# Mobile robotics
# Probabilistic localization and mapping

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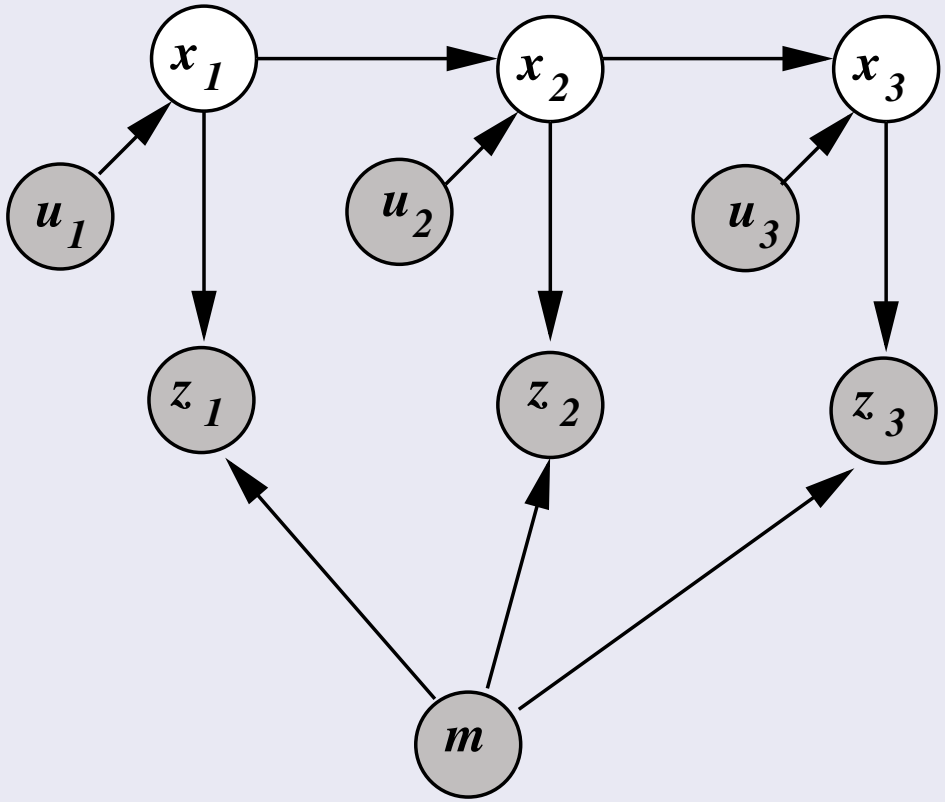
These slides contents base on a book Probabilistic Robotics (S. Thurn et al. )

