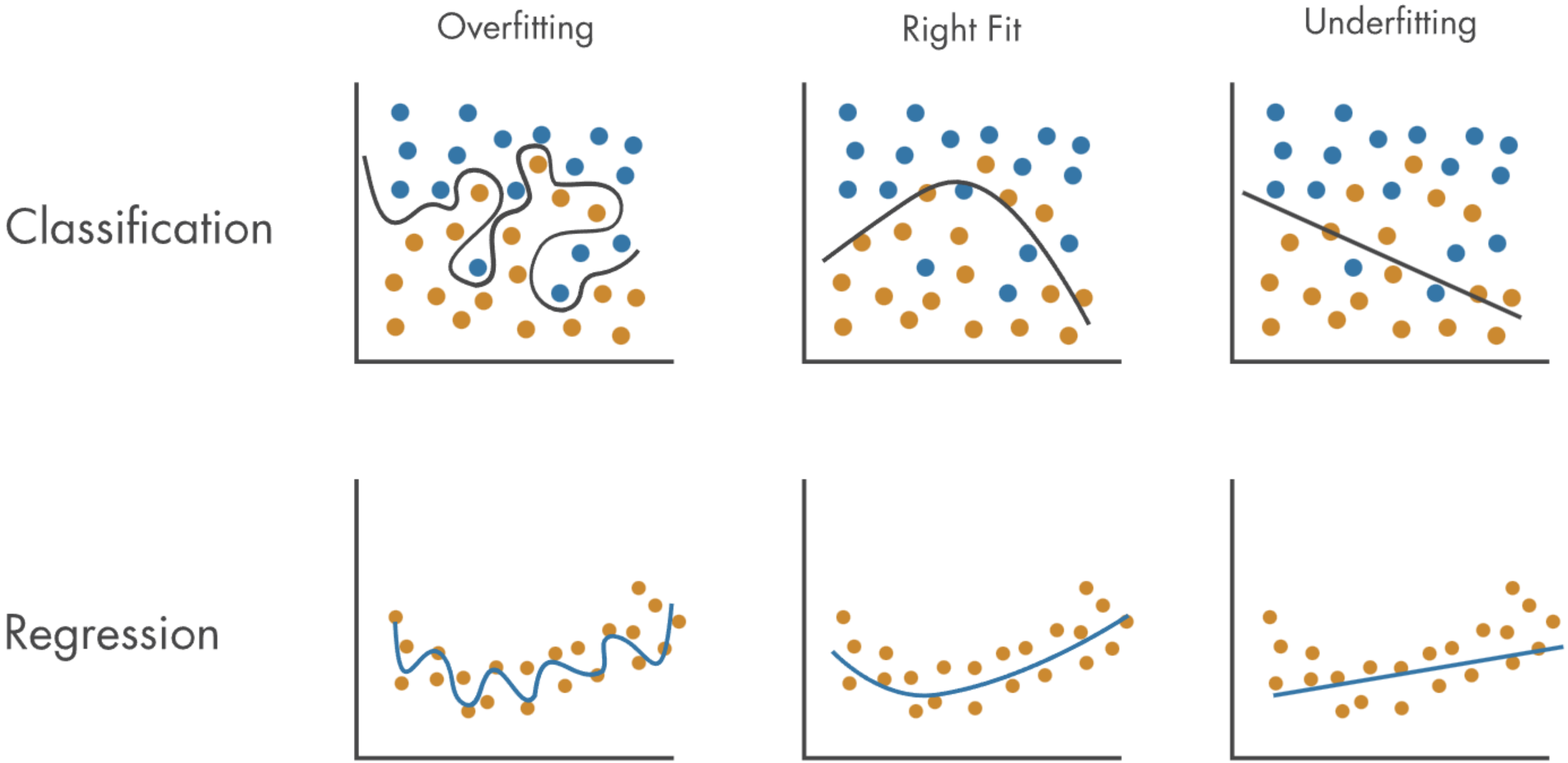
拟合问题



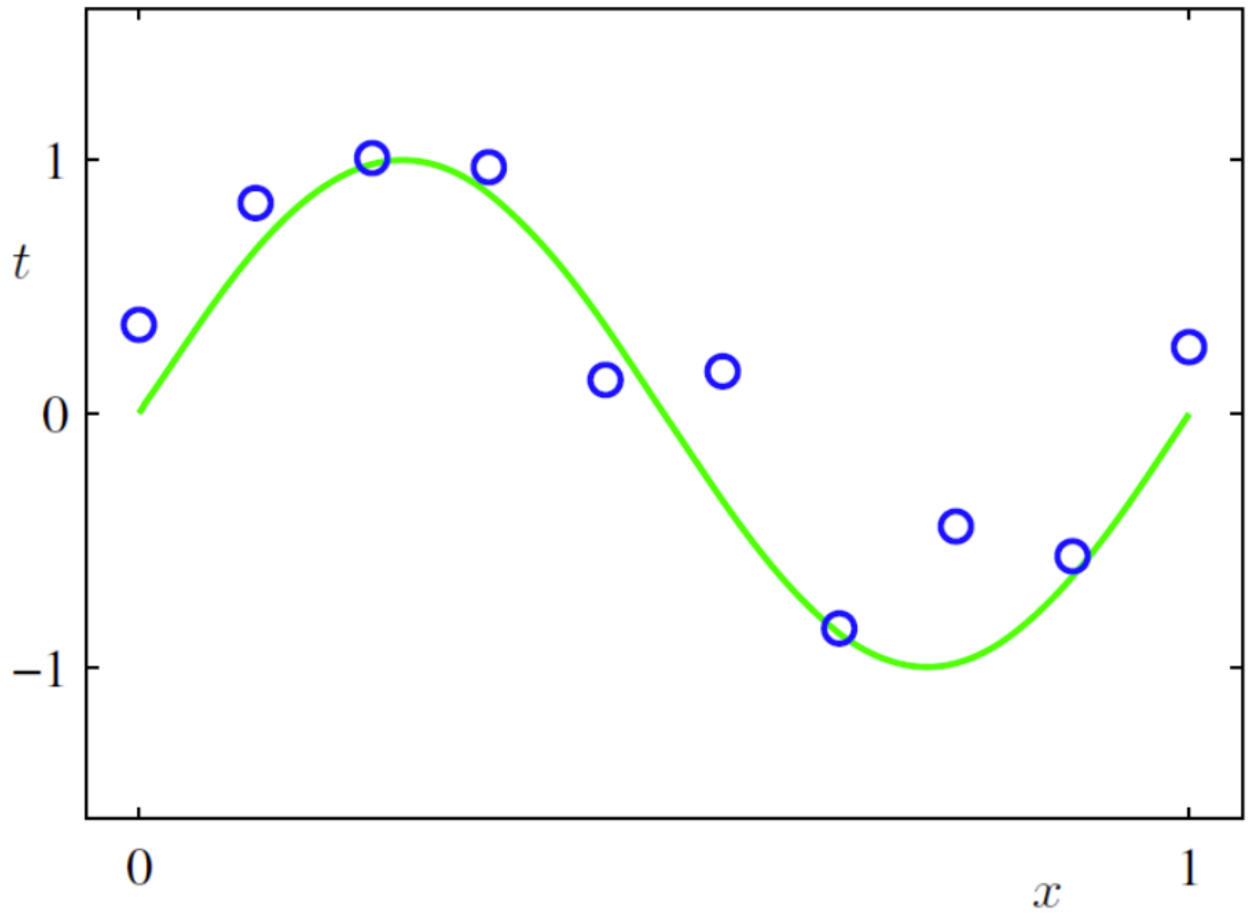


Can we predict *Income* using *Years of Education*?

