南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

考试科目： 概率论与数理统计 **开课单位：** 数学系

考试时长： 2023/6/9 10:30–12:30 **命题教师：** 概率统计教学组

题号	1	2	3	4	5	6	7	8
分值	20 分	20 分	10 分	10 分	10 分	10 分	10 分	10 分

本试卷共三大部分，满分（100）分
（考试结束后请将试卷、答题卡一起交给监考老师）

(附注：下分位数)

$u_{0.95} = 1.65, u_{0.975} = 1.96, u_{0.99} = 2.33, u_{0.995} = 2.58$
$\chi^2_{0.975}(15) = 27.49, \chi^2_{0.95}(15) = 25.00, \chi^2_{0.025}(15) = 6.26, \chi^2_{0.05}(15) = 7.26$
$\chi^2_{0.975}(16) = 28.85, \chi^2_{0.95}(16) = 26.30, \chi^2_{0.025}(16) = 6.91, \chi^2_{0.05}(16) = 7.96$
$t_{0.995}(15) = 2.95, t_{0.975}(15) = 2.13, t_{0.95}(15) = 1.75, t_{0.975}(16) = 2.12, t_{0.95}(16) = 1.746$

第一部分 选择题（每题4 分，总共20 分）
Part One – Single Choice (4 marks each question, 20 marks in total)

1. 设连续随机变量X的累积分布函数为：$F(x) = \begin{cases} a & \text{for } x \le 1 \\ bx\ln(x) + cx + d & \text{for } 1 < x < e \\ d & \text{for } x \ge e \end{cases}$.

   求a, b, c和d的值.

   Suppose the cumulative distribution function for a continuous random variable X is: $F(x) = \begin{cases} a & \text{for } x \le 1 \\ bx\ln(x) + cx + d & \text{for } 1 < x < e \\ d & \text{for } x \ge e \end{cases}$. Then the values of a, b, c, d are:

   A. $\mathrm{a} = 0,\ \mathrm{b} = 1, \mathrm{c} = -1,\ \mathrm{d} = 1$ **B.** $\mathrm{a} = 1,\ \mathrm{b} = -1, \mathrm{c} = 1,\ \mathrm{d} = 0$

   C. $\mathrm{a} = 0,\ \mathrm{b} = -1, \mathrm{c} = 1,\ \mathrm{d} = 1$ **D.** $\mathrm{a} = 0,\ \mathrm{b} = 1, \mathrm{c} = 1,\ \mathrm{d} = 1$

2. 已知X的概率密度函数为$f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$，其中$\lambda$是一个常数，则$E\left(\frac{1}{X}\right)$ =(　　).

   The probability density function of X is given by: $f(x) = \begin{cases} \lambda^2 x e^{-\lambda x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$, where λ is a constant. Find $E\left(\frac{1}{X}\right)$ in terms of λ.

   A. λ^2　**B.** $\dfrac{2}{\lambda}$　**C.** λ　**D.** $\dfrac{\lambda}{2}$

3. 设X和Y为联合正态随机变量, 其均值和方差分别为μ_X和σ_X^2, μ_Y和σ_Y^2, 且相关系数为ρ. 如果$W = aX - bY$, 其中a和b是常数. 则W的方差等于(　　)

   Suppose that X and Y are joint normal random variables with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and correlation coefficient ρ. If W = aX − bY, where a and b are constants, what is the variance of W?

   A. $a^2\sigma_X^2 - b^2\sigma_Y^2 - 2ab\rho\sigma_X\sigma_Y$　**B.** $a^2\sigma_X^2 + b^2\sigma_Y^2 - 2ab\rho\sigma_X\sigma_Y$

   C. $a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho\sigma_X\sigma_Y$　**D.** $a^2\sigma_X^2 + b^2\sigma_Y^2 - 2ab\rho$

4. 设$X_1, X_2, ..., X_n, ...$为独立同分布的随机变量序列，且均服从参数为λ的指数分布，记$\Phi(x)$为标准正态分布函数，则根据独立同分布的中心极限定理有(　　)

