现代信号处理

Lecture 14

唐晓颖
电子与电气工程系
南方科技大学

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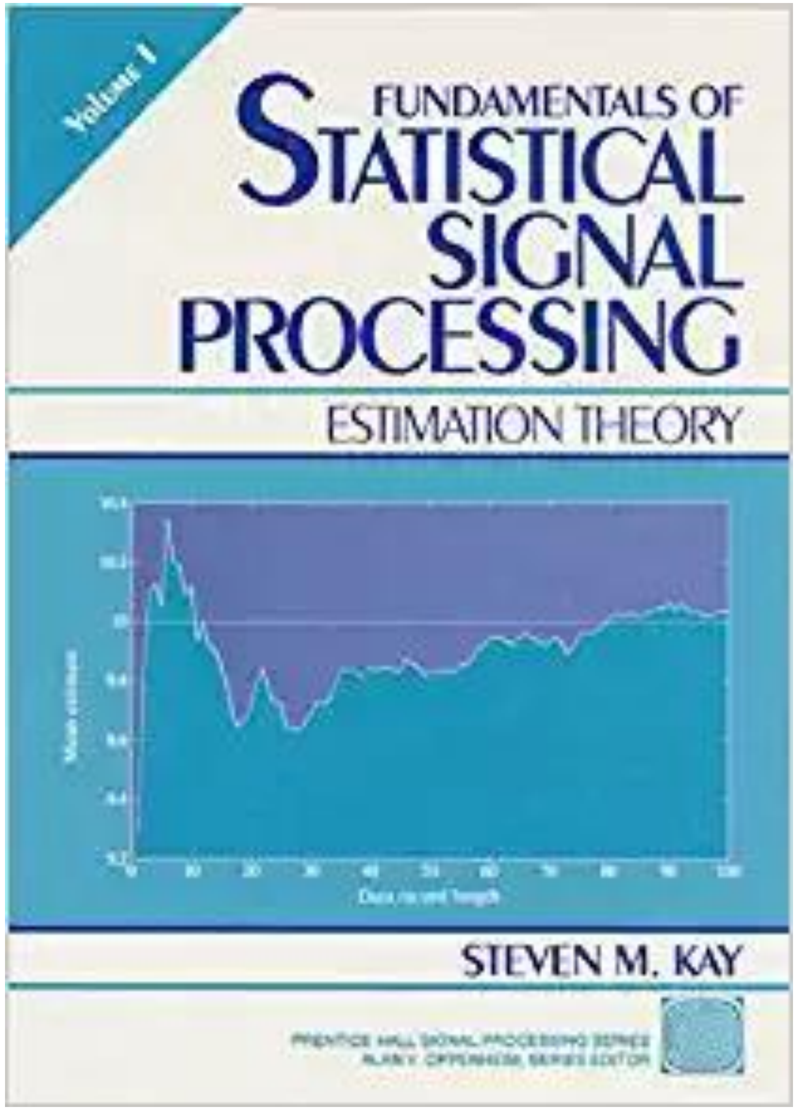
An Introduction to Signal Detection and Estimation

Textbook Information:

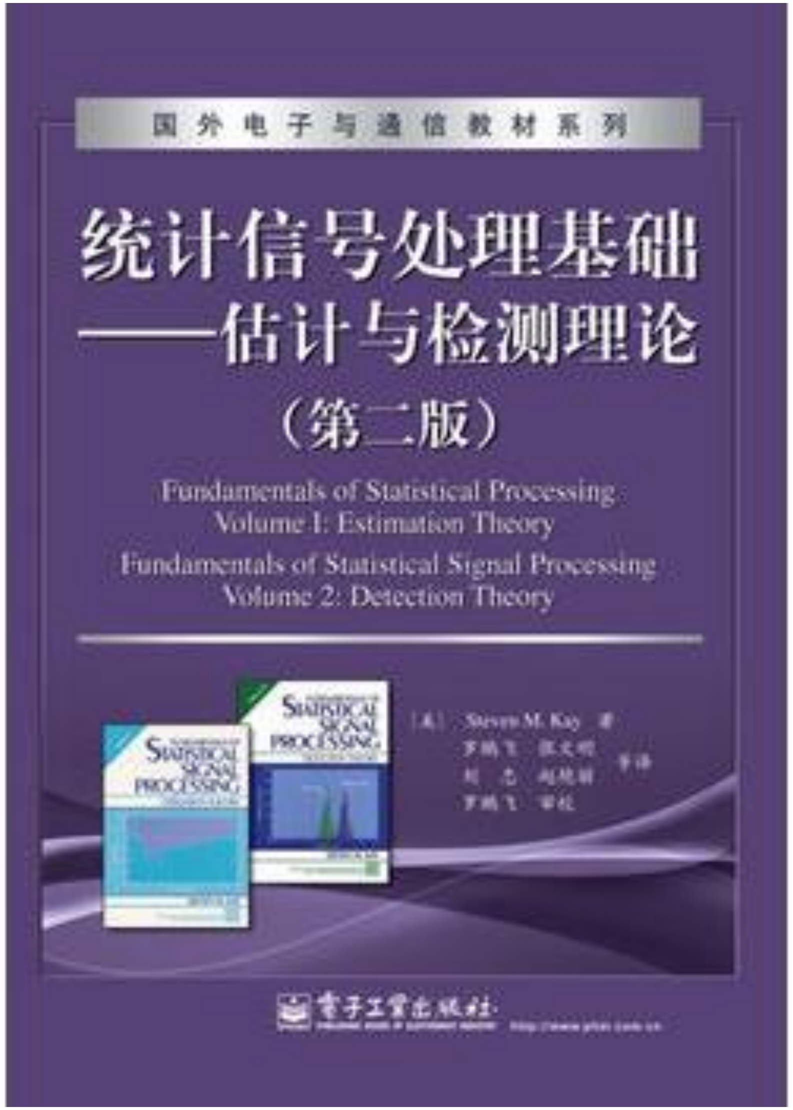
Fundamentals of Statistical Signal Processing : Estimation Theory

