



Ch 10.5 Euler and Hamilton Paths (Week 13)

Euler Paths and Circuits

1 An *Euler circuit* in a graph G is a simple circuit containing every edge of G . An *Euler path* in G is a simple path containing every edge of G .

- **Path/Walk:** edges travel from vertex to vertex
Simple Path/Trial: walk with no repeated edge (path \rightarrow trail with no repeated vertices)
(when trail is used)
- **Circuit/Cycle:** a path of length > 0 + start = end
Simple Circuit: no repeated edge
- **Euler circuit:**
simple circuit with every edge | start = end + no repeated edge
- **Euler path:**
simple path with every edge | start \neq end + no repeated vertex

1 **Theorem 1** A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree.

2 **Theorem 2** A connected multigraph has an Euler path but not an Euler circuit *iff* it has exactly two vertices of odd degree.

Hamilton Paths and Circuits

2 A simple path in a graph G that passes through every vertex exactly once is called a *Hamilton path*, and a simple circuit in a graph G that passes through every vertex exactly once is called a *Hamilton circuit*. That is, the simple path $x_0, x_1, \dots, x_{n-1}, x_n$ in the graph $G = (V, E)$ is a Hamilton path if $V = \{x_0, x_1, \dots, x_{n-1}, x_n\}$ and $x_i \neq x_j$ for $0 \leq i < j \leq n$, and the simple circuit $x_0, x_1, \dots, x_{n-1}, x_n, x_0$ (with $n > 0$) is a Hamilton circuit if $x_0, x_1, \dots, x_{n-1}, x_n$ is a Hamilton path.

- a graph with a vertex of degree one cannot have a Hamilton circuit
- the more edges a graph has, the more likely it is to have a Hamilton circuit

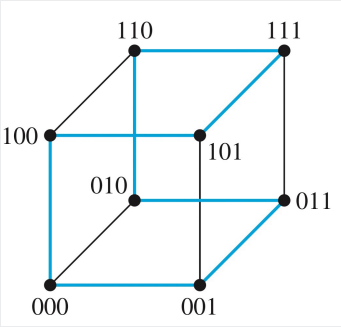
3 **Dirac's Theorem** If G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then G has a Hamilton circuit.

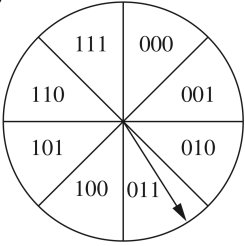
4 **Ore's Theorem** If G is a simple graph with n vertices with $n \geq 3$ such that $\deg(u) + \deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G , then G has a Hamilton circuit.

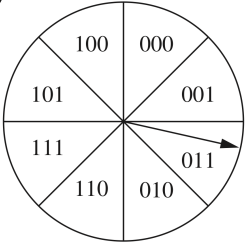
Applications of Hamilton Circuits

▼ Example 8

Gray Codes: a labeling of the arcs of the circle such that adjacent arcs are labeled with bit strings that differ in exactly one bit.

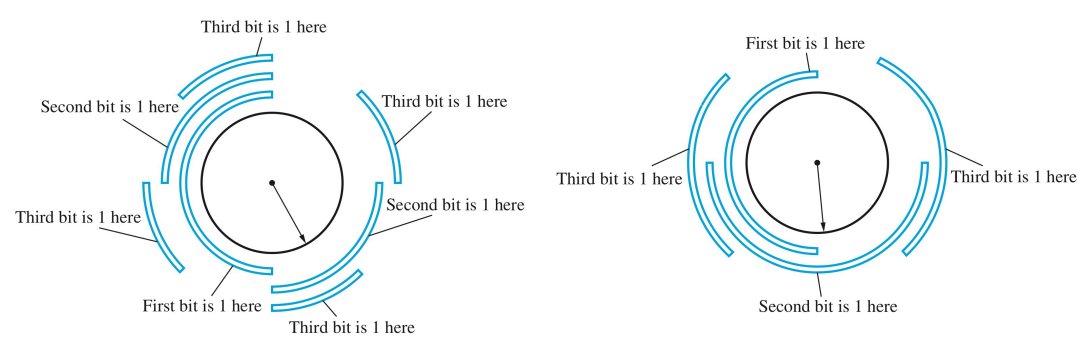


(a)

(b)

(b) is a Gray code

We can model this problem using the n -cube Q_n .



The left diagram shows three concentric rings. The innermost ring is labeled 'First bit is 1 here' at the bottom. The middle ring is labeled 'Second bit is 1 here' at the top-left and bottom-right. The outermost ring is labeled 'Third bit is 1 here' at the top-right and bottom-left. The right diagram shows a single ring divided into segments. The top segment is labeled 'First bit is 1 here'. The bottom segment is labeled 'Second bit is 1 here'. The left and right segments are labeled 'Third bit is 1 here'.

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