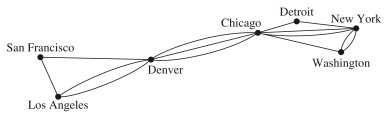
Ch10.1 Graphs and Graph Models (Week 13)

1 A *graph* $G = (V, E)$ consists of V, a nonempty set of *vertices* (or *nodes*) and E, a set of *edges*. Each edges has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.

Simple Graph

- A graph in which each edge connects two different vertices and no two edges connect the same pair of vertices



Multigraphs

Multigraphs

- Graphs that may have multiple edges connecting the same vertices
- *Loops*: edges that connect to a vertex to itself
- *Pseudographs*: graph that may include loops, multiple edges connecting the same pair of vertices

Simple Graph

2 A *directed graph* (or *digraph*) (V, E) consists of a nonempty set of vertices V and a set of *directed edges* (or *arcs*) E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair $\{u, v\}$ is said to *start* at u and *end* at v.

- **Simple directed graph:** a directed graph has no loops and no multiple directed edges
- **Directed multigraphs:** directed graphs that may have multiple directed edges from a vertex to a second one
- **Mixed graph:** a graph with both directed and undirected edges

TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

- Are the edges of the graph undirected or directed (or both)?
- If the graph is undirected, are multiple edges present that connect the same pair of vertices? If the graph is directed, are multiple directed edges present?
- Are loops present?

Three key questions to understand the stucture of a graph

Graph Models

- **Social Networks**

  Ex. Acquaintanceship and Friendship Graphs, Influence Graphs, Collaboration Graphs

- **Communication Networks**

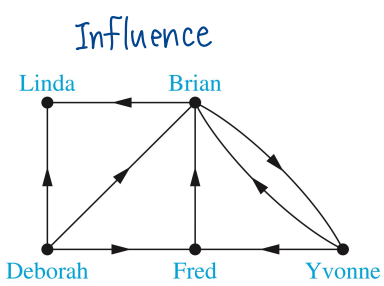
  Ex. Call Graphs

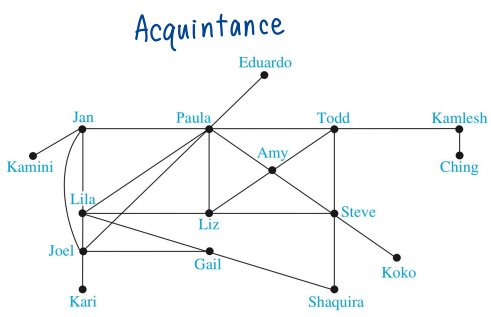
- **Information Networks**

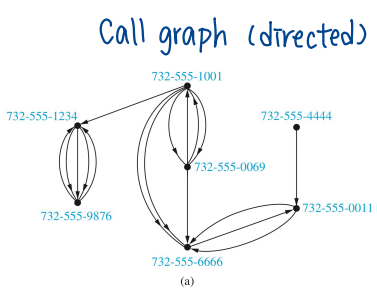
  Ex. The Web Graph, Citation Graphs

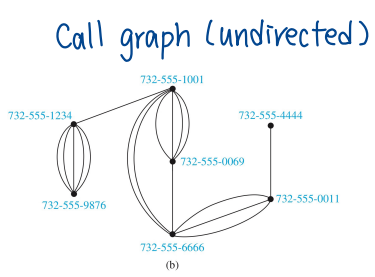
- **Software Design Applications**

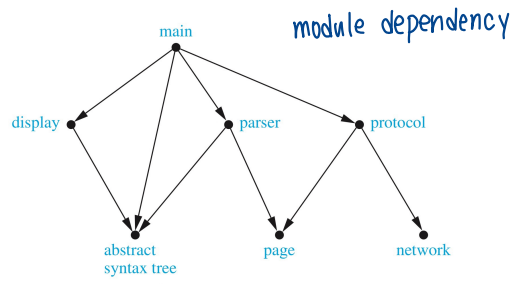
  Ex. Module Dependency Graphs







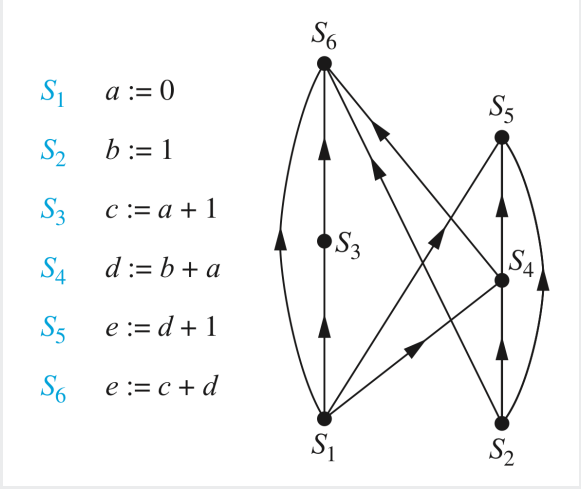




▼ Example 8 **Precedence Graphs and Concurrent Processing**

Computer programs can be executed more rapidly by executing certain statements concurrently. It is important not to execute a statement that requires of statements not yet executed.

For instance, S_5 cannot be executed before S_1, S_2, and S_4 are executed.



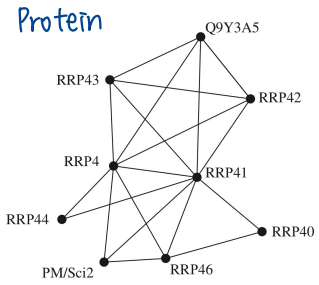
A Precedence Graph

- **Transportation Networks**

  Ex. Airline Routes, Road Networks

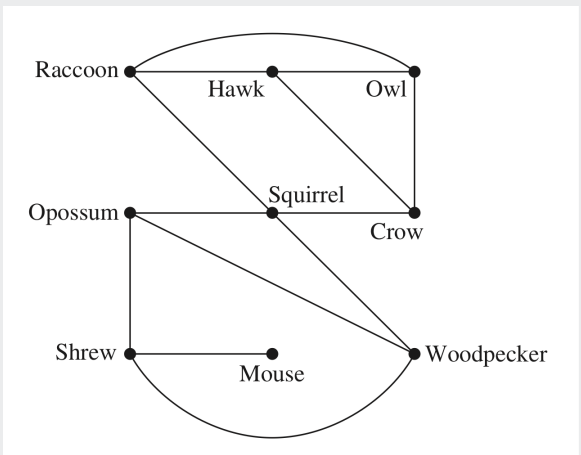
- **Biological Networks**

  Ex. Protein Interaction Graphs



▼ Example 11 **Niche Overlap Graphs in Ecology**

The competition between species in an ecosystem, an undirected edge connects two vertices if the two species compete (meaning some of the food sources are the same). A niche overlap graph is a simple graph because no loops or multiple edges are needed in this model.



The Niche Overlap Graph

- **Tournaments**

  Ex. Round-Robin Tournaments, Single-Elimination Tournaments