Real Analysis Midterm Exam 2025 Fall

Question 1

Show the following claims. Let $E \subset [0,1]$ be a measurable set.

1. If $m(E) = 1$, then $\bar{E} = [0,1]$.
2. If $m(E) = 0$, then $E^\circ = \emptyset$.

Question 2

Prove that a unit sphere has measure zero in $\mathbb{R}^n$.

Question 3

1. Prove: $E \subset \mathbb{R}^d$, E is measurable if and only if for any $\epsilon > 0$, there exists measurable sets A, B such that $A \subset E \subset B$ and $m(B \setminus A) < \epsilon$.
2. Let $A, B \subset \mathbb{R}^n$. Suppose that $A \subset B$, and A is measurable. If $m(A) = m_*(B) < \infty$, then show that B is also measurable.

Question 4

1. State the definition of rectangle.
2. State the definition of exterior measure.
3. Prove: for cube Q, $m_*(Q) = Q$ holds.

Question 5

Let $A, B \subset \mathbb{R}^n$ with $m_*(A), m_*(B) < \infty$. Then $m_*(A \cup B) = m_*(A) + m_*(B)$ if and only if there exist measurable sets E, F such that $A \subset E$, $B \subset F$, and $m(E \cap F) = 0$.