Scientific Computing Midterm Exam

Friday, October 31, 2025, 16:00–18:20

One "cheat sheet" (an A4 size piece of paper with any auxiliary materials handwritten on both sides) is allowed. No lecture notes, textbooks, calculators, cell phones, or anything else is allowed.

1. $(6 \times 3 = 18$ points) **State ONE advantage** (a very brief, but concrete and correct answer will give you a full credit) of:

   (a) Newton's method over the secant method;

   (b) the secant method over Newton's method;

   (c) the false position method over the secant method;

   (d) the secant method over the false position method;

   (e) the Muller method over Newton's methods;

   (f) Newton's method over the Muller method.

2. (20 points) Show the derivation of the fourth-order **compact finite-difference scheme** for the 2-point boundary value problem

$$\begin{cases} -\phi''(x) + c(x)\phi(x) = f(x), & x \in (0,1), \\ \phi(0) = \alpha, & \phi(1) = \beta, \end{cases}$$

   where c and f are given smooth functions and α, β are given real numbers.

3. (10 points) Give an example of a matrix norm, which is *not* a subordinate matrix norm. Prove that this norm is not a subordinate matrix norm.

4. $(2 \times 8 = 16$ points) Let $\alpha = g(\alpha)$. Prove the following properties of the fixed-point method $x_{n+1} = g(x_n)$ with x_0 being an initial guess:

   (a) $0 < g'(\alpha) < 1$: *monotone convergence*, that is, the error $x_n - \alpha$ maintains a constant sign as n varies;

   (b) $-1 < g'(\alpha) < 0$: *oscillatory convergence*, that is, $x_n - \alpha$ changes sign as n varies.

5. (16 points) Let f be a smooth function and $f(\alpha) = 0$. Define an iteration formula by

$$x_{n+1} = z_{n+1} - \frac{f(z_{n+1})}{f'(x_n)}, \qquad z_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

   subject to an initial guess x_0. Show that the order of convergence of $\{x_n\}$ to α is at least 3.

6. $(2 \times 10 = 20$ points)

$$A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$$

   (a) Find the LU factorization of A.

   (b) Find the Cholesky factorization of A.