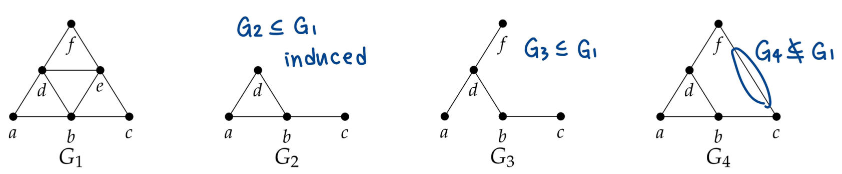
Ch 4.1 Definitions (Oscar)

Graph Theory Definitions

Graph: A collection of vertices, some of which are connected by edges.

Subgraph: every vertex and edge is also in the graph.

Induced subgraph: every vertex in the subgraph is a vertex in the graph and each pair of vertices in the subgraph are adjacent in the subgraph *iff* they are adjacent in the graph. [只要點在範圍內，連接的段也算在內]



Isomorphic: graphs that are basically the same (not equal).

An **isomorphism** between two graphs G_1 and G_2 is a bijection $f : V_1 \to V_2$ between the vertices of the graphs such that $\{a, b\}$ is an edge in G_1 *iff* $\{f(a), f(b)\}$ is an edge in G_2.
Two graphs are **isomorphic** if there is an isomorphism between them. In this case we write $G_1 \cong G_2$.

Simple: no pairs of vertices is connected more than once, no vertex is connected to itself. (*graphs*)

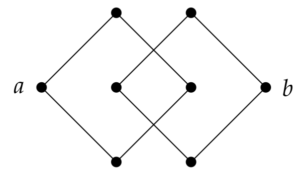
Multigraph: a graph contains multiple edges between two vertices + single edge loops.

Adjacent: Two vertices connected by an edge.

Connected: there is a path from any vertex to any other vertex

Degree of a vertex: number of edges.

Complete graph: every pair of vertices is adjacent. (K_n)



Not connected

Lemma 4.1.5 Handshake Lemma. In any graph, the sum of the degrees of vertices in the graph is always twice the number of edges.

- sometimes called *degree sum formula* → $\sum_{v \in V} d(v) = 2e.$

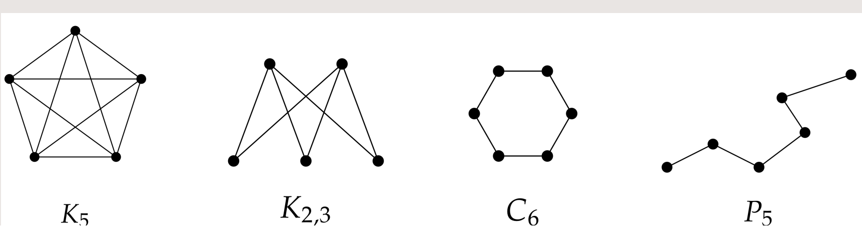
Proposition 4.1.8 In any graph, the number of vertices with odd degree must be even.

Bipartite graph: possible to divide the vertices into two disjoint sets that no edge is in the same set.

Complete bipartite graph: every vertex in the first set is adjacent to every vertex in the second set.

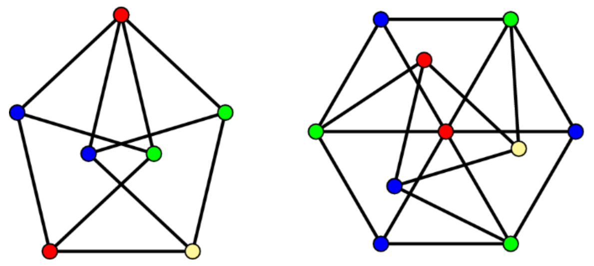
▼ Named Graphs

K_n	The complete graph on n vertices.
$K_{m,n}$	The complete bipartite graph with sets of m and n vertices.
C_n	The cycle on n vertices, just one big loop.
P_n	The path on $n+1$ vertices (so n edges), just one long path.



Vertex coloring: An assignment of colors to each of the vertices of a graph.

Chromatic number: minimum number of colors required in a proper vertex coloring of the graph.



Walk: consecutive vertices are adjacent. **Trial:** A walk with no edge repeated.

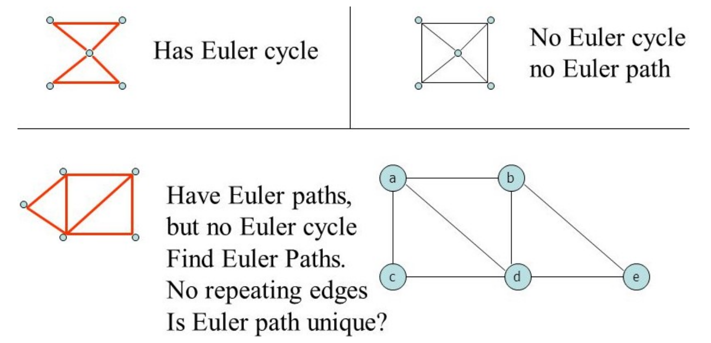
Path: A walk has no repeated vertices (or edges).

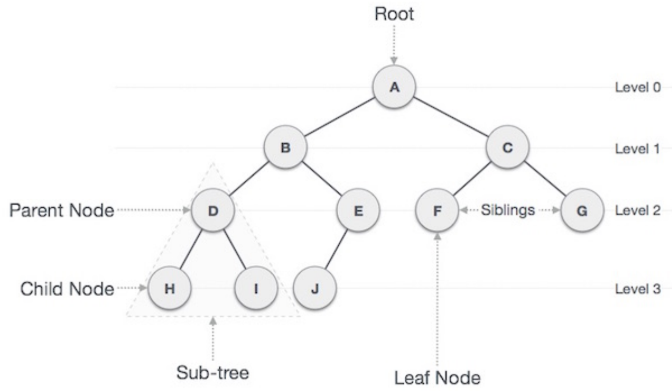
Cycle: A path start = end, no other repeated vertices.

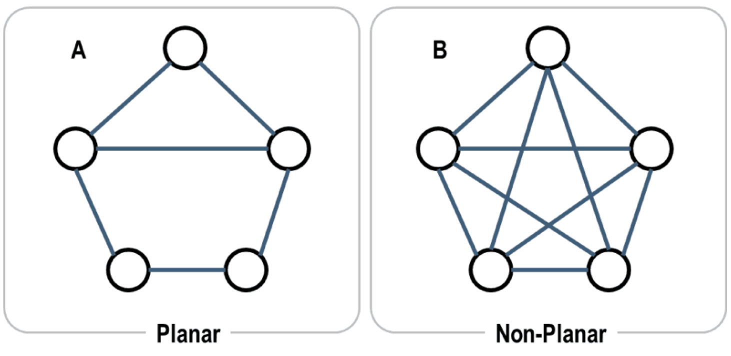
Euler path: A walk uses each edge exatly once.

Euler circuit: An Euler path start = end.

Planar: a graph can be drawn without any edges crossing.







Tree: A connected graph with no cycles. **Forest:** no need to be connected. **Leaves:** vertices in a tree with degree 1.