

MIDTERM ASSESSMENT



**UNIVERSITY
OF LONDON**

CM120

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathemaitcs

INSTRUCTIONS TO CANDIDATES:

This assignments consists of five questions. You should answer all five questions.

Full marks will be awarded for complete answers to a total of Five questions.
The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

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Question 1 Set Theory

- (a) i. Describe the following set using the listing method:

$$A = \{x : x \in \mathbb{Z} \text{ and } 0 \leq x^3 < 100\}$$

[2]

- ii. Rewrite the following set using the set builder method:

$$B = \{-1, 1/2, -1/3, 1/4, -1/5, 1/6, \dots\}$$

[2]

- (b) In a survey of 200 student, it was found that: 150 students took programming (P), 80 students took mathematics (M), 55 students took art (A), 60 students took mathematics and programming (M & P), 25 students took art and mathematics (A & M), 40 students took art and programming (A & P), and 15 students took art, mathematics and programming (A & M & P).

- i. Draw a Venn diagram to display this information.
- ii. Use Venn diagram to find the number of students that took
 1. programming only
 2. two modules only
 3. mathematics and programming but not art

[6]

- (c) Let A and B be two subsets of the universal set U . Prove or disprove that

$$\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B).$$

[4]

- (d) Let A and B be two subset of a universal set U . Show that:

$$A \subseteq B \Leftrightarrow \overline{A} \cup B = U$$

[6]

Question 2 Functions

- (a) Let $f : \mathbb{R} \rightarrow \mathbb{Z}$. where $f(x) = \lfloor \frac{x}{2} \rfloor$.
- i. Find $f(1)$ [1]
 - ii. What is the set of pre-images of 10 [1]
 - iii. Say whether or not $f(x)$ is injective(one-to-one), justifying your answer. [2]
 - iv. Say whether or not $f(x)$ is surjective(onto), justifying your answer. [2]
- (b) Given a function $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = 3x + 5$.
- i. Show that the function g is a bijection. [2]
 - ii. Find g^{-1} [2]
- (c) Let $f : D_f \rightarrow [0, = \infty)$ be a bijective function with $f(x) = \ln(x + 1)$.
- i. Find domain, D_f of this function. [1]
 - ii. Find the inverse function f^{-1} . [2]
 - iii. Plot the curves of both function, f and f^{-1} in the same graph. [2]
 - iv. What can you say about these two curves? [1]
- (d) Determine whether each of the following functions, defined from $\mathbb{Z} \times \mathbb{Z}$ to \mathbb{Z} , is one-to-one , onto, or both. Explain your answers.
- i. $f(x, y) = x^2 + 1$
 - ii. $g(x, y) = x + y + 2$

[4]

Question 3 Propositional Logic

- (a) Let p, q, r and s four propositions. Assuming that p and r are false and that q and s are true, find the truth value of each of the following propositions:

- i. $((p \wedge \neg q) \rightarrow (q \wedge r)) \rightarrow (s \vee \neg q)$
- ii. $((p \vee q) \wedge (q \vee s)) \rightarrow ((\neg r \vee p) \wedge (q \vee s))$

[2]

- (b) Let p and q be two propositions defined as follows: p means 'A student can take the algorithm module ' whereas q means 'Student passes discrete mathematics'

Express each of the three following compound propositions symbolically by using p, q and appropriate logical symbols.

- i. '*A sufficient condition for a student to take the algorithm module is that they pass discrete mathematics*'.
- ii. '*A student can take the algorithm module only if they pass discrete mathematics*'.
- iii. '*A student can takes the algorithm module if they pass discrete mathematics*'.
- iv. '*A student either passes discrete mathematics or can take the algorithm module*'

[4]

- (c) Write in words and express symbolically in terms p and q , defined in (a), the contrapositive, the converse and the inverse of the implication:

'A student can take the algorithm module if they pass discrete mathematics'

[6]

- (d) Consider the following three propositions:

s means "Samir goes to the party "

c means "Callum goes to the party"

j means "Jay goes to the party".