by Steven Kay. Volume I, Prentice Hall, 1993



统计信号处理基础:估计与检测理论

by罗鹏飞 张文明 刘忠 等, 电子工业出版社, 2011



Chapter 1 – Introduction

1.1 Estimation in Signal Processing

Modern estimation theory can be found at the heart of many electronic signal processing systems designed to extract information.

1. Radar
2. Sonar
3. Speech
4. Image analysis
5. Biomedicine
6. Communications
7. Control
8. Seismology

Estimate the values of a group of parameters

Example: Radar system

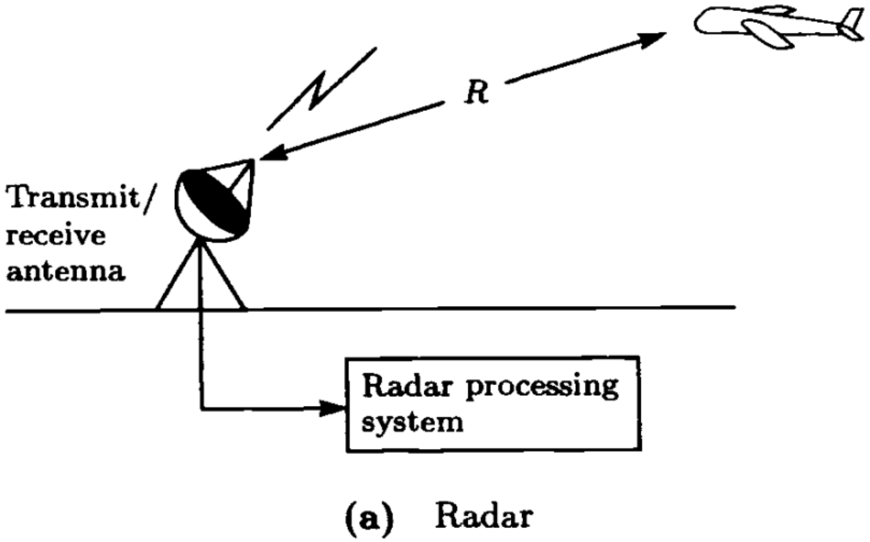
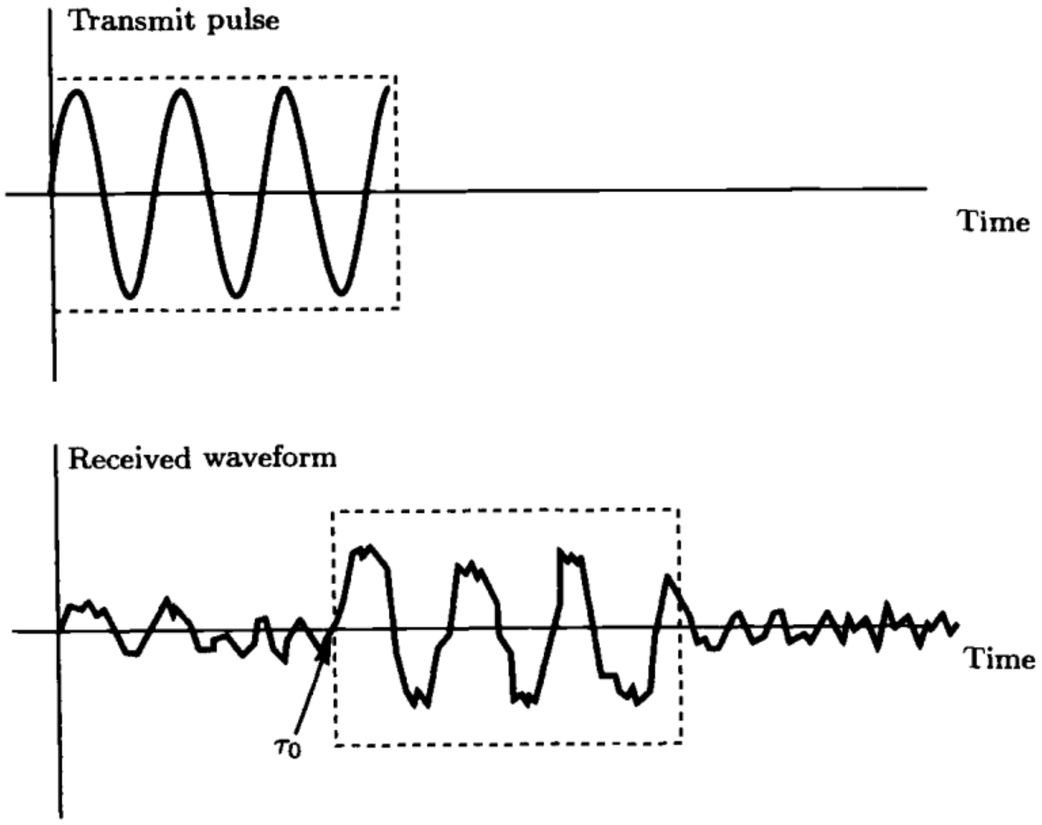


