Chap 4. #1, 12, 15, 16, 17, 18, 21, 30, 35, 43.
See attached R outputs

#1.

1. The data in Table 4.3 are a subset of the data obtained by Thomas and Simmons (1969), who investigated the relation of sputum histamine levels to inhaled irritants or allergens. The histamine content was reported in micrograms per gram of dry weight of sputum. The subjects for this portion of the study consisted of 22 smokers; 9 of them were allergics and the remaining 13 were asymptomatic (nonallergic) individuals. Care was taken to avoid people who carried out part of their daily work in an atmosphere of noxious gases or other respiratory toxicants. Table 4.3 gives the ordered sputum histamine levels for the 22 individuals in the study.

Test the hypothesis of equal levels versus the alternative that allergic smokers have higher sputum histamine levels than nonallergic smokers. Use the large-sample approximation.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics (Y_i) $m=9$	rank	Nonallergics (X_i) $n=13$	rank
1651.0	22	48.1	15
1112.0	21	48.0	14
102.4	20	45.5	13
100.0	19	41.7	12
67.6	18	35.4	10
65.9	17	34.3	9
64.7	16	32.4	8
39.6	11	29.1	6
31.0	7	27.3	5
		18.9	4
		6.6	3
		5.2	2
		4.7	1

Source: H. V. Thomas and E. Simmons (1969).

H_0: Allergic smokers have equal levels of sputum histamine as nonallergic smokers ($\Delta = 0$)

H_a: Allergic smokers have higher levels of sputum histamine as nonallergic smokers ($\Delta > 0$)

Using large-sample approximation: $N = m+n = 9+13 = 22$

By (4.7), (4.8):

$$E_0(W) = \frac{n(m+n+1)}{2} = \frac{13(9+13+1)}{2} = 149.5$$

$$\mathrm{Var}_0(W) = \frac{mn(m+n+1)}{12} = \frac{9(13)(9+13+1)}{12} = 224.25$$

Ranking the observations by increasing orders, we calculate W with the rank of Y

$$W = \sum_{j=1}^{n} S_j = 22+21+20+19+18+17+16+11+7$$
$$= 151$$

for $H_a: \Delta > 0$

By (4.9),

$$W^* = \frac{W - E_0(W)}{[Var_0(W)]^{\frac{1}{2}}} = \frac{151 - 148.5}{(224.25)^{\frac{1}{2}}} \approx 0.10017$$
$$< z_{0.05} = 1.645$$

$\therefore$ We fail to reject H_0 and conclude that Allegic workers have same levels of sputum histamine as non allegic workers

#12.

12. Show directly, or illustrate via an example, that the maximum value of W is $n(2m+n+1)/2$. What is the minimum value of W?

Via example:

Suppose we have data

X		Y	
5	(5)	2	(2)
4	(4)	7	(7)
6	(6)	9	(8)
3	(3)	10	(9)
1	(1)		

$n=4$

$m=5$

$\therefore W = 2+7+8+9 = 26.$

$$W_{max} = 6+7+8+9 = 30 = \frac{n(2m+n+1)}{2}$$

$$= \frac{4(2(5)+4+1)}{2} = 30$$

$$W_{min} = 1+2+3+4 = 10$$

#15.

15. Phadke et al. (2006) conducted a study to evaluate the soleus Hoffman reflex (H-reflex) for two different leg loading conditions on people who have not experienced spinal cord injuries (noninjured subjects) and people with incomplete spinal cord injuries (i-SCI subjects). The Phadke et al. (2006) paper was selected by Erin Easton (2006) for her term project in M. Hollander's 2006 Applied Nonparametric Statistics class. This problem is based on a portion of her analysis. Decreasing the load of weight on the leg is one way that patients with SCI undergo rehabilitation in order to relearn how to stand and walk. Leg loading is controlled through a body weight support (BWS) system that consists of a harness and a suspension system. The typical setting for BWS during rehabilitation for post-SCI patients is 60% leg loading (or 40% BWS). In the Phadke et al. study, 40% BWS was compared to 0% BWS (or 100% leg loading) for both i-SCI and noninjured subjects in order to determine whether a change in percent BWS changed the soleus H-reflex response for subjects in a standing position. Here, we focus on a portion of their data comparing noninjured to i-SCI subjects for 40% BWS.

