

Analysis of Host-Parasite Dynamics

How the reproduction rate affect the outcome

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Introduction

In nature several methods for reproduction exist, each with their pros and cons. Asexual reproduction, such as cloning, allows every individual in a population to reproduce and allows very rapid population growth.[1] Sexual reproduction, however, only allows a portion of the population (the females) to bear offspring and the base reproduction rate is therefore lower. Therefore, the purpose of sexual reproduction has long puzzled evolutionary biologists.[2] For sexual reproduction to survive it must provide some survival advantage that makes up for the lower reproduction rate. One theory that has been developed to explain this advantage deals with co-evolution of organisms that have a host - parasite relationship. The host evolves protection against parasite infection while the parasite seeks to evolve new ways to infect the host. In this scenario the diversity that comes with sexual reproduction is theorized to offer enough protection/innovation to make up for the lower reproduction rate, making sexual reproduction the more successful technique. [1]

Some organisms can practice both sexual and asexual reproduction and thus it is possible to have populations that are partially sexual.[2]

In this paper we will examine an article (Flatt *et al.* [3]) in which they have looked at how sexual and asexual reproduction affects host-parasite population dynamics. They assumed a sexually reproducing host and let the parasite sexual reproduction rate vary. A measure of how sexual a population is can be described by a mixing factor, a number which represents the degree individuals share genes with each other. Their conclusion was that as the parasite mixing rate increases the stability also increases. The question is now if it is possible to recreate the plots and see if the conclusions stay the same.

Method

Figure 1 is a reproduction of Figure 1 in the article "A Bit of Sex Stabilizes Host-Parasite Dynamics" written by Thomas Flatt, Nicolas Maire and Michael Doebeli (2001) [3]. Equations (2-6) from the article were used in MatLab to form the plots. Equation(s) (2) were used to calculate the host and parasite populations before mixing, using the Nicholson-Bailey model. Then equations (4) and (6) were used to calculate the genotype distribution after mixing for host and parasite respectively. Equations (3) and (5) are auxiliary equations used in equations (4) and (6). Equation (3) and equation(s) (4) are shown below:

$$p = \frac{H_{AA} + \left(\frac{H_{Aa}}{2}\right)}{H_{tot}} \quad \begin{aligned} H_{AA,t+1} &= (1 - v_h)H_{AA,t} + v_h \cdot p^2 \cdot H_{tot,t} \\ H_{Aa,t+1} &= (1 - v_h)H_{Aa,t} + v_h \cdot 2p(1 - p) \cdot H_{tot,t} \\ H_{aa,t+1} &= (1 - v_h)H_{aa,t} + v_h \cdot p(1 - p)^2 \cdot H_{tot,t} \end{aligned}$$

Where v_h is the mixing factor of the host. The equations for the parasite are similar. Starting values for the simulations were $H_0 = 10$, $P_0 = 1$, $p = 0.55$, $q = 0.48$, $K = 10$, $c = 1$, $a = 0.45$ with H_0 and P_0 being original host and parasite population densities respectively, p and q representing the original distribution of host and parasite genotype distributions, K is the host carrying capacity, c is parasite infection ratio per encountered host and a being

the parasite searching efficiency.

The populations were iterated for 1 000 generations and the 40 last values in each iteration were stored for different values of λ , the base growth factor for the host population. All the stored values were then plotted as a function of lambda. Every mentioned value above is taken from the article except for the number of iterations. In the referenced article they iterated over 100 000 generations and the reason why only 1 000 iterations were done in this paper is that the variations in the graphs for larger than 1 000 iterations were not that significant.

Even though every value and equation needed were given in the article and many steps were described it is impossible to know exactly how the authors made the simulations. Therefore some own approaches and techniques were used, such as conditionals to avoid division by zero, that may give changes in the outcome.

Result

In Figure 1 three plots on the distribution of final population densities for a fully sexual host and a parasite with different rates of sexual reproduction are shown. The host and parasite populations are labeled black and red respectively and the mixing factor of the parasite is different in each plot.