Figure 1.1 Radar system

In radar we are interested in determining the position of an aircraft, as for example, in airport surveillance radar.

To determine the range R we transmit an electromagnetic pulse that is reflected by the aircraft, causing an echo to be received by the antenna τ_0 seconds later. The range is determined by the equation $\tau_0 = 2R / c$, where c is the speed of electromagnetic propagation.

Clearly, if the round trip delay τ_0 can be measured, then so can the range R.



(b) Transmit and received waveforms

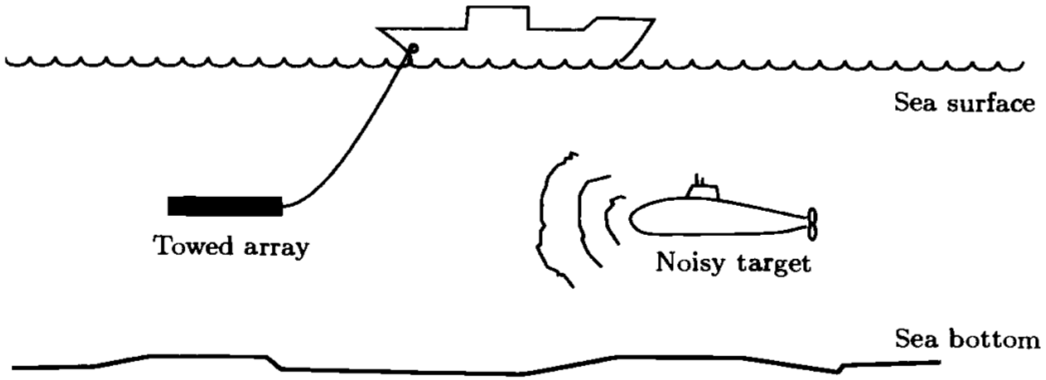
Figure 1.1 Radar system

The received echo is decreased in amplitude due to propagation losses and hence may be obscured by environmental noise. Its onset may also be perturbed by time delays introduced by the electronics of the receiver.

Determination of the round trip delay can therefore require more than just a means of detecting a jump in the power level at the receiver.

The value of R obtained from $R = \tau_0 c / 2$ is then only an estimate.