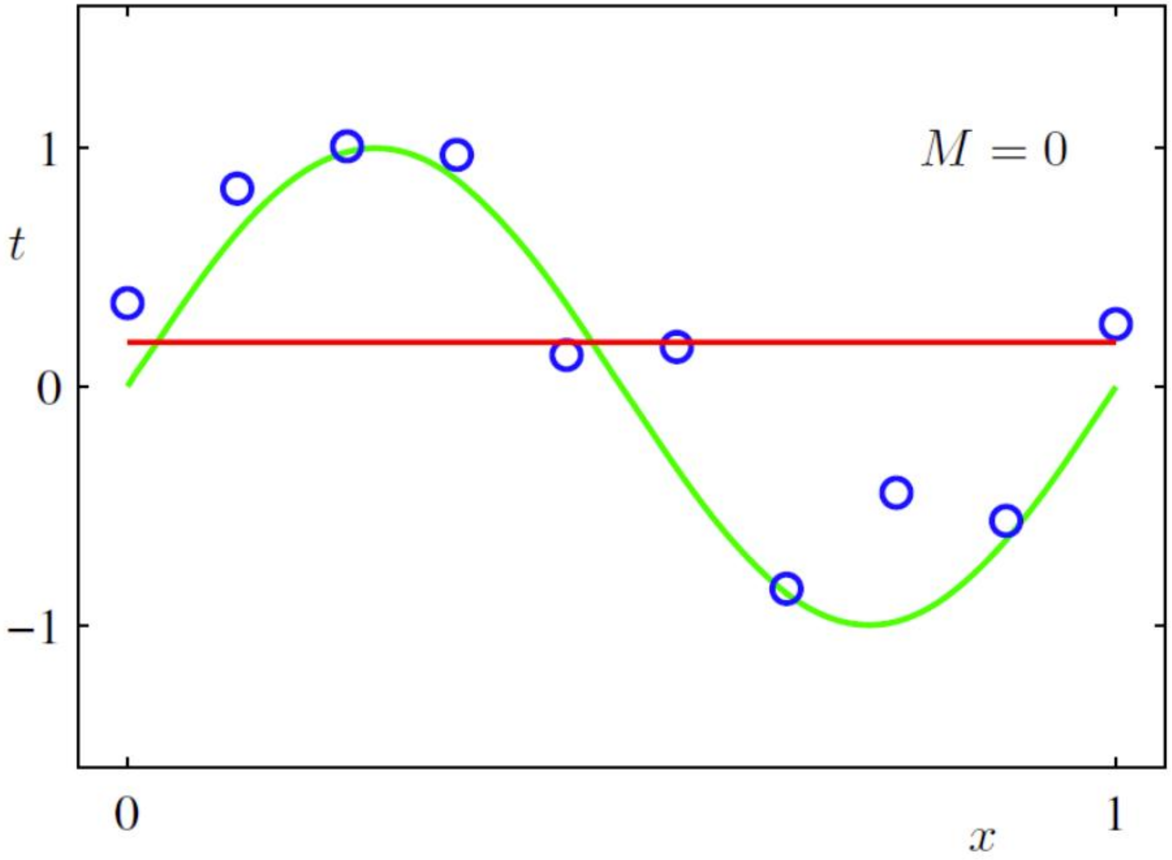
拟合问题


Overfitting
Right Fit
Underfitting
Classification
Regression

过拟合



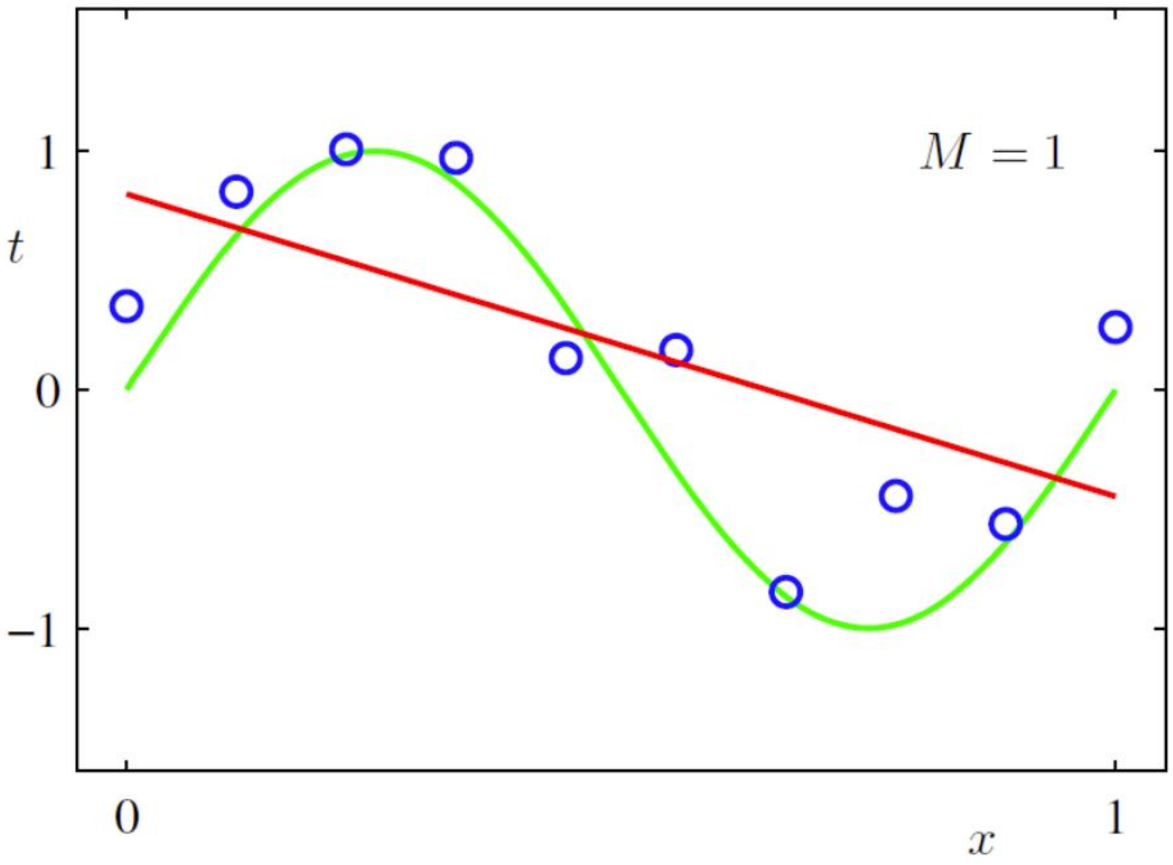
过拟合



Regression using polynomial of degree *M*

$$f(x) = a_0$$

过拟合

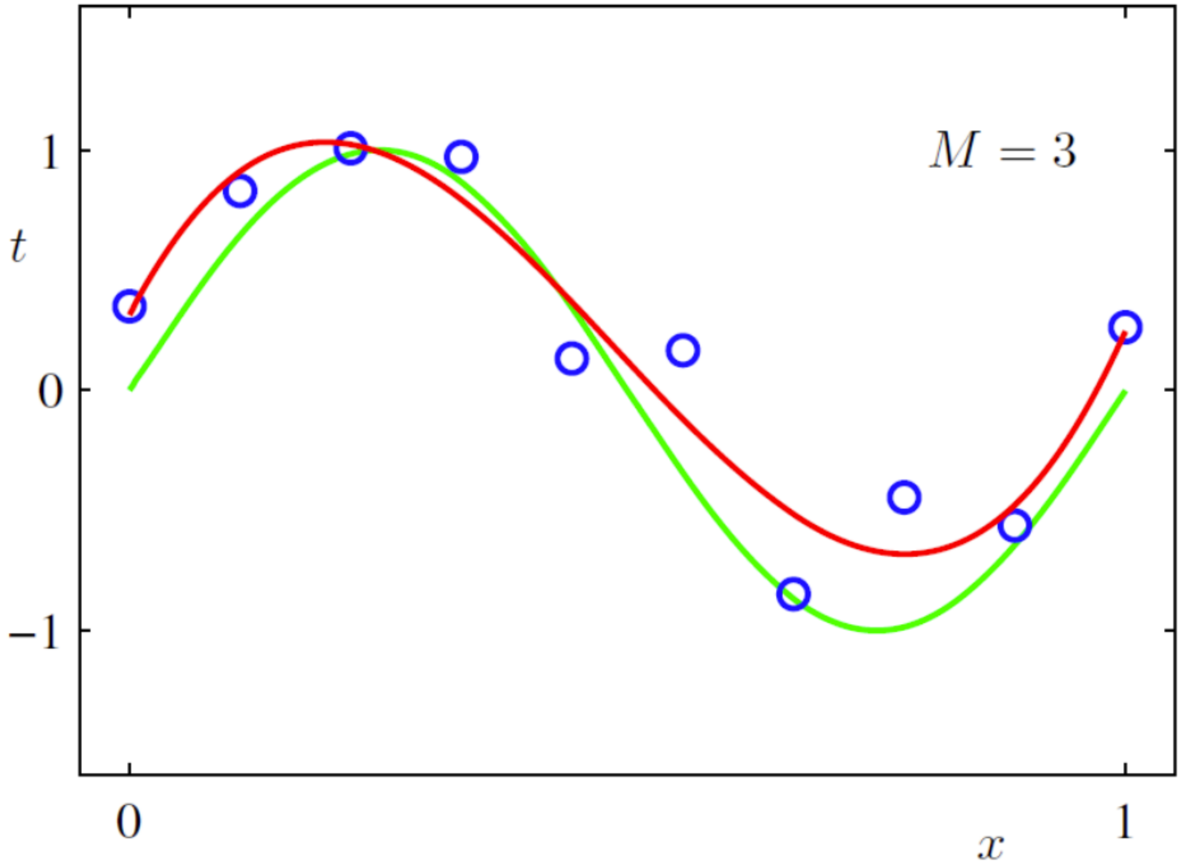


Regression using polynomial of degree *M*