# Robot localization task – definition

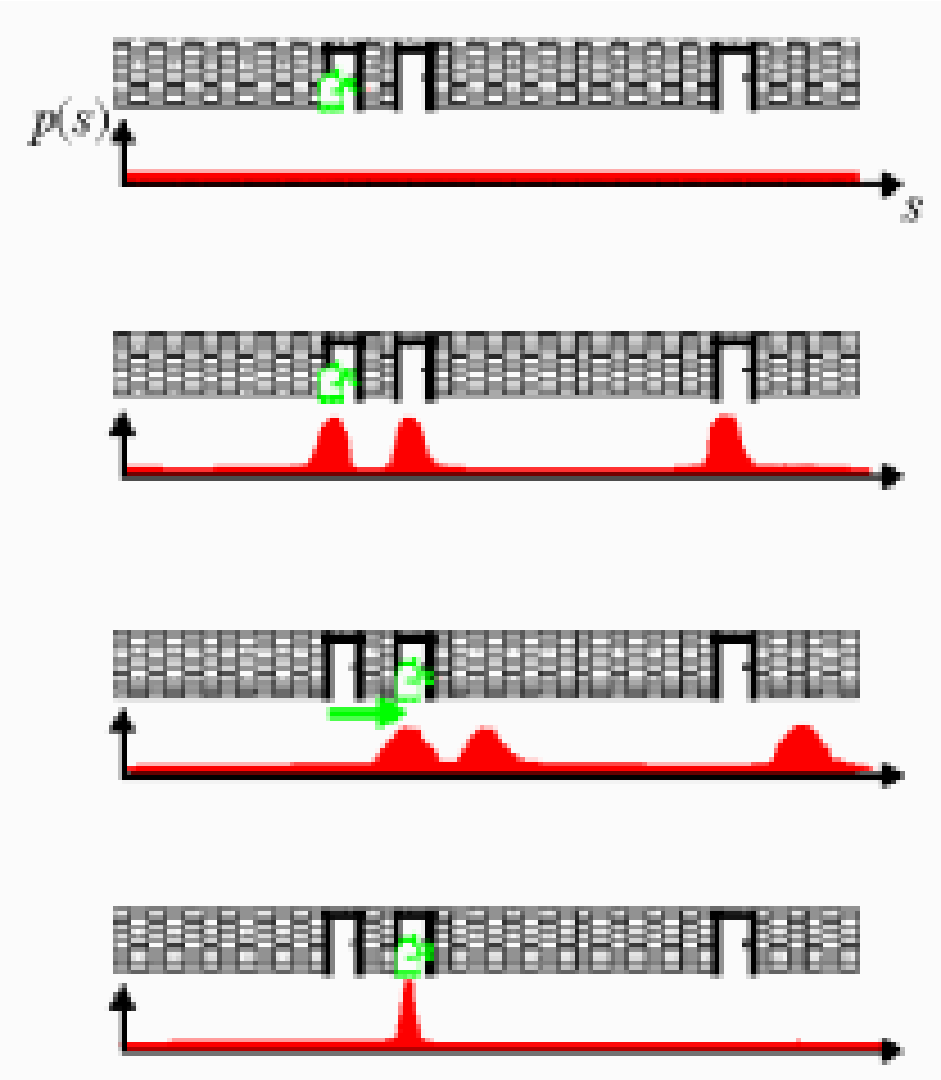



Having known: environment model $m$, controls $\mathcal{U} = \{u_{1:t}\}$ and observations $\mathcal{Z} = \{z_{1:t}\}$ determine $x_t$ (or $\mathcal{X} = \{x_{1:t}\}$).

# Localization task taxonomy

- local (position tracking) / global (incl. kidnapped robot problem)
- static/dynamic environment
- passive/active
- single-/multi-robot

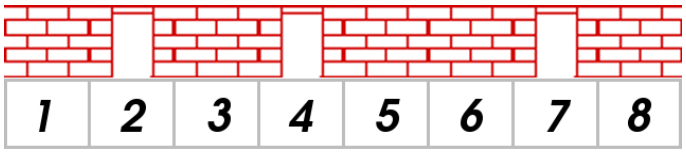
# Probabilistic localization – the idea




[0] http://www.cs.cmu.edu/ thrun/tutorial/sld027.htm

# Example

$$p(z_t = \text{door} \mid x_t = x_i) = \begin{cases} 1 & \text{for } x_i = 2, 4, 7, \\ 0 & \text{for } x_i = 1, 3, 5, 6, 8 \end{cases}$$




$$x_{t+1} = f(x_t, u_t = 1) = x_t + 1$$

$$Z = \{\text{door}, \text{wall}, \text{door}\}$$

$p(x_t = x_i | z_{1:t})$ :

| $i$, $z_i$ \ $x_i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 |
| 1 (door) | 0 | 1/3 | 0 | 1/3 | 0 | 0 | 1/3 | 0 |
| 2 (wall) | 0 | 0 | 1/3 | 0 | 1/3 | 0 | 0 | 1/3 |
| 3 (door) | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

# Markov model localization

parameters: $bel(x_{t-1}), u_t, z_t, m$

for all $x_t$ do

prediction phase: expected probability of the new state
$\overline{bel}(x_t) = \int p(x_t|u_t, x_{t-1}, m) bel(x_{t-1}) dx_{t-1}$

correction phase: estimate update based on observation
$bel(x_t) = \eta p(z_t|x_t, m) \overline{bel}(x_t)$

output: $bel(x_t)$

# Example – cont.

## Remark

in most cases: $p(z_t|x_t)$ and $p(x_{t+1}|x_t, u_t)$ are not binary, so the state change and output may suffer from noise.

## Questions

How would the solution change if

1. if $u_i = 0.9$, so the transfer function is given by

$$p(x_{t+1} = x_i + 1|x_t = x_i) = 0.9 \qquad p(x_{t+1} = x_i|x_t = x_i) = 0.1$$

2. the probability that door are open is 0.5 and the robot is unable to distinguish between a closed door and a wall?