Example: Passive sonar system

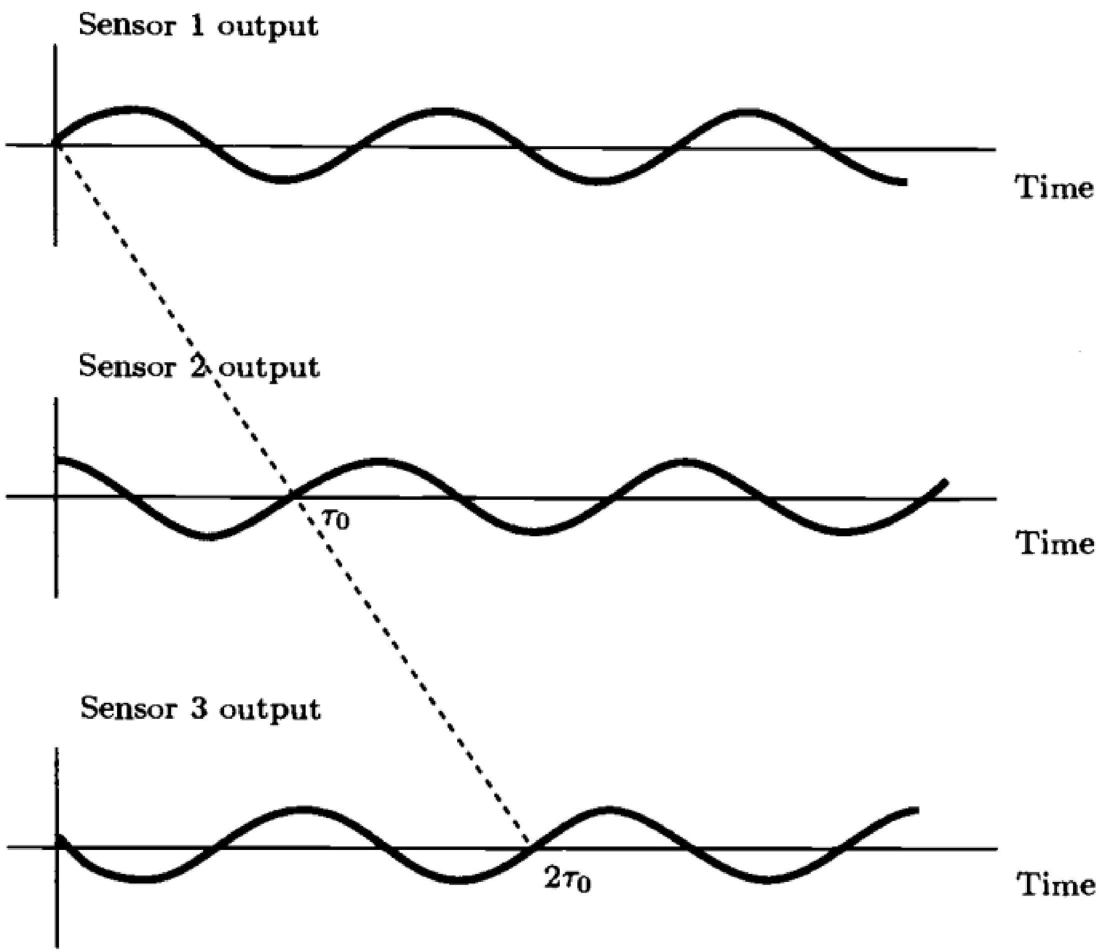


(a) Passive sonar

Figure 1.2 Passive sonar system

We are also interested in the position of a target, such as a submarine.

The target radiates noise due to machinery on board, propellor action, etc. This noise, which is actually the *signal* of interest, propagates through the water and is received by an array of sensors. The sensor outputs are then transmitted to a tow ship for input to a digital computer.



(b) Received signals at array sensors

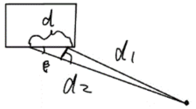
Figure 1.2 Passive sonar system

By measuring τ_0 , the delay between sensors, we can determine the bearing β from the expression

$$\beta = \arccos\left(\frac{c\tau_0}{d}\right)$$

where c is the speed of sound in water and d is the distance between sensors.

Again, however, the received waveforms are not "clean" but are embedded in noise, making the determination of τ_0 more difficult. The value of β obtained is then only an estimate.