The soleus muscle is one of the muscles that run from the just below the back of the knee down to the heel, and contraction of this muscle results in plantar flexion of the foot (pointing of the toes) and in maintenance of the body in a stable standing position. The H-reflex is an involuntary response (or flexion) in a muscle on electrical stimulation of the nerves that controls contraction and relaxation of the muscle. The tibialis anterior muscle is a muscle that runs along the front side of the tibia from below the knee to the top of the foot, and contraction of this muscle results in the dorsal flexion of the foot (rise of the foot toward the front of the leg). The tibial nerve runs along the entire back side of the leg, and it supplies electrical impulses to the muscles of the back of the leg, including the soleus. An electromyogram (EMG) is used to measure the electrical current in a muscle. The current is generally proportional to the activity level of the muscle, where an inactive muscle has no current. The H/M ratio is the ratio of the maximum soleus H-reflex to the maximum soleus muscle potential (or to a preset percentage of the maximum potential). Table 4.5 gives the H/M ratios for five noninjured subjects and eight i-SCI subjects for 40% BWS.

Is there evidence, for this 40% BWS situation, that the H/M ratios of the i-SCI subjects are significantly larger than the H/M ratios of the noninjured subjects? What is the approximate P-value achieved by your test.

Table 4.5 H/M Ratios of Noninjured Subjects and i-SCI Subjects For 40% BWS

Noninjured H/M ratios (X_i)	Ranks	i-SCI H/M ratios (Y_j)	Ranks
.19	4	.89	13
.14	3	.76	10
.02	1.5	.63	8
.44	6	.69	9
.37	5	.58	7
		.79	11.5
		.02	1.5
$m=5$		.79	11.5
		$n=8$	

Source: C.P. Phadke, S.S. Wu, F.J. Thompson, and A.L. Behrman (2006).

$H_0: \Delta = 0$

$H_a: \Delta > 0$

$$\therefore W = \sum_j S_j = 13+10+8+9+7+11.5+1.5+11.5 = 71.5$$

$$E_0(W) = \frac{8(5+8+1)}{2} = 56$$

$$Var_0(W) = \frac{5(8)(5+8+1)}{12} = 46.67$$

$$W^* = \frac{71.5 - 56}{\sqrt{46.67}} \approx 2.27 > z_{0.05} = 1.645.$$

$\therefore$ We reject H_0 to conclude that the H/M ratios of the i-SCI subjects are significantly larger than H/M ratios of the noninjured subjects.

The approximate p-value is 0.0116.

16

16. Apply the exact conditional test based on W (see Comment 5) to the H/M ratios data of Table 4.5. Compare your result with that obtained in Problem 15.

Table 4.5 H/M Ratios of Noninjured Subjects and i-SCI Subjects For 40% BWS

Noninjured H/M ratios	Ranks	i-SCI H/M ratios	Ranks
.19	4	.89	13
.14	3	.76	10
.02	1.5	.63	8
.44	6	.69	9
.37	5	.58	7
		.79	11.5
		.02	1.5
		.79	11.5

Source: C.P. Phadke, S.S. Wu, F.J. Thompson, and A.L. Behrman (2006).

From R:

$H_0: \Delta = 0$

$H_a: \Delta > 0$

```
> wilcox.test(c(noninjured, iSCI)~factor(c(0,0,0,0,0, 1,1,1,1,1,1,1,1)))

        Wilcoxon rank sum test with continuity correction

data:  c(noninjured, iSCI) by factor(c(0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1))
W = 4.5, p-value = 0.02768
alternative hypothesis: true location shift is not equal to 0

Warning message:
In wilcox.test.default(x = DATA[[1L]], y = DATA[[2L]], ...) :
  cannot compute exact p-value with ties
```

We have p-val $= 0.02768 < 0.05$

and reject H_0 to conclude that the H/M ratios of the i-SCI subjects are significantly larger than H/M ratios of the noninjured subjects

compared to Q15, p-value here is larger.

#17.

17. Apply van der Waerden's test to the H/M ratios data of Table 4.5. Compare your result with the results of Problems 15 and 16.

Table 4.5 H/M Ratios of Noninjured Subjects and i-SCI Subjects For 40% BWS

Noninjured H/M ratios	Ranks	i-SCI H/M ratios	Ranks
.19	4	.89	13
.14	3	.76	10
.02	1.5	.63	8
.44	6	.69	9
.37	5	.58	7
		.79	11.5
		.02	1.5
		.79	11.5

Source: C.P. Phadke, S.S. Wu, F.J. Thompson, and A.L. Behrman (2006).