(recall Viterbi algorithm)

# EKF localization

EKF localization is a special case of Markov model localization

## Assumptions

1. beliefs (*bel*) are represented by moments: $(\mu_t, \Sigma_t)$
2. map is represented by a collection of features $z_t = \{z_t^1, z_t^2, \ldots\}$
3. $(+)$ the features are identifiable by a correspondence $c_t = \{c_t^1, c_t^2, \ldots\}$

## Properties

- Gaussian is unimodal – suitable for position tracking and cases when the approximate pose may be restricted to a small region
- It does not allow hard constraints (like walls)
- Linearization is usually valid only close to the real pose
- It does not interpret negative information (missing features)

# General EKF algorithm

prediction phase:

$$\bar{\mu}_t = g(u_t, \mu_{t-1})$$
$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$$

observation phase:

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(z_t - h_t(\bar{\mu}_t))$$
$$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$$

# EKF localization algorithm

Unicycle robot equation in discrete form

$$x_t = x_{t-1} + g(x_{t-1}, u_t) + \mathcal{N}(0, R_t)$$

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_t}{\omega_t}\sin\theta + \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{v_t}{\omega_t}\cos\theta - \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

Controls with Gaussian noise

$$\begin{pmatrix} \hat{v} \\ \hat{\omega} \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} + \mathcal{N}(0, M_t) = \begin{pmatrix} v \\ \omega \end{pmatrix} + \begin{pmatrix} \varepsilon(\alpha_1 v_t^2 + \alpha_2 \omega_t^2) \\ \varepsilon(\alpha_3 v_t^2 + \alpha_4 \omega_t^2) \end{pmatrix}$$

# EKF localization algorithm – prediction phase

Auxiliary data calculated from the model:

$$\theta = \mu_{t-1,\theta}$$

$$G_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial \mu_{t-1}} = \begin{pmatrix} 1 & 0 & -\frac{v_t}{\omega_t}\cos\theta + \frac{v_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ 0 & 1 & -\frac{v_t}{\omega_t}\sin\theta - \frac{v_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$$

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

Update rule:

$$\bar{\mu}_t = \mu_{t-1} + g(\mu_{t-1}, u_t)$$

$$\bar{\Sigma}_t = G_t \Sigma_t G_t^T + V_t M_t V_t^T$$

# EKF localization algorithm – observation phase

Observation model for an observation $z_t^j = (r_t^j, \phi_t^j, s_t^j)$

$$z_t^j = h(x_t, j, m) + \mathcal{N}(0, Q_t)$$

$$\begin{pmatrix} r_t^j \\ \phi_t^j \\ s_t^j \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x_{t,x})^2 + (m_{j,y} - x_{t,y})^2} \\ \operatorname{atan2}(m_{j,y} - x_{t,x}, m_{j,x} - x_{t,x}) - x_{t,\theta} \\ m_{j,s} \end{pmatrix} + \mathcal{N}(0, Q_t)$$

with $Q_t = \operatorname{diag}\{\sigma_r, \sigma_\phi, \sigma_s\}$ and

$$H_t^j = \frac{\partial h(\hat{\mu}_t, j, m)}{\partial x_t} = \begin{pmatrix} -\frac{m_{j,x} - \hat{\mu}_{t,x}}{h_3} & -\frac{m_{j,y} - \hat{\mu}_{t,y}}{h_3} & 0 \\ \frac{m_{j,y} - \hat{\mu}_{t,y}}{h_3^2} & -\frac{m_{j,x} - \hat{\mu}_{t,x}}{h_3^2} & -1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{where} \quad h_3 = h(\hat{\mu}_t, j, m)_r$$

# EKF localization algorithm – observation phase

For each observation $z_t^j = (r_t^j, \phi_t^j, s_t^j)$

$$S_t^j = H_t^j \bar{\Sigma}_t (H_t^j)^T + Q_t$$

$$K_t^j = \bar{\Sigma}_t (H_t^j)^T (\bar{\Sigma}_t)^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^j (z_t^j - \hat{z}_t^j)$$

$$\bar{\Sigma}_t = (I - K_t^j H_t^j) \bar{\Sigma}_t$$

Finally:

$$\mu_t = \bar{\mu}_t$$

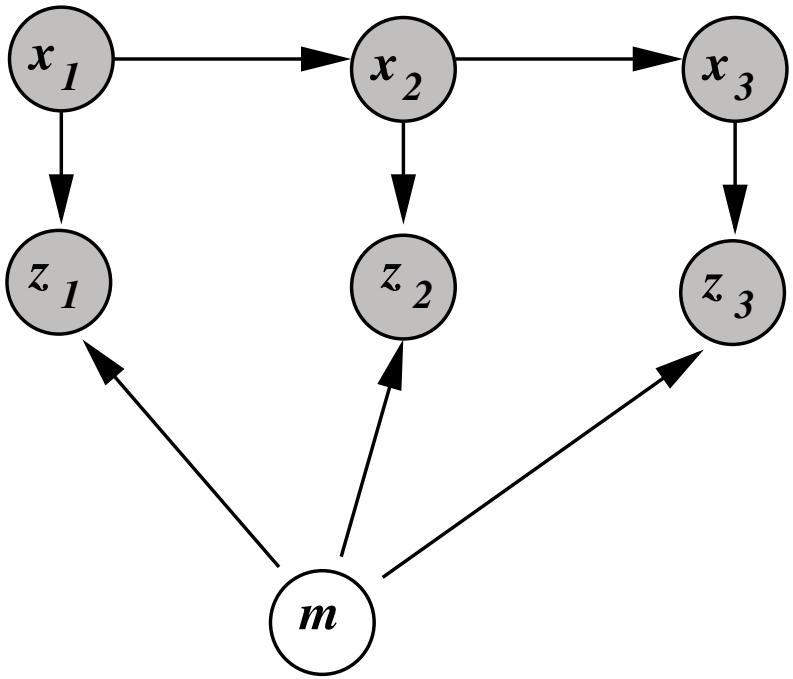
$$\bar{\Sigma}_t = \Sigma_t$$

$$p_{z_t} = \prod_j \frac{1}{\sqrt{\det 2\pi S_t^j}} \exp\left\{ -\frac{1}{2} (z_t^j - \hat{z}_t^j)^T (S_t^j)^{-1} (z_t^j - \hat{z}_t^j) \right\}$$

# Mapping

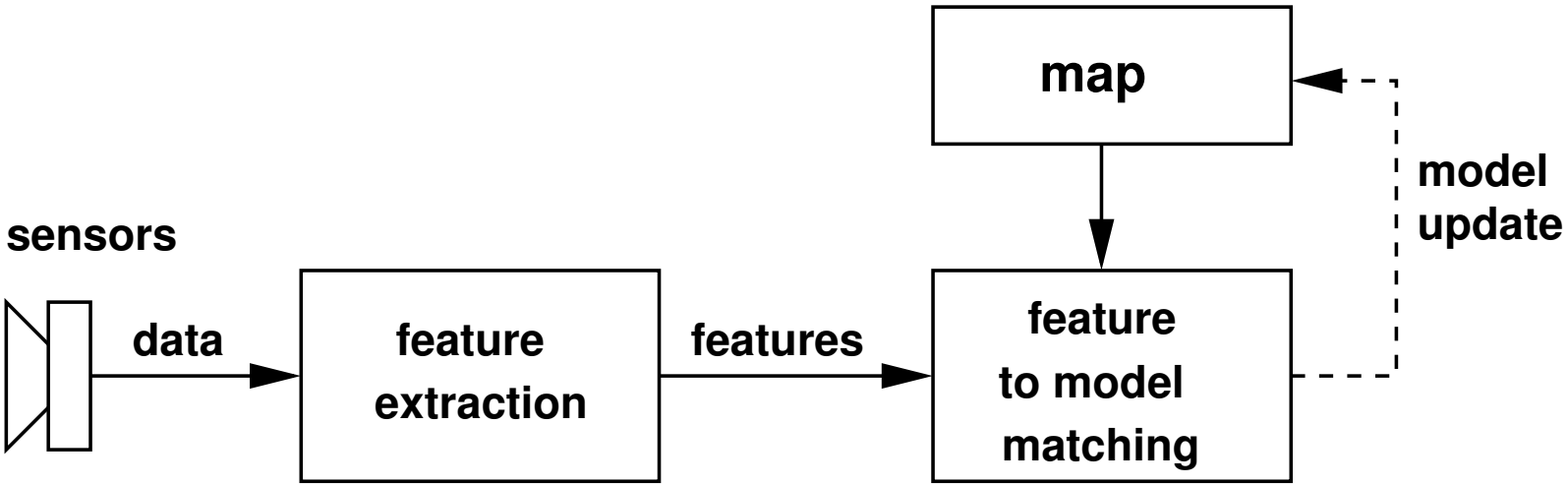
## Task definition

Based on **known** series of robot poses (*trajectory*) and sensor readings (*observations*) in those poses – determine obstacles' coordinates and free regions in the robot surrounding (a map *in chosen representation*).