$d_2 - d_1 = c\tau_0$

$\cos\beta = \frac{c\tau_0}{d}$

$\beta = \arccos\frac{c\tau_0}{d}$

d足够小

Example: Speech Sounds

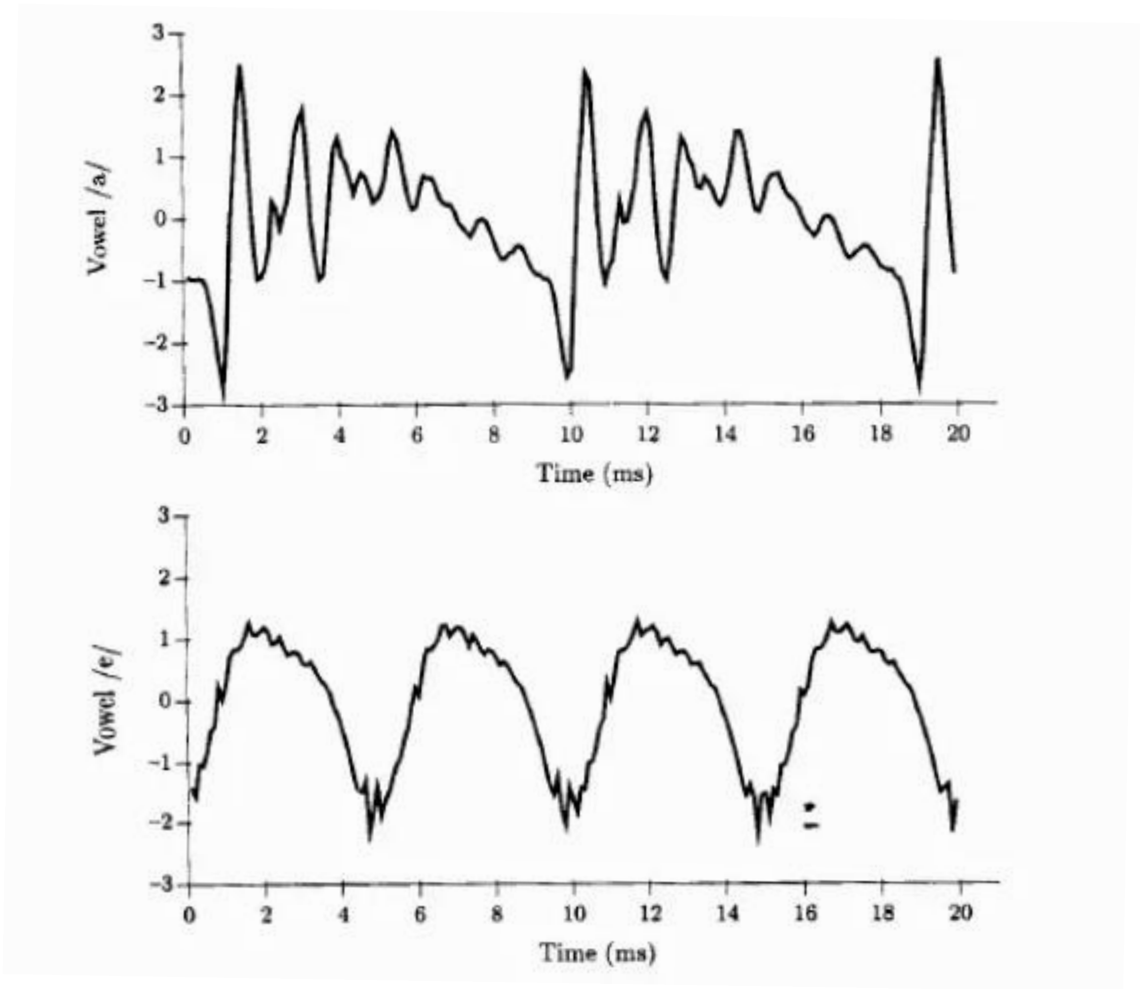
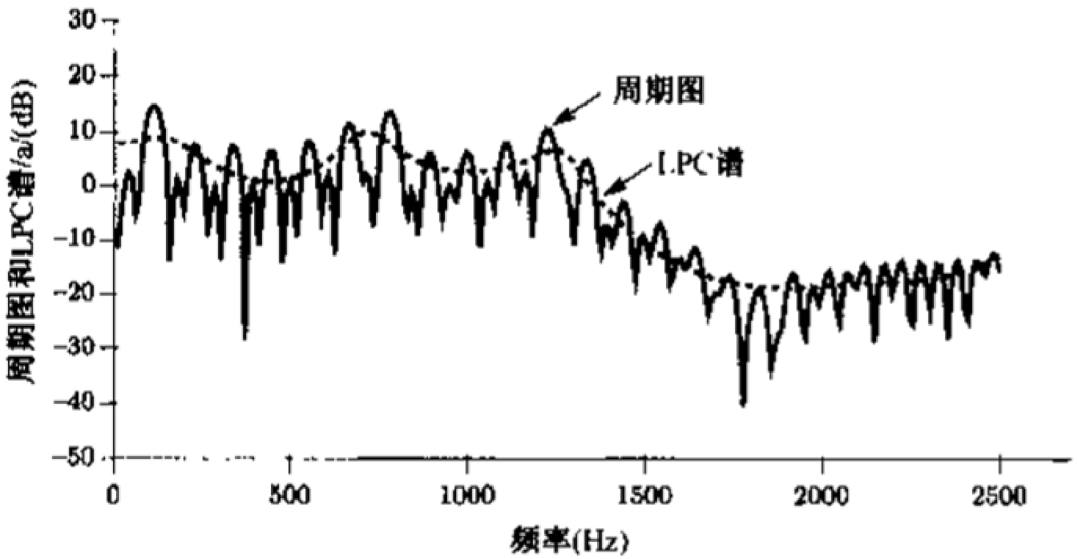


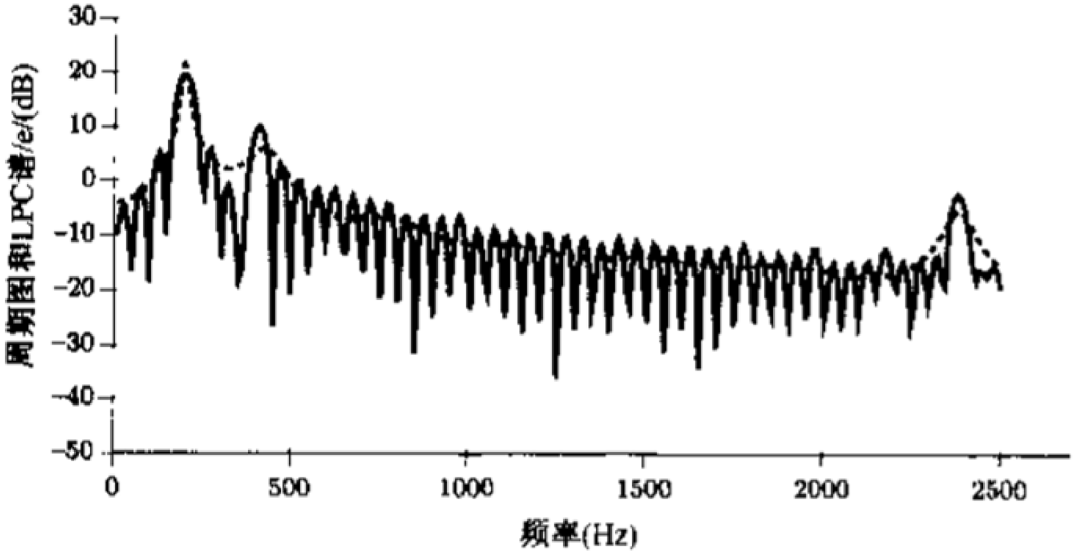
Figure 1.3 Examples of speech sounds

In speech processing a machine recognizes individual speech sounds or phonemes. As an example, the vowels /a/ and /e/ are shown in figure. Note that they are periodic waveforms whose period is called the pitch.

