CS 703 Project Proposal

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1 Motivation

2 Preliminaries

All graphs discussed in this proposal are undirected and simple. A set of vertices in a graph is a *clique* if there is an edge between each pair of vertices within the set. A clique is *maximal* if it is not properly contained in another clique. A *maximum clique* is a clique with the maximum number of vertices. Note that a graph may have multiple maximum cliques. Given a graph G, the size of the maximum clique of G is called the *clique number*, denoted by $\omega(G)$.

3 Problem definition

The problem this project attempts to address can be formulated as:

> Let $\mathcal{G}$ be a random graph generator and a random graph G with $G \sim \mathcal{G}$. Compute $\mathbf{E}[\omega(G)]$.

Multiple random graph models have already been carefully studied. One classical example is the Erdős-Rényi model [3] in which each pair of vertices in a graph with n vertices has equal probability p of being adjacent. For example, when $p = 1/2$, the clique number of G is at most $(2+\varepsilon)\log_2(n) + 1$ with high probability, for all $\varepsilon > 0$ [4]. Power Law graphs characterize the networks in which the number of vertices with a certain degree k is propotional to $1/k^c$ for some constant c, and such a model is studied in [1], but an estimation of the size of the maximum clique is not present in the work.

4 Action plan

We propose to investigate the power law graph model and derive a bound for the size of the maximum cliques in the power law graphs for different parameter c. Our expectation is that for smaller c the maximum clique size should be larger, as more vertices have large degrees and hence more likely larger cliques may be present. However, this still needs to be supported by proofs and experiments.

In addition to the power law graph model, we plan to find another appropriate random graph model for the programming by example setting. Hence we propose to research on the existing random graph models that have interesting properties[2], and study the input/output examples from real data to extract some succint models out of it.

5 Deliverables and milestones

The milestones for the projects are as follows.

1. Oct 21: Finish implementing the Power Law graph model.
2. Oct 23: Finish reviewing the literature[1].
3. Oct 26: Finish constructing new plausible models.
4. Nov 12: Proof sketch for power law graph model and new models if available.
5. Nov 18: Finish implementing and experiments on the new model if possible.
6. Dec 2: Finish report write-up and slides preparation.

While the experiments demonstrate the result in a Construction of new models

[1] A not so measurable milestone, but still critical.

References

[1] William Aiello, Fan Chung, and Linyuan Lu. A random graph model for massive graphs. In *Proceedings of the thirty-second annual ACM symposium on Theory of computing*, pages 171–180. ACM, 2000.

[2] David Easley and Jon Kleinberg. *Networks, crowds, and markets: Reasoning about a highly connected world*. Cambridge University Press, 2010.

[3] Paul Erdos and Alfréd Rényi. On the evolution of random graphs. *Publ. Math. Inst. Hung. Acad. Sci*, 5(1):17–60, 1960.

[4] Daniel A. Spielman. Random graphs: Markov's inequality, October 2007.