- $m$ – map
- $x_i$ – robot pose in time step $i$
- $z_i$ – observation in time step $i$

# Mapping – scheme




Mapping – phases [Crowley 1989]

1. Creating high level description based on current sensor readings and sensor models
2. Feature (detected by sensors) matching to current environment description
3. Map update (modification of object detection probabilities)

# Mapping – features and sensors

## Typical features

- lines (and line segments)
- corners
- markers

Detected features, even in static environment, must be tracked to verify and correct map if necessary.

## Sensors

In map building one usually uses

- data from robot self-localization (odometry and others)
- laser scanners
- sonars
- mono- and stereo-vision
- depth sensors (TOF, structural light)

# Map taxonomy

## Steadiness

- static
- semi-static
- dynamic

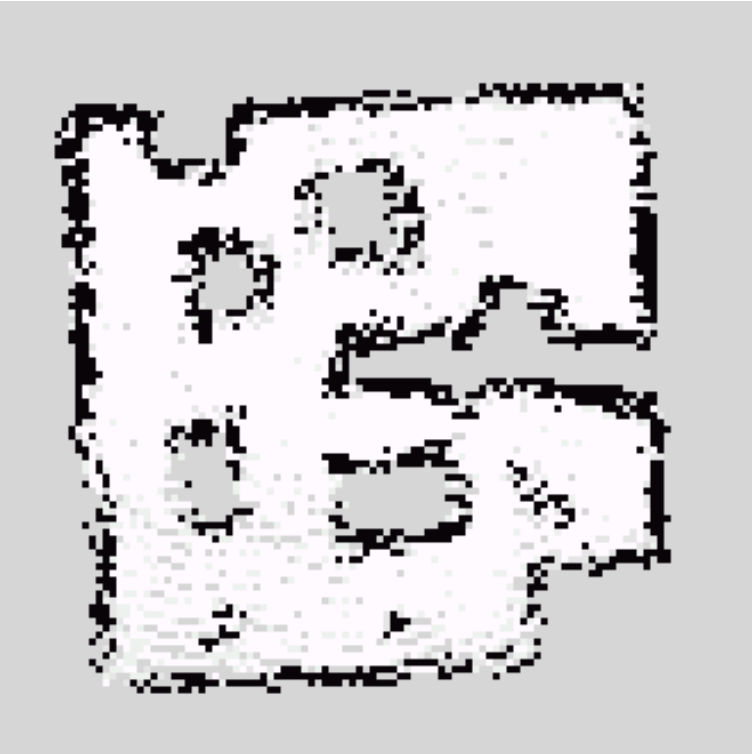
## Structure

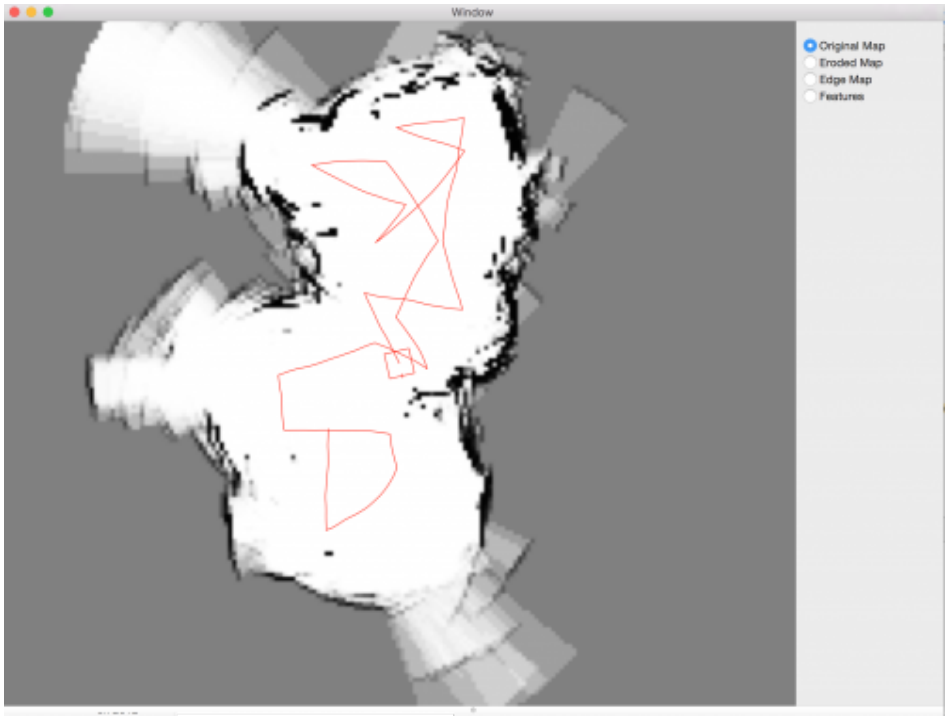
- metric
- topological
- hybrid

## Representation (of distances)

- grid
- vector

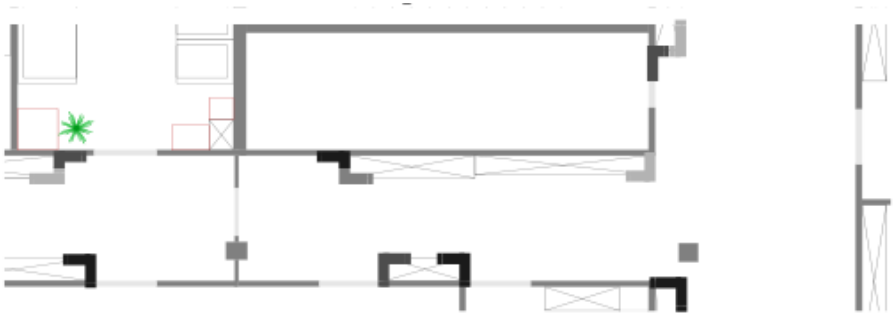
# Grid metric map

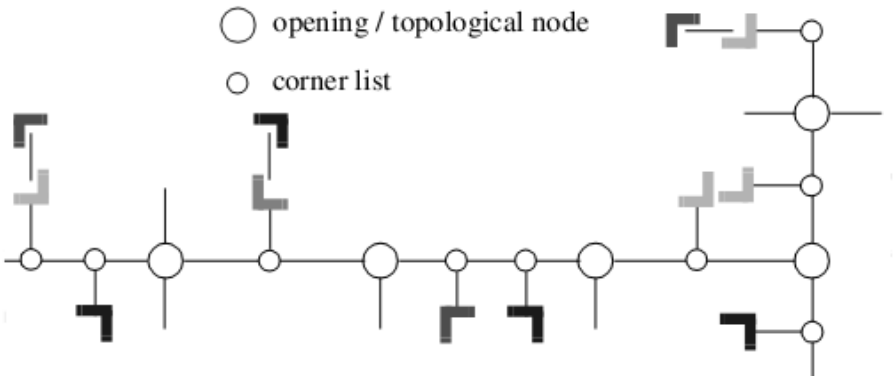


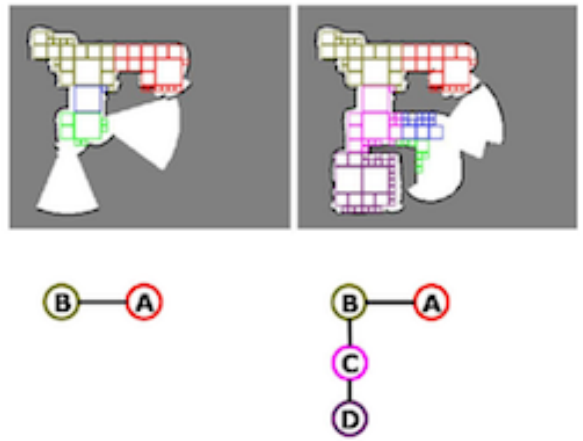


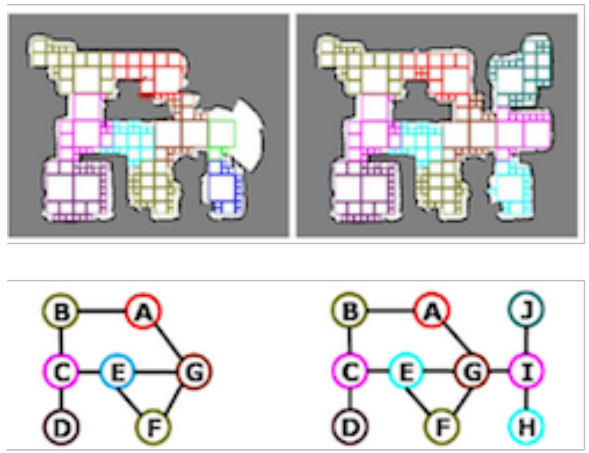

[0] http://emergent.brynmawr.edu/eprg/?page=PyroModuleMapping
http://letsmakerobots.com/robot/project/
mapping-rover-the-classic-rover-5-with-improved-3d-printed-axis-adaptors