$$f(x) = a_0 + a_1 x$$

过拟合

$t = \sin(2\pi x) + \epsilon$

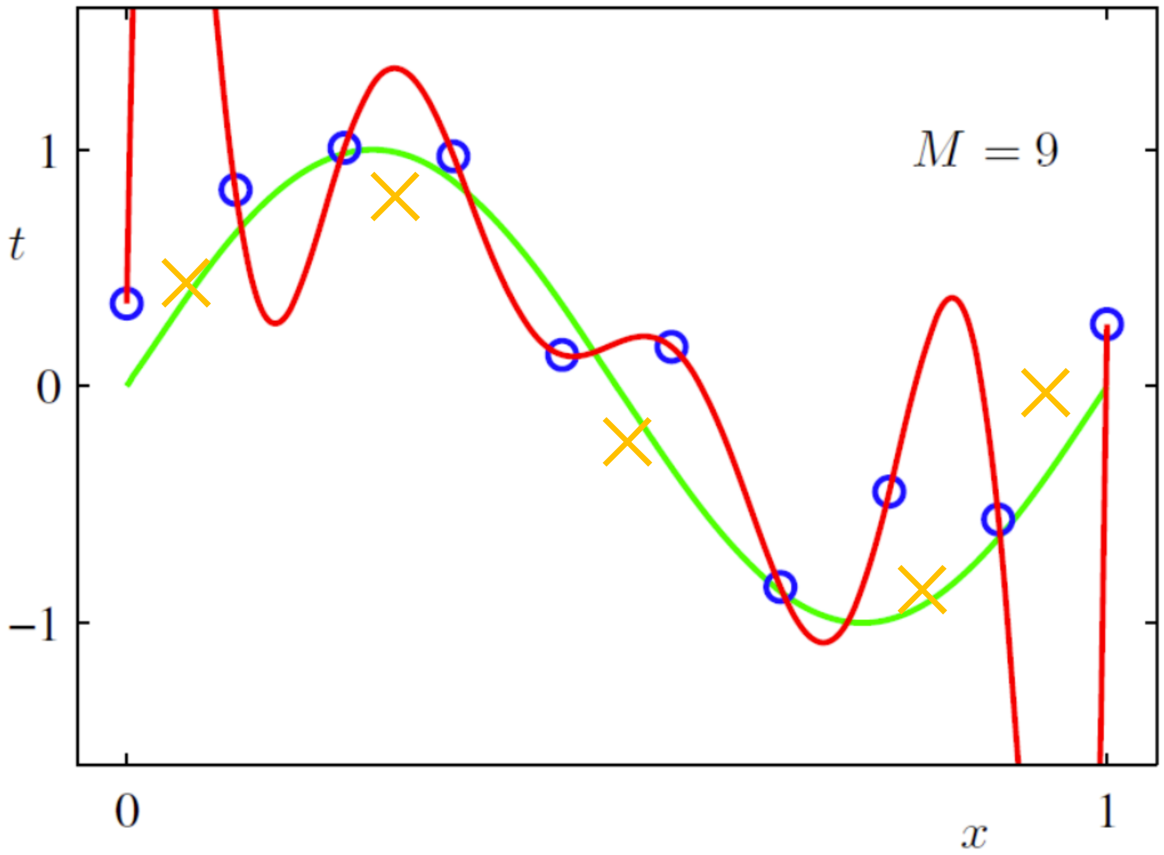


Regression using polynomial of degree *M*

$f(x) = a_0 + a_1 x + a_2 x^2$

过拟合

$$t = \sin(2\pi x) + \epsilon$$



Regression using polynomial of degree *M*

