

Topic 1: Sets

Ch2.1 Sets

| A set is countable when it is finite *or* has the same size as the set of positive integers.

→ The set of rational numbers is countable, the set of real numbers is not (bc there can be infinite 0.1, 0.01, 0.001...)

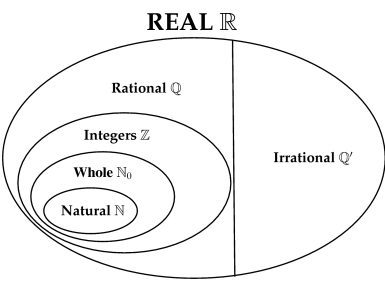


A *set* is an unordered collection of objects, called *elements* or *members* of the set. $a \in A$: a is an element of the set A; $a \notin A$: a is not an element of the set A.

Roster Method (Listing Method)

- Listing the elements inside braces, i.e. {2, 4, 6, 8}

$N = \{0, 1, 2, 3, \dots\}$, the set of **natural numbers**
 $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$, the set of **integers**
 $Z^+ = \{1, 2, 3, \dots\}$, the set of **positive integers**
 $Q = \{p/q \mid p \in Z, q \in Z, \text{ and } q \neq 0\}$, the set of **rational numbers**
 R , the set of **real numbers**
 R^+ , the set of **positive real numbers**
 C , the set of **complex numbers**



$[a, b]$ is called the **closed interval** (include); (a, b) is called the **open interval** (not include)



$A \subseteq B$: The set A is a subset of B *iff* every element of A is also an element of B; $A \subset B$: A is a subset of B, but $A \neq B$ (proper subset)



Two sets are *equal* *iff* they have the same elements. A and B are equal *iff* $\forall x(x \in A \leftrightarrow x \in B)$. To show that two sets A and B are equal, show that $A \subseteq B$ and $B \subseteq A$.



For every set S, (i) $\emptyset \subseteq S$ and (ii) $S \subseteq S$

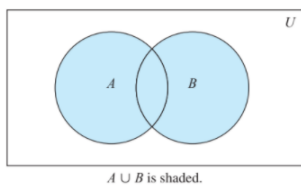


$|S|$: the cardinality of S, distinct elements in S. $\mathcal{P}(S)$: the power set of S, the set of all subsets of S.

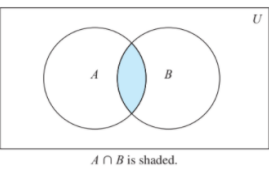


$A \times B$: is the set of all ordered pairs (a,b), where $a \in A$ and $b \in B$. $A \times B \neq B \times A$ unless $A = \emptyset$ or $B = \emptyset$ or $A = B$.

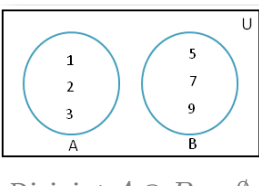
Ch2.2 Set Operations



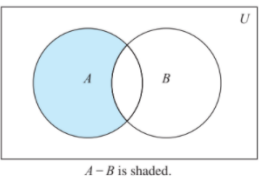
Union $A \cup B$



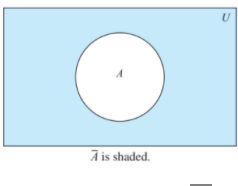
Intersection $A \cap B$



Disjoint $A \cap B = \emptyset$



Difference $A - B$



Complement \bar{A}

The principle of inclusion-exclusion (PIE)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Important Set identities

$$A - B = A \cap \bar{B}$$

Full table of Set Identities in Apendix A.