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Foundations1 assignment 2023
Submit by Monday of week 8 (30 October 2023)
by the deadline hour relevant to your campus.
Worth 15%
Submit only typed material
No handwritten material
NO EMAIL SUBMISSIONS
EACH CAMPUS WILL INFORM YOU WHERE
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Marcondes Browning, Lucca Anthony, H00369673

Questions

1. For each term A of the terms below, give its translation $\omega(A)$ from $\mathcal{M}$ to Λ showing all the steps, their number and underlining all the parts you are working on, just like we did in the above example:

   (a) $(\lambda xy.x)$. (1)

$$
\begin{aligned}
&\underline{\omega(\lambda xy.x)} =^{0}\\
&\underline{\omega_{[x,y,z]}(\lambda xy.x)} =^{3}\\
&\lambda \underline{\omega_{[x,x,y,z]}(\lambda y.x)} =^{3}\\
&\lambda\lambda \underline{\omega_{[y,x,x,y,z]}(x)} =^{1}\\
&\lambda\lambda\, 2
\end{aligned}
$$

   (b) $(\lambda xyz.xz(yz))$. (1)

$$
\begin{aligned}
&\underline{\omega(\lambda xyz.xz(yz))} =^{0}\\
&\underline{\omega_{[x,y,z]}(\lambda xyz.xz(yz))} =^{3}\\
&\lambda \underline{\omega_{[x,x,y,z]}(\lambda yz.xz(yz))} =^{3}\\
&\lambda\lambda \underline{\omega_{[y,x,x,y,z]}(\lambda z.xz(yz))} =^{3}\\
&\lambda\lambda\lambda \underline{\omega_{[z,y,x,x,y,z]}(xz(yz))} =^{2}\\
&\lambda\lambda\lambda \underline{\omega_{[z,y,x,x,y,z]}(xz)}(\omega_{[z,y,x,x,y,z]}(yz)) =^{2}\\
&\lambda\lambda\lambda \underline{\omega_{[z,y,x,x,y,z]}(x)}\omega_{[z,y,x,x,y,z]}(z)(\omega_{[z,y,x,x,y,z]}(yz)) =^{1}\\
&\lambda\lambda\lambda\, 3\, \underline{\omega_{[z,y,x,x,y,z]}(z)}(\omega_{[z,y,x,x,y,z]}(yz)) =^{1}\\
&\lambda\lambda\lambda\, 3\, 1\, (\underline{\omega_{[z,y,x,x,y,z]}(yz)}) =^{2}\\
&\lambda\lambda\lambda\, 3\, 1\, (\underline{\omega_{[z,y,x,x,y,z]}(y)}\omega_{[z,y,x,x,y,z]}(z)) =^{1}\\
&\lambda\lambda\lambda\, 3\, 1\, (2\, \underline{\omega_{[z,y,x,x,y,z]}(z)}) =^{1}\\
&\lambda\lambda\lambda\, 3\, 1\, (2\, 1)
\end{aligned}
$$

   (c) $xz(\lambda xy.z(\lambda z.zy)x)$. (1.5)

