Ch10.2 Graph Terminology and Special Types of Graphs (Week 13)

Basic Terminology

1 Two vertices u and v in an undirected graph G are called *adjacent* (or *neighbors*) in G if u and v are endpoints of an edge e of G. Such an edge e is called *incident with* the vertices u and v and e is said to *connect* u and v.

2 The set of all neighbors of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called the neighborhood of v. If A is a subset of V, we denote by $N(A)$ the set of all vertices in G that are adjacent to at least one vertex in A. So, $N(A) = \bigcup_{v \in A} N(A)$.

3 The *degree* of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by deg(v).

- **Isolated:** A vertex of degree zero
- **Pendant:** A vertex *iff* has degree one

1 **The Handshaking Theorem**
Let $G = (V, E)$ be an undirected graph with m edges. Then $2m = \sum_{v \in V} \deg(v)$.
(Note that this applies even if multiple edges and loops are present.)

2 **Theorem 2** An undirected graph has an even number of vertices of odd degree.

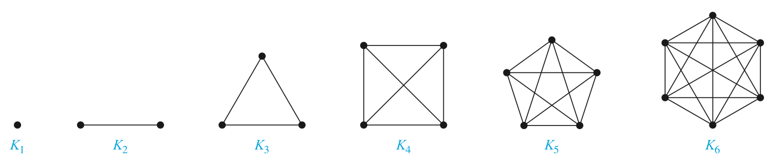
4 When (u, v) is an edge of the graph G with directed edges, u is said to be *adjacent to* v and v is said to be *adjacent from* u. The vertex u is called the *initial vertex* of (u, v), and v is called the *terminal* or *end vertex* of (u, v). The initial vertex and terminal vertex of a loop are the same.

5 In a graph with directed edges the *in-degree* of a vertex v, denoted by $\deg^{-}(v)$, is the number of edges with v as their terminal vertex. The *out-degree* of v, denoted by $\deg^{+}(v)$, is the number of edges with v as their initial vertex. (Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex._

3 **Theorem 3**
Let $G = (V, E)$ be a graph with directed edges. Then $\sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v) = |\mathrm{E}|$.

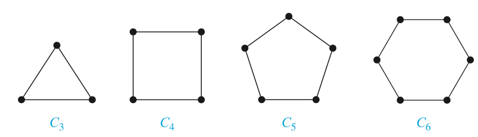
- **Underlying undirected graph:** results from ignoring directions of edges

Complete Graphs



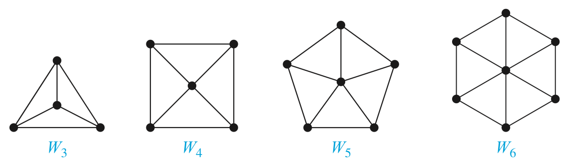
The Graphs K_n for $1 \leq n \leq 6$

Cycles



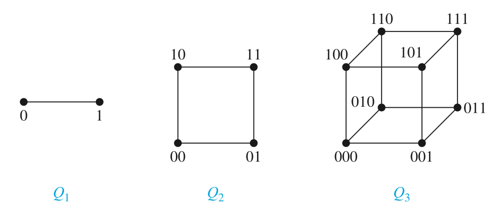
The Cycles C_3, C_4, C_5, and C_6

Wheels



The Wheels W_3, W_4, W_5, and W_6

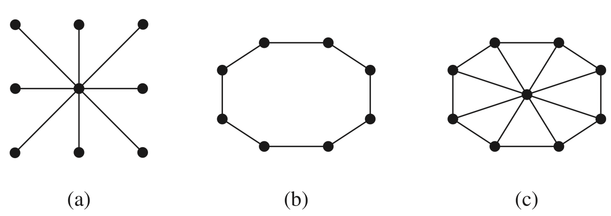
n-Cubes



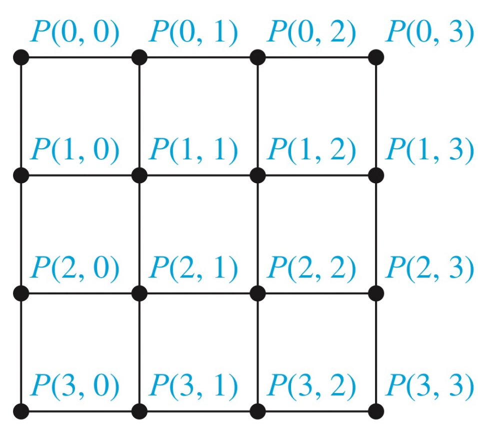
The n-cube Q_n, $n = 1, 2, 3$

Some Applications of Special Types of Graphs

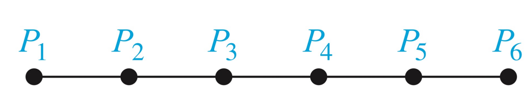
Local Area Networks



(a) Star Topology (b) Ring Topology (c) Hybrid Topology



Interconnection Networks for Parallel Computation



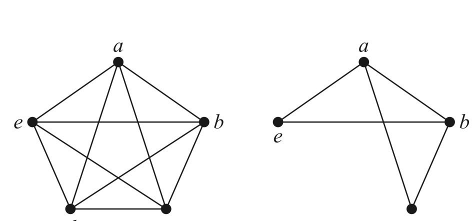
- **Serial:** algorithms written to solve problems one step at a time
- **Parallel processing/algorithms:** break a problem into a number of subproblems that can be solved concurrently

New Graphs from Old

7 A *subgraph* of a graph $G = (V, E)$ is a graph $H = (W, F)$, where $W \subseteq V$ and $F \subseteq E$. A subgraph H of G is a *proper subgraph* of G if $H \neq G$.

8 Let $G = (V, E)$ be a simple graph. The **subgraph induced** by a subset W of the vertex set V is the graph (W, F), where the edge set F contains an edge in E *iff* both endpoints of this edge are in W.

Removing or Adding Edges or a Graph



A Subgraph of K_5

Edge Contractions

- removes an edge e with endpoints u and v and merges u and w into a new single vetex w

Removing Vertices from a Graph

- remove a vertex v $\rightarrow$ subgraph $G - v$

Graph Unions

9 The *union* of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_! \cup G_2$.