20250311 – LnL: Dendritic Computation

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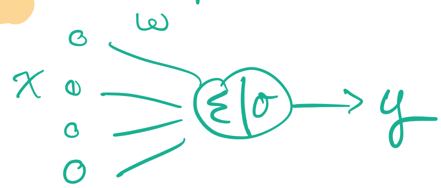
Objectives

- Motivation: what level of abstraction do we need to study computation in biological neural networks?
- What is a (simple model of) a biological neuron?
- Suprathreshold neuronal activity and ReLU
- Subthreshold neuronal activity and dendrites
  - ↳ Passive Properties
  - ↳ Active Properties
- Compound synaptic activity and clusters

Why? - Networks made of neurons, ANNs + BNNs

- ANNs powerful with very simple neurons



$y = \sigma(\Sigma w x)$

$\sigma = \begin{cases} \text{step} \rightarrow \\ \text{reLU} \rightarrow \end{cases}$

siLU
Swish
LReLU
etc

- BNNs powerful with pretty complex neurons

Q → What objectives might a BNN neuron have to meet (in a living system)?

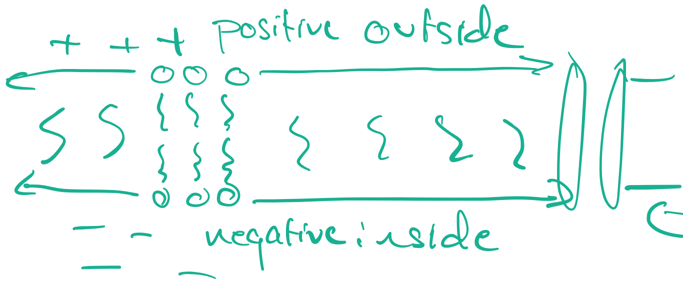
↳ * Other objectives: - energy efficiency
- homeostasis

↳ Is that complexity contributing to network computation? What parts?

→ All models are abstractions, what level of abstraction do we need to study computation in BNNs?

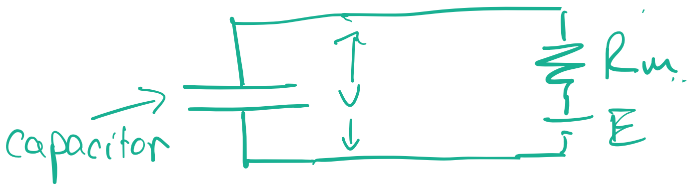
- One way to slice it: What is a biological neuron?
  - A cell

- Cell is electrical because of cell membrane + ions (+ organelles)



- Cell membrane is a capacitor

- in cell membrane are channels, which are resistors



voltage potential difference

- This allows you to build a simple model of a neuron, if you assume channel is always open

$$I_c + I_L = 0$$

$$C_m \frac{dV}{dt} = -\frac{V - E_L}{R}$$

Ohm's law $V = IR \rightarrow I = \frac{V}{R}$

Capacitor Equation $I_c = C_m \frac{dV}{dt}$

$Q = C_m V$

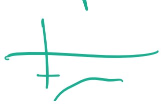
$\downarrow$

$\frac{dQ}{dt} = C_m \frac{dV}{dt}$

$I = C_m \frac{dV}{dt}$

when $\frac{dV}{dt} = 0$, $V = E_L$

- exponential decay



[further from E_L, stronger the pull]

If put external current into the system
(take a probe, stick it on a membrane)
then:

$$I_c + I_L = I_{ext}$$

$$I_c = -I_L + I_{ext}$$

$$C_m \frac{dV}{dt} = -\frac{V-E_L}{R} + I_{ext}$$

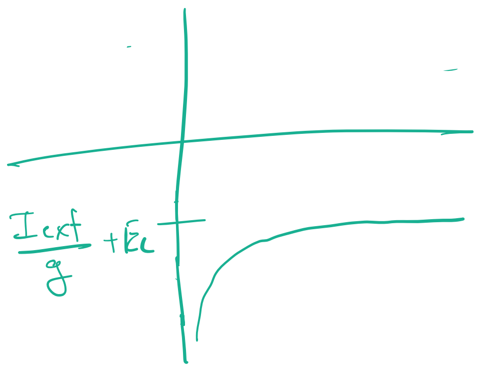
$$C_m \frac{dV}{dt} = -g(V-E_L) + I_{ext}$$