$\underline{\omega(xz(\lambda xy.z(\lambda z.zy)x))} =^0$
$\underline{\omega_{[x,y,z]}(xz(\lambda xy.z(\lambda z.zy)x))} =^2$
$\underline{\omega_{[x,y,z]}(xz)}\omega_{[x,y,z]}(\lambda xy.z(\lambda z.zy)x) =^2$
$\underline{\omega_{[x,y,z]}(x)}\omega_{[x,y,z]}(z)\omega_{[x,y,z]}(\lambda xy.z(\lambda z.zy)x) =^1$
$1\ \underline{\omega_{[x,y,z]}(z)}\omega_{[x,y,z]}(\lambda xy.z(\lambda z.zy)x) =^1$
$1\ 3\ \underline{\omega_{[x,y,z]}(\lambda xy.z(\lambda z.zy)x)} =^3$
$1\ 3\ \lambda\underline{\omega_{[x,x,y,z]}(\lambda y.z(\lambda z.zy)x)} =^3$
$1\ 3\ \lambda\lambda\underline{\omega_{[y,x,x,y,z]}(z(\lambda z.zy)x)} =^2$
$1\ 3\ \lambda\lambda\underline{\omega_{[y,x,x,y,z]}(z(\lambda z.zy))}\omega_{[y,x,x,y,z]}(x) =^2$
$1\ 3\ \lambda\lambda\underline{\omega_{[y,x,x,y,z]}(z)}(\omega_{[y,x,x,y,z]}(\lambda z.zy))\omega_{[y,x,x,y,z]}(x) =^1$
$1\ 3\ \lambda\lambda\ 5\ (\underline{\omega_{[y,x,x,y,z]}(\lambda z.zy)})\omega_{[y,x,x,y,z]}(x) =^3$
$1\ 3\ \lambda\lambda\ 5\ (\lambda\underline{\omega_{[z,y,x,x,y,z]}(zy)})\omega_{[y,x,x,y,z]}(x) =^2$
$1\ 3\ \lambda\lambda\ 5\ (\lambda\underline{\omega_{[z,y,x,x,y,z]}(z)}\omega_{[z,y,x,x,y,z]}(y))\omega_{[y,x,x,y,z]}(x) =^1$
$1\ 3\ \lambda\lambda\ 5\ (\lambda\ 6\ \underline{\omega_{[z,y,x,x,y,z]}(y)})\omega_{[y,x,x,y,z]}(x) =^1$
$1\ 3\ \lambda\lambda\ 5\ (\lambda\ 6\ 5)\underline{\omega_{[y,x,x,y,z]}(x)} =^1$
$1\ 3\ \lambda\lambda\ 5\ (\lambda\ 6\ 5)\ 2$

2. Assume the following translation function f from $\mathcal{M}$ to $\mathcal{M}'$ that will translate terms in $\mathcal{M}$ to terms in $\mathcal{M}'$:

$$\begin{array}{lll} 0. & f(v) & = & v \\ 1. & f(\lambda v.A) & = & [v]f(A) \\ 2. & f(AB) & = & \langle f(B)\rangle f(A) \end{array}$$

So for example:
$\underline{f((\lambda x.x)y)} =^2$
$\langle\underline{f(y)}\rangle f(\lambda x.x) =^0$
$\langle y\rangle\underline{f(\lambda x.x)} =^1$
$\langle y\rangle[x]\underline{f(x)} =^0$
$\langle y\rangle[x]x.$

Similarly you can show that: $f((\lambda x.(\lambda y.xy)z)(\lambda z'.z')) = \langle[z']z'\rangle[x]\langle z\rangle[y]\langle y\rangle x$.
Use this translation function f to translate all the terms in Question 1 above into terms of $\mathcal{M}'$. That is, give $f(\lambda xy.x)$ and $f(\lambda xyz.xz(yz))$ and $f(xz(\lambda xy.z(\lambda z.zy)x))$ showing all the steps and underlining all the parts you are working on. (3.5)

- $\underline{f(\lambda xy.x)} =^1$
  $[x]\underline{f(\lambda y.x)} =^1$

$[x][y]\underline{f(x)} =^0$
$[x][y]x$

- $\underline{f(\lambda xyz.xz(yz))} =^1$
$[x]\underline{f(\lambda yz.xz(yz))} =^1$
$[x][y]\underline{f(\lambda z.xz(yz))} =^1$
$[x][y][z]\underline{f(xz(yz))} =^2$
$[x][y][z]\langle\underline{f(yz)}\rangle f(xz) =^2$
$[x][y][z]\langle\langle\underline{f(z)}\rangle f(y)\rangle f(xz) =^0$
$[x][y][z]\langle\langle z\rangle \underline{f(y)}\rangle f(xz) =^0$
$[x][y][z]\langle\langle z\rangle y\rangle \underline{f(xz)} =^2$
$[x][y][z]\langle\langle z\rangle y\rangle\langle\underline{f(z)}\rangle f(x) =^0$
$[x][y][z]\langle\langle z\rangle y\rangle\langle z\rangle\underline{f(x)} =^0$
$[x][y][z]\langle\langle z\rangle y\rangle\langle z\rangle x$

