Ch5.2 Strong Induction and Well-Ordering

Introduction

- Mathematicl Induction $\equiv$ Strong Induction $\equiv$ Well-Ordering

Strong Induction

- sometimes called **second principle of mathematical induction** or **complete induction**
- when the latter is used, mathematical induction is called **incomplete induction**

▲ Strong Induction and the Infinite Ladder (refer to Ch5.1 → didn't put this example)

1. we can reach the first rung, and
2. for every integer k, if we can reach all the first k rungs, then we can reach the $(k+1)$st rung.

Examples of Proofs Using Strong Induction

- Attempt a proof by strong induction, unless the inductive step of a proof by mathematical induction is clear

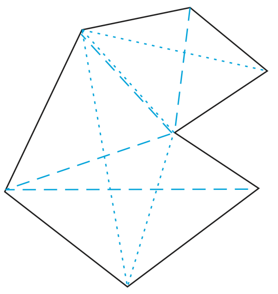
Alternative Form of Strong Induction

Basis Step : We varify that the proposition $P(b),\ P(b+1),\ \ldots,\ P(b+j)$ are true.

Inductive Step : We show that $[P(b) \wedge P(b+1) \wedge \ldots \wedge P(k)] \rightarrow P(k+1)$ is true for every integer $k \geq b+j$.

Using Strong Induction in Computational Geometry

- **Polygon**: closed geometric figure consists of sides
- **Sides**: a sequence of line segments
- **Vertex:** common endpoint
- **Simple:** no two nonconsecutive sides intersect
- **Interior:** points inside the curve; **Exterior:** points outside the curve
- **Convex:** every line segment connecting two points in the interior of the polygon lies entirely inside the polygon
- **Triangulation:** dividing a simple polygon into triangles by adding nonintersecting diagonals
- **Diagonal:** a line segment connecting two *nonconsecutive* vertices
- **Interior Diagonal:** the line (excluding endpoints) lies entirely inside the polygon



> **Theorem 1**
> A simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n-2$ triangles.

> **Lemma 1**
> Every simple polygon with at least four sides has an interior diagonal.

Proofs Using the Well-Ordering Property

Well-Ordering Property

- every nonempty set of nonnegative integers has a least element

Axioms for the Set of Positive Integers (Appendix 1)

☐ **Axiom 1** The number 1 is a positive integer.

☐ **Axiom 2** If n is a positive integer, then $n+1$, the successor of n, is also a positive integer.

☐ **Axiom 3** Every positive integer other than 1 is the successor of a positive integer.

☐ **Axiom 4** **The Well-Ordering Property** Every nonempty subset of the set of positive integers has a least element.

☐ **Mathematical induction axiom** If S is a set of positive integers such that $1 \in S$ and for all positive integers n if $n \in S$, then $n+1 \in S$, then S is the set of positive integers.