We found that moderate levels of sexual reproduction in the parasite population improves parasite survival and stabilizes a sexually reproducing host.

In Figure 2 examples of host and parasite populations are plotted for an asexual parasite (the plots to the left) and a sexual parasite (the plots to the right) and are shown for a few iterations. A λ of 15 was chosen so that the increased stability seen in Figure 1 could be visualized. Each sub population AA and BB (both red), Aa and Bb (both green), aa and bb (both blue) are included in the plot.

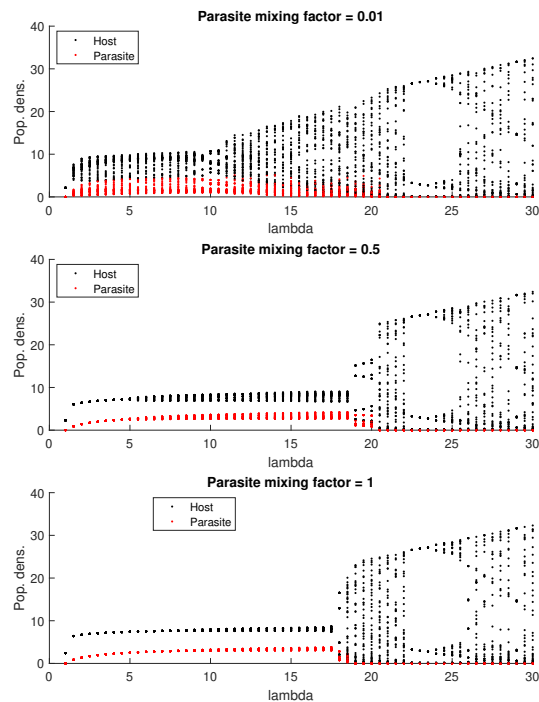


Figure 1: Top: asexual parasite, Middle: partially sexual parasite Bottom: sexual parasite. In all plots the host is completely sexual.

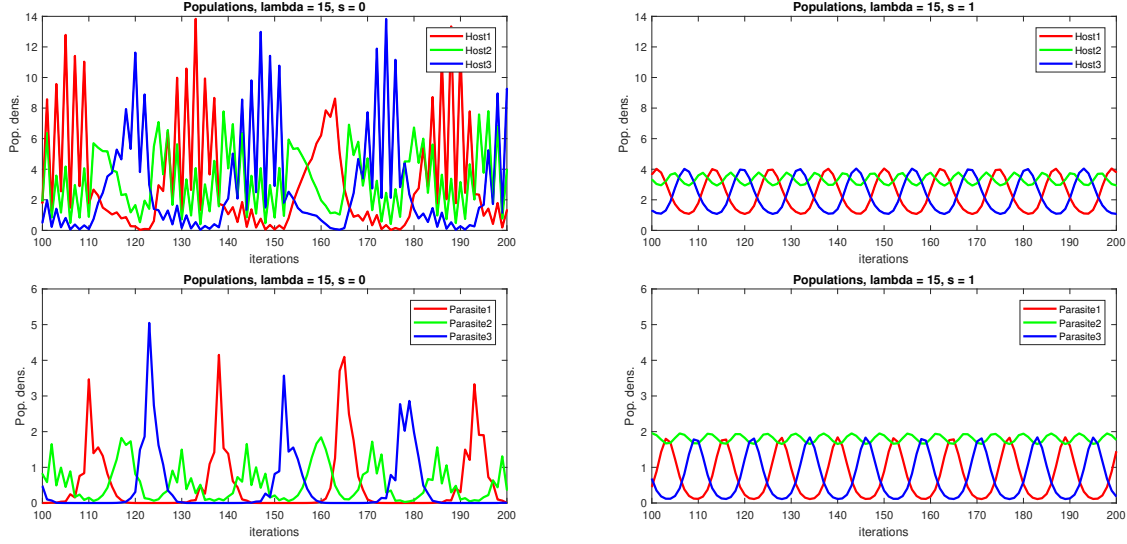


Figure 2: Left: asexual parasite. Right: Sexual parasite. Top: host population densities, Bottom: parasite population densities.

