考试科目： 概率论与数理统计　　　**开课单位：** 数学系

考试时长： 2 小时　　　**命题教师：** 概率统计教学组

<table>
<tr><td rowspan="2">题号</td><td rowspan="2">Part 1</td><td rowspan="2">Part 2</td><td colspan="6">Part 3</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>分值</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

本试卷共三大部分，满分 100 分（考试结束后请将试卷、答题本、草稿纸一起交给监考老师）

第一部分　选择题（每题 4 分，总共 20 分）

Part One – Single Choice (4 marks each question, 20 marks in total)

1. 设A,B,C三个事件两两独立，则A,B,C相互独立的充分必要条件是（　　）.

（A）A与BC独立　　（B）AB与$A\cup C$独立

（C）AB与AC独立　　（D）$A\cup B$与$A\cup C$独立

Let A,B,C be three events that are pairwise independent. Which of the following conditions must hold if A,B,C are to be mutually independent?

（A）A and BC are independent　　（B）AB and $A\cup C$ are independent

（C）AB are AC are independent　　（D）$A\cup B$ and $A\cup C$ are independent

2. 设$X{\sim}N(2,1)$, $Y{\sim}N(-1,1)$，且X,Y独立，记$Z=3X-2Y-6$，则$Z\sim$（　）.

（A）$N(2,1)$　　（B）$N(2,13)$

（C）$N(1,1)$　　（D）$N(1,5)$

Independent random variables X and Y have distributions $X{\sim}N(2,1)$, $Y{\sim}N(-1,1)$ respectively. Let $Z=3X-2Y-6$，then the distribution $Z\sim$（　）.

（A）$N(2,1)$；　　（B）$N(2,13)$；

（C）$N(1,1)$；　　（D）$N(1,5)$.

3. 随机变量X 和 Y的方差相等且不为零，则X 和 Y的相关系数为$\rho_{XY}=1$的充分必要条件是（　）.

（A）$Cov(X+Y,\ X)$=0　　（B）$Cov(X+Y,\ Y)$=0

（C）$Cov(X+Y,\ X-Y)$=0　　（D）$Cov(X-Y,\ X)$=0

Two random variables X and Y have the same variance. The correlation coefficient ρ_{XY} between X and Y equals 1 if（　　）holds.

（A）$Cov(X+Y,\ X)$=0　　（B）$Cov(X+Y,\ Y)$=0

（C）$Cov(X+Y,\ X-Y)$=0　　（D）$Cov(X-Y,\ X)$=0

4. 从总体$X\sim N(\mu,\sigma^2)$中抽取简单随机样本X_1,X_2,X_3，统计量

$$\hat{\mu}_1=\frac{1}{2}X_1+\frac{1}{3}X_2+\frac{1}{6}X_3 \qquad \hat{\mu}_2=\frac{1}{2}X_1+\frac{1}{4}X_2+\frac{1}{4}X_3$$

$$\hat{\mu}_3=\frac{1}{3}X_1+\frac{1}{3}X_2+\frac{1}{3}X_3 \qquad \hat{\mu}_4=\frac{1}{5}X_1+\frac{2}{5}X_2+\frac{2}{5}X_3$$

都是总体均值$EX=\mu$的无偏估计量，则有效的估计量是（　）.

（A）　$\hat{\mu}_1$;　　（B）$\hat{\mu}_2$;　　（C）$\hat{\mu}_3$;　　（D）$\hat{\mu}_4$.

Let X_1,X_2,X_3 be a random sample from a population $X\sim N(\mu,\sigma^2)$. For the following unbiased estimators, which is more efficient?

$$\hat{\mu}_1=\frac{1}{2}X_1+\frac{1}{3}X_2+\frac{1}{6}X_3 \qquad \hat{\mu}_2=\frac{1}{2}X_1+\frac{1}{4}X_2+\frac{1}{4}X_3$$

$$\hat{\mu}_3=\frac{1}{3}X_1+\frac{1}{3}X_2+\frac{1}{3}X_3 \qquad \hat{\mu}_4=\frac{1}{5}X_1+\frac{2}{5}X_2+\frac{2}{5}X_3$$

（A）　$\hat{\mu}_1$;　　（B）$\hat{\mu}_2$;　　（C）$\hat{\mu}_3$;　　（D）$\hat{\mu}_4$.

