Topic 4: Predicate Logic

Ch1.4 Predicates and Quantifiers

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

> $\forall$: universal quantifier; $\exists$: existential quantifier.

- $\forall$: "all of", "for each", "given any", "for arbitrary", and "for any"; $\exists$: "for some", "for at least one", or "there is".
- $\forall$ and $\exists$ have higher precedence than all logical operators

> $S \equiv T$: S and T (involving predicates and quantifiers) are logically equivalent.

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	*Equivalent Statement*	*When Is Negation True?*	*When False?*
$\neg\exists x P(x)$	$\forall x \neg P(x)$	For every x, $P(x)$ is false.	There is an x for which $P(x)$ is true.
$\neg\forall x P(x)$	$\exists x \neg P(x)$	There is an x for which $P(x)$ is false.	$P(x)$ is true for every x.

Ch1.5 Nested Quantifiers

The Order of Quantifiers

- $\forall x \forall y\ P(x,y)$ and $\forall y \forall x\ P(x,y)$ have the same meaning
- $\exists y \forall x\ P(x,y)$ and $\forall x \exists y\ P(x,y)$ are not logically equivalent

TABLE 1 Quantifications of Two Variables.

Statement	*When True?*	*When False?*
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair x, y.	There is a pair x, y for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every x there is a y for which $P(x, y)$ is true.	There is an x such that $P(x, y)$ is false for every y.
$\exists x \forall y P(x, y)$	There is an x for which $P(x, y)$ is true for every y.	For every x there is a y for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair x, y for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair x, y.

Ch1.3 Propositional Equivalences

> *tautology*: always true; *contradiction*: alwasy false; *contingency*: neither of the above.

> $p \equiv q$: p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. ($\Leftrightarrow$ is sometimes used instead of $\equiv$)

Ch1.6 Rules of Inference

Argument : A sequence of propositions; **premises:** All but the final proposition; **conclusion:** The final proposition; **valid:** if the truth of all its premises implies that the conclusion is true.

Rules of Inference for Propositional Logic & Quantified Statements

See Appendix D-1& 2 for the table.