Assignment 3 - Local Search for Selective TSP

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Github

> https://github.com/Luncenok/EvolutionaryComputing

Problem Description

This is a variant of the Traveling Salesman Problem where:

- Select exactly 50% of nodes (rounded up if odd)
- Form a Hamiltonian cycle through selected nodes
- Minimize: total path length + sum of selected node costs
- Distances are Euclidean distances rounded to integers

Algorithm Pseudocode

Local Search - Steepest Descent (Nodes Exchange)

```
1. Start with initial solution
2. Mark all nodes in solution as selected
3. While improvement found:
   a. bestDelta = 0
   b. For each pair of positions (i, j) in solution (intra-route):
      Calculate delta for swapping nodes at positions i and j
      If delta < bestDelta: remember this move
   c. For each position pos in solution and each non-selected node (inter-
route):
      Calculate delta for exchanging solution[pos] with non-selected node
      If delta < bestDelta: remember this move
   d. If bestDelta < 0:
      Apply best move
   e. Else:
      Stop (no improvement found)
4. Return solution
```

Local Search - Steepest Descent (Edges Exchange)

```
Similar to Nodes Exchange, but for intra-route moves:
- Instead of swapping two nodes, reverse the segment between positions i
and j
```

```
- This corresponds to removing two edges and adding two new edges (2-opt
move)
```

Local Search - Greedy (Nodes Exchange)

```
1. Start with initial solution
2. Mark all nodes in solution as selected
3. While improvement found:
   a. Calculate total number of moves:
      numInter = |solution| × (n - |solution|)  # inter-route moves
      numIntra = |solution| × (|solution| - 1) / 2  # intra-route moves
      totalMoves = numInter + numIntra
   b. Create random permutation of move indices [0, 1, ..., totalMoves-1]
   c. For each move index in random order:
      - If index < numInter: this is an inter-route move
        Decode index to (position, non-selected node)
        Calculate delta for node exchange
      - Else: this is an intra-route move
        Decode index to (pos1, pos2)
        Calculate delta for node swap
      - If delta < 0:
        Apply move and break (found improvement)
4. Return solution
```

Local Search - Greedy (Edges Exchange)

```
Similar to Greedy Nodes, but intra-route moves use segment reversal
instead of node swaps.
```

Randomization Strategy

In the greedy local search version, we use **random sampling without replacement** to ensure proper randomization:

```cpp
// Random sampling without replacement
std::vector<bool> tried(totalMoves, false);
std::uniform_int_distribution<> moveDist(0, totalMoves - 1);

while (!improved && triedCount < totalMoves) {
    // Pick random untried move
    int moveIdx = moveDist(rng);
    if (tried[moveIdx]) continue;  // Skip if already tried

    tried[moveIdx] = true;
    triedCount++;
```

```
    // Decode move index to specific move (inter or intra)
    // Evaluate delta and apply if improving
}
```

This approach:

1. Assigns unique index to each move (inter-route: 0 to numInter-1, intra-route: numInter to totalMoves-1)
2. Randomly samples move indices using uniform distribution
3. Tracks tried moves with boolean vector to ensure no repetition
4. Stops immediately when first improving move is found

Advantages over full shuffle:

- No need to create and shuffle 15,000-element vector
- Stops early when improvement found (doesn't evaluate remaining moves)
- True random mixing of inter-route and intra-route moves
- Memory efficient (only boolean tracking array)

Trade-off: As more moves are tried, collision probability increases, requiring more random number generation attempts