   Let $X_1, X_2, ..., X_n, ...$ be independent and identically distributed random variables, and all of them follow the exponential distribution with parameter λ. Note that $\Phi(x)$ is the standard normal distribution function, then according to the central limit theorem, we have (　　)

   A. $\lim\limits_{n\to\infty} P\left\{\dfrac{\sum_{i=1}^n X_i - n\lambda}{\lambda\sqrt{n}} \leqslant x\right\} = \Phi(x)$　**B.** $\lim\limits_{n\to\infty} P\left\{\dfrac{\sum_{i=1}^n X_i - n\lambda}{\sqrt{n\lambda}} \leqslant x\right\} = \Phi(x)$

   C. $\lim\limits_{n\to\infty} P\left\{\dfrac{\sum_{i=1}^n X_i - \lambda}{\sqrt{n\lambda}} \leqslant x\right\} = \Phi(x)$　**D.** $\lim\limits_{n\to\infty} P\left\{\dfrac{\lambda\sum_{i=1}^n X_i - n}{\sqrt{n}} \leqslant x\right\} = \Phi(x)$

5. 已知正态分布总体的标准差是5, 但其均值是未知的. 今有原假设$H_0: \mu \leqslant 80$和备择假设$H_1: \mu > 80$. 现在随机抽取4 个数据进行检验, 并将平均值与临界值84.9进行比较. 如果平均值大于等于84.9, 则零假设将被拒绝. 则显著性水平α 的值为(　　)

   A normally distributed population is known to have a standard deviation of 5, but its mean is in question. Now, we have the null hypothesis $H_0: \mu \leqslant 80$ and the alternative hypothesis $H_1: \mu > 80$, which will be tested using four randomly selected data and comparing the mean to the value 84.9. If the average is greater than or equal to 84.9, the null hypothesis will be rejected. Then the value of significance level α is (　　)

   A. 0.01　**B.** 0.025　**C.** 0.05　**D.** 0.1

第二部分　填空题（每空2 分，总共20 分）

Part Two – Blank Filling (2 marks each blank, 20 marks in total)

1. 设A和B为两事件，且$P(A) = 0.4, P(B) = 0.4, P(A|B) = 0.5$，则$P(A \cup B) =$ __________.

   Let A and B be two events with $P(A) = 0.4, P(B) = 0.4$ and $P(A|B) = 0.5$, then $P(A \cup B) =$ __________.

2. 假设有两枚硬币，其中一枚是均匀的，另一枚是有偏差的（总是正面朝上）。现从中随机选一枚硬币，抛掷此硬币一次后，再将其抛掷一次。若两次均为正面朝上，则此枚硬币为有偏差的概率为= __________.

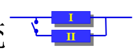
   Suppose that we have a fair coin and a biased coin that always comes up heads. One of the two coins is chosen at random, the coin is flipped once, and then it is flipped again. Suppose that the chosen coin comes up heads both times. The probability that the coin is biased = __________.

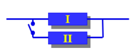
3. 设随机变量X服从参数为λ的泊松分布，若$E(X-1)(X+3) = 1$，则$\lambda =$ __________.

   The random variable X has an Poisson distribution with parameter λ. If $E(X-1)(X+3) = 1$, then $\lambda =$ __________.

4. 随机变量X和Y满足$D(X) = D(Y) = D(X+Y) \neq 0$，则相关系数$\rho_{XY} =$ __________.

   If X and Y are random variables with $D(X) = D(Y) = D(X+Y) \neq 0$, then the correlation $\rho_{XY} =$ __________.

5. 假设在系统  中，原件和备件的寿命都是10，如果原件失效，系统将自动用其备件替代，但替换出错的概率为0.1，则整个系统的寿命为__________.

   In the system , the life of both the original and spare parts is 10. If the original part fails, the system will replace it with its spare part, but the probability of replacement error is 0.1. What is the life of the entire system __________.