5. 已知两个独立的随机变量和分布是$X\sim N(\mu_1,\sigma_1^2)$，$Y\sim N(\mu_2,\sigma_2^2)$，为检验总体$X$的均值大于总体$Y$的均值，则应做检验假设是（ ）

（A）H_0: $\mu_1>\mu_2$; H_1:$\mu_1\le\mu_2$.　　（B）H_0: $\mu_1\ge\mu_2$; H_1:$\mu_1<\mu_2$.

（C）H_0: $\mu_1<\mu_2$; H_1:$\mu_1\ge\mu_2$.　　（D）H_0: $\mu_1\le\mu_2$; H_1:$\mu_1>\mu_2$.

Let X and Y be two independent random variables having distributions $X\sim N(\mu_1,\sigma_1^2)$ and $Y\sim N(\mu_2,\sigma_2^2)$. In order to test that the mean of population X is greater than the mean of population Y, the null and alternative hypotheses should be（ ）

（A）H_0: $\mu_1>\mu_2$; H_1:$\mu_1\le\mu_2$.　　（B）H_0: $\mu_1\ge\mu_2$; H_1:$\mu_1<\mu_2$.

（C）H_0: $\mu_1<\mu_2$; H_1:$\mu_1\ge\mu_2$.　　（D）H_0: $\mu_1\le\mu_2$; H_1:$\mu_1>\mu_2$.

第二部分 填空题（每空 2 分，总共 20 分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 设A, B是两事件，$P(A) = P(\bar{B})$，$P(A|B) = 0.2$，$P(B|A) = 0.3$，则$P(A)$ =_______.

   Let A, B be two events. If $P(A) = P(\bar{B})$, $P(A|B) = 0.2$, $P(B|A) = 0.3$, then $P(A)$ =_______.

2. 设$(X,Y) \sim N(\mu,\mu,\sigma^2,\sigma^2,0)$，则$P\{X < Y\}$ =_______.

   If $(X,Y) \sim N(\mu,\mu,\sigma^2,\sigma^2,0)$, then $P\{X < Y\}$ =_______.

3. 设 随 机 变 量 X 的 密 度 函 数 为 $f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, \quad x \le 0 \end{cases}$，则 条 件 概 率 $P\{X \le 2 | X \ge 1\}$ =_______.

   If the density function of a random variable X is $f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, \quad x \le 0 \end{cases}$, then the conditional probability $P\{X \le 2 | X \ge 1\}$ =_______.

4. 设随机变量X的密度函数为$f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, \quad 其它 \end{cases}$，则$P\{X \ge -1\}$ =_______.

   If the density function of a random variable X is $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{other} \end{cases}$, then $P\{X \ge -1\}$ =_______.

5. 设随机变量$X \sim U(0,1)$，$Y \sim U(0,1)$，且X与Y独立，记(X,Y)的概率密度为$f(x,y)$，则$f\left(\frac{1}{2},\frac{1}{2}\right)$ =_______.

   X and Y are independent random variables having distributions $X \sim U(0,1)$ and $Y \sim U(0,1)$. If the joint density function of (X,Y) is $f(x,y)$, then $f\left(\frac{1}{2},\frac{1}{2}\right)$ =_______.

6. 已知随机变量X服从二项分布$b(n,p)$，且$E(X) = 2.4, D(X) = 1.44$，则二项分布的参数n =_______，p =_______（这里每空各一分）.

Let the random variable X follow a Binomial distribution $b(n,p)$ where $E(X)=2.4$, $D(X)=1.44$. Calculate the value of the parameters $n=$____, $p=$____ .

(Here each blank earns 1 mark)

7. 设随机变量X_1,X_2,X_3相互独立，且都服从参数为λ的泊松分布，令$Y=\frac{1}{3}(X_1+X_2+X_3)$，则$Y^2$的数学期望等于_______.

Let X_1,X_2,X_3 be independent Poisson random variables each with parameter λ.