Key Results

Objective Function Values

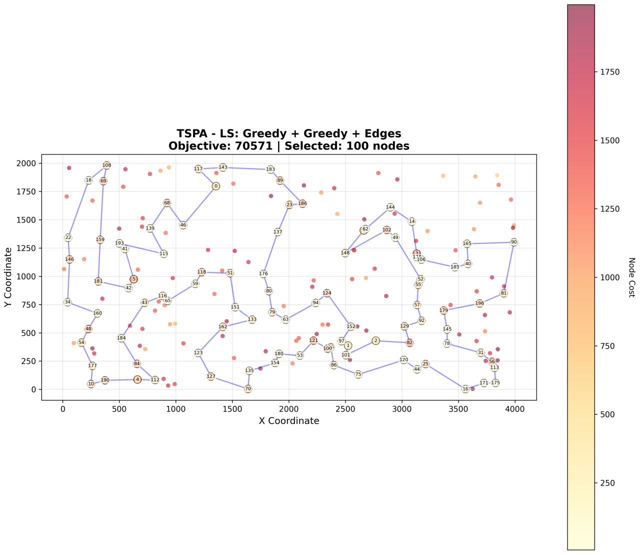
Method	TSPA	TSPB
Random	264638 (238611 - 287962)	213875 (190076 - 244960)
Nearest Neighbor (end only)	85108 (83182 - 89433)	54390 (52319 - 59030)
Nearest Neighbor (any position)	73178 (71179 - 75450)	45870 (44417 - 53438)
Greedy Cycle	72646 (71488 - 74410)	51400 (49001 - 57324)
Greedy 2-Regret	115474 (105852 - 123428)	72454 (66505 - 77072)
Greedy Weighted (2-Regret + BestDelta)	72129 (71108 - 73395)	50950 (47144 - 55700)
Nearest Neighbor Any 2-Regret	116659 (106373 - 126570)	73646 (67121 - 79013)
Nearest Neighbor Any Weighted	72401 (70010 - 75452)	47653 (44891 - 55247)
LS: Random + Steepest + Nodes	**87965 (81347 - 100053)**	**63288 (54775 - 78289)**
LS: Random + Steepest + Edges	**73828 (71559 - 78727)**	**48281 (45843 - 51140)**
LS: Random + Greedy + Nodes	**86336 (80068 - 94432)**	**60888 (53634 - 67187)**
LS: Random + Greedy + Edges	**74182 (71423 - 76992)**	**48675 (46143 - 52199)**
LS: Greedy + Steepest + Nodes	**71614 (70626 - 72950)**	**45414 (43826 - 50876)**
LS: Greedy + Steepest + Edges	**71460 (70510 - 72614)**	**44979 (43921 - 50629)**

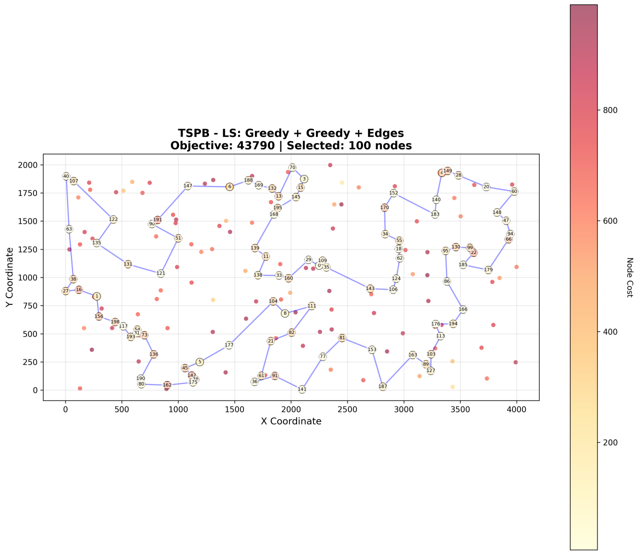
Method	TSPA	TSPB
LS: Greedy + Greedy + Nodes	**71658 (70687 - 73046)**	**45513 (43826 - 51140)**
LS: Greedy + Greedy + Edges	**71517 (70571 - 72780)**	**45033 (43790 - 50632)**

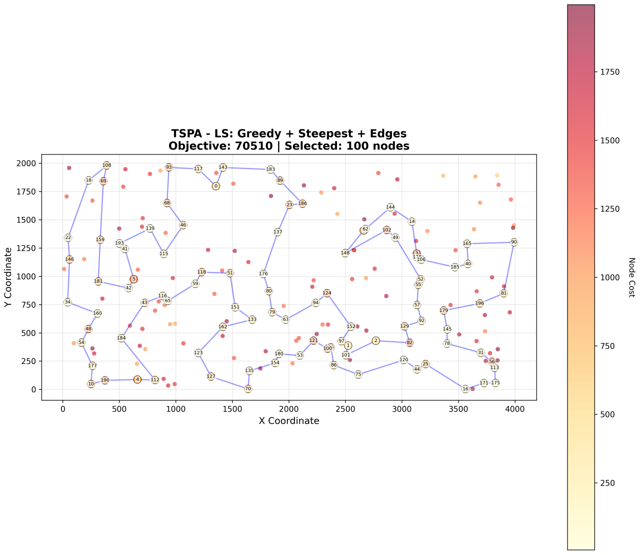
Running Times (ms)

