1.

$T(n)=4T(\frac{n}{2})+n^2\lg(n)$

$T(n)=n^2\lg(n)+4\cdot(\frac{n}{2})^2\lg(\frac{n}{2})+16\cdot(\frac{n}{4})^2\lg(\frac{n}{4})+\ldots.$

$=\sum_{i=0}^{\lg(n)} n^2\lg(\frac{n}{2^i})$

$=n^2\sum_{i=0}^{\lg(n)}(\lg(n)-\lg(2^i))$

$=n^2(\sum_{i=0}^{\lg(n)}\lg(n)-\sum_{i=0}^{\lg(n)} i)$

$=n^2((\lg^2(n)+\lg(n))-(\frac{1}{2}\lg^2(n)+\lg(n)))$

$=\frac{1}{2}n^2\lg^2(n)=O(n^2\lg^2(n))$

2.

$T(n)=5T(\frac{n}{3})+O(1)$

$=\sum_{i=0}^{log_3(n)} 5^i$

$=5^{log_3(n)}\cdot O(1)$

$=O(n^{log_3(5)})$

3.

$T(n)=6T(\frac{n}{2})+n^3$

$=T(n)= n^3+6(\frac{n}{2})^3+36(\frac{n}{4}))^3+\ldots$

$=\sum_{i=0}^{\lg(n)} 6^i\,(\frac{n}{2^i})^3$

$=n^3\sum_{i=0}^{\lg(n)}(\frac{6}{2^3})^i$

$=O(n^3)$

4.

$T(n)=4T(\frac{n}{4})+n$

$T(n)=n+4\frac{n}{4}+16\frac{n}{16}+\ldots.$

$=\sum_{i=0}^{log_4(n)} n$

$=\Theta(n\log_4(n))$

5.

T(n) = $n^2+(\frac{2}{3})^2 n^2+(\frac{2}{3})^4 n^2+(\frac{2}{3})^8 n^2(\frac{2}{3})^{16} n^2+\ldots$

height of the tree: h = $\log_{\frac{3}{2}} n$

T(n)=$n^2 \sum_{i=0}^{\log_{\frac{3}{2}} n} (\frac{2}{3})^{2^i}$

$\sum_{i=0}^{\log_{\frac{3}{2}} n} (\frac{2}{3})^{2^i} < \sum_{i=0}^{\log_{\frac{3}{2}} n} (\frac{2}{3})^i$

$\sum_{i=0}^{\log_{\frac{3}{2}} n} (\frac{2}{3})^{2^i} <$ (1- $(\frac{2}{3})^h$)/ (1-$\frac{2}{3}$) < 3

So

T(n) = O(n^2)

6.

T(n) = n+$(\frac{2}{3})^1$n+$(\frac{2}{3})^2$n+$(\frac{2}{3})^3 n$+$(\frac{2}{3})^4$n

T(n) = (1+$(\frac{2}{3})^1 + (\frac{2}{3})^2 + (\frac{2}{3})^3 + (\frac{2}{3})^4 + \cdots$)n

height of the tree : h = $\log_3 n$

T(n) = n$\sum_{i=0}^{\log_3 n}(\frac{2}{3})^i$

$\sum_{i=0}^{\log_3 n}(\frac{2}{3})^i$ = (1- $(\frac{2}{3})^h$)/ (1-$\frac{2}{3}$) < 3

So

T(n) = O(n)

7.

T(n) = nlg(n)+$\frac{5}{3}$nlg($\frac{5}{3}n$)+$(\frac{5}{3})^2$nlg$(\frac{5}{3})^2 n$+$(\frac{5}{3})^3$nlg$(\frac{5}{3})^3 n$+...

height of the tree : h = $\log_3 n$

T(n) = nlgn + $\frac{5}{3}$nlg($\frac{5}{3}$)+$\frac{5}{3}$n lgn+ $2(\frac{5}{3})^2$nlg$(\frac{5}{3})^1$+ $(\frac{5}{3})^2$nlgn+$3(\frac{5}{3})^3$nlg$(\frac{5}{3})^1$+ $(\frac{5}{3})^3$nlgn+...

T(n) = (nlgn)$(\sum_{i=0}^{\log_3 n}(\frac{5}{3})^i)$+$\left(\lg\frac{5}{3}\right)$ ∗n∗ $(\sum_{i=0}^{\log_3 n} i * (\frac{5}{3})^i)$

T(n) < n(lgn)∗ ((1-$(\frac{5}{3})^{\log_3 n}$)/(1-$\frac{5}{3}$)) + n*(lgn)* ((1-$(\frac{5}{3})^{\log_3 n}$)/(1-$\frac{5}{3}$))

T(n) < n(lgn)∗($(\frac{5}{3})^{\log_3 n}$)

So

T(n) = O(n(lgn)∗($(\frac{5}{3})^{\log_3 n}$))

8.

T(n) = lgn +2 lg(n-1)+4lg(n-2)+...+2^alg(n-a)+...

height of the tree : h = n-2,   0<=a<<n-2

T(n) < $(\text{lgn}) * \sum_{i=0}^{n-2} 2^i$

T(n) < (1-2^{n-2})/(1-2)lgn

T(n) < 2^{n-2}lgn

Tn=O(2^nlgn)

9.

T(n) = lg^2n+3lg^2(n-2)+9lg^2(n-4)+..+$3^a lg^2$(n-2a)

Height of the tree: n-2h = 2, h= (n-2)/2, so   0<= a <= (n-2)/2

T(n) < $(\text{lgn}) * \sum_{i=0}^{(n-2)/2} 3^i$

T(n) < (1-3^{n-2})/(1-3)lgn

T(n)=O($3^n lgn$)

10.

T(n) = 2lgn+4lg(n-2)+8lg(n-4)+16lg(n-6)+...

Height of the tree : n-2h = 2, h= (n-2)/2

T(n) < (lgn)* $\sum_{i=1}^{(n-2)/2} 2^i$

T(n) < 2((1-2^{n-2})/(1-2))lgn

T(n) = O(2^nlgn)