Ch12.4 Minimization of Circuits (Week 10)

Introduction

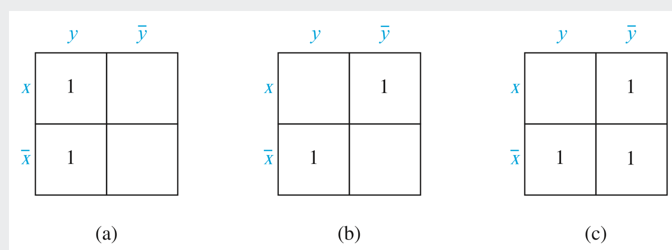
- Minimizing Boolean functions is an NP-complete problem.

Karnaugh Maps (K-maps)

- 1 is placed in the cell representing a minterm
- Cells are said to be **adjacent** if the minterms that they represent differ in exactly one literal

▼ Example 1

Find the K-maps for (a) $xy + \overline{x}y$, (b) $x\overline{y} + \overline{x}y$, and (c) $x\overline{y} + \overline{x}y + \overline{x}\,\overline{y}$.

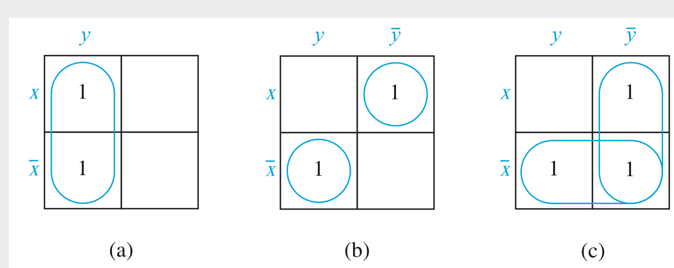


Sum-of-Products Expansions

▼ Example 2

Simplify the sum-of-products expansion given in Example 1.

Ans: (a) y, (b) $x\overline{y} + \overline{x}y$, and (c) $\overline{x} + \overline{y}$.



Simplify the Sum-of-Products Expansions

Implicant

- The product of literals corresponding to a block of all 1s in the K-map (Each cell with a 1 in it)

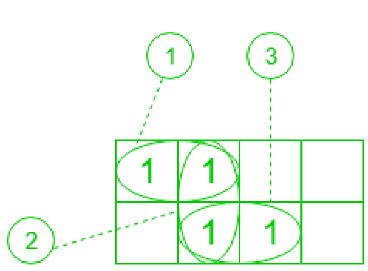
<u>Prime Implicant</u> (PI)

- This block of 1s is not contained in a larger block of 1s representing the product of fewer literals than this product
- (not in other blocks, by itself) OR (all possible groups formed)

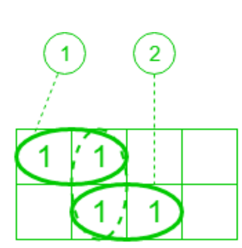
Essential Prime Implicant (EPI)

- The only block of 1s covering a 1 in the K-map.
- cover at least one minterm that can't be covered by any other prime implicant

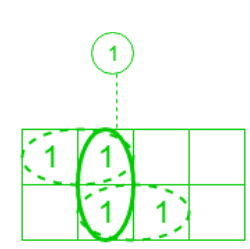
Redundant Prime Implicant (RPI)



No. of Prime Implicants = 3



No. of Essential Prime Implicants = 2



No. of Redundant Prime Implicants = 1

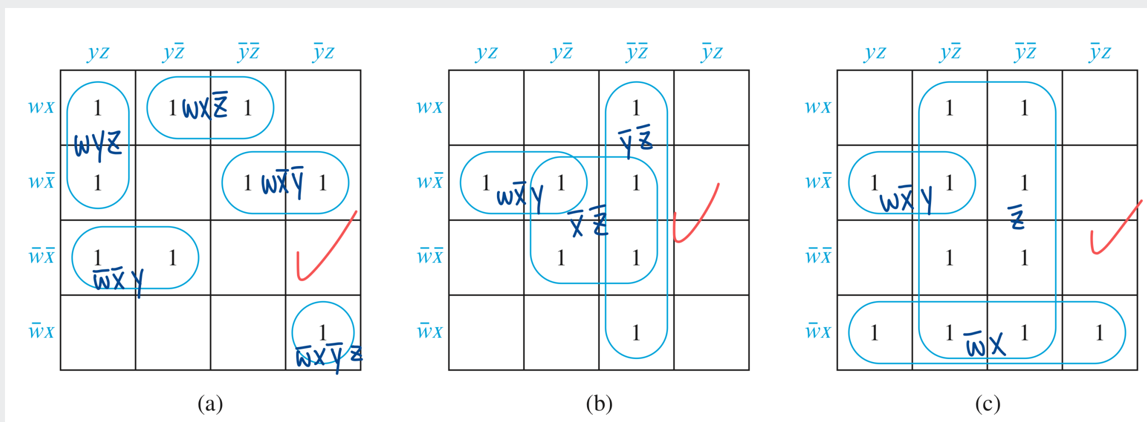
▼ Example 4'

Use K-maps to simplify these sum-of-products expansions.

(a) $wxyz + wxy\overline{z} + wx\overline{y}\,\overline{z} + w\overline{x}yz + w\overline{x}\,\overline{y}z + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}x\overline{y}z + \overline{w}\,\overline{x}yz + \overline{w}\,\overline{x}y\overline{z}$

(b) $wx\overline{y}\,\overline{z} + w\overline{x}yz + w\overline{x}y\overline{z} + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}x\overline{y}\,\overline{z} + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$

(c) $wxy\overline{z} + wx\overline{y}\,\overline{z} + w\overline{x}yz + w\overline{x}y\overline{z} + w\overline{x}\,\overline{y}\,\overline{z} + \overline{w}xyz + \overline{w}xy\overline{z} + \overline{w}x\overline{y}\,\overline{z} + \overline{w}x\overline{y}z + \overline{w}\,\overline{x}y\overline{z} + \overline{w}\,\overline{x}\,\overline{y}\,\overline{z}$



(a) (b) (c)