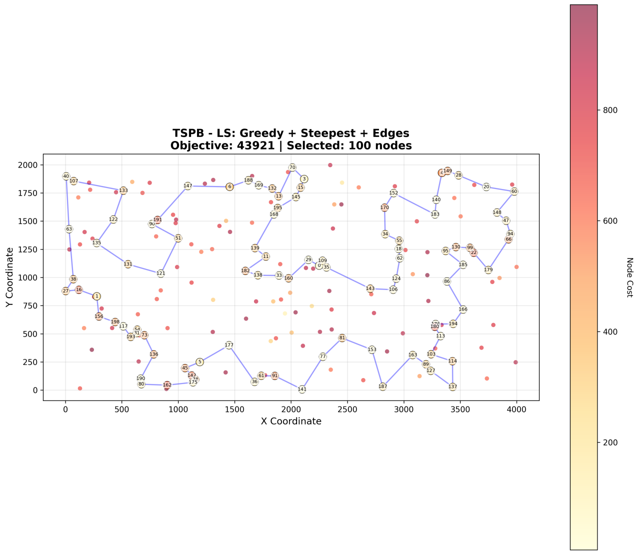
Method	TSPA	TSPB
Random	0.005 (0.004 - 0.015)	0.005 (0.004 - 0.007)
Nearest Neighbor (end only)	0.019 (0.014 - 0.056)	0.019 (0.015 - 0.022)
Nearest Neighbor (any position)	0.736 (0.706 - 1.050)	0.742 (0.717 - 1.049)
Greedy Cycle	0.698 (0.670 - 1.110)	0.718 (0.682 - 1.275)
Greedy 2-Regret	0.979 (0.936 - 1.344)	0.987 (0.944 - 1.302)
Greedy Weighted (2-Regret + BestDelta)	1.012 (0.949 - 1.971)	0.998 (0.959 - 1.429)
Nearest Neighbor Any 2-Regret	0.942 (0.863 - 1.675)	0.917 (0.874 - 1.224)
Nearest Neighbor Any Weighted	0.904 (0.873 - 1.209)	0.920 (0.889 - 1.269)
LS: Random + Steepest + Nodes	**5.677 (4.226 - 8.143)**	**5.708 (4.447 - 7.293)**
LS: Random + Steepest + Edges	**3.339 (2.855 - 3.793)**	**3.442 (3.010 - 3.986)**
LS: Random + Greedy + Nodes	**23.694 (12.277 - 85.151)**	**22.225 (12.500 - 45.449)**
LS: Random + Greedy + Edges	**16.215 (10.323 - 28.643)**	**16.047 (9.396 - 26.851)**
LS: Greedy + Steepest + Nodes	**1.158 (0.991 - 1.442)**	**0.942 (0.790 - 1.791)**
LS: Greedy + Steepest + Edges	**1.123 (0.990 - 1.356)**	**1.025 (0.809 - 3.370)**
LS: Greedy + Greedy + Nodes	**8.040 (4.389 - 13.077)**	**7.791 (3.984 - 17.071)**
LS: Greedy + Greedy + Edges	**8.218 (4.515 - 13.029)**	**8.835 (5.014 - 17.285)**