Have the person whose voice is to be recognized say each vowel three times and store the waveforms. To recognize the spoken vowel, compare it to the stored vowels and choose the one that is closest to the spoken vowel or the one that minimizes some distance measure.

Difficulties arise if the pitch of the speaker's voice changes from the time he or she recorded the sounds (the training session) to the time when the speech recognizer is used.





Attributes are chosen that are less susceptible to variation. For example, the spectral envelope will not change with pitch since the Fourier transform of a periodic signal is a sampled version of the Fourier transform of one period of the signal.

To extract the spectral envelope we employ a model of speech called linear predictive coding (LPC). The parameters of the model determine the spectral envelope.

It is interesting that in this example a human interpreter can easily discern the spoken vowel. The real problem then is to design a machine that is able to do the same.

- Image analysis - estimate the position and orientation of an object from a camera image, necessary when using a robot to pick up an object [Jain 1989]

- Biomedicine - estimate the heart rate of a fetus [Widrow and Stearns 1985]

- Communications - estimate the carrier frequency of a signal so that the signal can be demodulated to baseband [Proakis 1983]

- Control - estimate the position of a powerboat so that corrective navigational action can be taken, as in a LORAN system [Dabbous 1988]

- Seismology - estimate the underground distance of an oil deposit based on sound reflections due to the different densities of oil and rock layers [Justice 1985].

In all these systems we are faced with the problem of extracting values of parameters based on continuous-time waveforms.

Due to the use of digital computers to sample and store the continuous-time waveform,

$$x(t) \Longrightarrow x[n]$$

continuous-time → discrete-time or a dataset

Mathematically, we have the *N*-point data set $\{x[0], x[1], \ldots, x[N-1]\}$ which depends on an unknow parameter θ. We wish to determine θ based on the data or to define an estimator:

$$\theta = g\left(x[0], x[1], \ldots, x[N-1]\right)$$

where g is some function. This is the problem of parameter estimation, which is the subject of this course.

1.2 The Mathematical Estimation Problem

In determining good estimators the first step is to mathematically model the data.

Because the data are inherently random, we describe it by its probability density function (PDF) or $p\left(x[0], x[1], \ldots, x[N-1]; \theta\right)$. The PDF is parameterized by the unknown parameter θ

$$p\left(x[0] \mid \theta\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(x[0]-\theta)^2\right]$$

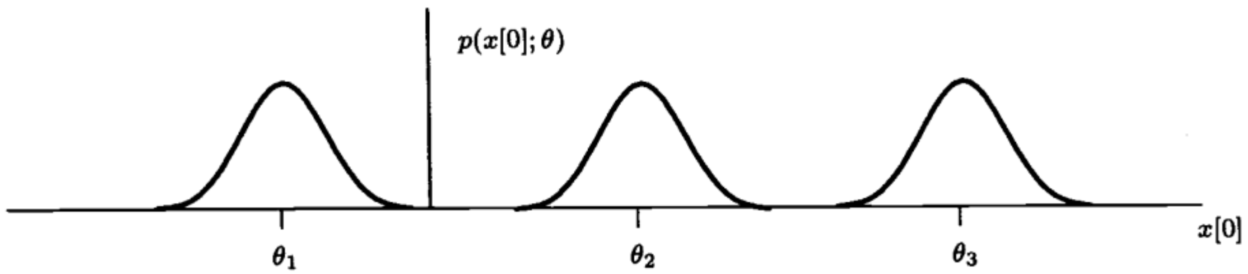


Figure 1.5 Dependence of PDF on unknown parameter

Because the value of θ affects the probability of $x[0]$, we should be able to ***infer*** the value of θ from the observed value of $x[0]$.

For example, if the value of $x[0]$ is negative, the value $\theta = \theta_1$ might be more reasonable than $\theta = \theta_2$.

This specification of the PDF is critical in determining a good estimate.

In an actual problem we are not given a PDF but must choose one that is not only consistent with the problem constraints and any prior knowledge, but one that is also mathematically tractable.

