现代信号处理: Homework 5

Due on Jan. 30, 2026

助教
学号 12345678

要求:latex
DDL:2026/1/4 下午23:59分前提交pdf电子版
电子版以"homework5-姓名-学号"形式发送到12332186@mail.sustech.edu.cn邮箱

Problem 1

Given observations $x[n]$ for $n = 0, 1, \ldots, N-1$, where the samples are i.i.d. and distributed according to $U[\theta_1, \theta_2]$, find a sufficient statistic for $\boldsymbol{\theta} = [\theta_1, \theta_2]^T$.

Problem 2

For $n = 0, 1, \ldots, N-1$, suppose $x[n] = Ar^n + w[n]$, where A is an unknown parameter, r is an unknown constant, and $w[n]$ is white noise with zero mean and variance σ^2. Find the BLUE of A and its minimum variance. Does the minimum variance tend to zero as $N \to \infty$?

Problem 3

The observed i.i.d. samples $\{x[0], x[1], \ldots, x[N-1]\}$ follow the distributions below:

a. Laplace:

$$p(x[n]; \mu) = \frac{1}{2} \exp\left[-|x[n] - \mu|\right]$$

b. Gaussian:

$$p(x[n]; \mu) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x[n] - \mu)^2\right]$$

Find the BLUE of the mean μ for both cases. Also, explain the MVU estimator of μ.

Problem 4

Assume that the observed signal is $x[n] = As[n] + w[n]$, for $n = 0, 1, \ldots, N-1$, where $w[n]$ is noise with zero mean and covariance matrix $\mathbf{C}$, and $s[n]$ is a known signal. The amplitude A is the parameter to be estimated. Find the BLUE of A. Discuss what happens if the characteristic vector of $\mathbf{C}$ is $\mathbf{s} = [s[0]\ s[1]\ \ldots\ s[N-1]]^T$. Also, find the minimum variance.

Problem 5

Prove the linearity property of the BLUE with respect to linear transformations of $\boldsymbol{\theta}$. Specifically, if we wish to estimate

$$\boldsymbol{\alpha} = \mathbf{B}\boldsymbol{\theta} + \mathbf{b},$$

where $\mathbf{B}$ is a known $p \times p$ invertible matrix and $\mathbf{b}$ is a known $p \times 1$ vector, prove that the BLUE of $\boldsymbol{\alpha}$ is given by

$$\hat{\boldsymbol{\alpha}} = \mathbf{B}\hat{\boldsymbol{\theta}} + \mathbf{b},$$

where $\hat{\boldsymbol{\theta}}$ is the BLUE of $\boldsymbol{\theta}$. Assume the data model $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $E(\mathbf{w}) = \mathbf{0}$ and $E(\mathbf{w}\mathbf{w}^T) = \mathbf{C}$. Hint: Substitute $\boldsymbol{\theta}$ for $\boldsymbol{\alpha}$ in the data model.

Problem 6

For the general linear model

$$\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{s} + \mathbf{w},$$

where $\mathbf{s}$ is a known $N \times 1$ vector, $E(\mathbf{w}) = \mathbf{0}$, and $E(\mathbf{w}\mathbf{w}^T) = \mathbf{C}$, find the BLUE of $\boldsymbol{\theta}$.

Problem 7

We observe N i.i.d. samples from the following PDFs:

a. Gaussian:

$$p(x;\mu) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x-\mu)^2\right]$$

b. Exponential:

$$p(x;\lambda) = \begin{cases} \lambda \exp(-\lambda x) & x > 0 \\ 0 & x < 0 \end{cases}$$

In each case, find the MLE of the unknown parameter and verify that it indeed maximizes the likelihood function. Is the estimator meaningful?

Problem 8

The following is the formal definition of a consistent estimator: If for any given $\epsilon > 0$, it satisfies

$$\lim_{N\to\infty} \Pr\left\{|\hat{\theta} - \theta| > \epsilon\right\} = 0,$$

then the estimator $\hat{\theta}$ is consistent.

Prove that for the problem of estimating a DC level A in white Gaussian noise with known variance, the sample mean is a consistent estimator. Hint: Use Chebyshev's inequality.