Scientific Computing Final Exam

1. State ONE advantage and ONE disadvantage of a method over another

2. Using Newton–Cotes quadrature to compute the integral

$$\int_0^1 \cos(10^4 \pi x) f(x)\, dx$$

requires $n >> 10^4$, where n is the number of subintervals. Explain why this is the case, and suggest an alternative numerical integration method that can compute the integral accurately with only 2 points.

3. Derive pratical method to estimate the error of a general composite quadrature rule, don't just state the error formula of a certain quadrature rule.

Hint: Perform the quadrature multiple times with different step sizes.

P.S. This question is actually asking about Richardson extrapolation.

4. Write down Gauss–Seidel method for solving linear system, derived from the PDE:

$$-\Delta u = f$$

5. Simple least square computation (vectors)

6. For matrix

$$A = \begin{pmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{pmatrix},$$

determine all values of a such that A is p.d. but the Jacobi method for solving the linear system $Ax = 0$ does not converge.