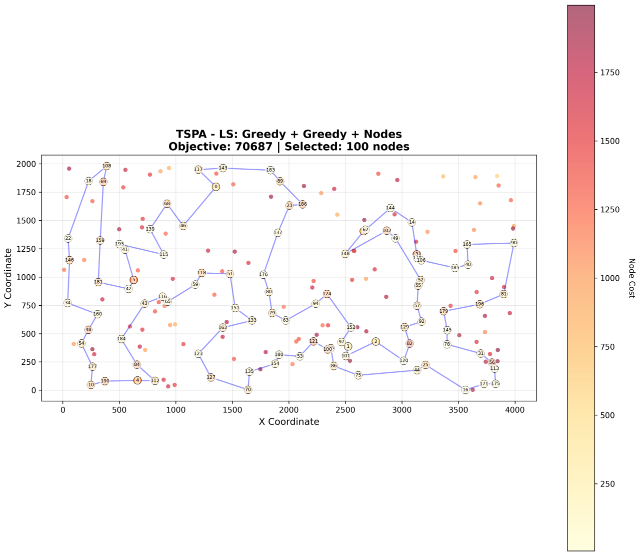
Visualizations

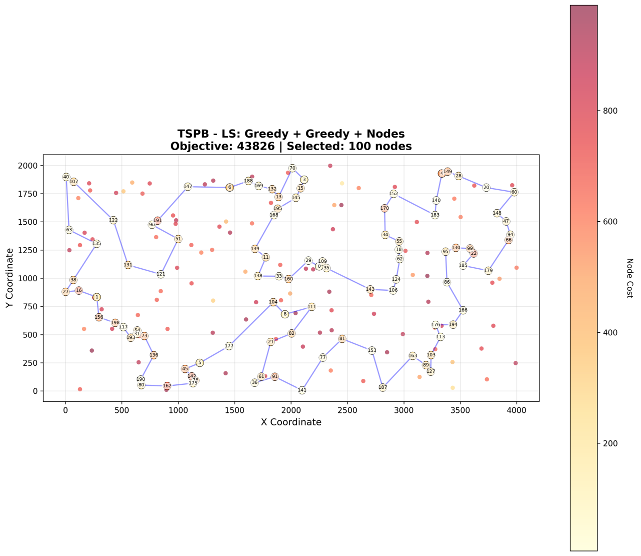

TSPA - LS: Greedy + Greedy + Edges
Objective: 70571 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

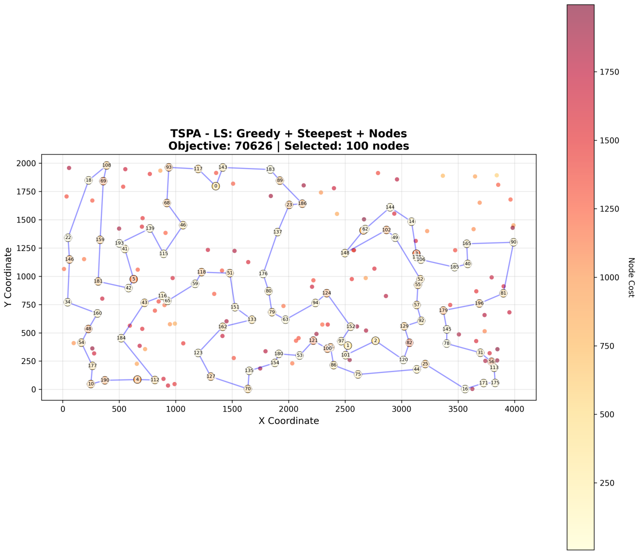

TSPB - LS: Greedy + Greedy + Edges
Objective: 43790 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

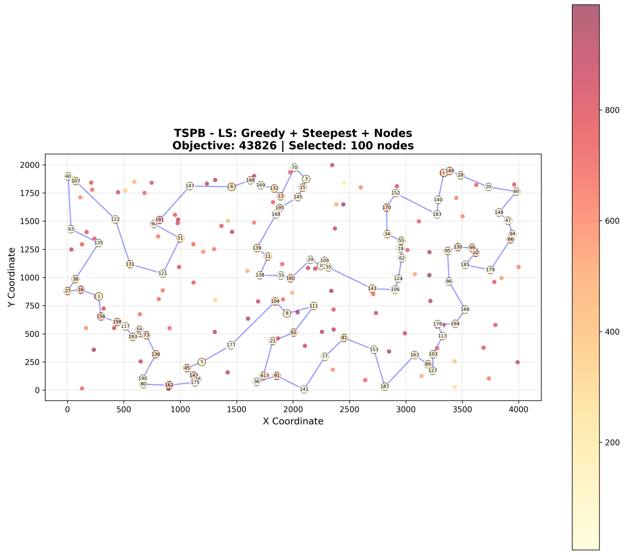

TSPA - LS: Greedy + Steepest + Edges
Objective: 70510 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

