#9.

Let $H_3(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

then $H_3'(x) = 3a_3x^2 + 2a_2x + a_1$

$H_3(-1) = -a_3 + a_2 - a_1 + a_0 \Rightarrow 1$ ① $A\begin{pmatrix}a_3\\a_2\\a_1\\a_0\end{pmatrix} = \begin{pmatrix}1\\1\\2\\1\end{pmatrix}$ $|A| = -1 \times \begin{vmatrix}3&-2&1\\3&2&1\\8&4&2\end{vmatrix} + 1\times\begin{vmatrix}-1&1&-1\\3&-2&1\\3&2&1\end{vmatrix}$

$H_3'(-1) = 3a_3 - 2a_2 + a_1 \Rightarrow 1$ ② $\Rightarrow A = \begin{pmatrix}-1&1&-1&1\\3&-2&1&0\\3&2&1&0\\8&4&2&1\end{pmatrix}$ $= -1\times(12-16+12-16+12-12)$

$H_3'(1) = 3a_3+2a_2+a_1 \Rightarrow 2$ ③ $+1\times(2-6+3-6+2-3)$

$H_3(2) = 8a_3+4a_2+2a_1+a_0 \Rightarrow 1$ ④ $= -1\times -8 + -8 = 0$

$\Rightarrow$ A is singular

$\Rightarrow$ check ②③ $4a_2 = 1$ $a_2 = \frac14$ $\Rightarrow 3a_3 - \frac12 + a_1 = 1$ $3a_3+a_1 = \frac32$

① $-a_3 - a_1 + a_0 = \frac34$ $\Updownarrow$ ✗

④ $8a_3+2a_1+a_0 = 0$ $9a_3+3a_1 = -\frac34$ $3a_3+a_1 = -\frac14$

∴ Hermite-Birkoff interpolating polynomial doesn't exist

#12 ∵ $\left(\frac{1}{1+b_2x^2}\right) = 1 - b_2x^2 + b_2^2x^4 - b_2^3x^6 + \cdots$

∴ $r(x) = (a_0 + a_2x^2 + a_4x^4)(1 - b_2x^2 + b_2^2x^4 - b_2^3x^6+\cdots)$

$\Rightarrow (x^0)$ $a_0 = 1$

(x^2) $a_2 - a_0b_2 = -\frac{1}{2!}$ $\Rightarrow a_2 = b_2 - \frac12$

(x^4) $a_4 - a_2b_2 + a_0b_2^2 = \frac{1}{4!}$ $\Rightarrow a_4 = a_2b_2 - b_2^2 + \frac1{24} = \frac1{24} - \frac12 b_2$

(x^6) $-a_4b_2 + a_2b_2^2 - a_0b_2^3 = -\frac1{6!}$ $\Rightarrow a_4b_2 = a_2b_2^2 - a_0b_2^3 + \frac1{720}$

$\left(\frac1{24} - \frac12 b_2\right)b_2 = \left(b_2 - \frac12\right)b_2^2 - b_2^3 + \frac1{720}$

$\frac1{24}b_2 = \frac1{720}$ $b_2 = \frac1{30}$

$\Rightarrow a_4 = \frac1{24} - \frac1{60} = \frac3{120} = \frac1{40}$

$a_2 = \frac1{30} - \frac12 = -\frac{14}{30} = -\frac7{15}$

$\Rightarrow a_0 = 1$ $a_2 = -\frac7{15}$ $a_4 = \frac1{40}$ $b_2 = \frac1{30}$