# Derivation of Energy Eigenvalues for a Particle in a Square Well Potential

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## 1 Introduction

In quantum mechanics, the behavior of particles in potential wells is described by the time-independent Schrödinger equation. Let's consider a one-dimensional square well potential with width $L$, where the potential energy is zero inside the well ($V(x) = 0$) and infinite outside ($V(x) = \infty$ for $x < 0$ and $x > L$).

## 2 Derivation of Energy Eigenvalues

The time-independent Schrödinger equation for this system is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

Inside the well ($0 < x < L$), where $V(x) = 0$, this simplifies to:

$$\frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{\hbar^2}\psi(x)$$

The general solution to this differential equation involves sine and cosine functions due to the boundary conditions $\psi(0) = 0$ and $\psi(L) = 0$. The eigenfunctions are given by:

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

The corresponding energy eigenvalues are derived as follows:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_n(x)}{dx^2} = E_n\psi_n(x)$$

$$\frac{n^2\pi^2\hbar^2}{2mL^2} = E_n$$

Therefore, the energy eigenvalues for the particle in the square well potential are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

where $n = 1, 2, 3, \ldots$ is the quantum number representing different energy levels.

## 3 Conclusion

The quantization of energy levels in a square well potential illustrates the discrete nature of energy in quantum mechanics. Understanding these energy eigenvalues and eigenfunctions is crucial in analyzing the behavior of particles in confined systems.