6. 设(X, Y)为从方形区域$0 \leq x \leq 2, 0 \leq y \leq 2$内随机均匀抽取的一点，令$Z = \max\{X, Y\}$，则$Z$的累积分布函数在1处的值$F_Z(1) =$ __________.

   Let (X, Y) be a point chosen uniformly at random inside the square $0 \leq x \leq 2, 0 \leq y \leq 2$. Define $Z = \max\{X, Y\}$, the cumulative distribution function (cdf) for Z at point 1 is $F_Z(1) =$ __________.

7. 设$X_1, X_2, \cdots, X_n$是来自均匀分布总体$X \sim U[\theta, 1]$的样本，则未知参数θ的最大似然估计$\hat{\theta} =$ __________.

   Let $X_1, X_2, \cdots, X_n$ be a random sample from a uniformly distributed population $X \sim U[\theta, 1]$, the maximum likelihood estimate of unknown parameter θ is $\hat{\theta} =$ __________.

8. 设$X_1, X_2, \cdots, X_n$为来自标准正态总体$X \sim N(0,1)$的一个样本, 则$c =$ _________ 时，统计量$\dfrac{c(X_1+X_2+X_3+X_4)}{\sqrt{X_5^2+X_6^2}}$服从自由度为2的$t$分布.

   Let $X_1, X_2, \cdots, X_n$ be a random sample from a standard normal population $X \sim N(0,1)$). When $c =$ _________, the statistic $\dfrac{c(X_1+X_2+X_3+X_4)}{\sqrt{X_5^2+X_6^2}}$ has a $t-$distribution with degree of freedom 2.

9. 设$X_1, X_2, \cdots, X_n$为正态总体$N(\mu, \sigma^2)$的一个样本, 则$n(\bar{X}-\mu)^2+(n-1)S^2$的方差为_________.

   Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal population $N(\mu, \sigma^2)$, then the variance of $n(\bar{X}-\mu)^2+(n-1)S^2$ is _________.

10. 设$X_1, X_2, \cdots, X_n$来自总体X的一个样本, 且有$E(X) = \mu$及$D(X) = \sigma^2$。记$\bar{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$, $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$，则$c =$ _________时，$(\bar{X})^2 - cS^2$是未知参数μ^2的无偏估计量.

    Let $X_1, X_2, \cdots, X_n$ be a random sample from population X with $E(X) = \mu$ and $D(X) = \sigma^2$. Define $\bar{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$ and $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$. When $c =$ _________, $(\bar{X})^2 - cS^2$ is an unbiased estimate of the unknown parameter μ^2.

第三部分　解答题（每题10分，总共60 分）
Part Three–Question Answering (10 marks each question, 60 marks in total)

1. 已知某班有90%的学生考试及格, 10%的学生不及格. 经发现在考试及格的学生中有90%按时交作业, 而在不及格的学生中只有10%按时交作业. 现从中随机抽取一位学生.

(1) 求抽到的这位学生是按时交作业的概率;

(2) 若已知抽到的这位学生是按时交作业的, 求他考试及格的概率.

In a class, 90% students passed the exam and 10% students failed. It was found that among the students who passed the exam, 90% delivered the homework on time; and among the students who failed, only 10% delivered the homework on time. Now, pick a student from the class randomly.

(1) Calculate the probability that the student delivered the homework on time;

(2) If the student delivered the homework on time, calculate the probability that he/she passed the exam.

2. 设随机变量(X,Y)的联合频率函数为

$X\backslash Y$	1	2	3
0	0.1	0.1	0.2
1	0.1	0.2	0.3

. 求以下三个问题:

(1) 求(X,Y)的边际频率函数并判断X与Y是否相互独立;

(2) 求$Z=\mathrm{Max}(X,Y)$的频率函数; (3) 求Z的方差$D(Z)$.

Suppose the joint frequency function of (X,Y) is

$X\backslash Y$	1	2	3
0	0.1	0.1	0.2
1	0.1	0.2	0.3

.

