Ch10.2 (con't) (Week 14)

Bipartite Graphs

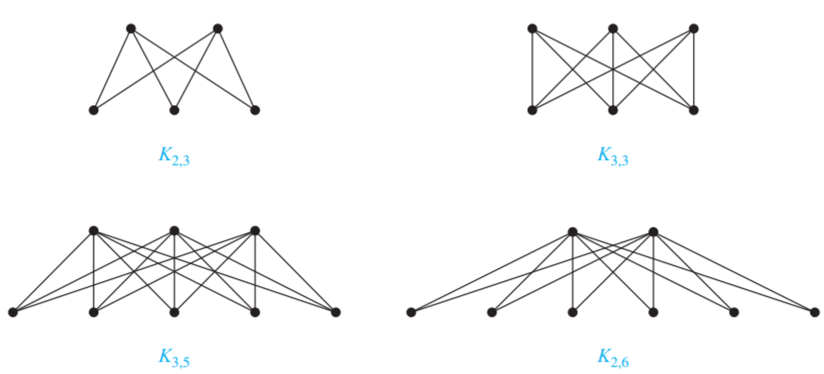
> **6** A simple graph G is called *bipartite* if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that evrey edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair (V_1, V_2) a *bipartition* of the vertex set V of G.

> **4** **Theorem 4** A simple graph is bipartite *iff* it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

- A graph is bipartitie *iff* it is **not** possible to start = end vertex, and pass through odd number of distinct edges.

Complete Bipartite Graphs $K_{m,n}$

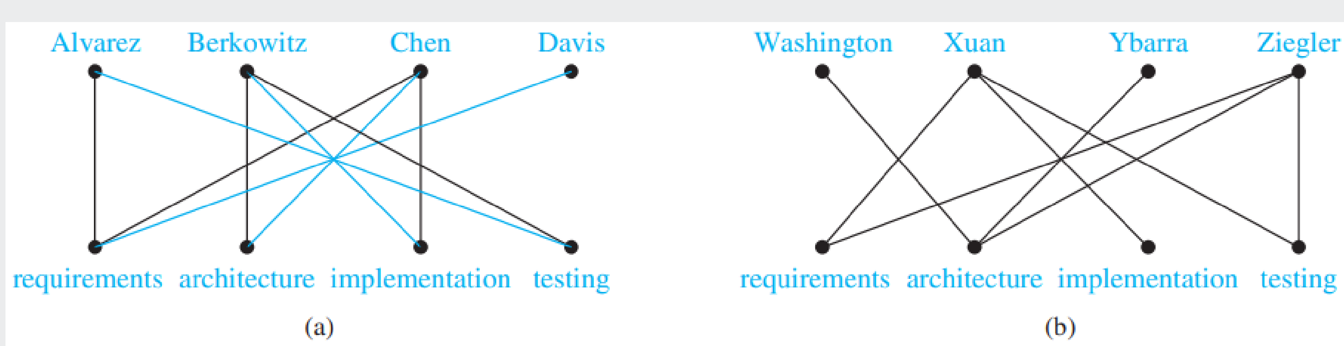
- an edge between two vertices *iff* one vertex is in the first subset and the other vertex is in the second subset.
  (an edge can only connect two vertices in the two subsets.)



Bipartite Graphs and Matchings

▼ Example 14

Job Assignments



(a) blue line: assigned job (a&b) black line: capable jobs (b) not possible

- **mathcing:** no two edges are incident with the same vertex
- **maximum mathcing:** matching with the largest number of edges
- **complete mathching from V_1 to V_2:** every vertex in V_1 is the endpoint of an edge in the matching ($|M| = |V_1|$)

> **5** **Hall's Marriage Theorem** The bipartite graph $G = (V, E)$ with bipartition (V_1, V_2) has a complete matching from V_1 to V_2 *iff* $|N(A)| \geq |A|$ for all subsets A of V_1.