Topic 2: Functions

Ch2.3 Functions

1 input $\rightarrow >$ 1 outputs: not a function
domain: possible inputs
codomain: possible outputs
image: input
preimage: output
range: all outputs

How to do injective/surjective proofs:

injective

$$f(x) = f(y)$$
$$mx + b = my + b$$
$$mx = my$$
$$x = y \ \checkmark$$

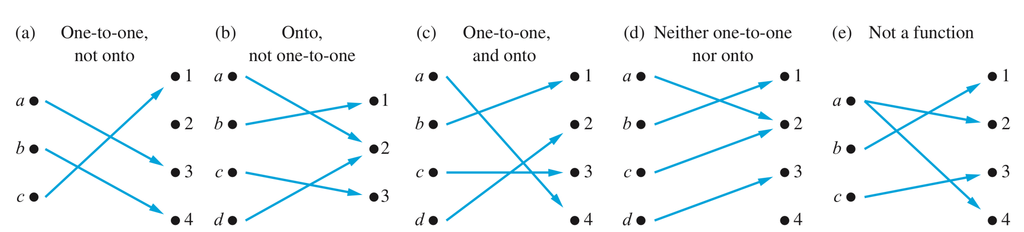
surjective

$$y = mx + b$$
$$y - b = mx$$
$$\underbrace{\boxed{\frac{y-b}{m}}}_{\text{inverse}} = x \ \checkmark$$

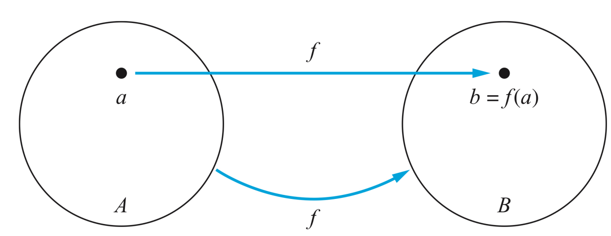
How to prove not injective/surjective:

not injective: find a counterexample of 2 different inputs map to the same output.

not surjective: find a preimage/output (within the codomain) that doesn't have an image/input (within the domain).



Examples of Different Types of Correspondences



The function f Maps A to B.

one-to-one/injective: *iff* $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.
onto/surjective: *iff* for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.
bijection/one-to-one correspondence: both injective and surjective.

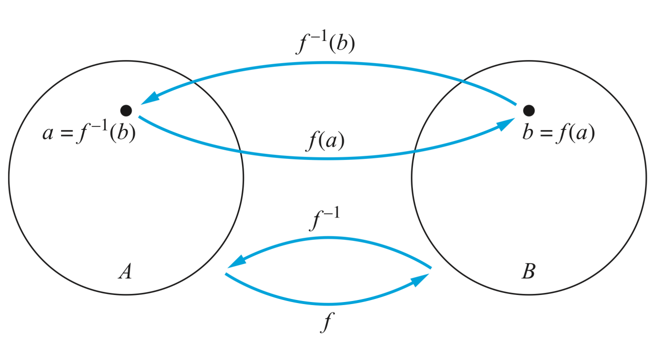
Inversefunctions

Inverse function: (be bijective) $f^{-1}(b) = a$ when $f(a) = b$.

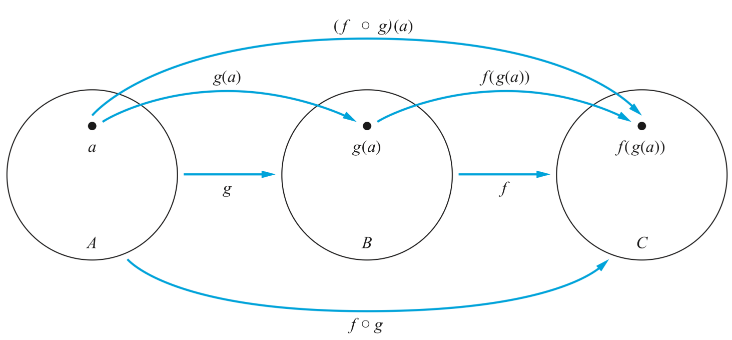
- **invertible**: Can define an inverse of this function (bijective);
  not invertible: The inverse of such a function does not exist.

Composition: $f \circ g$, $(f \circ g)(a) = f(g(a))$.

- $f \circ g$ and $g \circ f$ are not equal.



The function f^{-1} is the inverse of function f.



The composition of the functions f and g.

Some Important Functions

floor: $\lfloor x \rfloor$, largest integer $\leq x$; *ceiling*: $\lceil x \rceil$, smallest integer $\geq x$.

Appendix B-1 & 2: floor & ceiling function graphs; B-3: useful properties of floor and ceiling function.