(1) Find the marginal frequency function of X and Y, and determine whether they are independent;

(2) Find the frequency function of $Z=\mathrm{Max}(X,Y)$;

(3) Find the variance of Z.

3. 设连续型随机变量X和Y相互独立, 且密度函数分别为$f_X(x)=\begin{cases}1, & \text{for } 0\leqslant x\leqslant 1\\ 0, & \text{otherwise}\end{cases}$ 和$f_Y(x)=\begin{cases}e^{-y}, & \text{for } y>0\\ 0, & \text{otherwise}\end{cases}$. 求以下三个问题:

(1) (X,Y)的联合密度函数$f(x,y)$; (2) 期望$E(XY)$; (3) $Z=X+Y$的密度函数.

Suppose the continuous r.v. X and Y are independent, and their probability density functions are $f_X(x)=\begin{cases}1, & \text{for } 0\leqslant x\leqslant 1\\ 0, & \text{otherwise}\end{cases}$ and $f_Y(x)=\begin{cases}e^{-y}, & \text{for } y>0\\ 0, & \text{otherwise}\end{cases}$, respectively. Please find:

(1) The joint probability density function $f(x,y)$ of (X,Y);

(2) The expectation $E(XY)$;

(3) Let $Z=X+Y$, find the probability density function $f_Z(z)$ of Z.

4. 已知总体X的概率密度为$f(x)=\begin{cases}\frac{2x}{\alpha^2}, & 0\leqslant x\leqslant\alpha,\\ 0, & 其他,\end{cases}$ 其中$\alpha>2$ 未知，$X_1,X_2,\cdots,X_n$ 是来自总体X 的随机样本. 求以下两个问题:

(1) 求α 的矩估计$\hat{\alpha}_M$ 与极大似然估计$\hat{\alpha}_{MLE}$; (2) 已知$k\hat{\alpha}_{MLE}$ 是α 的无偏估计，求k.

Suppose the probability density function of the population X is: $f(x) = \begin{cases} \frac{2x}{\alpha^2}, & 0 \leqslant x \leqslant \alpha, \\ 0, & \text{otherwise}, \end{cases}$ where $\alpha > 2$ is unknown, and $X_1, X_2, \cdots, X_n$ are random samples from the population X.

(1) Find the Moment estimator $\hat{\alpha_M}$ and the Maximum Likelihood estimator $\hat{\alpha}_{MLE}$ for α.

(2) Given that $k\hat{\alpha}_{MLE}$ is an unbiased estimator for α, find the value of k.

5. 设某机床加工的零件的长度$X \sim N(\mu, \sigma^2)$,今抽查16个零件长度(单位：mm)，测得相关的样本均值和样本方差为: $\bar{x} = 12,\ s^2 = 4$.

(1) 求σ^2 的置信度为95% 的置信区间 (结果精确到小数点后两位);

(2) 在5% 的显著性水平下，能否认为该机床加工的零件长度为13mm.

Given that the length of the parts produced by a machine follows a normal distribution $X \sim N(\mu, \sigma^2)$. The sample of 16 parts (unit: mm) was collected, and their length data was obtained as follows: $\bar{x} = 12,\ s^2 = 4$.

(1) Find the confidence interval for σ^2 with a confidence level of 95% (The result is accurate to two decimal places);

(2) At a significance level of 5%, can we claim that the length of parts processed by the machine is 13 mm?

6. 根据数据显示用旧安眠药A时平均睡眠时间为20.8小时，标准差为1.6小时。今有一种新安眠药B，据说在一定剂量下能达到新的疗效：比旧安眠药A的睡眠时间多于3小时。为了验证这个说法是否正确，今收集到一组使用新安眠药B的睡眠时间为：$22.7, 23.7, 24.7, 25.7$ (小时). 请问：从这组数据能否说明新安眠药B达到了新的疗效? (假定睡眠时间服从正态分布，$\alpha = 0.05$)
According to the data, when the old sleep medicine A is used, the average sleeping duration is 20.8 hours with a standard deviation of 1.6 hours. Now, there is a new sleep medicine that is claimed to have a new therapeutic effect at a certain dosage: more than 3 hours of sleeping time is obtained compared to an old sleep medicine A. In order to verify this claim, the sleeping durations by using the new sleep medicine were collected as follows: $22.7, 23.7, 24.7, 25.7$ (hours).