If $Y=\frac{1}{3}(X_1+X_2+X_3)$, then the mathematical expectation of Y^2 is _______.

8. 设X_1,X_2,X_3,X_4为来自总体$X\sim N(1,\sigma^2)$的样本，则统计量$\frac{X_1-X_2}{|X_3+X_4-2|}$的分布为__________.（请写出分布类型及其参数）

Let X_1,X_2,X_3,X_4 be a random sample from a population $X\sim N(1,\sigma^2)$. The distribution of the statistic $\frac{X_1-X_2}{|X_3+X_4-2|}$ is __________.

(List the distribution type and its degrees of freedom)

9. 设$X_1,X_2,\cdots,X_n(n\geq 2)$为来自总体$X\sim N(0,1)$的样本，$\bar{X}$和$S^2$分别为样本均值和样本方差，则$\frac{(n-1)X_1^2}{\sum_{i=2}^{n}{X_i}^2}$服从的分布为__________.（请写出分布类型及其参数）

Let $X_1,X_2,\cdots,X_n(n\geq 2)$ be a random sample from a population $X\sim N(0,1)$. If $\bar{X}$ and S^2 are the sample mean and sample variance respectively, then $\frac{(n-1)X_1^2}{\sum_{i=2}^{n}{X_i}^2}$ follows the __________ distribution.

(List the distribution type and its degrees of freedom)

10. 考虑两个总体$N(\mu_1,{\sigma_1}^2),N(\mu_2,{\sigma_2}^2)$的假设问题:

$$H_0:{\sigma_1}^2\leq{\sigma_2}^2;\quad H_1:{\sigma_1}^2>{\sigma_2}^2,$$

在各总体中分别抽取容量为$m=21,n=27$的样本，S_1^2,S_2^2分别为样本方差，且设两组样本相互独立. 则当${\sigma_1}^2={\sigma_2}^2$时，统计量$\frac{S_1^2}{S_2^2}\sim$__________.（请写出分布类型及其参数）

The hypothesis for two populations $N(\mu_1,{\sigma_1}^2), N(\mu_2,{\sigma_2}^2)$ is as follows:

$$H_0: {\sigma_1}^2 \le {\sigma_2}^2; \quad H_1: {\sigma_1}^2 > {\sigma_2}^2,$$

Independent samples of size $m = 21$ and $n = 27$ are taken from the two populations. The sample variances are denoted by S_1^2, S_2^2 respectively. If ${\sigma_1}^2 = {\sigma_2}^2$, then the statistic $\frac{S_1^2}{S_2^2}$ follows ________ distribution?

(List the distribution type and its degrees of freedom)

第三部分 大题（每题 10 分，总共 60 分）

Part Three – Question Answering (10 marks each question, 60 marks in total)

1. 同时掷两个不同颜色的骰子，观察其朝上的点数，记事件A =“两个骰子的点数和等于 3”，B =“两个骰子的点数和等于 7”，C =“至少有一个骰子的点数为 1”，求：

   (1) $P(A|C)$;

   (2) $P(B|C)$;

   (3) A和C是否独立？说明理由。

Roll two dice at the same time and observe the results. Let event A be “the sum of the numbers from the two dice is 3”. Let event B be “the sum of the numbers from the two dice is 7”. Let event C be “at least one of the numbers is 1”.

(1) What is $P(A|C)$?
(2) What is $P(B|C)$?
(3) Are events A and C dependent ? Why?

2. 设离散型随机变量X的期望为$\frac{11}{18}$, 且其频率函数如下, 其中a和b为常数:

X	a	1	b
p	b	1/2	a

   (1) 求a^2+b^2的值;

   (2) 求$Z=\frac{1}{2}X-\frac{1}{4}$的方差$D(Z)$.

Let X be a discrete random variable with expectation $\frac{11}{18}$. Suppose the probability frequency function of X is as follows, where a and b are constants.

X	a	1	b
p	b	1/2	a

(1) Find the value of a^2+b^2;

(2) Calculate the variance $D(Z)$ of $Z=\frac{1}{2}X-\frac{1}{4}$.

