

# Assignment 7 - Large Neighborhood Search for Selective TSP

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## Github

<https://github.com/Luncenok/EvolutionaryComputing>

## Problem Description

This is the same variant of the Traveling Salesman Problem as in previous assignments:

- Select exactly 50% of nodes (rounded up if odd)
- Form a Hamiltonian cycle through selected nodes
- Minimize: total path length + sum of selected node costs
- Distances are Euclidean distances rounded to integers

Instances:

- **TSPA, TSPB** with 200 nodes, selecting 100 nodes.

## Goal

Implement Large Neighborhood Search (LNS) in two versions:

1. **LNS with Local Search:** Apply local search after each destroy-repair iteration
2. **LNS without Local Search:** Only apply destroy-repair without subsequent local search

For both versions:

- Use a random starting solution with local search applied initially
- Run **20 times** per instance
- Use a **time limit equal to the average MSLS time** from Assignment 6
- Report the number of main loop iterations

## Algorithm Pseudocode

### Large Neighborhood Search (LNS)

LNS iteratively destroys and repairs a solution to explore larger neighborhoods.

```
LNS(timeLimit, useLocalSearch=True):
    # Initialize with random solution
    current = generateRandomSolution()
```

```

current = localSearch(current) # Always apply to initial

best = current
bestObjective = objective(current)

startTime = now()
iterations = 0

while (now() - startTime) < timeLimit:
    iterations += 1

    # Destroy: remove ~30% of nodes
    partial = destroy(current)

    # Repair: rebuild using greedy heuristic
    repaired = repair(partial)

    # Optional local search
    if useLocalSearch:
        candidate = localSearch(repaired)
    else:
        candidate = repaired

    obj = objective(candidate)

    # Update best
    if obj < bestObjective:
        bestObjective = obj
        best = candidate

    # Accept if better (greedy acceptance)
    if obj < objective(current):
        current = candidate

return best, iterations

```

## Destroy Operator

The destroy operator removes approximately 30% of nodes from the current solution using weighted random selection. Nodes connected by longer edges have higher probability of removal:

```

destroy(solution, destroyFraction=0.30):
    numToRemove = solSize * destroyFraction

    # Calculate weights based on adjacent edge costs
    weights = []
    for i in range(len(solution)):
        prev = (i - 1) % len(solution)
        next = (i + 1) % len(solution)
        edgeCost = distance[solution[prev]][solution[i]] +
distance[solution[i]][solution[next]]

```

```

        weight = edgeCost + costs[solution[i]]
        weights.append(weight)

    # Weighted random selection of nodes to remove
    toRemove = weightedRandomSelect(numToRemove, weights)

    # Return remaining nodes (preserving tour order)
    return [node for i, node in enumerate(solution) if i not in toRemove]

```

### Destroy rationale:

- Weighted selection targets "bad" edges (longer connections) and costly nodes
- 30% removal provides significant diversification while retaining solution structure
- Preserving tour order helps maintain good partial structures
- Randomization prevents deterministic cycling

### Repair Operator

The repair operator rebuilds the solution to full size using the weighted 2-regret heuristic (the best-performing greedy heuristic from previous assignments):

```

repair(partial, selectCount):
    solution = partial

    while len(solution) < selectCount:
        bestNode = None
        bestPos = None
        bestScore = -infinity

        for each unselected node:
            # Find best and second-best insertion positions
            best1, best2 = findTwoBestInsertions(node, solution)

            regret = best2 - best1
            score = wRegret * regret - wBest * best1

            if score > bestScore:
                bestScore = score
                bestNode = node
                bestPos = bestInsertPosition

        solution.insert(bestPos, bestNode)

    return solution

```

### Repair rationale:

- Weighted 2-regret achieves best results among greedy heuristics
- Balances urgency (regret) with quality (insertion cost)
- Efficiently rebuilds while maintaining good tour quality

## Experimental Setup

- **Instances:** TSPA, TSPB (200 nodes, 100 selected)
- **Objective:** Minimize path length + sum of selected node costs
- **Local search:** Steepest descent with edge exchange (from Assignment 3)
- **Destroy fraction:** 30% of nodes removed
- **Repair heuristic:** Weighted 2-regret ( $w_{regret}=1.0$ ,  $w_{best}=1.0$ )
- **Evaluation:**
  - Run both LNS versions **20 times** per instance
  - Use **time limit = average MSL time** (~1090 ms)
  - Report min, max, and average objective values and running times
  - Report average number of destroy-repair iterations