$H_0: \Delta = 0$

$H_a: \Delta > 0$.

By R:

```
> results = waerden.test(iSCI, noninjured)
> results
$statistics
     Chisq Df   p.chisq
  3.593444  5 0.6092979

$parameters
     test     name.t ntr alpha
  Waerden noninjured   6  0.05

$means
          iSCI normalScore       std r  Min  Max   Q25  Q50  Q75
0    0.5333333 -0.01454693 0.4445597 3 0.02 0.79 0.405 0.79 0.79
0.02 0.6300000 -0.43164424        NA 1 0.63 0.63 0.630 0.63 0.63
0.14 0.7600000  0.14083537        NA 1 0.76 0.76 0.760 0.76 0.76
0.19 0.8900000  1.22122722        NA 1 0.89 0.89 0.890 0.89 0.89
0.37 0.5800000 -0.76545610        NA 1 0.58 0.58 0.580 0.58 0.58
0.44 0.6900000 -0.14083537        NA 1 0.69 0.69 0.690 0.69 0.69
```

p-val = 0.6093 > 0

∴ We fail to reject H_0 to conclude that the H/M ratio of the i-SCI subjects are equal to the H/M ratios of the noninjured subjects.

The p-val is larger than Q15. Q16

#18. **18.** Consider the data of Table 4.3. Associate the Y's (X's) with the allergies (nonallergies) and estimate Δ of model (4.2) using $\widehat{\Delta}$.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics Y_i	Nonallergics X_i
1651.0	48.1
1112.0	48.0
102.4	45.5
100.0	41.7
67.6	35.4
65.9	34.3
64.7	32.4
39.6	29.1
31.0	27.3
	18.9
	6.6
	5.2
	4.7
$n=9$	$m=13$

Source: H. V. Thomas and E. Simmons (1969).

for mn differences $Y_j - X_i$ for $i=1,\dots,m$
$j=1,\dots,n$

$n=9$

$m=13$

$$k = \frac{mn}{2} = \frac{13(9)}{2} = 58.5.$$

$\therefore$ $\hat{\Delta}$ is the average of $Y-X$ differences that occupy the position 59, from R:

```
> X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
> Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)
> # store the difference
> n = 9
> m = 13
> diff = NULL
>
> for (i in 1:m){
+   for (j in 1:n){
+     diff = c(diff, X[i]-Y[j])
+ }
+   }
> # sort the difference by increasing order
> sort(diff)[59]
[1] -54.3
```

$\hat{\Delta} = -54.3$

From Wilcoxon test output, $\hat{\Delta} = -54.3$

```
> wilcox.test(X, Y, conf.int = T)

        Wilcoxon rank sum exact test

data:  X and Y
W = 11, p-value = 0.000772
alternative hypothesis: true location shift is not equal to 0
95 percent confidence interval:
 -95.8 -22.1
sample estimates:
difference in location
                 -54.3
```

#21. **21.** Consider the data of Table 4.3. Estimate $\delta = P(X < Y)$ and determine an approximate 90% confidence interval for δ.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

(rank)	Allergics X	(rank)	Nonallergics Y	
(22)	1651.0	(15)	48.1	2
(21)	1112.0	(14)	48.0	2
(20)	102.4	(13)	45.5	2
(19)	100.0	(12)	41.7	2
(18)	67.6	(10)	35.4	1
(17)	65.9	(9)	34.3	1
(16)	64.7	(8)	32.4	1
(11)	39.6	(6)	29.1	
(7)	31.0	(5)	27.3	
		(4)	18.9	
		(3)	6.6	
		(2)	5.2	
		(1)	4.7	

$m = 9$ $\qquad$ $n = 13$

Source: H. V. Thomas and E. Simmons (1969).

To estimate $\hat{\delta} = P(X<Y) = \dfrac{U}{mn}$.