3. 假设连续型随机变量X和Y的联合概率密度函数为

$$f(x,y)=\begin{cases} be^{-3x-2y}, x>0, y>0, \\ \quad 0, \quad \text{其他}. \end{cases}$$

(1) 确定常数b;

(2) 求X和Y的边缘概率密度函数$f_X(x)$和$f_Y(y)$，并确定(给出原因)它们是否独立.

Assume X and Y are two continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases} be^{-3x-2y}, x>0, y>0, \\ \quad 0, \quad \text{otherwise}. \end{cases}$$

(1) Find the value of b;

(2) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$ of X and Y respectively. Determine with reasons whether X and Y are independent.

4. 令 $\{X_1, X_2, \cdots\}$ 是独立同分布的$N(2,5)$-随机变量序列. 对于正整数n, 定义

$$S_n=\sum_{i=1}^{n} X_i,\ \bar{X}_n=S_n/n.$$

(1) S_{10}，S_{20}，与$\bar{X}_{20}$分别服从何种分布?

(2) S_{10}与S_{20}是否相关? 求出S_{10}与S_{20}的相关系数$\rho(S_{10}, S_{20})$来证明你的结论.

Assume $\{X_1, X_2, \cdots\}$ is a sequence of independent and identically distributed $N(2,5)$ random variables. Denote $S_n=\sum_{i=1}^{n} X_i$ and $\bar{X}_n=S_n/n$.

(1) What distributions do S_{10}, S_{20} and $\bar{X}_{20}$ follow?

(2) Are S_{10} and S_{20} correlated? Find the correlation coefficient $\rho(S_{10}, S_{20})$ of S_{10} and S_{20} to support your conclusion.

5. 设总体X的频率函数为

X	1	2	3
p_k	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

其中$\theta(0<\theta<1)$为未知参数。已知取得了样本值$x_1=1, x_2=2, x_3=1$。

(1) 求θ的最大似然估计值。

(2) 求θ的矩估计值；

Let the frequency function of a random variable X be as follows:

X	1	2	3
p_k	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

where θ is an unknown parameter $(0<\theta<1)$. The observed sample $x_1=1$, $x_2=2$, $x_3=1$ has been obtained to estimate θ.

(1) Compute the Maximum Likelihood estimator of θ;
(2) Compute the Moment Estimator of θ.

6. 求解以下两个问题：

(1) 设雷达速度测量值$X\sim N(\mu,\sigma^2)$，且雷达没有系统误差. 今用雷达测得飞机的 9 个飞行数据（样本）的平均值为 760，方差为 64. 求飞机飞行速度的置信区间 $(1-\alpha=95\%)$.

(2) 从甲地发送一个信号到乙地，由于存在线路噪声干扰，使得甲地发送 一个幅值为μ的信号，而乙地收到的信号是一个服从 $N(\mu,4)$ 分布的随机变量. 在测试中，甲地将同一信号发送了 4 次，乙地收到的信号值为

6　　7　　9　　10

接收方有某种理由猜测甲地发送的信号值为 10，问这种猜测是否正确? $(\alpha=0.05)$

(附注: $u_{0.95}=1.65$； $u_{0.975}=1.96$； $t_{0.95}(9)=1.833$； $t_{0.95}(10)=1.812$;

$t_{0.95}(8)=1.860$; $t_{0.975}(8)=2.306$; $t_{0.975}(9)=2.262$; $t_{0.975}(10)=2.228$;

$\chi^2_{0.95}(9)=16.92$； $\chi^2_{0.95}(10)=18.31$; $\chi^2_{0.975}(9)=19.02$； $\chi^2_{0.975}(10)=20.48$.)

Answer the following two questions:

(1) Assume the speed of a plane measured by radar follows $X \sim N(\mu, \sigma^2)$ with no systematic error. Nine random measurements are obtained with sample mean 760 and variance 64. Find a 95% confidence interval for the average plane speed ?