Can this data set demonstrate that the new sleep medicine achieves the new therapeutic effect? (Assume sleeping durations follow a normal distribution, with $\alpha = 0.05$).

答案

一、选择题

A C B D B

二、填空题

1. 0.6

2. $\frac{4}{5}$

3. 1

4. $-\frac{1}{2}$

5. 19

6. $F_Z(z) = \begin{cases} 0, & z < 0 \\ z^2/4, & 0 \le z \le 2 \\ 1, & z > 2 \end{cases} \longrightarrow F_Z(1) = \frac{1}{4}$

7. $\min\{X_1, X_2, \cdots, X_n\}$ 或者 $X_{(1)}$

8. $c = \sqrt{2}/2$

9. $2n\sigma^4$

10. $c = \frac{1}{n}$

1. 已知某班有90%的学生考试及格, 10%的学生不及格. 经发现在考试及格的学生中有90%按时交作业, 而在不及格的学生中只有10%按时交作业. 现从中随机抽取一名学生.

(1) 求抽到的这位学生是按时交作业的概率;

(2) 若已知抽到的这位学生是按时交作业的, 求他考试及格的概率.

[分析] 考察全概率公式和贝叶斯概率公式.

[解析] 设 A =“学生考试及格”， B =“学时按时交作业”，

则 $P(A)=0.9, P(\bar{A})=0.1, P(B\mid A)=0.9, P(B\mid \bar{A})=0.1$ 2'

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(1) 由全概率公式 $P(B)=P(A)P(B\mid A)+P(\bar{A})P(B\mid \bar{A})$ $=0.9\times0.9+0.1\times0.1=0.82$ 3' 1'

(2) 由贝叶斯公式 $P(A\mid B)=\dfrac{P(A)P(B\mid A)}{P(B)}=\dfrac{0.81}{0.82}\approx 0.99$ 3' 1'

2. 设随机变量(X,Y)的联合频率函数为

$X\backslash Y$	1	2	3
0	0.1	0.1	0.2
1	0.1	0.2	0.3

. 求以下三个问题:

(1) 求(X,Y)的边际频率函数并判断X与Y是否相互独立;

(2) 求$Z = \text{Max}(X,Y)$的频率函数; (3) 求Z的方差$D(Z)$.

解：(1) (X,Y)的边缘分布律分别为:

3

X	0	1
P	0.4	0.6

1'

和

Y	1	2	3
P	0.2	0.3	0.5

1'

因为$P\{X=0,Y=1\} \neq P\{X=0\}P\{Y=1\}$ 1'
所以X与Y不独立.

3 (2) $Z = \text{Max}(X,Y)$的分布律:

$Z = \text{Max}(X,Y)$	1	2	3
P	0.2	0.3	0.5

3'

4 (3) 可求得Z^2的频率函数为

Z^2	1	4	9
P	0.2	0.3	0.5

1'

且有

$$EZ = 2.3 \quad 1'$$

$$E(Z^2) = 5.9 \quad 1'$$

从而

$$DZ = E(Z^2) - \{EZ\}^2 = 0.61 \quad 1'$$

3. 设连续型随机变量X和Y相互独立，且密度函数分别为$f_X(x) = \begin{cases} 1, & \text{for } 0 \leqslant x \leqslant 1 \\ 0, & \text{otherwise} \end{cases}$ 和$f_Y(x) = \begin{cases} e^{-y}, & \text{for } y > 0 \\ 0, & \text{otherwise} \end{cases}$. 求:

3 (1) (X, Y)的联合密度函数;

3 (2) 期望$E(XY)$;

4 (3) $Z = X + Y$的密度函数.

