ASSIGNMENT 5

Due date Monday 30th of November

(1) The flows ϕ and ψ of two vector fields X and Y on a smooth manifold M do not commute in general. By differentiating, show that $\phi_t \circ \psi_s = \psi_s \circ \phi_t$ implies that the Lie derivative of X with respect to Y is zero.

(2) Problem 7.14 of Lee.

(3) Problem 7.15 of Lee.

(4) Think of S^3 as the unit sphere in $\mathbb{C}^2$. The Hopf action of S^1 on S^3 is defined by $z \cdot w = zw$, where $z \in S^1$ and $w = (w_1, w_2) \in \mathbb{C}^2$.

   - Show that the Hopf action is smooth and that the orbits are disjoint unit circles in $\mathbb{C}^2$ whose union is S^3
   - Consider the flow $\Theta : \mathbb{R} \times S^3 \to S^3$ defined by $\Theta_t(w) = e^{it}w$. Explicitly calculate the infinitesimal generator V of Θ. Your formula should give to each $w \in S^3$ a vector $V_w \in T_w S^3$, where $T_w S^3$ is thought of as a subspace of $T_w \mathbb{C}^2$
   - True or false: the vector field on S^3 just obtained is nowhere vanishing.

   This map $S^3 \to S^2$ is called the Hopf fibration. A video visualising it can be found here https://nilesjohnson.net/hopf.html