(2) A signal is sent from location A to location B. Due to noise, a signal with amplitude μ sent out from location A, will be received at location B with distribution $N(\mu, 4)$.
In a test, a signal was sent out from location A four times. The values that location B received were　6　7　9　10.
The receiver guesses the amplitude value of the signal sent from location A is 10. Is this guess acceptable?　$(\alpha = 0.05)$

($u_{0.95} = 1.645$; $u_{0.975} = 1.96$; $t_{0.95}(9) = 1.833$; $t_{0.95}(10) = 1.812$;
$t_{0.95}(8) = 1.860$; $t_{0.975}(8) = 2.306$; $t_{0.975}(9) = 2.262$; $t_{0.975}(10) = 2.228$;
$\chi^2_{0.95}(9) = 16.92$; $\chi^2_{0.95}(10) = 18.31$; $\chi^2_{0.975}(9) = 19.02$; $\chi^2_{0.975}(10) = 20.48$.)

答案：

第一部分

A　B　D　C　D

第二部分

1. 0.4

2. $\frac{1}{2}$

3. $1-e^{-1}$

4. 1

5. 1

6. 6, 0.4

7. $\lambda^2+\frac{1}{3}\lambda$

8. $t(1)$

9. $F(1,n-1)$

10. $F(20,26)$

第三部分

1. 同时掷两个不同颜色的骰子，观察其朝上的点数，记事件A="两个骰子的点数和等于 3"，B="两个骰子的点数和等于 7"，C="至少有一个骰子的点数为 1"，求：

(1) $P(A|C)$;

(2)$P(B|C)$;

(3)A和C是否独立？说明理由。

Solution. Sample space

$$\Omega = \{(1,1),(1,2),(1,3),\ldots,(6,6)\} = \{(i,j) \mid i,j = 1,2,3,4,5,6\}$$

(Each outcome is equally likely, with probability 1/36.)

$$\begin{aligned} A &= \{(1,2),(2,1)\} \\ B &= \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\} \\ C &= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\} \end{aligned}$$

(1) $P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{2/36}{11/36} = \frac{2}{11}$.
(2) $P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{2/36}{11/36} = \frac{2}{11}$.
(3) $P(A) = 2/36 \neq P(A \mid C)$, so they are not independent.

2. 设离散型随机变量X的期望为$\frac{11}{18}$，且其频率函数如下，其中a和b为常数:

X	a	1	b
p	b	$1/2$	a

(i)求a^2+b^2的值；[6 分]

(ii)求$Z=\frac{1}{2}X-\frac{1}{4}$ 的方差$D(Z)$．[4 分]

(i) By the property of frequency function, $b+a+\frac{1}{2}=1$ [**2 marks**]. Moreover, by the definition of the expectation, we have $ab+1\cdot\frac{1}{2}+ba=\frac{11}{18}$ [**2 marks**]. Solving these two equalities, we obtain $a=\frac{1}{3}$, $b=\frac{1}{6}$ or $a=\frac{1}{6}$, $b=\frac{1}{3}$ [**1 mark**]. In both these cases, $a^2+b^2=\frac{5}{36}$ [**1 mark**].

(ii) Direct calculations yield that $D(X)=EX^2-(EX)^2=a^2b+1^2\cdot\frac{1}{2}+b^2\cdot a-(\frac{11}{18})^2=\frac{1}{36}+\frac{1}{2}-\frac{121}{324}=\frac{25}{162}$ [**2 marks**]. By the property of the variance, we have $D(Z)=\frac{1}{4}D(X)=\frac{25}{648}$ [**2 marks**].

3. 假设连续型随机变量X和Y的联合概率密度函数为

$$f(x,y)=\begin{cases} be^{-3x-2y}, x>0, y>0, \\ 0, \quad \text{其他}. \end{cases}$$

(a) 确定常数b;

(b) 求X和Y的边缘概率密度函数$f_X(x)$和$f_Y(y)$，并确定(给出原因)它们是否独立.

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases} be^{-3x-2y}, x>0, y>0, \\ 0, \quad \text{otherwise}. \end{cases}$$

(a) Find the value of b;

(b) Find the marginal probability density functions $f_X(x)$ and $f_Y(y)$ of X and Y, respectively, and determine with reasons whether X and Y are independent.

