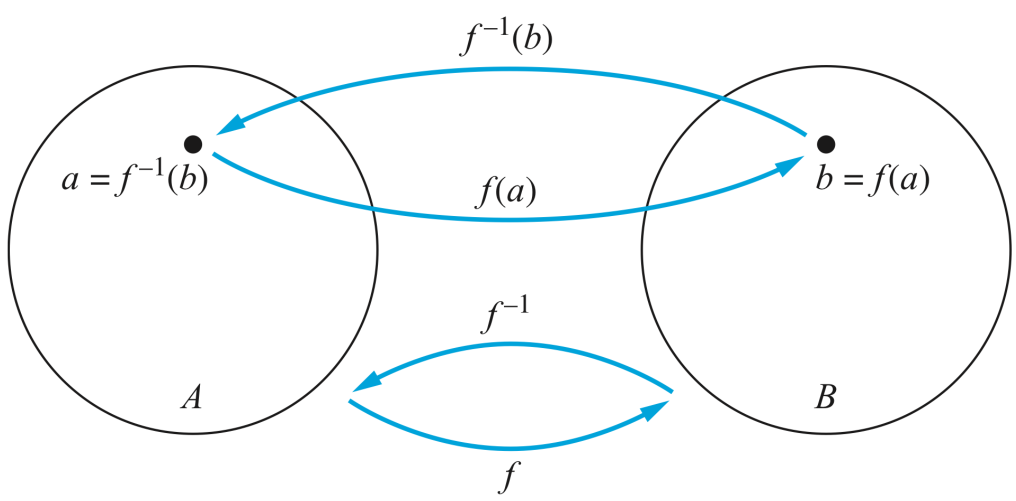
Ch2.3 part 2

Inverse Functions and Compositions of Functions

> **9** Let f be a one-to-one correspondence from the set A to the set B. The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1}. Hence, $f^1(b) = a$ when $f(a) = b$.

- Not to confuse f^{-1} with $1/f$.

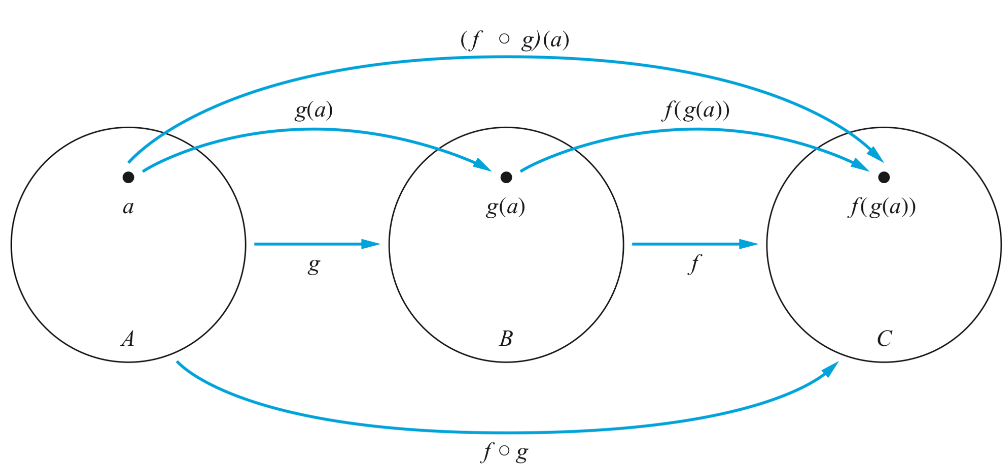


The function f^{-1} is the inverse of function f.

- If a function f is not a one-to-one correspondence, we cannot define an inverse function of f.
- **invertible**: Can define an inverse of this function.
- **not invertible:** The inverse of such a function does not exist.

> **10** Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The *composition* of the functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$.

- The composition $f \circ g$ cannot be defined unless the range of g is a subset of the domain of f.



The composition of the functions f and g.

- $f \circ g$ and $g \circ f$ are not equal.

- When the composition of a function and its inverse is formed, an identity function is obtained.