[解析] (1) 因X 与Y 独立, 故(X, Y) 的联合概率密度

$$f(x, y) = f_X(x) f_Y(y) = \begin{cases} e^{-y}, & \text{for } 0 \leqslant x \leqslant 1,\ y > 0 \\ 0, & \text{otherwise} \end{cases}$$ 3'

(2)

$$\begin{aligned} E(XY) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f(x, y) dx dy \\ &= \int_0^{\infty} \int_0^1 xy e^{-y} dx dy = \int_0^1 x dx \int_0^{\infty} y e^{-y} dy \\ &= \end{aligned}$$ 3'

(3) X 与Y 独立, 由卷积公式$Z = X + Y$ 的概率密度

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x) \cdot f_Y(z - x) dx$$ 1'

被积函数不为0 的区域为:
$0 \leq x \leq 1$ 且$y = z - x > 0$, 即$0 \leq x \leq 1$ 且$x < z$, 故

$$f_Z(z) = \begin{cases} 0, & z \leq 0, \\ \int_0^z e^{-(z-x)} dx = 1 - e^{-z}, & 0 < z \leq 1, \\ \int_0^1 e^{-(z-x)} dx = e^{1-z} - e^{-z}, & z > 1. \end{cases}$$ 3'

4. 已知总体X 的概率密度为

$$f(x)=\begin{cases}\frac{2x}{\alpha^2}, & 0\leqslant x\leqslant \alpha,\\ 0, & 其他,\end{cases}$$

其中$\alpha>2$ 未知，$X_1, X_2, \cdots, X_n$ 是来自总体X 的随机样本。

1. 求α 的矩估计$\hat{\alpha}_M$ 与极大似然估计$\hat{\alpha}_{MLE}$;
2. 已知$k\hat{\alpha}_{MLE}$ 是α 的无偏估计，求k。

(a)

$$EX=\int_0^\alpha \frac{2x^2}{\alpha^2}\mathrm{d}x=\frac{2\alpha}{3},$$

由矩法方程$EX=\bar{X}$，即$\frac{2\alpha}{3}=\bar{X}$，得到$\alpha$ 的矩估计为$\hat{\alpha}_M=\frac{3\bar{X}}{2}$

$$\begin{aligned}L(\alpha)&=f(x_1)f(x_2)\cdots f(x_n)\\ &=\frac{2^n}{\alpha^{2n}}x_1x_2\cdots x_n I\{0\leqslant x_1,\cdots,x_n\leqslant\alpha\}\\ &=\frac{2^n}{\alpha^{2n}}x_1x_2\cdots x_n I\{0\leqslant x_{(1)}\leqslant x_{(n)}\leqslant\alpha\}\end{aligned}$$

5'

显然$L(\alpha)$ 关于α 单调减少，且$\alpha\geqslant x_{(n)}$，其中$x_{(n)}=max\{x_1,\cdots,x_n\}$ 为顺序统计量，则α 的最大似然估计量为$\hat{\alpha}_{MLE}=X_{(n)}$。

(b)考虑顺序统计量$X_{(n)}$的分布函数：

$$F_{X_{(n)}}(x)=P(X_{(n)}\leqslant x)=P^n(X\leqslant x)=F^n(x)$$

1'

$$求导得到: f_{X_{(n)}}(x)=nF^{n-1}(x)f(x)$$

$$F(x)=\int_0^x\frac{2s}{\alpha^2}\mathrm{d}s=\frac{x^2}{\alpha^2}$$

1'

$$f_{X_{(n)}}(x)=n\frac{x^{2(n-1)}}{\alpha^{2(n-1)}}\frac{2x}{\alpha^2}=2n\frac{x^{2n-1}}{\alpha^{2n}}$$

1'

所以

$$E(X_{(n)})=\int_0^x 2n\frac{x^{2n}}{\alpha^{2n}}\mathrm{d}x=\frac{2n}{2n+1}\alpha$$

1'