(i) By the property $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy=1$, we have

$$1=\int_0^{\infty}\int_0^{1} be^{-3x-2y}dxdy=\frac{b}{6} \quad [\mathbf{2\ marks}] \quad \Rightarrow \mathbf{b=6}. \quad [\mathbf{2\ marks}]$$

(ii) If $x<0$, $f_X(x)=0$.
If $x>0$, $f_X(x)=\int_0^{\infty} f(x,y)dy=\int_0^{\infty} 6e^{-3x-2y}dy=3e^{-3x}$. Hence,
$$f_X(x)=\begin{cases} 3e^{-3x}, & x>0 \\ 0, & x\le 0 \end{cases} \quad [\mathbf{2\ marks}]$$

Similarly, if $y<0$, $f_Y(y)=0$.
If $y>0$, $f_Y(y)=\int_0^{\infty} f(x,y)dx=\int_0^{\infty} 6e^{-3x-2y}dx=2e^{-2y}$. Hence,
$$f_Y(y)=\begin{cases} 2e^{-2y}, & y>0 \\ 0, & y\le 0 \end{cases} \quad [\mathbf{2\ marks}]$$
Since $f_X(x)f_Y(y)=f(x,y)$, X are Y independent. [**2 marks**]

4. 令 $\{X_1, X_2, \cdots\}$ 是独立同分布的$N(2,5)$-随机变量序列. 对 于正整数n，定义 $S_n = \sum_{i=1}^{n} X_i$，$\bar{X}_n = S_n/n$.

(a) S_{10}，S_{20}，与$\bar{X}_{20}$分别服从何种分布?

(b) S_{10}与S_{20}是否相关? 求出S_{10}与S_{20}的相关系数$\rho(S_{10}, S_{20})$来证明你的结论.

Assume $\{X_1, X_2, \cdots\}$ is a sequence of independent identically distributed $N(2,5)$ random variables. Denote $S_n = \sum_{i=1}^{n} X_i$ and $\bar{X}_n = S_n/n$.

(a) What distributions do S_{10}, S_{20} and $\bar{X}_{20}$ follow?

(b) Are S_{10} and S_{20} uncorrelated or correlated? Find the correlation coefficient $\rho(S_{10}, S_{20})$ of S_{10} and S_{20} to support your conclusion.

(i) The independence of $\{X_1, X_2, \ldots\}$ and $X_i \sim N(2,5)$ imply that S_{10}, S_{20}, and $\overline{X}_{20}$ are both normally distributed. **[3 marks]** Moreover, by the properties of expectation and variance, we have $S_{10} \sim N(20, 50)$, **[1 mark]** $S_{20} \sim N(40, 100)$, **[1 mark]** and $\overline{X}_{20} \sim N(2, 1/4)$. **[1 mark]**

(ii) By the (i) we know $D(S_{20}) = 100$, $D(S_{10}) = 50$, then

$$\begin{aligned}\sigma(S_{10}, S_{20}) &= \frac{Cov(S_{10}, S_{20})}{\sqrt{D(S_{10})}\sqrt{D(S_{20})}} \quad \textbf{[1 mark]}\\ &= \frac{Cov\left(S_{10}, S_{10} + \sum_{i=11}^{20} X_i\right)}{50\sqrt{2}}\\ &= \frac{D(S_{10})}{50\sqrt{2}} = \frac{\sqrt{2}}{2} > 0. \quad \textbf{[1 mark]}\end{aligned}$$

So S_{10} and S_{20} are (positively) correlated. **[2 marks]**

5. 设总体X的频率函数为

X	1	2	3
p_k	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

其中$\theta(0<\theta<1)$为未知参数。已知取得了样本值$x_1=1,x_2=2,x_3=1$。

(1) 求θ的矩估计值；

(2) 求θ的最大似然估计值。

【解】

(1) 由

$$\mu_1=\theta^2+2\times 2\theta(1-\theta)+3\times(1-\theta)^2=3-2\theta$$

解得

$$\theta=\frac{1}{2}(3-\mu_1)$$

故得θ 的矩估计为

$$\hat{\theta}=\frac{1}{2}(3-\bar{X})$$

矩估计值为

$$\hat{\theta}=\frac{1}{2}(3-\bar{x})$$

今

$$\bar{x}=\frac{1}{3}(x_1+x_2+x_3)=\frac{1}{3}(1+2+1)=\frac{4}{3}$$

故θ 的矩估计值为$\hat{\theta}=\frac{5}{6}$.

