Ch12.1 Boolean Functions (Week 9)

Introduction

- Boolean sum: $+$ or OR
  $1+1=1,\ \ 1+0=1,\ \ 0+1=1,\ \ 0+0=0$
- Boolean product: $\cdot$ or AND
  $1\cdot 1=1,\ \ 1\cdot 0=0,\ \ 0\cdot 1=0,\ \ 0\cdot 0=0$

Precedence

1 Complement 2 Products 3 Sums

Boolean to Logical operator

- Complement → $\neg$
- Sum → $\vee$
- Product → $\wedge$
- 0 → F(false)
- 1 → T(true)

Boolean Expressions and Boolean Functions

Boolean variable

- Assumes values only from 0 and 1

Boolean function of degree n

- A function from B^n to B

Different ways to express Boolean functions

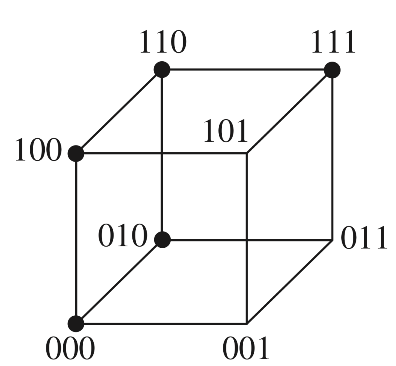

TABLE 2

x	y	z	xy	$\bar{z}$	$F(x, y, z) = xy + \bar{z}$
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	1	1

Number of Boolean functions of degree n

- 2^{2^n}

Identities of Boolean Algebra

TABLE 5 Boolean Identities.

Identity	*Name*
$\bar{\bar{x}} = x$	Law of the double complement
$x + x = x$ $x \cdot x = x$	Idempotent laws
$x + 0 = x$ $x \cdot 1 = x$	Identity laws
$x + 1 = 1$ $x \cdot 0 = 0$	Domination laws
$x + y = y + x$ $xy = yx$	Commutative laws
$x + (y + z) = (x + y) + z$ $x(yz) = (xy)z$	Associative laws
$x + yz = (x + y)(x + z)$ $x(y + z) = xy + xz$	Distributive laws
$\overline{(xy)} = \bar{x} + \bar{y}$ $\overline{(x + y)} = \bar{x}\,\bar{y}$	De Morgan's laws
$x + xy = x$ $x(x + y) = x$	Absorption laws
$x + \bar{x} = 1$	Unit property
$x\bar{x} = 0$	Zero property

Duality

- The **dual** of a Boolean expression is obtained by interchanging $+$ and $\cdot$ and interchanging 0s and 1s.

▼ Example 11

Find the duals of $x(y+0)$ and $\overline{x} \cdot 1 + (\overline{y} + z)$

Ans: $x + (y \cdot 1)$ and $(\overline{x} + 0) \cdot (\overline{y} \cdot z)$

Duality principle

- The result of a dual function F^d that does not depend on F, useful for obtaining new identities.

▼ Example 12

Construct an identity from the obsorption law $x(x+y) = x$ by taking duals.

$\Rightarrow x + xy = x$ is also an obsorption law

The Abstract Definition of a Boolean Algebra

1 A *Boolean algebra* is a set B with two binary operations $\vee$ and $\wedge$, elements 0 and 1, and a unary operation $\overline{}$ such that these properties hold for all $x,\ y$, and z in B:

Identity Laws
$$\begin{cases} x \vee 0 = x \\ x \wedge 1 = x \end{cases}$$

Complement laws
$$\begin{cases} x \vee \overline{x} = 1 \\ x \wedge \overline{x} = 0 \end{cases}$$

Associative laws
$$\begin{cases} (x \vee y) \vee z = x \vee (y \vee z) \\ (x \wedge y) \wedge z = x \wedge (y \wedge x) \end{cases}$$

Commutative laws
$$\begin{cases} x \vee y = y \vee x \\ x \wedge y = y \wedge x \end{cases}$$

Distributive laws
$$\begin{cases} x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \\ x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \end{cases}$$