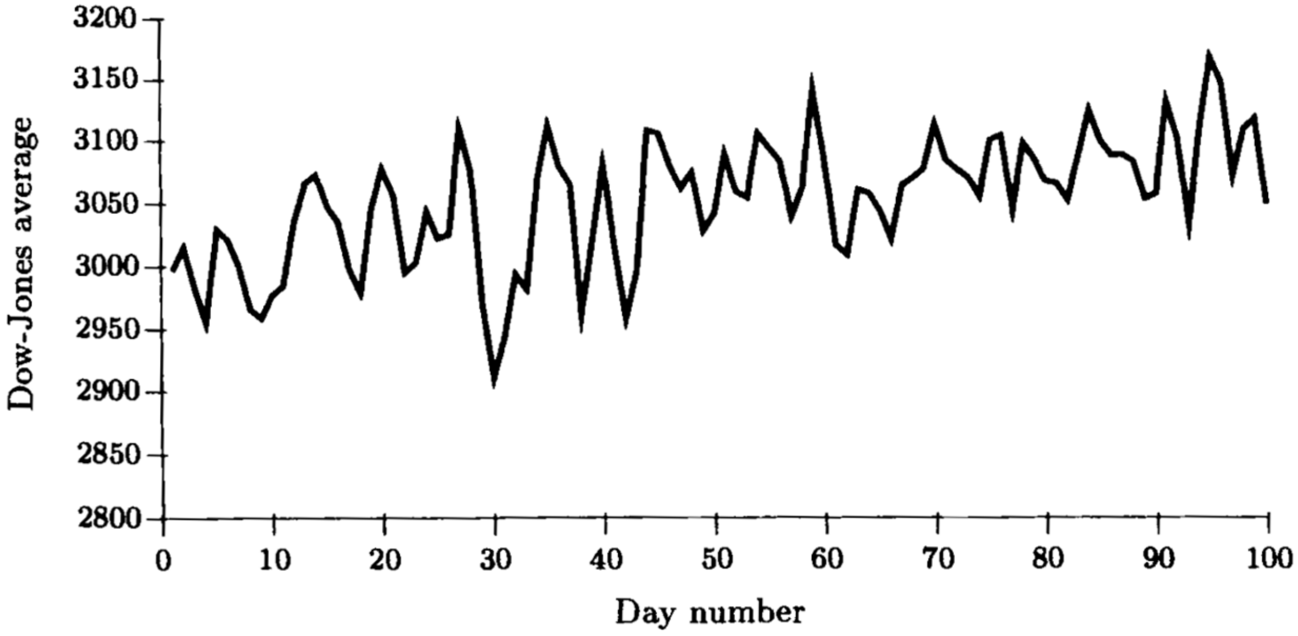


Figure 1.6 Hypothetical Dow-Jones average

What pattern do you observe?

This data, although appearing to fluctuate wildly, actually is "on the average" increasing.

