ASSIGNMENT 3

Due date Monday 27th of October

(1) In Example 1.5, page 6 of Lee, coordinate charts for the real n-dimensional projective plane RP^n are given. These actually determine an atlas of smooth charts for RP^n. Using ϕ_i, find a basis for $T_p RP^n$, and give a formula for each of the basis vectors with respect to the coordinate chart ϕ_j, assuming p is in the intersection of the domains of the coordinate charts ϕ_i and ϕ_j.

(2) Consider the smooth manifold $O(n)$ consisting of all $n \times n$ orthogonal matrices. Show that the tangent space to $O(n)$ at the identity matrix I is isomorphic to the vector space of $n \times n$ skew symmetric matrices. What is the dimension of the manifold $O(n)$? Hint: Consider a smooth curve $\gamma(t)$ in $O(n)$, with $\gamma(0) = I$. Then $\gamma^T(t)\gamma(t) = I$, for all t. Take the derivative of this at $t = 0$.

(3) Problem 3.2 of Lee

(4) Problem 3.6 of Lee