(2) 由给定的样本值, 得似然函数为

$$\begin{aligned}L&=\prod_{i=1}^{3}P\{X_i=x_i\}=P\{X_1=1\}P\{X_2=2\}P\{X_3=1\}\\&=\theta^2\cdot 2\theta(1-\theta)\cdot\theta^2=2\theta^5(1-\theta),\end{aligned}$$

$$\ln L=\ln 2+5\ln\theta+\ln(1-\theta)$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\ln L=\frac{5}{\theta}-\frac{1}{1-\theta}=0,$$

得θ 的最大似然估计值为$\hat{\theta}=\frac{5}{6}$.

6. 求解以下两个问题：

(1) 设雷达速度测量值$X \sim N(\mu, \sigma^2)$, 且雷达没有系统误差. 今用雷达测得飞机的 9 个飞行数据（样本）的平均值为 760，方差为 64. 求飞机飞行速度的置信区间$(1-\alpha = 95\%)$.

(2) 从甲地发送一个信号到乙地, 由于存在线路噪声干扰, 使得甲地发送 一个幅值为μ的信号, 而乙地收到的信号是一个服从 $N(\mu, 4)$ 分布的随机变量. 在测试中, 甲地将同一信号发送了 4 次，乙地收到的信号值为

$$6 \quad 7 \quad 9 \quad 10$$

接收方有某种理由猜测甲地发送的信号值为 10 ，问这种猜测是否正确? (显著性水平$\alpha = 0.05$)

(附注$u_{0.95} = 1.65$； $u_{0.975} = 1.96$；$t_{0.95}(9) = 1.833$；$t_{0.95}(10) = 1.812$

$$t_{0.95}(8) = 1.860; t_{0.975}(8) = 2.306; t_{0.975}(9) = 2.262; t_{0.975}(10) = 2.228$$

$$\chi^2_{0.95}(9) = 16.92；\chi^2_{0.95}(10) = 18.31; \chi^2_{0.975}(9) = 19.02；\chi^2_{0.975}(10) = 20.48$$

【解】

（1）由枢轴法可求得μ 的$1-\alpha$ 的置信区间是

$$\left(\bar{X} - \frac{S}{\sqrt{n}} t_{1-\alpha/2}(n-1), \bar{X} + \frac{S}{\sqrt{n}} t_{1-\alpha/2}(n-1)\right)$$

由题给数据, 得

$$n = 9, \alpha = 0.05, t_{1-\frac{\alpha}{2}}(n-1) = t_{0.975}(8) = 2.306$$
$$\bar{x} = 760, s = 8, \frac{s}{\sqrt{n}} t_{0.975}(8) = 6.15$$

故巡航导弹飞行速度μ 的95% 置信度的置信区间是

$$(\bar{x} - 6.15, \bar{x} + 6.15) = (766.15, 753.85)$$

（2）解: 已知$X \sim N(\mu, 4)$ ，其中σ_0^2 已知，且$\sigma_0 = 2,\ n = 4, \alpha = 0.05$
根据提议，原假设和备择假设取为：

$$\mathbf{H}_0 : \mu = \mu_0 = 10, \qquad \mathbf{H}_1 : \mu \neq \mu_0$$

采用u 检验法,求得$\mathbf{H}_0$ 的拒绝域为

$$\left|\bar{X} - \mu_0\right| > \frac{\sigma_0}{\sqrt{n}} u_{1-\frac{\alpha}{2}} = \frac{2}{\sqrt{4}} \times u_{0.975} = 1.96$$

又算得$x = 8$, 因为

$$|\bar{x} - \mu_0| = |8 - 10| = 2 > 1.96$$

故拒绝$\mathbf{H}_0$, 不能认为甲地发送的信号值为10。