# Topological map






[0]Tomatis et al. Combining topological and metric: a natural integration for SLAM, `http://www.autonomousrobotsblog.com/tag/mapping/`

# Comparison of topological and metric maps

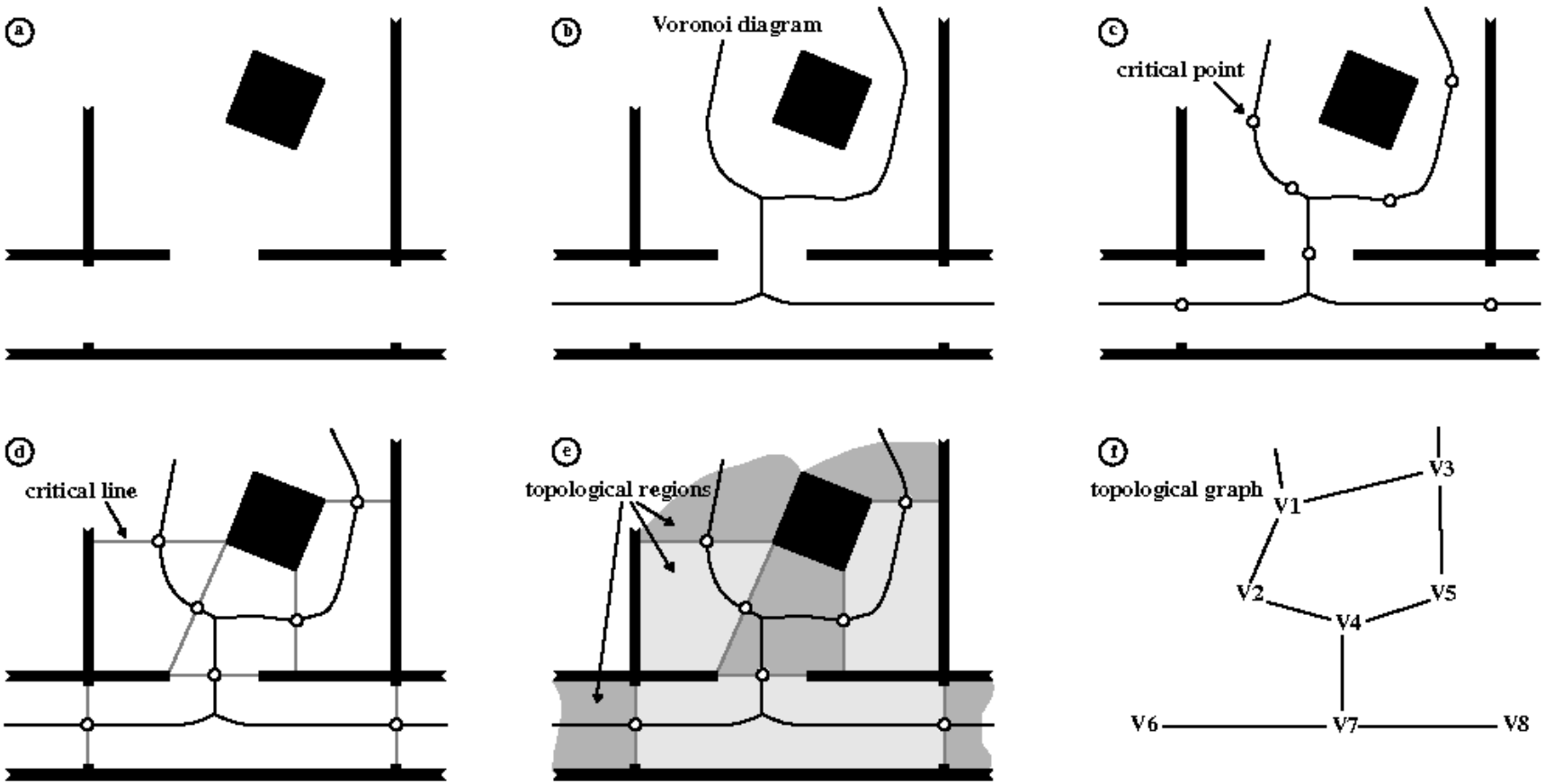
## Metric maps advantages

- ease of creation, representation, updates
- recognition independent of robot pose
- optimization of path lengths possible

## Topological maps advantages

- complexity dependant on region, not its size
- high precision of robot localization is not necessary
- simpler high level processing

# Conversion of metric map to topological




[0]Thrun. Learning maps for indoor mobile robot navigation

# Mapping – complexity

## Main factors influencing complexity:

- size – environment area with respect to sensors range
- noise in measurements and localization
- ambiguity of perception (multiple hypotheses matching)
- closed cycles (with localization errors)

# Practical limitations

- available measurements types
- sensor ranges
- measurement errors
- ambiguity of interpretation
- localization errors
- environment dynamics
- real time processing requirement

# Occupancy grid map

Searching for a (static) map to maximize probability

$$p(m|z_{1:t}, x_{1:t}).$$

Divide a map into cells $m = \{m_i\}$, each one marked with occupancy probability (1 – occupied, 0 – empty).

