10.208 Combinatorics

Find the number of positive integers not exceeding $2,310$ that are either the square or the cube of an integer.

Hint: use the subtraction rule, also known as the principle of inclusion-exclusion.

Let A be the set of all positive integers not exceeding $2,310$

And A_1: the set of all positive integers, perfect square and not exceeding $2,310$

And A_2: the set of all positive integers, perfect cube and not exceeding $2,310$

$A = \{1, 2, 3, \cdots, 2309, 2310\}$ → 2310 numbers

$A_1 = \{1^2, 2^2, 3^3, \cdots, 47^2, 48^2\}$ → 48 numbers

$A_2 = \{1^3, 2^3, 3^3, \cdots, 12^3, 13^3\}$ → 13 numbers

$A_1 \cap A_2 = \{1, 2^6, 3^6\} \Rightarrow A_1 \cup A_2 = 48 + 13 - 3 = 58$ elements.