## Key Results

### Summary Comparison

| Instance | ILS Avg | LNS+LS Avg | LNS Avg | LNS+LS vs ILS | LNS vs ILS |
|----------|---------|------------|---------|---------------|------------|
| TSPA     | 69340   | 69689      | 69817   | +0.50%        | +0.69%     |
| TSPB     | 43674   | 44239      | 44294   | +1.29%        | +1.42%     |

### Iteration Count Table

| Instance | LNS with LS | LNS without LS | ILS (LS runs) |
|----------|-------------|----------------|---------------|
| TSPA     | 1476.9      | 1683.4         | 3544.5        |
| TSPB     | 1438.95     | 1620.15        | 3566.7        |

### Comparison with All Previous Methods

| Method                                 | TSPA                     | TSPB                     |
|--|--------------------------|--------------------------|
| Random                                 | 264638 (238611 – 287962) | 213875 (190076 – 244960) |
| Nearest Neighbor (end only)            | 85108 (83182 – 89433)    | 54390 (52319 – 59030)    |
| Nearest Neighbor (any position)        | 73178 (71179 – 75450)    | 45870 (44417 – 53438)    |
| Greedy Cycle                           | 72646 (71488 – 74410)    | 51400 (49001 – 57324)    |
| Greedy 2-Regret                        | 115474 (105852 – 123428) | 72454 (66505 – 77072)    |
| Greedy Weighted (2-Regret + BestDelta) | 72129 (71108 – 73395)    | 50950 (47144 – 55700)    |
| Nearest Neighbor Any 2-Regret          | 116659 (106373 – 126570) | 73646 (67121 – 79013)    |
| Nearest Neighbor Any Weighted          | 72401 (70010 – 75452)    | 47653 (44891 – 55247)    |
| LS Random + Steepest + Nodes           | 88011 (81817 – 97630)    | 62848 (55928 – 70479)    |
| LS Random + Greedy + Nodes             | 93267 (86375 – 101454)   | 65388 (57842 – 76707)    |

| Method                       | TSPA                         | TSPB                         |
|------------------------------|------------------------------|------------------------------|
| LS Random + Greedy + Edges   | 81101 (76362 – 87763)        | 54088 (50858 – 59045)        |
| LS Greedy + Steepest + Nodes | 71614 (70626 – 72950)        | 45414 (43826 – 50876)        |
| LS Greedy + Steepest + Edges | 71460 (70510 – 72614)        | 44979 (43921 – 50629)        |
| LS Greedy + Greedy + Nodes   | 71908 (71093 – 73048)        | 45584 (43917 – 51165)        |
| LS Greedy + Greedy + Edges   | 71825 (70977 – 72706)        | 45376 (43845 – 51170)        |
| LS Random + Steepest + Edges | 73965 (71371 – 78984)        | 48252 (45823 – 51965)        |
| LM Random + Steepest + Edges | 74981 (72054 – 79520)        | 49325 (45965 – 52805)        |
| Candidates (k=5)             | 84726 (78843 – 91459)        | 49873 (47117 – 53865)        |
| Candidates (k=10)            | 77773 (72851 – 84000)        | 48450 (45669 – 51178)        |
| Candidates (k=15)            | 75510 (72276 – 83040)        | 48295 (45582 – 51938)        |
| Candidates (k=20)            | 74416 (71292 – 80264)        | 48221 (45338 – 51285)        |
| LM Candidates (k=10)         | 75157 (72331 – 80832)        | 49219 (46145 – 52021)        |
| LM Candidates (k=20)         | 74976 (72054 – 79520)        | 49302 (45965 – 52805)        |
| MSLS (200 iterations)        | 71306 (70748 – 71959)        | 45741 (45356 – 46168)        |
| ILS                          | 69340 (69107 – 69861)        | 43674 (43473 – 44056)        |
| <b>LNS with LS</b>           | <b>69689 (69185 – 70194)</b> | <b>44239 (43566 – 45964)</b> |
| <b>LNS without LS</b>        | <b>69817 (69496 – 70217)</b> | <b>44294 (43630 – 45602)</b> |

