

Quantum Communication via Closed Timelike Curves: A Hypothetical Scenario

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1 Introduction

In this document, we explore a hypothetical scenario where closed timelike curves (CTCs) are incorporated into the transmission process for quantum communication. We will mathematically describe the encoding, transmission, and transformation of quantum information as it traverses CTCs in spacetime.

2 Encoding with Qubits

Let q_i denote the qubit encoding the i -th element of the Quantum Encoding Matrix (QEM). The qubits are encoded as follows:

- q_1 : Encodes $Q_{1,1} = 1$ as $|0\rangle$
- q_2 : Encodes $Q_{1,2} = 0$ as $|1\rangle$
- q_3 : Encodes $Q_{2,1} = 0$ as $|0\rangle$
- q_4 : Encodes $Q_{2,2} = -1$ as $|-1\rangle$

3 Transmission through Closed Timelike Curves

The encoded qubits are transmitted through closed timelike curves (CTCs) in spacetime. Let $CTC(t, x, y, z)$ represent the equation of the CTCs in spacetime.

Let's duplicate the Gott time machine metric equation on both sides:

$$-dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

This equation represents the duplication of the Gott time machine metric equation on both sides of the equation, indicating that both sides are equal to each other. In other words, the left-hand side (LHS) of the equation is equal to the right-hand side (RHS), implying that the spacetime interval described by

the metric is invariant under duplication. However, it's important to note that this duplication is a mathematical operation and does not imply any physical change in the underlying spacetime structure.

The equation for the metric describing a spacetime with closed timelike curves (CTCs) can vary depending on the specific solution or scenario being considered. One example of a spacetime metric that permits closed timelike curves is the Gott time machine metric, proposed by physicist J. Richard Gott III.

The Gott time machine metric can be expressed mathematically using the line element ds^2 as follows:

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

In this equation: - ds^2 represents the spacetime interval or line element. - dt , dr , $d\theta$, and $d\phi$ are differentials representing increments in time, radial distance, and angular coordinates, respectively. - r represents radial distance from the center of rotation. - θ and ϕ represent angular coordinates.

This metric describes a rotating cosmic string that could permit closed time-like curves to form around it. However, it's important to note that the physical interpretation and feasibility of CTCs described by such metrics are highly speculative and subject to ongoing research and debate in theoretical physics.

Closed timelike curves (CTCs) are theoretical solutions to Einstein's field equations in general relativity that would allow for closed paths through spacetime, effectively enabling the possibility of time travel. While CTCs are theoretically allowed by certain solutions to Einstein's equations, their physical implications are highly speculative and subject to significant debate within the scientific community.

Mathematically, CTCs can be described using the metric tensor $g_{\mu\nu}$ in the context of general relativity. A spacetime containing CTCs would have a metric that permits closed timelike paths. However, constructing a specific mathematical representation of CTCs involves specifying a metric tensor that satisfies Einstein's field equations under conditions that permit closed timelike curves.

One example of a spacetime metric that could give rise to CTCs is the Gott time machine, proposed by physicist J. Richard Gott III. The Gott time machine metric is a solution to the Einstein field equations that describes a rotating cosmic string. The metric has the property that it allows closed timelike curves to form around the cosmic string.

The Gott time machine metric can be expressed mathematically using the line element ds^2 as follows:

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

where t represents time, r represents radial distance from the center of rotation, and θ and ϕ represent angular coordinates. This metric describes a rotating cosmic string that could, in principle, permit closed timelike curves to form around it.

However, it's important to note that the physical interpretation and feasibility of CTCs, as described by such metrics, are highly speculative and raise numerous paradoxes and issues, including the potential for violating causality and the chronology protection conjecture proposed by physicist Stephen Hawking. Therefore, while CTCs can be described mathematically within the framework of general relativity, their physical implications remain uncertain and subject to ongoing research and debate in theoretical physics.

4 Transformation along CTC

The transformation T applies to the qubits as they traverse the CTCs:

$$T(|q_1\rangle \otimes |q_2\rangle \otimes |q_3\rangle \otimes |q_4\rangle) = T(|q_1\rangle \otimes |q_2\rangle \otimes |q_3\rangle \otimes |q_4\rangle)$$

The transformation matrix T has the general form:

$$T = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

After applying the transformation T , the qubits are transformed to new states $|q'_i\rangle$:

$$\begin{aligned} |q'_1\rangle &= a|q_1\rangle + b|q_2\rangle + c|q_3\rangle + d|q_4\rangle \\ |q'_2\rangle &= e|q_1\rangle + f|q_2\rangle + g|q_3\rangle + h|q_4\rangle \\ |q'_3\rangle &= i|q_1\rangle + j|q_2\rangle + k|q_3\rangle + l|q_4\rangle \\ |q'_4\rangle &= m|q_1\rangle + n|q_2\rangle + o|q_3\rangle + p|q_4\rangle \end{aligned}$$

5 Conclusion

This document provides a hypothetical mathematical framework for incorporating closed timelike curves into the transmission process for quantum communication. While speculative, it allows us to explore the concept mathematically, disregarding its alignment with established physics principles.