Ch 1.5 Nested Quantifiers (Week 7)

Introduction

▼ example

$\forall x \exists y(x+y=0)$ is the same thing as $\forall x\ Q(x)$ is $\exists y\ P(x,y)$ where $P(x,y)$ is $x+y=0$

Understanding Statements Involving Nested Quantifiers

- Think of Quantification as Loops

The Order of Quantifiers

- $\forall x \forall y\ P(x,y)$ and $\forall y \forall x\ P(x,y)$ have the same meaning
- $\exists y \forall x\ P(x,y)$ and $\forall x \exists y\ P(x,y)$ are not logically equivalent

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Translating Mathematical Statements into Statements Involving Nested Quantifiers

▼ Example 6

"The sum of two positive integers is always positive. "
$\forall x \forall y((x>0) \wedge (y>0) \rightarrow (x+y>0))$

Translating from Nested Quantifiers into English

▼ Example 9

$\forall x(C(x) \vee \exists y(C(y) \wedge F(x,y)))$
$C(x)$ is "x has a computer" , $F(x,y)$ is "x and y are friends" , domain is "all students in your school"
⇒ "Every student in your school has a computer or has a friend who has a computer."

▼ Example 10

$\exists x \forall y \forall z\ ((F(x,y) \wedge (y \neq z)) \rightarrow \neg F(y,z))$
$F(a,b)$ means "a and b are friends" , domain is "all students in your school"
⇒ "There is a student none of whose friends are also friends with each other."

Translating English Sentences into Logical Expressions

▼ Example 12

"Everyone has exactly one best friend"
⇒ $\exists y(B(x,y) \wedge \forall z((z \neq y) \rightarrow \neg B(x,z)))$ ⇒ $\forall x \exists y(B(x,y) \wedge \forall z((z \neq y) \rightarrow \neg B(x,z)))$ OR $\forall x \exists ! y B(x,y)$

Negating Nested Quantifiers

▼ Example 14

$\forall x \exists y(xy=1)$ ⇒ $\neg \forall x \exists y(xy=1)$ ⇒ $\exists x \neg \exists y(xy=1)$ ⇒ $\exists x \forall y \neg(xy=1)$ ⇒ $\exists x \forall y(xy \neq 1)$