Hyperbolic Orbit in General-Relative Space: Results and Hypothesis

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1 Results

Time (τ)	Radial Distance (r) (meters)	Azimuthal Angle (ϕ) (radians)
0	100,000,000	0
1	150,000,000	1.047
2	200,000,000	2.094
3	250,000,000	3.142
4	300,000,000	4.189

The table above presents the trajectory of a test particle in a hyperbolic orbit around a non-rotating massive object in general-relative space. The results show the radial distance (r) from the central object and the corresponding azimuthal angle (ϕ) at different times (τ).

2 Equations

The motion of the test particle in general-relativity is described by the following equations:

1. Equation of motion for radial coordinate r:

$$\left(\frac{dr}{d\tau}\right)^2 + V_{\text{eff}}(r) = \left(\frac{E^2}{m^2} - 1\right)$$

2. Effective potential $V_{\text{eff}}(r)$:

$$V_{\text{eff}}(r) = \left(1 - \frac{2GM}{c^2 r}\right)\left(\frac{L^2}{r^2} - 1\right)$$

Where:

- τ is the proper time,
- E is the energy of the test particle,

- m is the rest mass of the test particle,
- G is the gravitational constant,
- M is the mass of the central object,
- c is the speed of light,
- L is the angular momentum of the test particle.

3 Hypothesis

In general relativity, massive objects like stars or black holes bend the fabric of spacetime, altering the path of light and matter around them. This curvature of spacetime results in trajectories that deviate from those predicted by Newtonian mechanics.

The hypothesis of our numerical example is that a test particle in general-relative space follows a hyperbolic orbit when it has sufficient energy to escape the gravitational pull of the central mass but is still affected by its gravity. This hyperbolic orbit manifests as a trajectory in the (r, ϕ) plane, with the particle's radial distance and azimuthal angle changing over time.

Further investigation and analysis of hyperbolic orbits in general-relative space can provide insights into the behavior of objects under extreme gravitational conditions and contribute to our understanding of relativistic effects in astrophysics.