Simulation of Wa-Tor

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1 Introduction

Predator and prey behaviour and dynamics is of importance not only for ecology and the human understanding of interactions between organisms, but also other fields of research. Thus variety of predator-prey models have been studied in mathematical biology and have become utilized to map ecological systems as well as wage-employment ratio in economic theory. In the making of these models, somewhat unrealistic assumptions have to be made, or chaotic dynamics emerge[5] [4].

In this report we study the chaotic behaviour of predator-prey models using the rather simplistic Wa-Tor model; a modified cellular automaton system of shark-fish population on a torus.

2 Predator-prey dynamics

In this section we introduce the Nicholson-Bailey model together with the Wa-Tor system. The former is then used to understand and compare with the latter in the following discussion.

2.1 Nicholson-Bailey model

The Nicholson-Bailey model (NB) was developed by Alexander John Nicholson and Victor Albert Bailey in 1935 to model the behaviour of host-parasitoid systems. The model can however also be used to model predator-prey behaviour, as is done in this report. The model is defined in discrete time, using density dependence, as [2]:

$$\begin{cases} H_{t+1} = H_t e^{r(1-H_t/k)} e^{-aP_t} \\ P_{t+1} = cH_t(1-e^{-aP_t}) \end{cases} \tag{NB}$$

where H is the number of host animals, and P is the number of parasitoids, $e^{r(1-H_t/k)}$ is the reproduction rate of the host animals, which decreases as the population increases, a is the searching efficiency of the parasitoid while c is the average number of viable eggs that a parasitoid lays on a single host [5] [2].

The model (NB) was iterated in Matlab, using the parameters in table 1 below.

Table 1: Parameters used in the simulation of NB

Parameter	Host	Parasitoid
r	1.37	-
a	-	$5 \cdot 10^{-5}$
K	$4 \cdot 10^{4}$	-
c	-	2

2.2 Wa-Tor

Wa-Tor is an imaginary system, in which fishes(prey) and sharks(predators) live in a torus-shaped sea. The sea is implemented as a square subdivided into smaller squares. The edges of the large square have periodic boundary conditions which means that they be seen as 'glued' together to form a torus. Each of the subsquares then represent a position in the sea, and can be filled with water, a shark or a fish. At each timestep an animal will decide what to do according to the following set of rules:

1. Fish
   - (a) Move to a random adjacent subsquare filled with water. If there are no such subsquares avaliable, don't move.
   - (b) If the fish has survived a specified amount of time it will reproduce, leaving a new fish at the previous location.
2. Shark
   - (a) Move to a random adjacent subsquare that contains a fish, killing the fish and gaining energy.
   - (b) If no such square exists, move to a random adjacent subsquare filled with water. If no such square exists, don't move.
   - (c) If the shark has energy above a certain threshold it will reproduce, leaving a new shark at the previous location.
   - (d) Lose a certain amount of energy. If the sharks energy reaches 0, the shark dies.

This system can approximate the equations defining the NB model, since the fishes(hosts) will reproduce exponentially as long as they don't clump up, which they do when they start to fill the entire torus. When they clump up they will not be able to spawn new fishes, since they don't move. The sharks represent the parasitoids, and are dependent on how many fish there are, as it will increase the reproduction rate of the sharks.

This system is implemented in the processing.py environment [3], giving visual insight to the dynamical developments. The parameters used for the simulation are listed in table 2.

Table 2: Parameters used in the simulation. The values are true for individual animals, not the entire population. Units are per timestep, unless stated otherwise.

Parameter	Fishes	Sharks
Reproduction rate	$\sqrt[13]{2}$	-
Maximum energy	-	1
Energy gain per meal	-	0.15
Energy loss rate	-	0.04
Energy for reproduction	-	> 0.98

The torus was divided into 40000 subsquares, and the squares were iterated through row by row. However, the direction of the iteration was randomized for every pass over the torus, as the animals behave in an anisotropic manner otherwise. The amount of fishes and sharks were recorded during the simulation and plotted versus each other. The results i seen in the following section.

