Lab 11 - Apply Penalty Methods to Evolutionary Algorithms to Solve Constrained Optimization Problems

CSE, SUSTech

Outline of This Lab

- Teaching Assistant
- What is Constrained Optimization Problem?
- What are Penalty Methods in Evolutionary Algorithms?
- Test Problems
- Experimental Setup
- Illustrate the Results!

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What is Constrained Optimization Problem?

The general problem we considered here can be described as:

$$\min_{x}\{f(x)\}$$

subject to

$$g_i(x) \le 0,\ i = 1, 2, \ldots, m$$

$$h_j(x) = 0,\ j = 1, 2, \ldots, p$$

where

- x is the n-dimensional vector, $x = (x_1, x_2, \ldots, x_n)$;
- f(x) is the objective function;
- $g_i(x)$ is the inequality constraint;
- $h_j(x)$ is the equality constraint.

Denote the whole search space as *S* and the feasible space as *F*, $F \subset S$. It is important to note that the global in *F* might not be the same as that in *S*.

What are Penalty Methods in Evolutionary Algorithms? *-Introduction*

NewObjectiveFunction = OriginalObjectiveFunction + PenaltyCoefficient * DegreeOfConstraintViolation

The general form of the exterior penalty function method:

$$\psi(\mathbf{x}) = f(\mathbf{x}) + \left(\sum_{i=1}^{m} r_i G_i(\mathbf{x}) + \sum_{j=1}^{p} c_j H_j(\mathbf{x}) \right),$$

where

- ✓$\psi(x)$ is the new objective function to be minimised,
- ✓$f(x)$ is the original objective function,
- ✓r_i and c_j are penalty factors (coefficients),
- ✓$G_i(x) = (\max(0, g_i(x)))^{\beta}$ and $H_j(x) = \max(0, |h_j(x)|^{\gamma})$.
- ✓β and γ are usually chosen as 1 or 2.

What are Penalty Methods in Evolutionary Algorithms?
-*Static Penalty Functions*

$$\psi(\mathbf{x}) = f(\mathbf{x}) + \sum_{i=1}^{m} r_i \left(G_i(\mathbf{x})\right)^2$$

where r_i's are pre-defined and fixed .

- ✓Equality constraints can be converted into inequality ones:

  $$h_j(x) \Rightarrow |h_j(x)| - \varepsilon \leq 0$$

  where ε > 0 is a small number.
- ✓Simple and easy to implement.
- ✓Requires rich domain knowledge to set r_i's.
- ✓r_i's can be divided into a number of different levels. When to use which is determined by a set of heuristic rules.

Test Problems I

The first optimization problem G1 consists of minimizing:

$$f(\boldsymbol{x}) = 5x_1 + 5x_2 + 5x_3 + 5x_4 - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i$$

subject to

$$\begin{aligned}
2x_1 + 2x_2 + x_{10} + x_{11} &\le 10 \\
2x_1 + 2x_3 + x_{10} + x_{12} &\le 10 \\
2x_2 + 2x_3 + x_{11} + x_{12} &\le 10 \\
-8x_1 + x_{10} &\le 0 \\
-8x_2 + x_{11} &\le 0 \\
-8x_3 + x_{12} &\le 0 \\
-2x_4 - x_5 + x_{10} &\le 0 \\
-2x_6 - x_7 + x_{11} &\le 0 \\
-2x_8 - x_9 + x_{12} &\le 0
\end{aligned}$$

where

$$\begin{aligned}
&0 \le x_i \le 1, && i = 1,\dots,9; \\
&0 \le x_i \le 100, && i = 10,11,12; \\
&0 \le x_i \le 1, && i = 13
\end{aligned}$$

Test Problems II

The second optimization problem minimizing:

$$
\begin{aligned}
f(\boldsymbol{x}) = {} & x_1^2 + x_2^2 + x_1 x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 \\
& + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 \\
& + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45
\end{aligned}
$$

subject to

$$105 - 4x_1 - 5x_2 + 3x_7 - 9x_9 \geq 0$$

$$-3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2x_3^2 + 7x_4 + 120 \geq 0$$

$$-10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0$$

$$-x_1^2 - 2x(x_2 - 2)^2 + 2x_1 x_2 - 14x_5 + 6x_6 \geq 0$$

$$8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0$$

$$-5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 \geq 0$$

$$3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} \geq 0$$

$$-0.5(x_1 - 8)^2 - 2(x_2 - 4) - 3x_5^2 + x_6 + 30 \geq 0$$

where

$$-10 \leq x_i \leq 10, \qquad i = 1, \cdots, 10$$

Experimental Setup

Implement an evolutionary algorithm with penlty functions to solve the above constrained optimization problems with 30 runs for each.

✓population size: 50

✓maximum genaration number: 200

Illustrate The Results!

- Provide the obtained avergae best, mean, and standard deviation values of the two constrained optimization problems;
- Plot 2 figures to display the average fitness values over generations for the two constrained optimization problems, respectively .
  - ✓ x-axis: generation number
  - ✓ y-axis: avergae best fitness value

Note:

- ✓ The global minimum of the Problem 1 is known to be

  x*= (1,1,1,1,1,1,1,1,1,3,3,3,1) with f(x*) = -15.
- ✓ The global minimum of the Problem 2 is known to be

  x*= (2.171996,2.363683,8.773926,5.095984,0.9906548,1.430574,1.321644,9.828726,8.280092,8.375927)

  with f(x*) = 24.3062091.