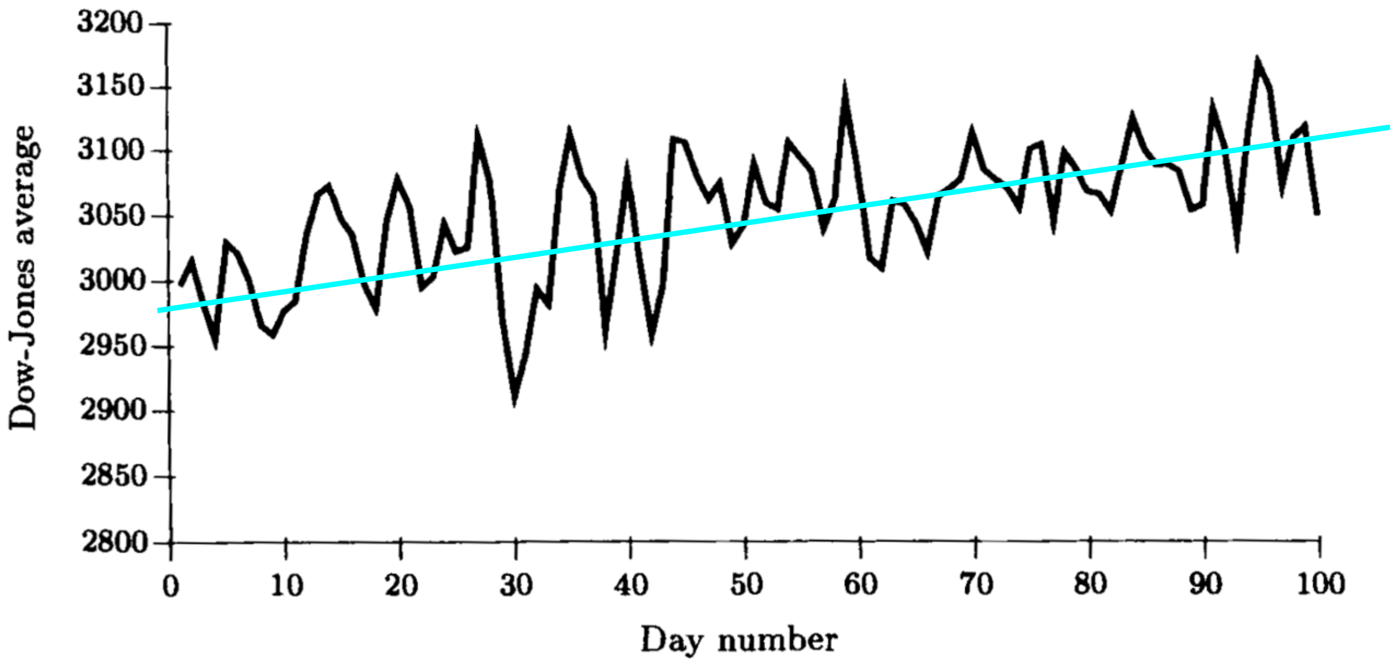


Figure 1.6 Hypothetical Dow-Jones average

We could assume that the data actually consist of a straight line embedded in random noise

Linear regression model

$$\boxed{x[n] = A + Bn + w[n] \qquad n = 0,1,\ldots,N-1}$$

$w[n] \sim N(0,\sigma^2)$ white Gaussian noise (WGN)

$w[n]$ is uncorrelated with all the other samples

In this case, we have $\theta = [A\ B]^T$

Letting $x = [x[0]x[1]\ldots x[N-1]]^T$, the PDF is

$$\boxed{p(x\,|\,\theta) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}}\exp\left[-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A-Bn)^2\right]}$$

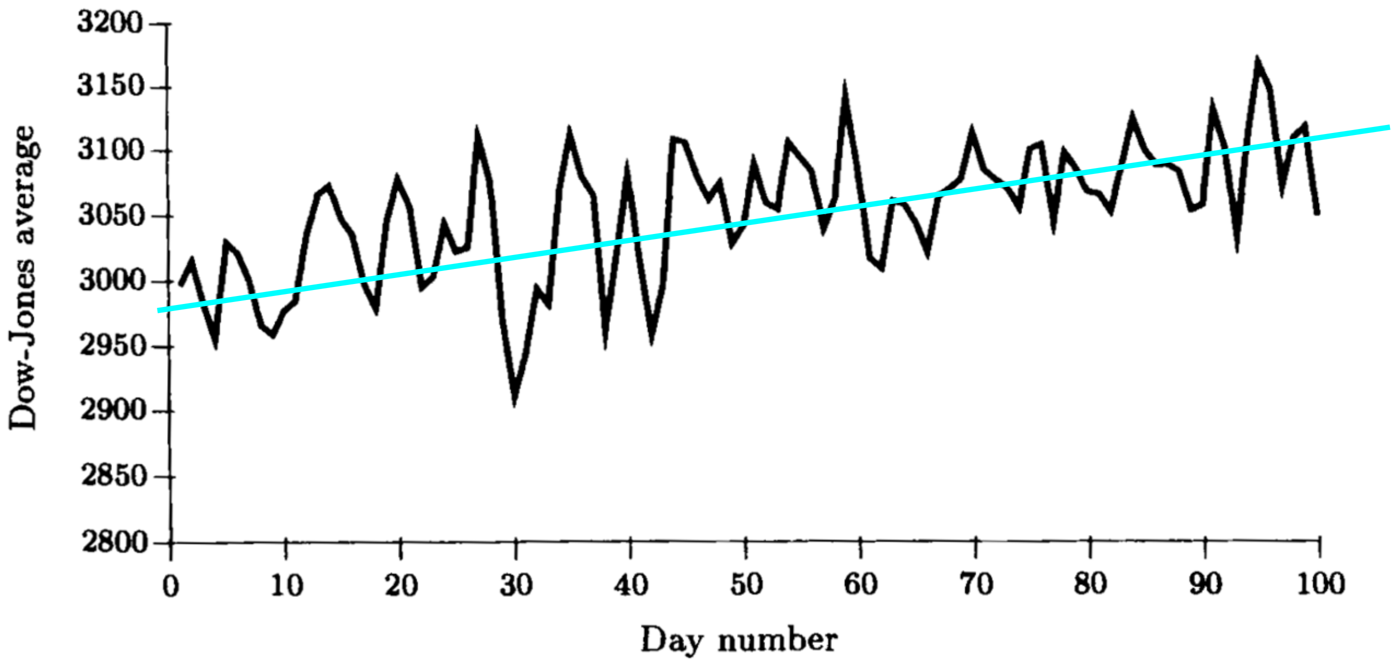


Figure 1.6 Hypothetical Dow-Jones average

Why is it reasonable

$x[n] = A + Bn + w[n] \qquad n = 0, 1, \ldots, N-1$

$w[n] \sim N(0, \sigma^2)$ WGN

$w[n]$ is uncorrelated with all the other samples

- The choice of a straight line for the signal component is consistent with the knowledge that the Dow-Jones average is hovering around 3000 (A models this).
- The conjecture that it is increasing ($B > 0$ models this)
- The assumption of WGN is justified by the need to formulate a mathematically tractable model so that closed form estimators can be found.
- Also, it is reasonable unless there is strong evidence to the contrary, such as highly correlated noise.