#1 $E_0(f) = \frac{h^3}{3} f''(\xi_0) \quad h = \frac{b-a}{2} \quad \xi_0 \in (a,b) \quad \therefore |E_0(f)| = \frac{(b-a)^3}{24} |f''(\xi_0)|$

$E_1(f) = -\frac{h^3}{12} f''(\xi_1) \quad h = b-a \quad \xi_1 \in (a,b) \quad \therefore |E_1(f)| = \frac{(b-a)^3}{12} |f''(\xi_1)|$

$\therefore |E_1(f)| = 2 \cdot |E_0(f)| \cdot \frac{|f''(\xi_1)|}{|f''(\xi_0)|} \quad \Rightarrow \frac{|f''(\xi_1)|}{|f''(\xi_0)|} \approx 1$

$\therefore |E_1(f)| \simeq 2 \cdot |E_0(f)|$

#3 (a)

for $f(x) = 1$

$I_2(f) = \frac{2}{3}(2 - 1 + 2) = 2$

$\int_{-1}^{1} 1\,dx = 2$

for $f(x) = x$

$I_2(f) = \frac{2}{3}(-1 - 0 + 1) = 0$

$\int_{-1}^{1} x\,dx = 0$

for $f(x) = x^2$

$I_2(f) = \frac{2}{3}(2 \cdot \frac{1}{4} - 0 + 2\frac{1}{4}) = \frac{2}{3}$

$\int_{-1}^{1} x^2 dx = \frac{1}{3}x^3\Big|_{-1}^{1} = \frac{2}{3}$

for $f(x) = x^3$

$I_2(f) = \frac{2}{3}(2(-\frac{1}{8}) - 0 + 2(\frac{1}{8})) = 0$

$\int_{-1}^{1} x^3 dx = 0$

for $f(x) = x^4$

$I_2(f) = \frac{2}{3}(2 \cdot (\frac{1}{16}) - 0 + 2(\frac{1}{16})) = \frac{1}{6}$

$\int_{-1}^{1} x^4 dx = \frac{1}{5}x^5\Big|_{-1}^{1} = \frac{2}{5}$

$r = 3$

(b)

for $f(x) = 1$

$I_4(f) = \frac{1}{4}(1 + 3 + 3 + 1) = 2$

$\int_{-1}^{1} 1\,dx = 2$

for $f(x) = x$

$I_4(f) = \frac{1}{4}(-1 - 1 + 1 + 1) = 0$

$\int_{-1}^{1} x\,dx = 0$

for $f(x) = x^2$

$I_4(f) = \frac{1}{4}(1 + \frac{1}{3} + \frac{1}{3} + 1) = \frac{2}{3}$

$\int_{-1}^{1} x^2 dx = \frac{2}{3}$

for $f(x) = x^3$

$I_4(f) = \frac{1}{4}(-1 + (-\frac{1}{9}) + (\frac{1}{9}) + 1) = 0$

$\int_{-1}^{1} x^3 dx = 0$

for $f(x) = x^4$

$I_4(f) = \frac{1}{4}(1 + \frac{1}{27} + \frac{1}{27} + 1) = \frac{14}{27}$

$\int_{-1}^{1} x^4 dx = \frac{2}{5}$

$r = 3$

$\Rightarrow$ I_2 and I_4 are both close Newton-Cotes

by thm, Infinitesimal p of I_2 and I_4 are 5

#5 for $f(x)=1$

$$G(f) = a\cdot 1 = \int_0^1 \sqrt{x}\cdot 1\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3} \qquad a = \frac{2}{3}$$

for $f(x)=x$

$$G(f) = a\cdot x_1 = \int_0^1 \sqrt{x}\cdot x\,dx = \int_0^1 x^{\frac{3}{2}}dx = \frac{2}{5}x^{\frac{5}{2}}\Big|_0^1 = \frac{2}{5} \qquad \frac{2}{3}\cdot x_1 = \frac{2}{5} \quad x_1 = \frac{3}{5}$$

for $f(x)=x^2$

$$G(f) = a\cdot x_1^2 = \frac{2}{3}\times\frac{9}{25} = \frac{6}{25}$$

$$I_w(x^2) = \int_0^1 \sqrt{x}\cdot x^2\,dx = \int_0^1 x^{\frac{5}{2}}dx = \frac{2}{7}x^{\frac{7}{2}}\Big|_0^1 = \frac{2}{7} \quad \neq$$

$\therefore\ r=1 \quad a = \frac{2}{3} \quad x_1 = \frac{3}{5}$

#6 for $f(x)=1$

$$G(f) = \alpha_1 + \alpha_2 + 0 = \int_0^1 1\,dx = 1 \qquad \Rightarrow \alpha_1 + \alpha_2 = 1 \qquad \alpha_1 = \frac{2}{3}$$

for $f(x)=x$

$$G(f) = \alpha_1\cdot 0 + \alpha_2\cdot 1 + \alpha_3\cdot 1 = \int_0^1 x\,dx = \frac{1}{2} \Rightarrow \alpha_2 + \alpha_3 = \frac{1}{2} \qquad \alpha_3 = \frac{1}{6}$$

for $f(x)=x^2$

$$G(f) = \alpha_1\cdot 0 + \alpha_2\cdot 1 + 0 = \int_0^1 x^2\,dx = \frac{1}{3} \Rightarrow \alpha_2 = \frac{1}{3} \qquad \therefore\ \alpha_1 = \frac{2}{3}\ \ \alpha_2 = \frac{1}{3}\ \ \alpha_3 = \frac{1}{6}$$