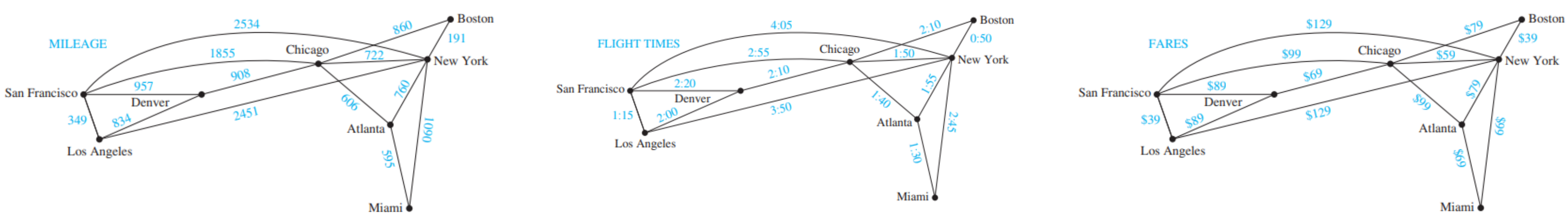
Ch10.6 Shortest-Path Problems

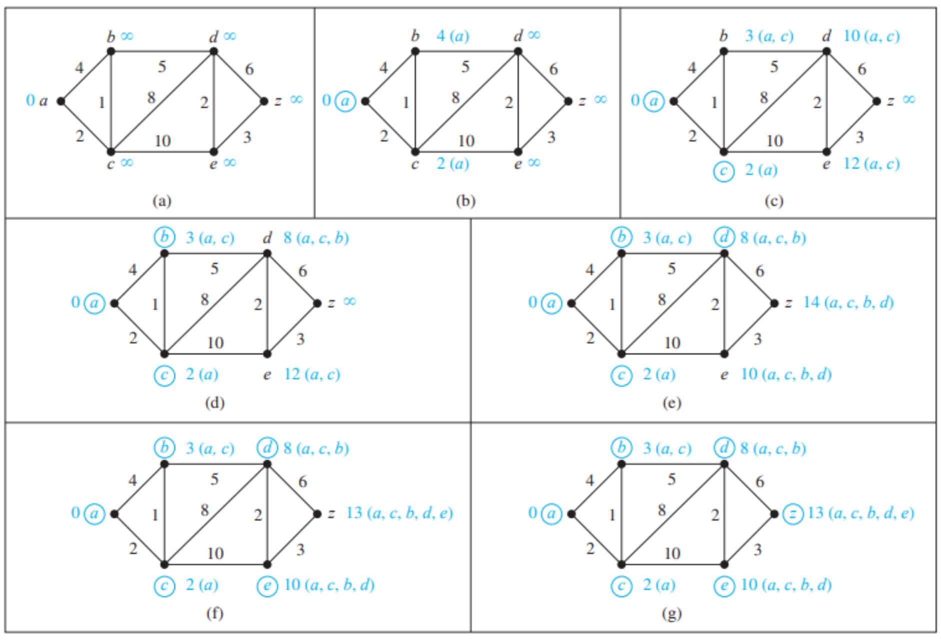
- Weighted graphs with different assignments



A Shortest-Path Algorithm

ALGORITHM 1 Dijkstra's Algorithm.

procedure *Dijkstra*(G: weighted connected simple graph, with
  all weights positive)
{G has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$
  where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in G}
for $i := 1$ **to** n
  $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is 0 and all
  other labels are ∞, and S is the empty set}
while $z \notin S$
  $u :=$ a vertex not in S with $L(u)$ minimal
  $S := S \cup \{u\}$
  for all vertices v not in S
    if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
    {this adds a vertex to S with minimal label and updates the
    labels of vertices not in S}
return $L(z)$ {$L(z) =$ length of a shortest path from a to z}



Using Dijkstra's Algorithm to Find a Shortest Path from a to z

> **1** **Theorem 1** Dijkstra's algorithm finds the length of a shortest path between two vertices in a connected simple undirected weighted graph.

> **2** **Theorem 2** Dijkstra's algorithm uses $O(n^2)$ operations (additions and comparisons) to find the length of a shortest path between two vertices in a connected simple undirected weighted graph with n vertices.

The Traveling Salesperson Problem

Asking for the circuit of minimum total weight in a weighted, complete, undirected graph.

(visits each vertex once, start = end)

$\equiv$ a Hamilton circuit with minimum total weight in teh complete graph

Approximation Algorithm

- Comes from: Impractical to solve a (TS) problem with a few dozen vertices
- Don't necessarily produce the exact solution, but guaranteed to produce one that's close