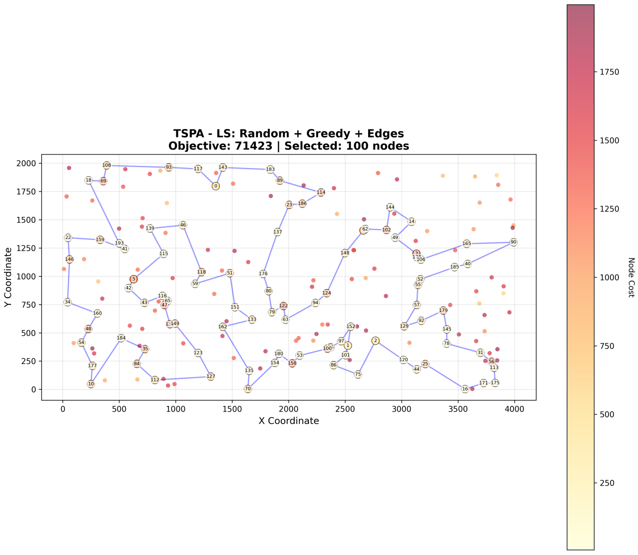

TSPB - LS: Greedy + Steepest + Edges
Objective: 43921 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

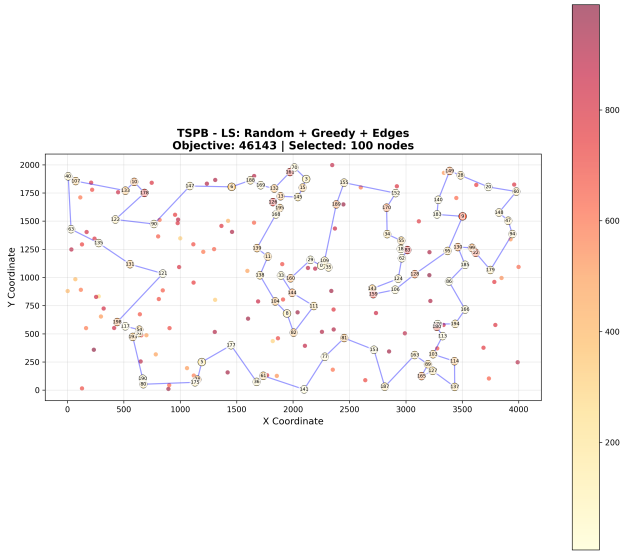

TSPA - LS: Greedy + Greedy + Nodes
Objective: 70687 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

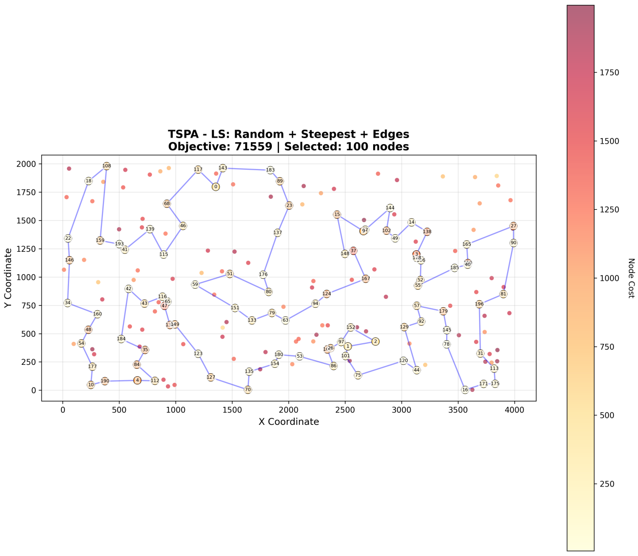

TSPB - LS: Greedy + Greedy + Nodes
Objective: 43826 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

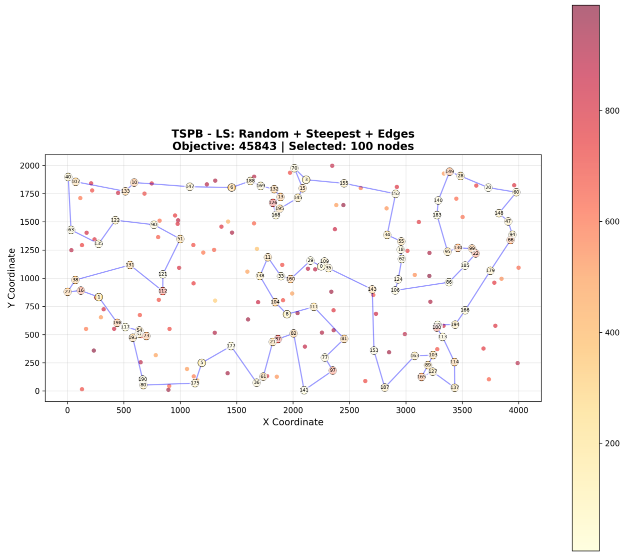

TSPA - LS: Greedy + Steepest + Nodes
Objective: 70626 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

