DM Final Exam

Part A

1. Let $\mathcal{R}$ be a relation on the set of positive integers with $x\mathcal{R}y \iff xy = 1$.

   Which of the following statements are **Not** True about the relation $\mathcal{R}$?

   Select one or more:

   - $\mathcal{R}$ is transitive $x\mathcal{R}y, y\mathcal{R}z \to x\mathcal{R}z$ ❌ ex. (3, 1/3), (1/3, 3) → (3, 3)
   - $\mathcal{R}$ is reflexive $x\mathcal{R}x$ ❌ other than (1, 1), all others are not
   - $\mathcal{R}$ is anti-symmetric not transitive, all symmetric
   - $\mathcal{R}$ is symmetric $x\mathcal{R}y \to y\mathcal{R}x$ ⭕

2. Let f_1, f_2, f_3, f_4 be four functions defiend as follows:

   $f_1 : \mathbb{Z}^+ \to \mathbb{Z}^+$ with $f_1(x) = x$. only in the first quadrant, both injective and surjective ⭕

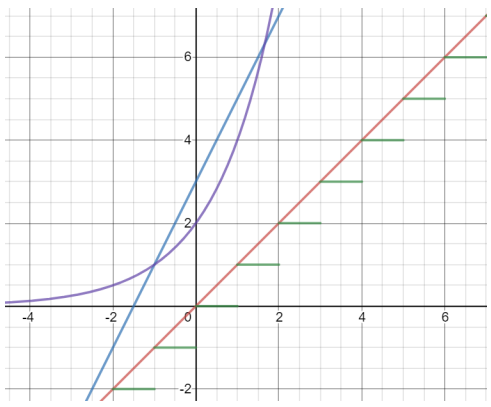
   $f_2 : \mathbb{Z} \to \mathbb{Z}$ with $f_2(x) = 2x + 3$. can't produce all integer outputs, ex. no preimage for 4, 2x+3=4 → 2x=1 → x=...(not an integer) ❌ not surjective

   $f_3 : \mathbb{Z} \to \mathbb{Z}$ with $f_3(x) = \lfloor x \rfloor$. since it's restricted to integers only, it becomes a linear funciton like f(x) = x. ⭕

   $f_4 : \mathbb{R} \to \mathbb{R}^+$ with $f_4(x) = 2^{x+1}$. no matter how small the inputs are, like negative, the outputs are all positive. ⭕

   Which one of these functions is **not** invertible?

   - f_1
   - f_2
   - f_3
   - f_4

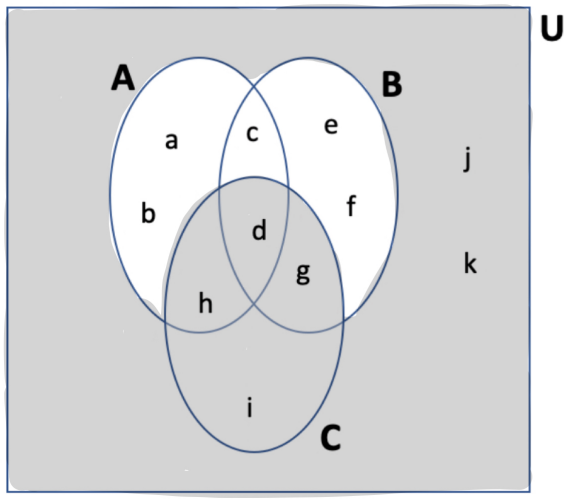


3. Which one of the following degree sequence **cannot** represent a simple graph?

   - 2, 2, 2
   - 3, 3, 3, 3
   - 1, 1, 0, 0
   - 3, 2, 1 a simple graph can't have loops or parallel edges, hence not possible to have a degree 3 with only 3 vertices

4. Given the followign Venn diagram representing three sets A, B, C, subsets of the universal set U:

   Which one of the following sets represents $\overline{(A \cup B) - C}$?



- none of the other options is correct
- $\{d, g, h, i, j, k\}$
- $\{a, b, c, e, f\}$
- $\{d, j, h, i\}$

5. How many strings of length 4 starting with the letter B can be formed using the letters ABCDE if repetitions are **not** allowed?

   Process: B _ _ _ → 1 x 4 x 3 x 2 = 24

   - 24
   - 96
   - 120
   - 625

6. Let p and q be two propositions. Which one of the following logical expressions is equivalent to $\neg(p \to q)$?

   Process: $\neg(p \to q) = \neg(\neg p \vee q) = p \wedge \neg q$

   - $p \wedge \neg q$
   - $\neg p \vee q$
   - $p \vee \neg q$
   - $\neg p \wedge q$

7. What is the number of edges in complete graph K_{10}?

   Process: 10 x 9 (each vertex has 9 connections) = 90 (sum of degree sequence), 90/2 = 45 (# of edges)

   - 100
   - 200
   - 450 probably wrong options but the closest
   - 900

8. Which of the following represent the set:

   $\{1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}, \frac{1}{64}, -\frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}\}$ probably missing a minus in 1/512

   Select one or more:

   - $\{(-1)^n 2^{-n} : n \in \mathbb{Z} \text{ and } 0 \leq n \leq 10\}$
   - $\{(-\frac{1}{2})^n : n \in \mathbb{Z} \text{ and } 0 \leq n \leq 10\}$
   - $\{(-\frac{1}{2})^n : n \in \mathbb{Z} \text{ and } 0 < n \leq 10\}$
   - $\{\frac{(-1)^n}{2n} : n \in \mathbb{Z} \text{ and } 0 \leq n \leq 10\}$

9. Let x be a real number and $P(x)$ be the statement $x > 0$. Select the right statement from the following.

   Select one:

   - P is not a proposition, as its truth value depends on the value of x.
   - The truth value of $P(2)$ is False.

- P can be expressed using propositional logic
- The truth value of $P(-2)$ is True.

10. Which one of the following correctly defines an Eulerian path?

    Select one:

    - An Eulerian path in graph G is a path that visits each vertex exactly once.
    - An Eulerian path in walk in which no edge is repeated.
    - An Eulerian path in graph G is a path that uses each edge in G precisely once.
    - An Eulerian path in a trial in which neither vertices nor edges are repeated.