Quantum Tunneling in a Confined Space

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1 Introduction

Quantum tunneling is a phenomenon in quantum mechanics where a particle passes through a potential barrier that it classically shouldn't be able to overcome. In this document, we'll consider the specific case of a particle confined within a cube, encountering a potential barrier at one end.

2 Mathematical Formulation

The time-independent Schrödinger equation for the particle within the cube is given by:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = E\Psi \tag{1}$$

with the boundary conditions:

$$\Psi(0) = \Psi(L) = 0 \tag{2}$$

Inside the cube ($0 < x < L$), the potential energy $V(x)$ is zero. Therefore, the solution to the Schrödinger equation is a linear combination of sines:

$$\Psi(x) = A\sin(kx) \tag{3}$$

where $k = \sqrt{\frac{2mE}{\hbar^2}}$. (For detailed derivation, see [**?**])

Outside the cube, the potential energy is infinite, so the wave function decays exponentially:

$$\Psi(x) = Be^{-\alpha x} \tag{4}$$

where $\alpha = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$.

Applying the boundary conditions gives us:

$$A = 0 \tag{5}$$

$$\alpha L = n\pi \tag{6}$$

where n is an integer.

The transmission coefficient T is given by:

$$T = \frac{\sqrt{E(V_0 - E)}}{E} \tag{7}$$

This equation describes the probability of the particle tunneling through the potential barrier. (For more information on quantum tunneling, see [**?**])

3 Conclusion

In this document, we've analyzed the quantum tunneling of a particle confined within a cube encountering a potential barrier. The transmission coefficient T quantifies the probability of tunneling, which depends on the energy of the particle and the height of the potential barrier.