### Running Times (ms)

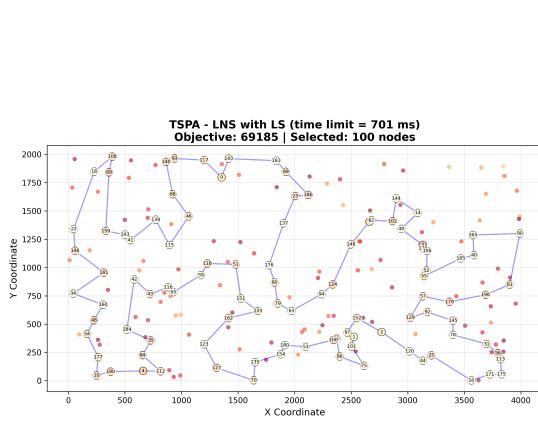
| Method                                 | TSPA                     | TSPB                    |
|--|--------------------------|-------------------------|
| Random                                 | 0.0001 (0.00004 – 0.003) | 0.00003 (0.00 – 0.0001) |
| Nearest Neighbor (end only)            | 0.0198 (0.0149 – 0.063)  | 0.0204 (0.0152 – 0.062) |
| Nearest Neighbor (any position)        | 0.878 (0.691 – 2.102)    | 0.720 (0.692 – 0.862)   |
| Greedy Cycle                           | 0.683 (0.654 – 0.967)    | 0.681 (0.657 – 0.796)   |
| Greedy 2-Regret                        | 0.952 (0.920 – 1.263)    | 0.941 (0.920 – 1.057)   |
| Greedy Weighted (2-Regret + BestDelta) | 0.957 (0.920 – 1.302)    | 0.946 (0.918 – 1.052)   |
| Nearest Neighbor Any 2-Regret          | 0.893 (0.852 – 1.175)    | 1.105 (0.844 – 21.87)   |
| Nearest Neighbor Any Weighted          | 0.905 (0.857 – 1.405)    | 0.894 (0.852 – 1.287)   |
| LS Random + Steepest + Nodes           | 6.297 (4.872 – 66.19)    | 5.700 (4.538 – 7.177)   |
| LS Random + Greedy + Nodes             | 4.173 (2.613 – 7.520)    | 3.813 (2.502 – 7.189)   |

| Method                       | TSPA                            | TSPB                            |
|------------------------------|---------------------------------|---------------------------------|
| LS Random + Greedy + Edges   | 3.375 (2.184 – 8.683)           | 3.275 (2.243 – 5.501)           |
| LS Greedy + Steepest + Nodes | 1.188 (0.993 – 3.055)           | 0.937 (0.803 – 1.608)           |
| LS Greedy + Steepest + Edges | 1.139 (1.000 – 1.856)           | 0.915 (0.808 – 1.441)           |
| LS Greedy + Greedy + Nodes   | 2.840 (2.182 – 4.170)           | 2.556 (1.909 – 4.489)           |
| LS Greedy + Greedy + Edges   | 2.877 (2.169 – 5.164)           | 2.628 (1.967 – 4.344)           |
| LS Random + Steepest + Edges | 3.356 (2.852 – 3.959)           | 3.515 (2.990 – 5.310)           |
| LM Random + Steepest + Edges | 3.858 (2.957 – 5.073)           | 3.949 (3.062 – 10.59)           |
| Candidates (k=5)             | 8.697 (7.449 – 9.838)           | 9.417 (8.245 – 10.69)           |
| Candidates (k=10)            | 10.03 (8.757 – 11.80)           | 11.05 (9.389 – 49.22)           |
| Candidates (k=15)            | 12.03 (9.876 – 50.62)           | 12.01 (10.41 – 13.79)           |
| Candidates (k=20)            | 12.55 (11.18 – 14.29)           | 13.30 (11.60 – 16.14)           |
| LM Candidates (k=10)         | 4.316 (3.698 – 5.795)           | 4.124 (3.478 – 4.605)           |
| LM Candidates (k=20)         | 15.04 (12.35 – 43.77)           | 14.85 (12.92 – 17.07)           |
| MSLS (200 iterations)        | 701.38 (665.26 – 788.26)        | 690.25 (672.52 – 749.00)        |
| ILS                          | 701.46 (701.38 – 701.57)        | 690.37 (690.25 – 690.60)        |
| <b>LNS with LS</b>           | <b>701.59 (701.38 – 701.84)</b> | <b>690.52 (690.32 – 690.67)</b> |
| <b>LNS without LS</b>        | <b>701.54 (701.39 – 701.76)</b> | <b>690.40 (690.26 – 690.66)</b> |