Discussion

As seen in Figure 1 Top) the (nearly) asexual parasite produces an unstable host population for all lambdas plotted. For the parasite populations with some sexual reproduction in Figure 1 Middle) and Bottom) the host population oscillations are much smaller. However, the difference between a half-sexual parasite (Middle) and a completely sexual parasite (Bottom) is small. Qualitatively, our results therefore agrees with the findings in Flatt *et al.* [3]. That being said, there are differences between our results. In our simulations the parasite dies out at lambdas above ~ 22 , something which does not happen in Flatt *et al.* In their study there are regions of two and four periodic oscillations for lambdas between 20-40, something which we lack, perhaps due to the parasite dying out. It is not known why the parasite dies out for higher λ in our case but not in the reference study.

The referenced article iterated over 100 000 generations to exclude the affect of the initial conditions. In this paper the populations were iterated over 1 000 generations. This was because the plots did not change that significantly, and the behaviour was still the same, between 1 000 iterations and a larger number of iterations. Therefore it is not this change between the article and our calculations that makes the plots vary.

Three parasite populations with different genotypes and three host populations susceptible to the respective parasites were used in the simulations, as in the article. The equations describing these genotypes were formed using Mendelian segregation [3]. This relation and behaviour between the different genotypes can however not be seen in Figure 1. To get a closer look at the behaviour the values of the populations at $\lambda = 15$ were plotted in separate Figures (Figure 2).

Each of the three sub populations of the host and the parasite (Figure 2 right) are oscillating periodically. This periodicity shows that during these iteration values the system is stable and shows no chaotic behaviour. [4]

The oscillations for the parasite mixing factor 0.01 shows a seemingly more chaotic behaviour than for the two other parasite mixing factors (0.5 and 1). This can be noted in Figure 1 when comparing the variation in the population densities for the different parasite mixing factors. Since the variation of the parasite mixing factor 0.01 is the largest its oscillations are also the most drastic.

By studying the sub populations of hosts and parasites in Figure 2 an explanation to the increased stability can be found. In the asexual case in Figure 2 (left) the host population densities oscillate strongly until the relevant parasite decimates the host. Then another host becomes dominant and the cycle repeats. The different parasite population densities periodically reach very low values, which allows the corresponding host to increase in number very quickly.

In the sexual case in Figure 2 (right) the parasite (and host) population densities never reach as low values as in Figure 2 (left). Thus, when a specific host population starts growing the corresponding parasite is numerous enough to limit the growth quickly. This prevents overpopulation and the subsequent crash in host population density. The reason a specific parasite (for example bb) can continue to have a relatively high population density even when its respective host population is low is genetic mixing. Besides ordinary reproduction of bb parasites from other bb parasites new bb parasites can also be produced from mixing of Bb parasites. Each Bb-Bb parasite pairing has a $1/4$ chance to produce bb offspring.

In this way the sexual parasite keeps "backups" of rare genotypes which would not have survived in significant numbers if it reproduced asexually.

Conclusion

Even though our plots varies from the ones in the article ([3]) the same features and behaviour can be seen. Both have a more chaotic behaviour when the parasite mixing factor is 0.01 and more stable systems appears when the parasite mixing factor is increased to 0.5 and 1. The changes between the two last systems (parasite mixing factor 0.5 and 1) is also not as big as the changes between them and the first (parasite mixing factor 0.01). Still, the main conclusion from the article can be drawn here too; sex has a stabilizing effect on host-parasite dynamics. Also, moderate and high levels of genetic mixing in the parasite populations have similar effects on the stability of the host population, like in Flat *et al.*[3]

References

- [1] Lively, C. (2010). *Antagonistic Coevolution and Sex*.
- [2] Ridley, M. (2003). *The Red Queen: Sex and the Evolution of Human Nature*.
- [3] Flatt, T., Maire, N. and Doebeli, M. (2001). *A Bit of Sex Stabilizes Host–Parasite Dynamics*.
- [4] Ohlén G., Åberg S. and Östborn P. (2007). *Chaos*.