Express each of the three following compound propositions symbolically by using c, j, s and appropriate logical symbols.

- i. "Samir goes to the party only if both Callum and 'Jay aren't going to the party.
- ii. "Either both Samir and Jay go to the party or Callum goes to the party, but not both ".

[4]

(e) A tautology is a proposition that is always true. Let p and q be two propositions, show that $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$ is a tautology.

[4]

Question 4 Predicate logic

- (a) Let $P(x, y)$ be a boolean function. Assume that $\forall x \exists y P(x, y)$ is *True* and that the domain of discourse is nonempty. Which of the following must also be true? If the statement is true, explain; otherwise, give a counter-example.

- i. $\forall x \forall y P(x, y)$
- ii. $\exists x \forall y P(x, y)$
- iii. $\exists x \exists y P(x, y)$

[6]

- (b) Given the following argument:

"If it rains then the concert will be cancelled"

"The concert was cancelled, therefore it rained"

Assume p means "it rains" whereas q means "concert cancelled"

- i. Translate this argument to a symbolic form.
- ii. Construct the truth table.
- iii. Determine if this argument is a valid argument or not.

[8]

- (c) Let p, q, r, s and t be statements variables. Use the valid argument forms to deduce the conclusion, $\neg q$, from the premises, giving a reason for each step.

- (a) $\neg p \vee q \rightarrow r$
- (b) $s \vee \neg q$
- (c) $\neg t$
- (d) $p \rightarrow t$
- (e) $\neg p \wedge r \rightarrow \neg s$

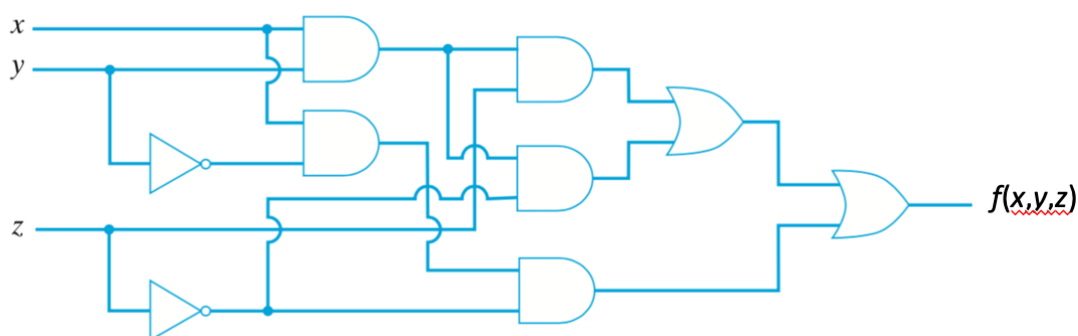
(f) $\therefore \neg q$

[6]

Question 5 Boolean Algebra

(a) What is the value of the boolean expression $(x + y)(\bar{x}.\bar{y})$? [3]

(b) Consider the following combinatorial circuit with three inputs x, y and z , and one output $f(x, y, z)$:



i. Write the output $f(x, y, z)$ in its disjunctive normal form. [2]

ii. Fill in the missing output value in the following table:

x	y	z	f(x,y,z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

[4]

iii. Show that $f(x, y, z)$ can be simplified to $f(x, y, z) = x(y + \bar{z})$ [2]

iv. Draw the simplified circuit equivalent to $f(x, y, z)$. [2]

(c) i. What is the advantage of using Karnaugh map (K-map)?

[2]

ii. Fill in the following K-map for the Boolean function

$$F(x, y, z) = \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.y.\bar{z} + x.\bar{y}.\bar{z}$$

<div> <div>z</div> <div>xy</div> </div>		00	01	11	10
		0			
	1				

[2]

- iii. Use the previous K-map and find a minimisation, as the sum of three terms, of the expression

$$F(x, y, z) = \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.y.\bar{z} + x.\bar{y}.\bar{z}$$

[3]

END OF PAPER