We need to calculate U, the Mann-Whitney form of the rank sum statistics.

$$U = \sum_{i=1}^{m}\sum_{j=1}^{n} \phi^*(X_i, Y_j) \text{ where}$$

$$\phi^*(X_i, Y_j) = \begin{cases} 1, & \text{if } X_i < Y_j \\ \frac{1}{2}, & \text{if } X_i = Y_j \\ 0, & \text{if } X_i > Y_j \end{cases}$$

From data, $U = 2+2+2+2+1+1+1 = 11$

$$\therefore \hat{\delta} = \frac{U}{mn} = \frac{11}{9(13)} = 0.094$$

$\therefore$ The mean rank for X is $\bar{R} = \dfrac{\sum_{i=1}^{m} R_i}{m}$

$$= \frac{22+21+\cdots+7}{9} = 16.78$$

The mean rank for Y is $\bar{S} = \dfrac{\sum_{j=1}^{n} S_j}{n}$

$$= \frac{15+14+13+\cdots+1}{13} = 7.85$$

$$\therefore S_{10}^2 = \frac{\sum_i (R_i - i)^2 - m(\bar{R} - \frac{m+1}{2})^2}{(m-1)n^2}$$

$$= \frac{[(22-1)^2 + (21-2)^2 + \cdots + (7-9)^2] - 9(16.78 - \frac{9+1}{2})^2}{8(13)^2}$$

$$= \frac{1700 - 1248.9}{1352} = 0.334$$

$$S_{01}^2 = \frac{\sum_j (S_j - j)^2 - n(\bar{S} - \frac{n+1}{2})^2}{(n-1)m^2}$$

$$= \frac{[(15-1)^2 + (14-2)^2 + \cdots + (9-6)^2] - 13(7.85 - \frac{14}{2})^2}{12(9)^2}$$

$$= 0.849$$

$\therefore$ when $\alpha = 0.1$, $Z_{\alpha/2} = 1.645$

$$\delta_L^S = \hat{\delta} - z_{\frac{\alpha}{2}} \sqrt{\frac{n S_{10}^2 + m S_{01}^2}{mn}}$$

$$= 0.094 - 1.645 \sqrt{\frac{13(0.334) + 9(0.849)}{13(9)}} = -0.432$$

$$\delta_U^S = \hat{\delta} + z_{\frac{\alpha}{2}} \sqrt{\frac{n S_{10}^2 + m S_{01}^2}{mn}}$$

$$= 0.094 + 1.645 \sqrt{\frac{13(0.334) + 9(0.849)}{13(9)}} = 0.620$$

$\therefore$ The 90% approximate CI for δ is

$[-0.432, 0.620]$

#30

30. Refer to Problem 18 and obtain a confidence interval for Δ with approximate confidence coefficient .95.

18. Consider the data of Table 4.3. Associate the Y's (X's) with the allergies (nonallergies) and estimate Δ of model (4.2) using $\widehat{\Delta}$.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics	Nonallergics
1651.0	48.1
1112.0	48.0
102.4	45.5
100.0	41.7
67.6	35.4
65.9	34.3
64.7	32.4
39.6	29.1
31.0	27.3
	18.9
	6.6
	5.2
	4.7

Source: H. V. Thomas and E. Simmons (1969).

By R:

```
> wilcox.test(X, Y, conf.int = T, conf.level = 0.95)

        Wilcoxon rank sum exact test

data:  X and Y
W = 11, p-value = 0.000772
alternative hypothesis: true location shift is not equal to 0
95 percent confidence interval:
 -95.8 -22.1
sample estimates:
difference in location
                 -54.3
```

The CI for Δ with approximate confidence coefficient 0.95 is $[-95.8, -22.1]$.

#35.

35. Consider the data of Table 4.3. Obtain an approximate 95% confidence interval for Δ using the large-sample approximation of this section. Compare your result with the approximate 95% confidence interval obtained in Problem 20.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics	Nonallergics
1651.0	48.1
1112.0	48.0
102.4	45.5
100.0	41.7
67.6	35.4
65.9	34.3
64.7	32.4
39.6	29.1
31.0	27.3
	18.9
	6.6
	5.2
	4.7

$m=9$ $\quad$ $n=13$

Source: H. V. Thomas and E. Simmons (1969).