- $\underline{f(xz(\lambda xy.z(\lambda z.zy)x))} =^2$
$\langle\underline{f(\lambda xy.z(\lambda z.zy)x)}\rangle f(xz) =^1$
$\langle[x]\underline{f(\lambda y.z(\lambda z.zy)x)}\rangle f(xz) =^1$
$\langle[x][y]\underline{f(z(\lambda z.zy)x)}\rangle f(xz) =^2$
$\langle[x][y]\langle\underline{f(x)}\rangle f(z(\lambda z.zy))\rangle f(xz) =^0$
$\langle[x][y]\langle x\rangle\langle\underline{f(\lambda z.zy)}\rangle f(z)\rangle f(xz) =^1$
$\langle[x][y]\langle x\rangle\langle[z]\underline{f(zy)}\rangle f(z)\rangle f(xz) =^2$
$\langle[x][y]\langle x\rangle\langle[z]\langle\underline{f(y)}\rangle f(z)\rangle f(z)\rangle f(xz) =^0$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle\underline{f(z)}\rangle f(z)\rangle f(xz) =^0$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle z\rangle\underline{f(z)}\rangle f(xz) =^0$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle z\rangle z\rangle\underline{f(xz)} =^2$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle z\rangle z\rangle\langle\underline{f(z)}\rangle f(x) =^0$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle z\rangle z\rangle\langle z\rangle\underline{f(x)} =^0$
$\langle[x][y]\langle x\rangle\langle[z]\langle y\rangle z\rangle z\rangle\langle z\rangle x$

3. Calculate $\omega_1(\lambda 1(\lambda 21)3)$ showing, numbering and underlining all the steps you carry out in the calculations. (4)

$FV(\lambda 1(\lambda 2\,1)3) = \{2\}$ and $max(FV(\lambda 1(\lambda 2\,1)3)) = 2$.
Hence $\underline{\omega_1(\lambda 1(\lambda 2\,1)3)} =^0$
$\omega_1'(3, \underline{lel(2, \textbf{listorder})}, (\lambda 1(\lambda 2\,1)3)) =$
$\underline{\omega_1'(3, [y], \lambda 1(\lambda 2\,1)3)} =^2$

$$
\begin{array}{l}
\lambda z.\omega_1'(4,[z,y],1(\lambda 2\,1)3) =^3 \\
\lambda z.\omega_1'(4,[z,y],1(\lambda 2\,1))\omega_1'(4+lams(1(\lambda 2\,1)),[z,y],3) =^3 \\
\lambda z.\omega_1'(4,[z,y],1)(\omega_1'(4+lams(1),[z,y],\lambda 2\,1))\omega_1'(4+lams(1(\lambda 2\,1)),[z,y],3) =^1 \\
\lambda z.el(1,[z,y])(\omega_1'(4+lams(1),[z,y],\lambda 2\,1))\omega_1'(4+lams(1(\lambda 2\,1)),[z,y],3) = \\
\lambda z.z(\omega_1'(4+lams(1),[z,y],\lambda 2\,1)\omega_1'(4+lams(1(\lambda 2\,1)),[z,y],3) = \\
\lambda z.z(\omega_1'(4,[z,y],\lambda 2\,1))\omega_1'(4+lams(1(\lambda 2\,1)),[z,y],3) =^2 \\
\lambda z.z(\lambda x'.\omega_1'(5,[x',z,y],2\,1))\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) =^3 \\
\lambda z.z(\lambda x'.\omega_1'(5,[x',z,y],2)\omega_1'(5+lams(2),[x',z,y],1))\omega_1'(4+lams(1(\lambda 21)),[x',z,y],3) =^1 \\
\lambda z.z(\lambda x'.el(2,[x',z,y])\omega_1'(5+lams(2),[x',z,y],1))\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) = \\
\lambda z.z(\lambda x'.z\omega_1'(5+lams(2),[x',z,y],1))\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) = \\
\lambda z.z(\lambda x'.z\omega_1'(5,[x',z,y],1))\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) =^1 \\
\lambda z.z(\lambda x'.zel(1,[x',z,y]))\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(4+lams(1(\lambda 2\,1)),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(4+lams(1)+lams(\lambda 2\,1),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(4+lams(\lambda 2\,1),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(4+1+lams(2\,1),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(5+lams(2)+lams(1),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(5+lams(1),[x',z,y],3) = \\
\lambda z.z(\lambda x'.zx')\omega_1'(5,[x',z,y],3) =^1 \\
\lambda z.z(\lambda x'.zx')el(3,[x',z,y]) = \\
\lambda z.z(\lambda x'.zx')y
\end{array}
$$