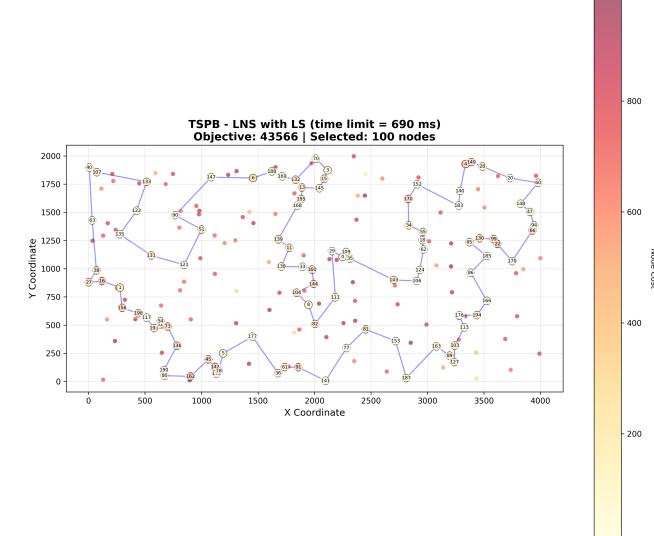
## Visualizations

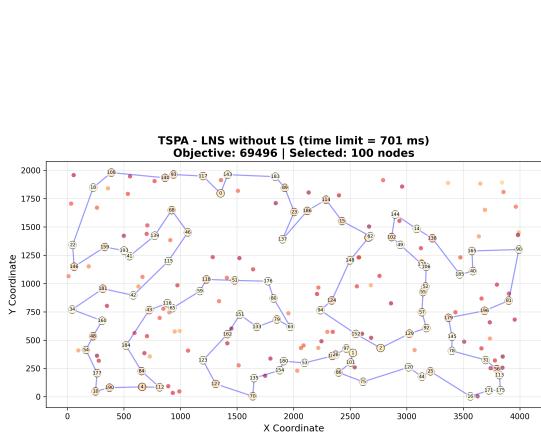
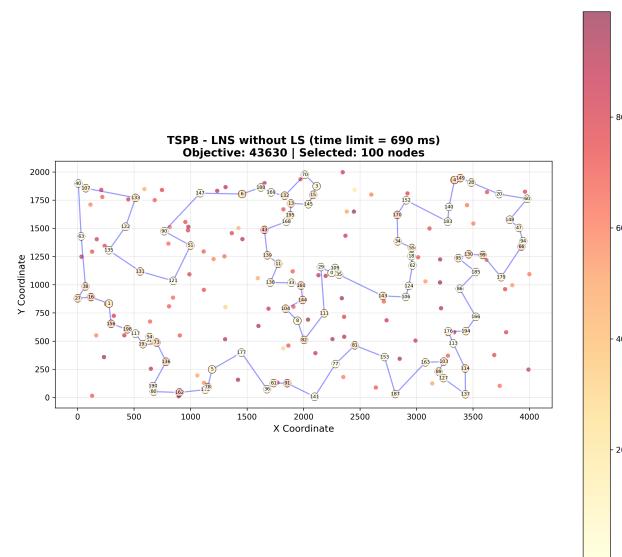
Best solutions found by LNS visualized on both instances:

**LNS with LS - TSPA**



**LNS with LS - TSPB**



**LNS without LS - TSPA****LNS without LS - TSPB**

## Analysis and Conclusions

### LNS vs ILS Comparison

#### **Key findings:**

- ILS outperforms LNS** on both instances with the same time budget
- TSPA: ILS achieves 69340 avg vs LNS+LS's 69689 (+0.50% worse)
- TSPB: ILS achieves 43674 avg vs LNS+LS's 44239 (+1.29% worse)
- ILS performs ~2.4x more local search runs than LNS iterations (3545 vs 1477 for TSPA)

#### **LNS characteristics:**

- LNS's large neighborhood (30% destruction) provides more diversification
- Each LNS iteration is more expensive due to greedy repair ( $O(n^2)$  per iteration)
- LNS may be better for escaping very deep local optima

#### Effect of Local Search in LNS

| Metric          | LNS with LS | LNS without LS | Difference  |
|-----------------|-------------|----------------|-------------|
| TSPA Avg Obj    | 69689       | 69817          | +0.18%      |
| TSPB Avg Obj    | 44239       | 44294          | +0.12%      |
| TSPA Iterations | 1477        | 1683           | +14.0% more |
| TSPB Iterations | 1439        | 1620           | +12.6% more |

#### **Observations:**

- LNS with LS is slightly better** in solution quality
- LNS without LS completes ~12-14% more iterations
- The quality gain from local search outweighs the iteration count loss
- The greedy repair alone produces solutions close to local optima, but LS provides final refinement