For $\alpha=0.05$, $z_{\alpha/2}=1.96$

By (4.45), for large-sample approximation:

$$C_{0.05} \approx \frac{mn}{2} - z_{\alpha/2}\left\{\frac{mn(m+n+1)}{12}\right\}^{\frac{1}{2}}$$

$$= \frac{9(13)}{2} - 1.96\left(\frac{9(13)(9+13+1)}{12}\right)^{\frac{1}{2}}$$

$$= 29.15.$$

$\therefore mn+1-C_\alpha = 9(13)+1-29.15 = 88.85.$

By R:

```
> X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
> Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)
> # store the difference
> n = 9
> m = 13
> diff = NULL
>
> for (i in 1:m){
+   for (j in 1:n){
+     diff = c(diff, X[i]-Y[j])
+ }
+   }
>
> # find the number of position 29 and 89
> sort(diff)[29]
[1] -95.8
> sort(diff)[89]
[1] -22.1
```

$\therefore$ The large sample approximated 95% CI is $[-95.8, -22.1]$

Compared with Q20:

20. Consider the data of Table 4.3. Use display (4.35) to obtain an approximate 95% confidence interval for Δ.

```
> wilcox.test(X, Y, conf.int = T, conf.level = 0.95)

        Wilcoxon rank sum exact test

data:  X and Y
W = 11, p-value = 0.000772
alternative hypothesis: true location shift is not equal to 0
95 percent confidence interval:
 -95.8 -22.1
sample estimates:
difference in location
                 -54.3
```

The results are same.

#43.

43. Apply the test based on $\widehat{U}$ to the data of Table 4.3. Compare your results with those of Problem 1.

Table 4.3 Sputum Histamine Levels ($\mu g/g$ Dry Weight Sputum)

Allergics (X_i)	Nonallergics (Y_j)
1651.0	48.1
1112.0	48.0
102.4	45.5
100.0	41.7
67.6	35.4
65.9	34.3
64.7	32.4
39.6	29.1
31.0	27.3
	18.9
	6.6
	5.2
	4.7

Source: H. V. Thomas and E. Simmons (1969).

$m=9$ $\qquad n=13$

$H_0: \theta_x = \theta_y$

$H_1: \theta_x < \theta_y$

$P_1 = 13 \qquad Q_1 = 2 \Rightarrow \bar{P} = \frac{106}{9}$

$P_2 = 13 \qquad Q_2 = 2 \qquad \bar{Q} = \frac{11}{13}$

$P_3 = 13 \qquad Q_3 = 2$

$P_4 = 13 \qquad Q_4 = 2 \qquad V_1 = (13-\frac{106}{9})^2 + \cdots (6-\frac{106}{9})^2 = \frac{464}{9}$

$P_5 = 13 \qquad Q_5 = 1$

$P_6 = 13 \qquad Q_6 = 1 \qquad V_2 = (2-\frac{11}{13})^2 + \cdots (0-\frac{11}{13})^2 = \frac{126}{13}$

$P_7 = 13 \qquad Q_7 = 1$

$P_8 = 9 \qquad Q_8 = 0 \qquad \therefore \hat{U} = \frac{\sum_j Q_j - \sum_i P_i}{2(V_1+V_2+\bar{P}\bar{Q})^{\frac{1}{2}}}$

$P_9 = 6 \qquad Q_9 = 0$

$Q_{10} = 0 \qquad = \frac{106-11}{\left[2\left(\frac{464}{9}+\frac{126}{13}+\frac{106}{9}\left(\frac{11}{13}\right)\right)\right]^{\frac{1}{2}}}$

$Q_{11} = 0$

$Q_{12} = 0 \qquad = 5.6287$

$Q_{13} = 0$

By R :

```
> pFligPoli(X, Y)
Number of X values: 13 Number of Y values: 9
Fligner-Policello U Statistic: 5.6287
Monte Carlo (Using 10000 Iterations) upper-tail probability: 5e-04
Monte Carlo (Using 10000 Iterations) two-sided p-value: 0.001
```

$\therefore \hat{U} = 5.6287$, matches the hand calculations

ph1855_hw3_ygu5

Yue Gu

2024-02-03

Hollander et al. Nonparametric Statistical Methods-Chapter 4

P133 Q1

```r
qnorm(0.95) #1.645
```

```
## [1] 1.644854
```

P135 Q15

```r
1-pnorm(2.27)
```

```
## [1] 0.01160379
```

P136 Q16

```r
# read data
noninjured = c(.19, .14, .02, .44, .37)
iSCI = c(.89, .76, .63, .69, .58, .79, .02, .79)

wilcox.test(c(noninjured, iSCI)~factor(c(0,0,0,0,0, 1,1,1,1,1,1,1,1)))
```

```
## Warning in wilcox.test.default(x = DATA[[1L]], y = DATA[[2L]], ...): cannot
## compute exact p-value with ties
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  c(noninjured, iSCI) by factor(c(0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1))
## W = 4.5, p-value = 0.02768
## alternative hypothesis: true location shift is not equal to 0
```