又

$$E(kX_{(n)})=\alpha$$

得到

$$k=\frac{2n+1}{2n}$$

1'

5. 设某机床加工的零件的长度$X \sim N\left(\mu, \sigma^2\right)$,今抽查16个零件长度(单位：mm)，测得相关的样本均值和样本方差为:

$$\bar{x}=12,\ s^2=4.$$

(1) 求σ^2 的置信度为95% 的置信区间；

(2) 在5% 的显著性水平下，能否认为该机床加工的零件长度为13mm。

(1)

$$\bar{x}=12,\ s^2=4,\ s=2,\ 1-\alpha=0.95,\ n=16$$

由于均值μ 未知，故选取统计量

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$P\{\chi^2_{\frac{\alpha}{2}}(n-1)<\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)<\chi^2_{1-\frac{\alpha}{2}}(n-1)\}=1-\alpha$$

查表得知：$\chi^2_{0.975}(15)=27.49,\ \chi^2_{0.025}(15)=6.26,$
从而置信区间为：

$$\begin{aligned}
&\left[\frac{(n-1)\,S^2}{\chi^2_{1-\frac{\alpha}{2}}\,(n-1)}, \frac{(n-1)\,S^2}{\chi^2_{\frac{\alpha}{2}}\,(n-1)}\right] \\
=&\left[\frac{(n-1)\,s^2}{\chi^2_{0.975}\,(15)}, \frac{(n-1)\,s^2}{\chi^2_{0.025}\,(15)}\right] \\
=&\left[\frac{15\times 4}{27.49}, \frac{15\times 4}{6.26}\right] \\
=&\left[\frac{60}{27.49}, \frac{60}{6.26}\right]=[2.18, 9.58]
\end{aligned}$$

6

(2) 本问题是方差未知的条件下，对均值$\mu=13$的假设检验，故

(i) $H_0: \mu=13, H_1: \mu \neq 13, \alpha=0.05, n=16$

(ii) $T=\frac{\bar{X}-13}{S/\sqrt{n}} \sim t\,(n-1)$

(iii) H_0 的拒绝域为

$$|T|=\left|\frac{\bar{X}-13}{S/\sqrt{16}}\right| \geq t_{0.975}\,(15)=2.13$$

4

(vi) 所以

$$\left|t\right|=\left|\frac{\bar{x}-13}{s/\sqrt{16}}\right|=\left|\frac{12-13}{2/4}\right|=2<2.13.$$

所以不拒绝H_0，即有95%的把握认为该机床加工的零件长度为13mm

6. 有一种新安眠药，据说在一定剂量下能达到新的疗效：比某种旧安眠药A的睡眠时间多于3小时。根据资料,用旧安眠药A时平均睡眠时间为20.8 小时，标准差为1.6小时，为了验证这个说法是否正确，收集到一组使用新安眠药的睡眠时间为：$25.7, 22.7, 24.7, 23.7$.(小时)

试问：从这组数据能否说明新安眠药达到新的疗效?(假定睡眠时间服从正态分布，$\alpha = 0.05$)

解： 可知$n = 4$, 并经计算可得$\bar{x} = 24.2$ 1'

(i) $H_0 : \mu \leq 23.8,\ \ H_1 : \mu > 23.8$ 2'

(ii) $U = \frac{\bar{X} - 23.8}{\sigma/\sqrt{n}} \sim N(0, 1)$ 2'

(iii) 查表得到$u_{0.95} = 1.65$
从而拒绝域为$(u_{0.95}, +\infty) = (1.65, +\infty)$ 2'

(vi) 计算得到

$$\begin{aligned} U &= \frac{\bar{X} - 23.8}{\sigma/\sqrt{n}} \\ &= \left| \frac{24.2 - 23.8}{1.6/\sqrt{4}} \right| \\ &= \cancel{0.48}\ 0.5 < 1.65 \end{aligned}$$

2'

显然不在拒绝域内，因此不拒绝H_0，即以95%的把握认为新安眠药达不到新的疗效。 1'