Rewrite $\frac{1}{R} = g$

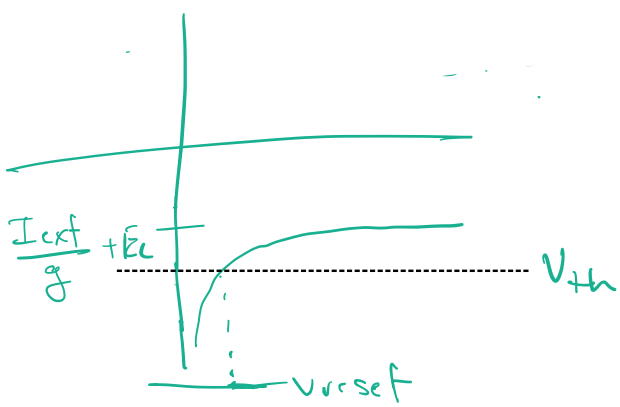
When $\frac{dV}{dt} = 0$, $V = \frac{I_{ext}}{g} + E_L$

$\frac{I_{ext}}{g} + E_L$

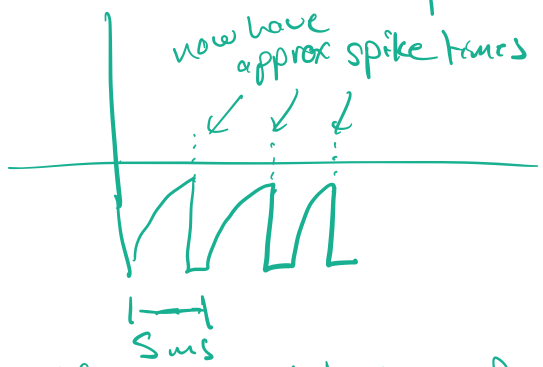
Analysis: this equation describes exponential decay

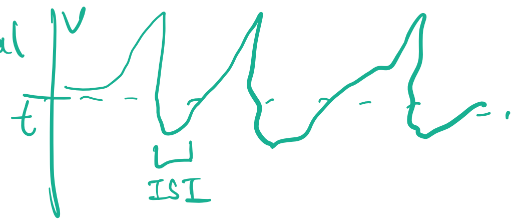


- Add in the action potential spike via thresholding

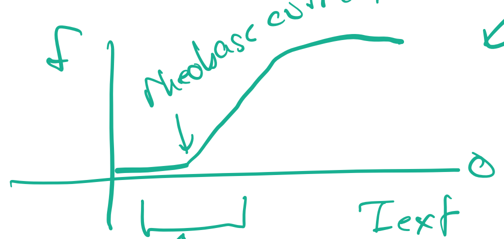


- Also add interspike interval





- You can plot the frequency of spiking as you increase I_{ext} ← FI curve

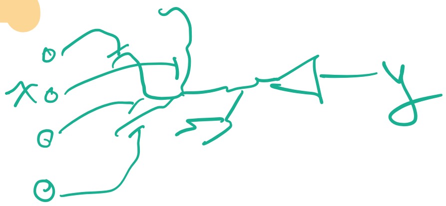


This is like ReLU!

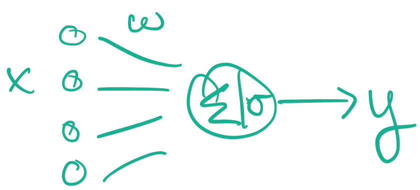
ReLU is an effective analogy for describing spike rates of thresholded electrical activity of neurons!