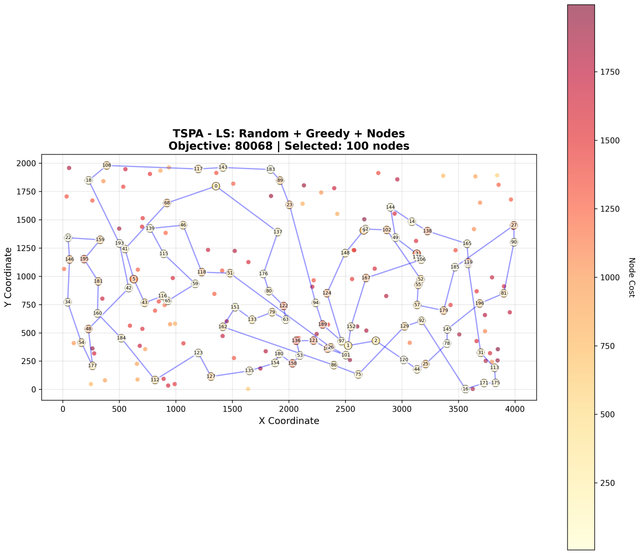

TSPB - LS: Greedy + Steepest + Nodes
Objective: 43826 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

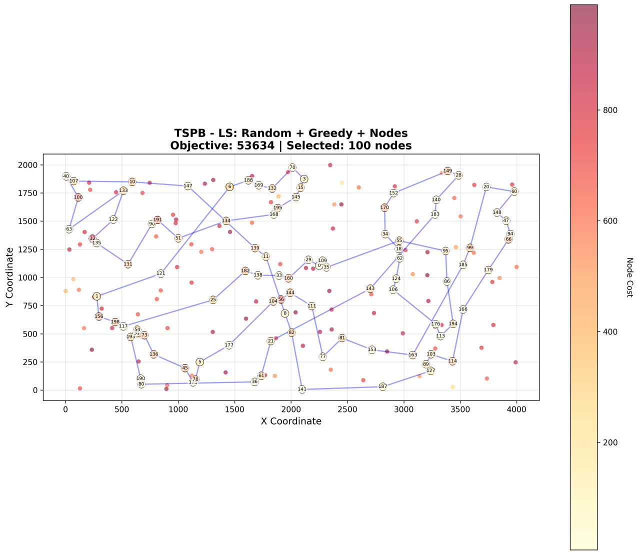

TSPA - LS: Random + Greedy + Edges
Objective: 71423 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

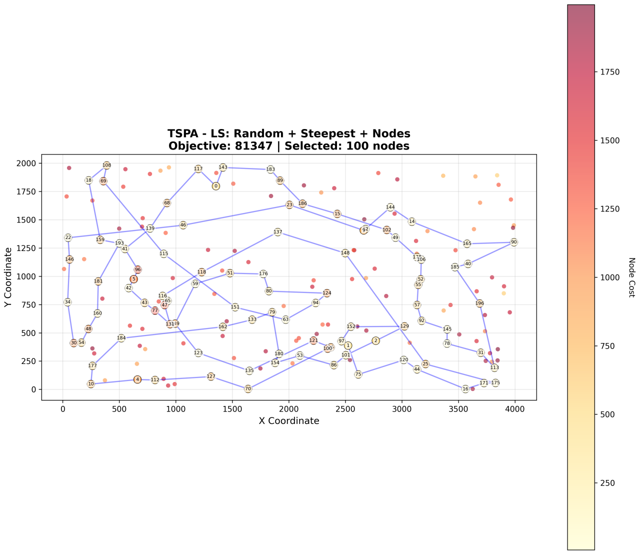

TSPB - LS: Random + Greedy + Edges
Objective: 46143 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost

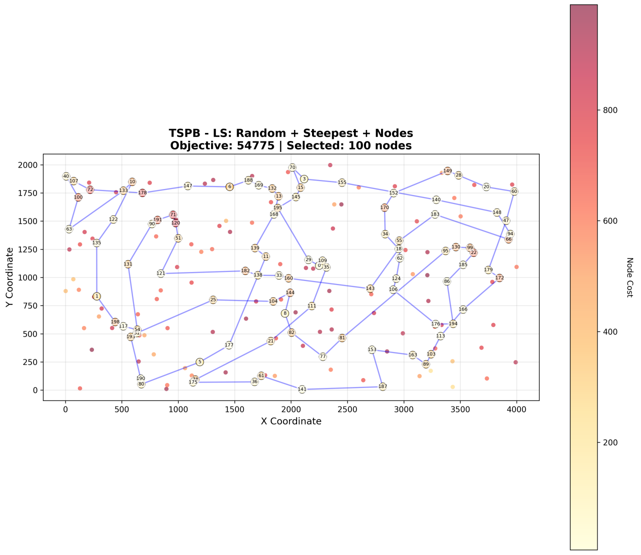

TSPA - LS: Random + Steepest + Edges
Objective: 71559 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost


TSPB - LS: Random + Steepest + Edges
Objective: 45843 | Selected: 100 nodes
X Coordinate
Y Coordinate
Node Cost









Conclusions

Overall Performance

Local search significantly improves solution quality compared to construction heuristics alone. The best results for both instances were achieved by **LS: Greedy + Steepest + Edges**:

- **TSPA**: Average objective improved from 72,129 (Greedy Weighted) to **71,460** (0.9% improvement)
- **TSPB**: Average objective improved from 45,870 (NN Any) to **44,979** (1.9% improvement)

The local search methods with greedy starting solutions consistently produced the best results, achieving objectives competitive with or better than the best construction heuristics, while also significantly reducing variance (smaller max-min range).

Impact of Local Search

Local search dramatically improves solution quality from random solution. More importantly, local search starting from the **best construction heuristics** improves results:

- **TSPA**: Greedy Weighted avg 72,129 → LS Greedy+Steepest+Edges avg 71,460 **(0.9% improvement)**
- **TSPB**: NN Any avg 45,870 → LS Greedy+Steepest+Edges avg 44,979 **(1.9% improvement)**

Steepest vs Greedy Local Search

Steepest descent consistently outperforms greedy local search in solution quality:

Starting	Move Type	TSPA Steepest	TSPA Greedy	TSPB Steepest	TSPB Greedy
Random	Nodes	87,965	86,336	63,288	60,888
Random	Edges	73,828	74,182	48,281	48,675
Greedy	Nodes	71,614	71,658	45,414	45,513
Greedy	Edges	**71,460**	71,517	**44,979**	45,033

Time-quality tradeoff (with improved randomization):

- **Steepest**: ~1-6 ms average (explores entire neighborhood, picks best move)
- **Greedy with random start**: ~16-24 ms average (random sampling, stops at first improvement)
- **Greedy with greedy start**: ~8-10 ms average (fewer iterations needed from good start)

Key insight: Steepest makes larger improvements per iteration (best move), requiring fewer iterations overall. Even though it evaluates all moves each iteration, the faster convergence makes it more efficient than greedy's "first improvement" strategy.

Nodes Exchange vs Edges Exchange

Edges exchange (2-opt) consistently outperforms nodes exchange:

TSPA (Greedy start + Steepest):

- Nodes: 71,614 average
- Edges: **71,460 average** (0.2% better)

TSPB (Greedy start + Steepest):

- Nodes: 45,414 average
- Edges: **44,979 average** (1.0% better)

This makes sense because:

1. **Edges exchange (segment reversal)** changes the order of multiple nodes, creating larger structural changes
2. **Nodes exchange (swap)** only affects two positions and their immediate neighbors
3. Edge exchange explores a fundamentally different neighborhood structure

Both methods have the same computational complexity $O(n^2)$, but edges exchange is slightly faster in practice (1.17 ms vs 1.21 ms) due to simpler delta calculations.

Starting Solution Quality

Good starting solutions are crucial for local search performance:

TSPA (Steepest + Edges):

- Random start: 73,828 average (range: 71,559-78,727)
- Greedy start: **71,460 average** (range: 70,510-72,614)
- Improvement: 3.2% better, 42% smaller range

TSPB (Steepest + Edges):

- Random start: 48,281 average (range: 45,843-51,140)
- Greedy start: **44,979 average** (range: 43,921-50,629)
- Improvement: 6.8% better, slightly smaller variance

Key insights:

1. Local search from greedy solutions reaches better local optima
2. Starting quality significantly impacts final quality
3. Even poor random solutions improve dramatically (random avg 264,638 → 73,828 after LS on TSPA)
4. The best greedy heuristic (Greedy Weighted) provides excellent starting points