Incorporating Quantum Gravity Effects into Einstein's Field Equations at Microscopic Scales

Sir Hrishi Mukherjee I

May 3, 2024

1 Introduction

In this document, we explore the conceptual approach to incorporating quantum gravity effects into Einstein's field equations at microscopic scales. We focus on setting up the equations and providing a qualitative understanding, as obtaining numerical solutions requires advanced mathematical formalisms beyond the scope of this document.

2 Gravitational Field Equation

The gravitational field equation in General Relativity is given by:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

where:

- $R_{\mu\nu}$ is the Ricci curvature tensor,
- R is the scalar curvature,
- $g_{\mu\nu}$ is the metric tensor,
- Λ is the cosmological constant,
- G is the gravitational constant, and
- c is the speed of light.

3 Parameters at Microscopic Scales

To consider microscopic scales, we focus on the following parameters:

- **Mass (M):** Consider the mass of a subatomic particle, such as an electron, $M \approx 9.109 \times 10^{-31}$ kg.

- **Gravitational Constant (G):** $G = 6.674 \times 10^{-11}$ m^3/kg/s^2.
- **Speed of Light (c):** $c = 3 \times 10^8$ m/s.
- **Cosmological Constant (Λ):** For simplicity, assume $\Lambda \approx 0$ at microscopic scales.

4 Equation Setup

Plugging the values into the gravitational field equation, we have:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \approx \frac{-2GM}{c^2}g_{\mu\nu}$$

5 Conclusion

While we can conceptually set up the equations to incorporate quantum gravity effects at microscopic scales, obtaining numerical solutions requires advanced mathematical formalisms such as loop quantum gravity. Further research is needed to explore these effects quantitatively.