University of London

Assessment Coversheet

Complete this coversheet and read the instructions below carefully.

Candidate Number:
WT0072

Degree Title:
BSc Computer Science

Course/Module Title:
Discrete Mathematics

Course/Module Code:
CM 1020

Enter the numbers, and sub-sections, of the questions in the order in which you have attempted them:

1.(a), 1.(b), 1.(c), 1.(d), 1.(e), 1.(f), 1.(g).
2.(a), 2.(b), 2.(c).

Date:

Instructions to Candidates

1. Complete this coversheet and begin typing your answers on the page below, or, submit the coversheet with your handwritten answers (where handwritten answers are permitted or required as part of your online timed assessment).
2. Clearly state the question number, and any sub-sections, at the beginning of each answer and also note them in the space provided above.
3. For typed answers, use a plain font such as Arial or Calibri and font size 11 or larger.
4. Where permission has been given in advance, handwritten answers (including diagrams or mathematical formulae) must be done on light coloured paper using blue or black ink.
5. Reference your diagrams in your typed answers. Label diagrams clearly.

The Examiners will attach great importance to legibility, accuracy and clarity of expression.

CM1020 Discrete Mathematics Final Exam

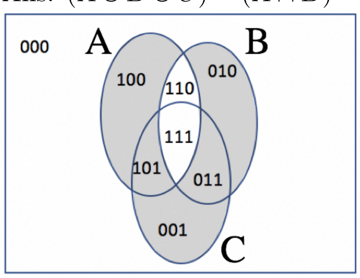
March 8, 2021

Part B

Question 1

(a)
Ans: $|A \cup B| = |A| + |B| - |A \cap B| \Rightarrow 45 = 30 + 35 - |A \cap B| \Rightarrow |A \cap B| = 20$

(b)
Ans: $(A \cup B \cup C) - (A \cap B)$



(c)
i. Ans: $p \leftrightarrow (q \rightarrow r) = T \leftrightarrow (T \rightarrow F) = T \leftrightarrow F = F$.
ii. Ans: $p \rightarrow (r \rightarrow q) = T \rightarrow (F \rightarrow T) = T \rightarrow T = T$.
iii. Ans: $(p \oplus r) \rightarrow \neg q = (T \oplus F) \rightarrow \neg T = T \rightarrow F = F$.
iv. Ans: $p \wedge (r \rightarrow q) = T \wedge (F \rightarrow T) = T \wedge T = T$.

(d)
i. Ans: False, if $x = 5 \Rightarrow 5^2 > 1$ but $5 + 1 \not< 4$.
ii. Ans: True, ex. $x = 2 \Rightarrow 2^2 > 1$ and $2 + 1 < 4$.
iii. Ans: False, if $x = 1 \Rightarrow 1^2 \not> 1$ but $1 + 1 < 4$.
iv. Ans: True, ex. $x = 3 \Rightarrow 3^2 > 1$ but $3 + 1 \not> 4$.

(e)
i. Ans: $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p \Rightarrow$ modus tollens.
ii. Ans: $((p \rightarrow q) \wedge q) \rightarrow p \Rightarrow$ fallacy of affirming the conclusion.

(f)

Ans:

$$\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C}) \equiv \overline{A} \cup \overline{B} \cup \overline{C}$$

$$\Rightarrow (\overline{A} \cup \overline{B}) \cup (A \cap B \cap \overline{C}) \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Associative Law}$$

$$\Rightarrow [(\overline{A} \cup \overline{B}) \cup A] \cap [(\overline{A} \cup \overline{B}) \cup B] \cap [(\overline{A} \cup \overline{B}) \cup \overline{C}] \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Distributive Law}$$

$$\Rightarrow (\overline{A} \cup \overline{B} \cup A) \cap (\overline{A} \cup \overline{B} \cup B) \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Associative Law}$$

$$\Rightarrow (U \cup \overline{B}) \cap (\overline{A} \cup U) \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Complement Law}$$

$$\Rightarrow U \cap U \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Domination Law}$$

$$\Rightarrow \overline{A} \cup \overline{B} \cup \overline{C} \equiv \overline{A} \cup \overline{B} \cup \overline{C} \quad \textbf{Domination Law}$$

(g)

Ans: $C_1^6 + C_2^6 + C_3^6 + C_4^6 + C_5^6 + C_6^6 = 6 + 15 + 20 + 15 + 6 + 1 = 63.$
Or, All possibilities - ways to attempt 0 questions $= 2^6 - 2^0 = 63.$

Question 2

(a)

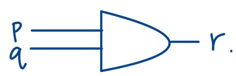
i. Ans: 1 NOT gate, 3 AND gates, and 1 OR gate.

ii. Ans: $(pq + p\overline{q})q$

iii. Ans:

$$\begin{aligned} & (pq + p\overline{q})q & \\ &= [p(q + \overline{q})]q & \textbf{Distributive Law} \\ &= (p \cdot 1)q & \textbf{Unit Property} \\ &= pq & \textbf{Identity Law} \end{aligned}$$

iv. Ans:



(b)

i. Ans: Injective & Surjective.

ii.

(1) Ans:

injective proof.

$f(a) = f(b)$

$2^{a+3} = 2^{b+3}$

$\log 2^{a+3} = \log 2^{b+3}$

$a + 3 = b + 3$

$a = b$

Hence injective.

surjective proof.

$y = 2^{x+3}$

$\log_2 y = x + 3$

$\log_2(y) - 3 = x$

because $Co - D_f = R^+$, hence surjective.

Therefore bijective.

(2) According to surjective proof above, $f^{-1}(x) = \log_2(x) - 3$.

(c)

i. Ans: $S_2 = 1 \cdot 1! + 2 \cdot 2! = 5,\ S_3 = S_2 + 3 \cdot 3! = 5 + 18 = 23.$

ii. Ans:

Proof.

Base step: $S_1 = 1 \cdot 1! = (1+1)! - 1 = 1$, hence true.

Inductive step:

Assume $n = k$. $S_k = (k+1)! - 1$.

We will show that $S_{k+1} = [(k+1)+1]! - 1 = (k+2)! - 1$.

$$
\begin{aligned}
S_{k+1} &= S_k + (k+1)(k+1)! && \text{by definition} \\
&= (k+1)! - 1 + (k+1)(k+1)! && S_k = (k+1)! - 1 \\
&= (k+1)![(k+1)+1] - 1 \\
&= (k+1)!(k+2) - 1 \\
&= (k+2)! - 1
\end{aligned}
$$

Hence, $S_n = (n+1)! - 1$ for all $n \in Z^+$.