  $(f^{-1} \circ f)(a) = f^{-1}(f(a)) = f^{-1}(b) = a,$

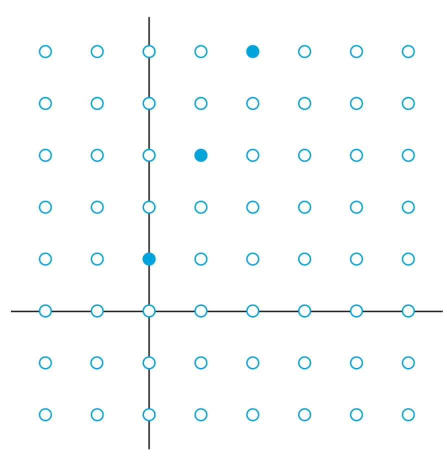
  and

  $(f \circ f^{-1})(b) = (f(f^{-1}(b)) = f(a) = b.$

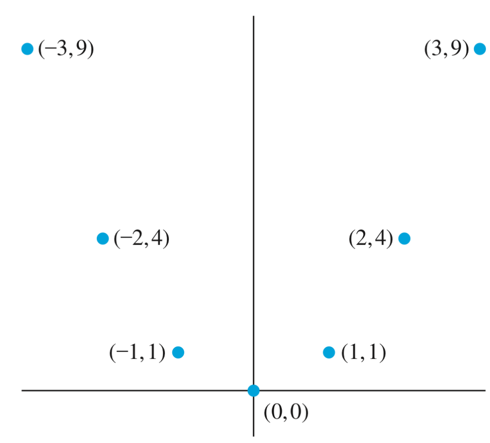
  $\Rightarrow f^{-1} \circ f = \iota_A$ and $f \circ f^{-1} = \iota_B$ where ι_A and ι_B are identity functions.

The Graphs of Functions

11 Let f be a fucntion from the set A to the set B. The *graph* of the function f is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } f(a) = b\}$.



The graph of $f(n) = 2n + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$.



The graph of $f(x) = x^2$ from $\mathbb{Z}$ to $\mathbb{Z}$.

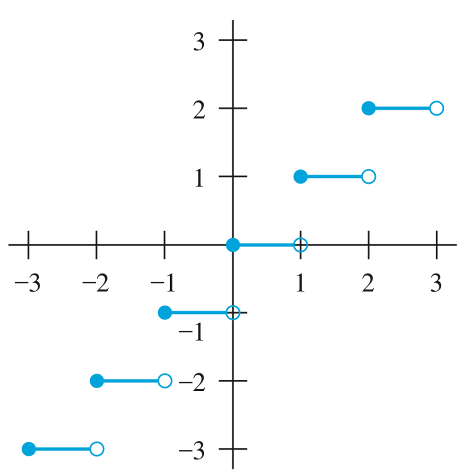
Some Important Functions

12 The *floor function* assigns to the real number x the largest integer that is less than or equal to x. The value of the floor function at x is denoted by $\lfloor x \rfloor$. The *ceiling function* assigns to the real number x the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted by $\lceil x \rceil$.

- The floor function is also called the greatest integer function. Often denoted by $[x]$.

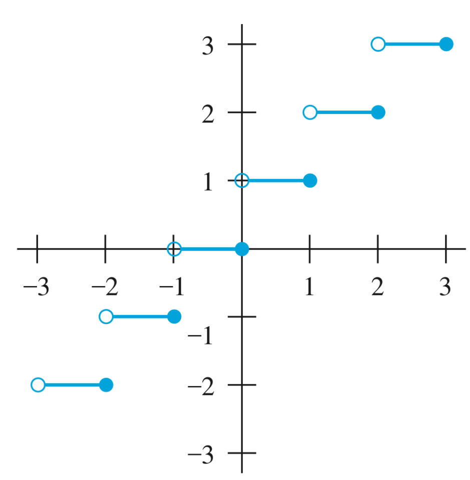
  Examples:

  $\lfloor \frac{1}{2} \rfloor = 0,\ \lceil \frac{1}{2} \rceil = 1,\ \lfloor \frac{-1}{2} \rfloor = -1,\ \lceil \frac{-1}{2} \rceil = 0,\ \lfloor 3.1 \rfloor = 3,\ \lceil 3.1 \rceil = 4,\ \lfloor 7 \rfloor = 7,\ \lceil 7 \rceil = 7.$



(a) $y = [x]$

Graph os (a) Floor Functions



(b) $y = [x]$

Graph of (b) Ceiling Functions

TABLE 1 Useful Properties of the Floor and Ceiling Functions. (n is an integer, x is a real number)	
(1a)	$\lfloor x \rfloor = n$ if and only if $n \leq x < n+1$
(1b)	$\lceil x \rceil = n$ if and only if $n-1 < x \leq n$
(1c)	$\lfloor x \rfloor = n$ if and only if $x-1 < n \leq x$
(1d)	$\lceil x \rceil = n$ if and only if $x \leq n < x+1$
(2)	$x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$
(3a)	$\lfloor -x \rfloor = -\lceil x \rceil$
(3b)	$\lceil -x \rceil = -\lfloor x \rfloor$
(4a)	$\lfloor x+n \rfloor = \lfloor x \rfloor + n$
(4b)	$\lceil x+n \rceil = \lceil x \rceil + n$

Partial Functions

13 A *partial function* f from a set A to a set B is an assignment to each element a in a subset of A, called the *domain of definition* of f, of a unique element b in B. The sets A and B are called the domain and codomain of f, respectively. We say that f is *undefined* for elements in A that are not in the domain of definition of f. When the domain of definition of f equals A, we say that f is a *total function*.

- We write $f : A \to B$ to denote that f is a partial function from A to B. Note that this is the same notation as is used for functions. The context in which the notation is used determines whether f is a partial function or a total function.