3 Results

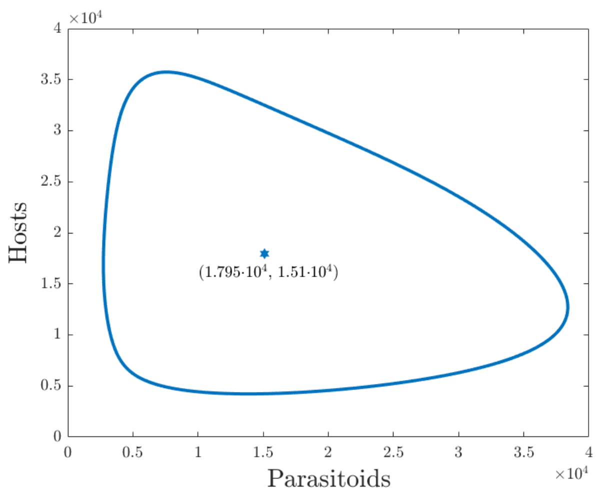
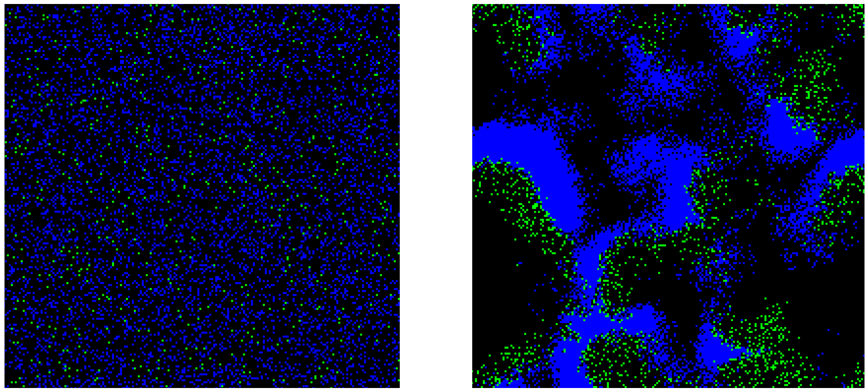


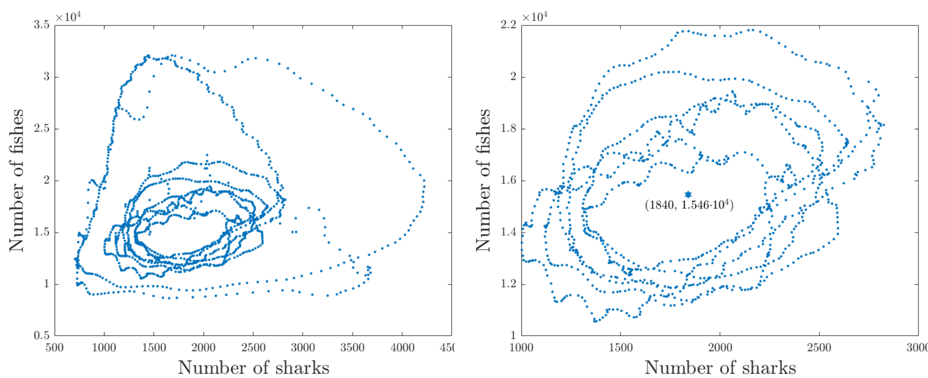
Figure 1: Iteration of the Nichelson-Baiely model.



(a) Initial condition.

(b) The system after many time steps.

Figure 2: Simulation of an initial condition with 18.8% fishes(blue) and 1.2% sharks(green) randomly placed on the 40000 subsquares.



(a) The transient behaviour.

(b) The attractor with dynamic life-like behaviour. The middle-point of the attractor is shown as a star.

Figure 3: Plots of the amount of fishes versus sharks as the system evolves.

4 Discussion

Both the NB iteration and the Wa-Tor system showed similar attractors as seen in figures (1) and (3b). This shows that some basic dynamics of the NB-model appears from simple algorithms of individual animals' behaviour. However, there are some important differences between the two models. The NB model can be viewed as an average behaviour of the animals movements over a predetermined timestep. The Wa-Tor model simulates single animals at a much higher time resolution, introducing more fluctuations to the system. Because of this the Wa-Tor model is spacially depedent, while the NB model is not. This has big implications for initial conditions, as well as the periodicity of the phase-space portrait. The Wa-Tor model seems to only have two three attractors namely $(0, 0)$, $(\infty, 0)$ and the approximately quasi-perodic shown in figure (3b). This may be attributed to the noise introduced by the random variables of the model, which pushes the system into the attractors with the largest basins of attraction. The NB model on the other hand has several types of attractors, depending on the parameters [2, p.86]. All of these can be reached through iteration since there is no random noise that disturbs the dynamics of the system. There has been prevoius work on a spatially dependent NB model[1], which shows similar behaviour to the Wa-Tor model used here.

The simulations of Wa-Tor were all using initial conditions that randomized the animals' positions. The basin of attraction for the approximately quasi-periodic behaviour then seemed rather small. Slightly different percentages for the fishes or sharks ended up in $(0, 0)$ or $(\infty, 0)$. However, it is possible that this is different for non-random starting locations. A further analysis of different initial conditions would be of interest.

References

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[4] Miscellaneous. *Goodwin model (economics)*. `https://en.wikipedia.org/wiki/Goodwin_model_(economics)`. [Online; accessed 1-June-2018].

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