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_M x^M$$

过拟合

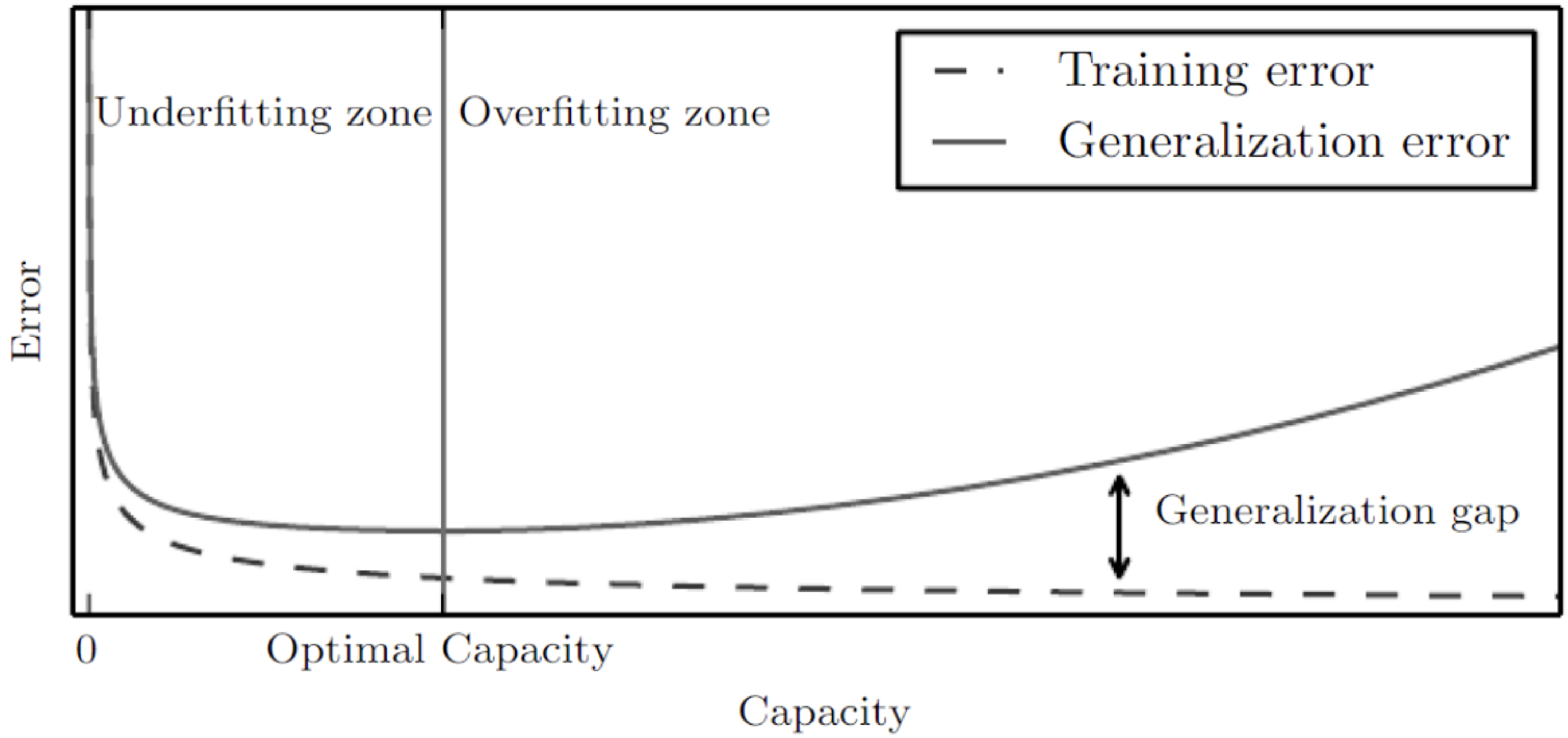


Figure from *Deep Learning*, Goodfellow, Bengio and Courville

Supplementary: Overfitting in statistical/machine learning

10 techniques to avoid overfitting

1. Train with more data

With the increase in the training data, the crucial features to be extracted become prominent. The model can recognize the relationship between the input attributes and the output variable. The only assumption in this method is that the data to be fed into the model should be clean; otherwise, it would worsen the problem of overfitting.