- But what about subthreshold activity?
  - can't do spike averaging w/o spikes
  - must consider what electrical dynamics are happening within the cell
  - If we are effectively describing the nonlinear transformation of In to Out

then what is happening before threshold



Neurons have dendrites.
If dendrites were
linear then the
computation would look
like, because w is
linear



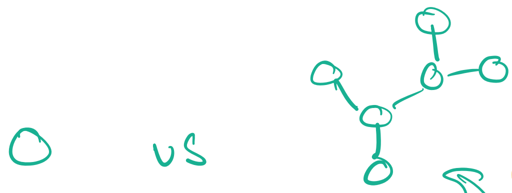
ANN neuron
has no dendrites

– So how do you wrap your head around
subthreshold nonlinearity?

$$I_c = -I_L + I_{ext}$$

$$C_m \frac{dV}{dt} = -g(V - E_L) + I_{ext}$$

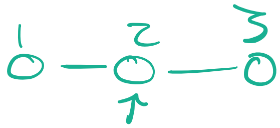
← This describes
a point neuron
OR
a patch of
cell membrane



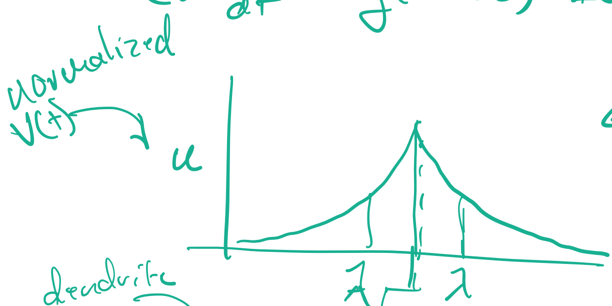
Multicompartment models

Wilfred Rall introduced w/ coupling factor g_{ax}

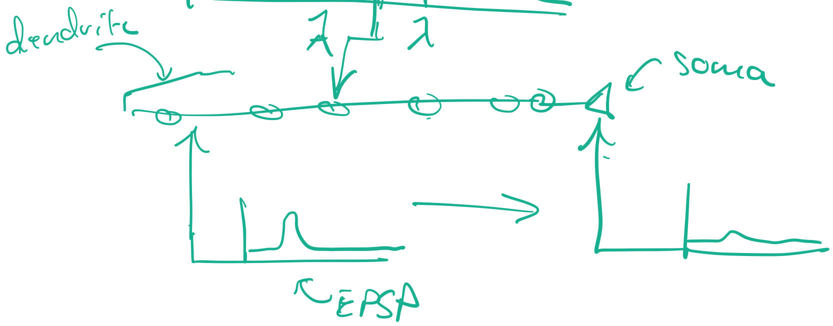
Let's take



$$C_m \frac{dV}{dt} = -g(V-E_L) + I_{ext} + \overbrace{g_{12}(V_2-V_1) + g_{23}(V_2-V_3)}$$



voltage attenuates away from input site



- How do far away synapses even affect the soma to reach threshold?

- Look at eq again, consider other ion channel

$I_c + I_L = I_{ext}$

$I_c + I_L + I_{ion} = I_{ext}$

ion = $\{Na^+, Ca^+, K^+\}$

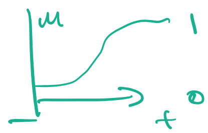
$$I_{ion} \Rightarrow I_{Na^+} = \underbrace{\bar{g}_{Na}}_{\substack{\text{max conductance,}\\ \text{scalar}}} \; \underbrace{g(V,t)}_{\substack{\text{kinetics}\\ \downarrow \\ [0,1]}} \; \underbrace{(V - E_{Na})}_{\Delta V}$$

- The special thing here is E_{Na} is positive → +50mV
- E_L is → -70mV

These, when ungated, pull V(t) toward them!

- Suddenly, with Na^+ channels, you can get larger EPSPs ← key to APs, but also key to spikes in dendrites too!

