Ch12.2 Representing Boolean Functions (Week 9)

Sum-of-Products Expansions

- Find a Boolean expression given the values of a Boolean function

> A *literal* is a Boolean variable or its complement. A *minterm* of the Boolean variables $x_1, x_2, \ldots, x_n$ is a Boolean product $y_1 y_2 \ldots y_n$, where $y_i = x_i$ or $y_i = \overline{x_i}$. Hence, a minterm is a product of n literals, with one literal for each variable.

Sum-of-Products or **Disjunctive normal form**

- The sum of minterms that represents the function

Product-of-sums expansion or **Conjunctive normal form**

- A Boolean expression that represents a Boolean function by taking a Boolean product of Boolean sums.
- Can be found from sum-of-product expansions by taking duals.

▼ Example 3

Find the sum-of-products expansion of $F(x, y, z) = (x + y)\overline{z}$.

1. Boolean identities

$$
\begin{aligned}
F(x,y,z) &= (x+y)\,\overline{z} & \\
&= x\,\overline{z} + y\,\overline{z} & \text{Distributive law} \\
&= x1\overline{z} + 1y\overline{z} & \text{Identity law} \\
&= x\,(y+\overline{y})\,\overline{z} + (x+\overline{x})\,y\,\overline{z} & \text{Unit property} \\
&= xy\overline{z} + x\,\overline{y}\,\overline{z} + xy\overline{z} + \overline{x}y\overline{z} & \text{Distributive law} \\
&= xy\overline{z} + x\,\overline{y}\,\overline{z} + \overline{x}y\overline{z} & \text{Idempotent law}
\end{aligned}
$$

2. Table values

TABLE 2

x	y	z	$x + y$	$\overline{z}$	$(x + y)\overline{z}$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

$F(x,y,z) = x\,y\,\overline{z} + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,\overline{z}$ → The function that is all 1s.

Functional Completeness

Functionally complete

- every Boolean function can be represented using these operators
- $\{\,\cdot\,,\ +,\ \overline{}\,\}, \{\,\cdot\,,\ \overline{}\,\}, \{+\,,\ \overline{}\,\}$
- $\{\,|\,\}$ and $\{\downarrow\}$

$|$ **or** $NAND$ **operator**

- $1 \,|\, 1 = 0,\ 1 \,|\, 0 = 0 \,|\, 1 = 0 \,|\, 0 = 1$

$\downarrow$ **or** NOR **operator**

- $1 \downarrow 1 = 1 \downarrow 0 = 0 \downarrow 1 = 0,\ 0 \downarrow 0 = 1$