4. In the SML files, you were given the following LEXP terms (which implement terms of $\mathcal{M}$):

```
val vx = (ID "x");
val vy = (ID "y");
val vz = (ID "z");
val t1 = (LAM("x",vx));
val t2 = (LAM("y",vx));
val t3 = (APP(APP(t1,t2),vz));
val t4 = (APP(t1,vz));
val t5 = (APP(t3,t3));
val t6 = (LAM("x",(LAM("y",(LAM("z",
                     (APP(APP(vx,vz),(APP(vy,vz)))))))));
val t7 = (APP(APP(t6,t1),t1));
val t8 = (LAM("z", (APP(vz,(APP(t1,vz))))));
val t9 = (APP(t8,t3));
```

For example vx implements the variable x. The SML term t1 implements the $\mathcal{M}$

term $\lambda x.x$.

Give the **full** term of $\mathcal{M}$ implemented by the SML LEXP term t9. (0.5)

The SML term t9 implements the $\mathcal{M}$ term $\cdots$

t9 $= (\lambda z.z((\lambda x.x)(z))(((\lambda x.x)(\lambda y.x))(z))$

5. Give the term It8 of IEXP that correspond to t8 of LEXP. (0.5)

```
val It8 = (ILAM("z",(IAPP((IID "z"),(IAPP(ILAM("x",(IID "x")),IID "z"))))));
```

6. Give the term Bt2 of BEXP that corresponds to t2 of LEXP. (0.5)

```
val Bt2 = (BLAM(BID 1));
```

7. Remote login to your university account (or do whatever you usually do to get to the university machines, or if you have SML on your own computer then do the work on your own computer).
On university machines, in the same directory in which you have the files assign21-help.sml, type the following line (and hit return):
poly
You will get the prompt

```
>
```

Type at the prompt the following:

```
>use "assign21-help.sml";
```

If you want, read and understand the messages you get, but don't bother if you don't want to, or you think you don't udnerstand.

Then, test the commands below (in poly) and write the output of the following: (1)

```
subs vy "x" t2;
subs vx "y" t2;
```

```
>subs vy "x" t2;
val it = LAM ("x1",ID "y") : LEXP
>subs vx "y" t2;
val it = LAM ("y",ID "x") : LEXP
```

8. Give $(\lambda y.x)[x := y]$ and compare with <u>subs vy "x" t2</u> above. (1.5)

Due to rule 6 of substitution in Lambda Calculus...

$(\lambda y.x)[x := y] \equiv^6$

$(\lambda x1.x)[y := x1][x := y] \equiv$

$(\lambda x1.y)$

Furthermore, when the LEXP term (subs vy "x" t2) is printed using (printLEXP (subs vy "x" t2)), the term (\x1.y) is returned. This means that both terms $(\lambda y.x)[x := y]$ and (subs vy "x" t2) are syntactically equivalent.