- How to evoke Na^+ spike? Need to open the gates, $g(V,t)$, which increases with move toward positive

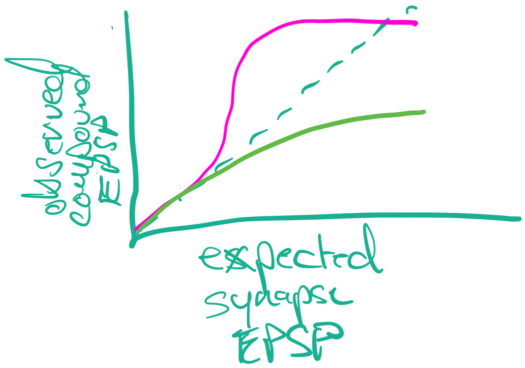


$$\frac{dm}{dt} = \frac{m_\infty(V) - m}{\tau_m(V)}$$

so need more EPSPs to open the gates!

- Summing compound synapses
  - 10,000 synapse in a PC
  - 1,000 on at any time
  - "binary"

– We can assume they linearly sum

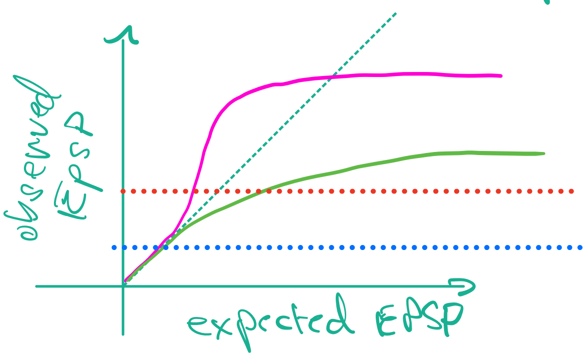


– But when sum reaches Na^+ spike threshold, becomes superlinear!

– Can expect with passive properties (attenuation, reduced driving force) get sublinear compound EPSP as well

– when distributed, follow the linear regime

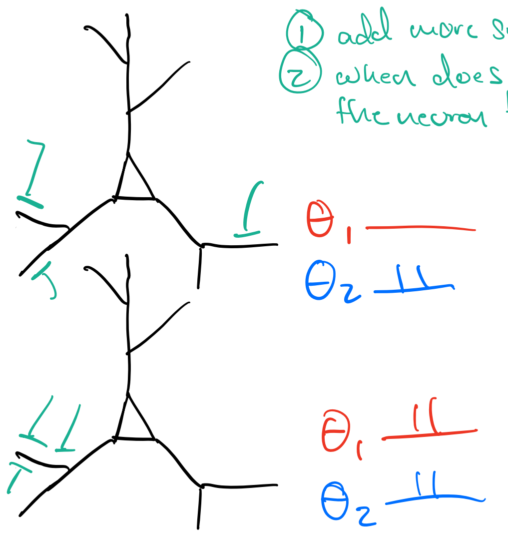
– Let's get to the computations:

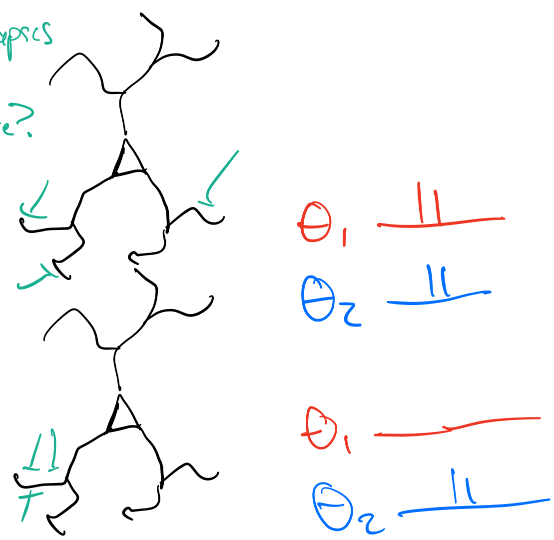


assuming we are stimulating 1 cluster of synapses

AP thresholds

(1) add more synapses
(2) when does the neuron fire?





(Tran-Van-Minh, et. al. 2015, Fig 1)

Cluster Sensitive | Scatter Sensitive

- So we see that multiple properties affect subthreshold nonlinearities in synaptic integration in a single neuron
  - location and activity of synaptic inputs
  - AP thresholds
  - passive cell properties
  - active cell properties
  - dendritic morphology

- How do we think about nonlinearity of neurons, in ANNs if we consider the impact of dendritic properties?