现代信号处理: Homework 3

Due on Nov.30, 2025

某某某
学号 这里写学号

要求:latex
DDL:2025/11/30 下午 23: 59 分前提交 pdf 电子版
电子版以"homework3-姓名-学号" 形式发送到 12332151@mail.sustech.edu.cn 邮箱

Problem 1

An unknown parameter θ influences the outcome of an experiment which is modeled by the random variable x. The PDF of x is

$$p(x;\theta) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(x-\theta)^2\right]$$

A series of experiments is performed, and x is found to always be in the interval [97, 103]. As a result, the investigator concludes that θ must have been 100. Is this assertion correct?

Problem 2

It is desired to estimate the value of a DC level A in WGN or

$$x[n] = A + w[n] \quad n = 0, 1, \ldots, N-1$$

where $w[n]$ is zero mean and uncorrelated, and each sample has variance $\sigma^2 = 1$. Consider the two estimators

$$\hat{A} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$$

$$\check{A} = \frac{1}{N+2}\left(2x[0] + \sum_{n=1}^{N-2} x[n] + 2x[N-1]\right).$$

Which one is better? Does it depend on the value of A ?

Problem 3

The data $\{x[0], x[1], ..., x[N-1]\}$ are observed where the $x[n]'s$ are independent and identically distributed (IID) as $N(0, \sigma^2)$. We wish to estimate the variance σ^2 as

$$\hat{\sigma^2} = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Is this an unbiased estimator? Find the variance of $\hat{\sigma^2}$ and examine what happens as $N \to \infty$.

Problem 4

Two samples $\{x[0], x[1]\}$ are independently observed from a $N(0, \sigma^2)$ distribution. The estimator

$$\hat{\sigma^2} = \frac{1}{2}(x^2[0] + x^2[1])$$

is unbiased. Find the PDF of $\hat{\sigma^2}$ to determine if it is symmetric about σ^2.

Problem 5

Independent bivariate Gaussian samples $\{x[0], x[1], ..., x[N-1]\}$ are observed. Each observation is a 2×1 vector which is distributed as $x[n] \sim \mathcal{N}(0, C)$ and

$$C = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

Find the CRLB for the correlation coefficient ρ.

Problem 6

If $x[n] = r^n + w[n]$ for $n = 0, 1, ..., N-1$ are observed, where $w[n]$ is WGN with variance σ^2 and r is to be estimated, find the CRLB. Does an efficient estimator exist and if so find its variance?

Problem 7

Using the results of Example 3.13, determine the best range estimation accuracy of sonar if

$$s(t) = \begin{cases} 1 - 100\,|t - 0.01| & 0 \le t \le 0.02 \\ 0 & otherwise. \end{cases}$$

Let $N_0/2 = 10^{-6}$ and $c = 1500m/s$.

Problem 8

功率信号自相关函数的性质：

1. 若 $x(n)$ 是周期的, 周期是 N, 则

$$r_x(m) = r_x(m+N)$$

2. 若 $x(n)$是实的, 则 $r_x(m) = r_x(-m)$

HW: 证明该4点性质

3. $r_x(0)$取最大值, $r_x(0) = P_x$ 为信号能量

4. 若 $x(n)$是复信号, 则 $r_x(m) = r_x^*(-m)$