P136 Q17

```r
noninjured = c(.19, .14, .02, .44, .37, 0, 0, 0)
iSCI =       c(.89, .76, .63, .69, .58, .79, .02, .79)

#install.packages("agricolae")
library(agricolae)
results = waerden.test(iSCI, noninjured)
results
```

```
## $statistics
##      Chisq Df   p.chisq
##   3.593444  5 0.6092979
##
## $parameters
##      test     name.t ntr alpha
##   Waerden noninjured   6  0.05
##
## $means
##           iSCI normalScore       std r  Min  Max   Q25  Q50  Q75
## 0    0.5333333 -0.01454693 0.4445597 3 0.02 0.79 0.405 0.79 0.79
## 0.02 0.6300000 -0.43164424        NA 1 0.63 0.63 0.630 0.63 0.63
## 0.14 0.7600000  0.14083537        NA 1 0.76 0.76 0.760 0.76 0.76
## 0.19 0.8900000  1.22122722        NA 1 0.89 0.89 0.890 0.89 0.89
## 0.37 0.5800000 -0.76545610        NA 1 0.58 0.58 0.580 0.58 0.58
## 0.44 0.6900000 -0.14083537        NA 1 0.69 0.69 0.690 0.69 0.69
##
## $comparison
## NULL
##
## $groups
##            score groups
## 0.19  1.22122722      a
## 0.14  0.14083537      a
## 0    -0.01454693      a
## 0.44 -0.14083537      a
## 0.02 -0.43164424      a
## 0.37 -0.76545610      a
##
## attr(,"class")
## [1] "group"
```

P141 Q18

```r
# input nonallergics (baseline) vs allergics
X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)
# store the difference
n = 9
m = 13
```

```r
diff = NULL

for (i in 1:m){
  for (j in 1:n){
    diff = c(diff, X[i]-Y[j])
}
  }
# sort the difference by increasing order
sort(diff)[59]
```

```
## [1] -54.3
```

```r
# check output from the test results
wilcox.test(X, Y, conf.int = T)
```

```
##
##  Wilcoxon rank sum exact test
##
## data:  X and Y
## W = 11, p-value = 0.000772
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
##  -95.8 -22.1
## sample estimates:
## difference in location
##                  -54.3
```

P144 Q30

```r
X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)
wilcox.test(X, Y, conf.int = T, conf.level = 0.95)
```

```
##
##  Wilcoxon rank sum exact test
##
## data:  X and Y
## W = 11, p-value = 0.000772
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
##  -95.8 -22.1
## sample estimates:
## difference in location
##                  -54.3
```

P145 Q35

```r
X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)
# store the difference
n = 9
m = 13
diff = NULL

for (i in 1:m){
  for (j in 1:n){
    diff = c(diff, X[i]-Y[j])
}
  }

# find the number of position 29 and 89
sort(diff)[29]
```

```
## [1] -95.8
```

```r
sort(diff)[89]
```

```
## [1] -22.1
```

```r
# compare with Q20
wilcox.test(X, Y, conf.int = T, conf.level = 0.95)
```

```
##
##  Wilcoxon rank sum exact test
##
## data:  X and Y
## W = 11, p-value = 0.000772
## alternative hypothesis: true location shift is not equal to 0
## 95 percent confidence interval:
##  -95.8 -22.1
## sample estimates:
## difference in location
##                  -54.3
```

P149 Q43

```r
X =  c(48.1, 48.0, 45.5, 41.7, 35.4, 34.3, 32.4, 29.1, 27.3, 18.9, 6.6, 5.2, 4.7)
Y =  c(1651.0, 1112.0, 102.4, 100.0, 67.6, 65.9, 64.7, 39.6, 31.0)

library(NSM3)
pFligPoli(X, Y)
```

```
## Number of X values: 13 Number of Y values: 9
## Fligner-Policello U Statistic: 5.6287
## Monte Carlo (Using 10000 Iterations) upper-tail probability: 4e-04
## Monte Carlo (Using 10000 Iterations) two-sided p-value: 8e-04
##
```