## Scale problem:

Map with 100x100 cells means $10^5$ cells, then number of possible maps: $2^{10000}$.

## Standard solution:

Assume independence of map cells and calculate each cell separately

$$p(m|z_{1:t}, x_{1:t}) \simeq \Pi_i \; p(m_i|z_{1:t}, x_{1:t})$$

Map is updated with use of binary Bayes filter and some representation of probability

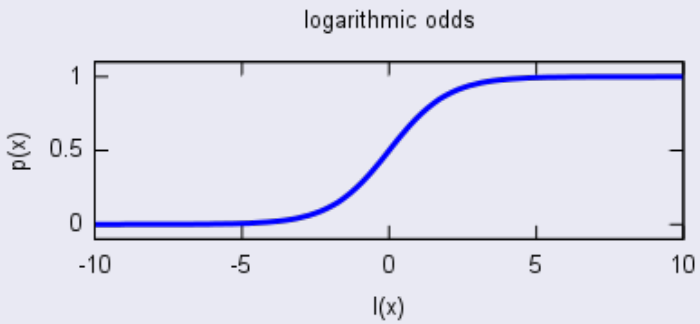
# Grid maps – representation of cell value

1. probability $p(x) \in [0, 1]$
2. logarithmic $l(x) \in (-\infty, +\infty)$

$$l(x) = \log \frac{p(x)}{p(\neg x)} = \log \frac{p(x)}{1 - p(x)}$$

with probabilities recovered by

$$p(x) = 1 - \frac{1}{1 + \exp\{l(x)\}}$$




3. hits/misses $(h(x), f(x))$

# Occupancy grid map – algorithm

$l_0 = \log \frac{p(m_i=1)}{p(m_i=0)} = \log \frac{p(m_i)}{1-p(m_i)}$

foreach $m_i$ do

  if $m_i$ in perception_field$(z_t)$

    $l_{t,i} = l_{t-1,i} + inverse\_sensor\_model(m_i, x_t, z_t) - l_0$

  *else*

    $l_{t,i} = l_{t-1,i}$

  endif

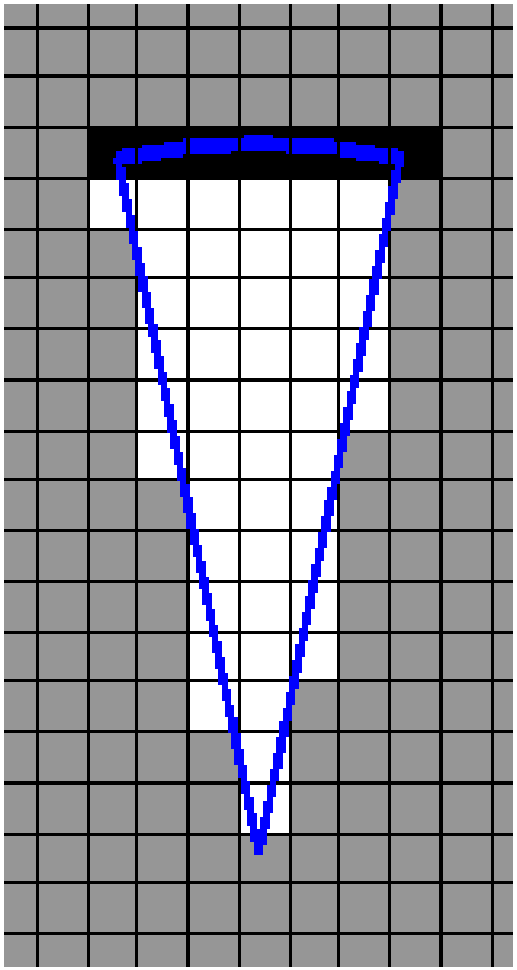
endfor

$p(m_i|z_{1:t}, x_{1:t}) = 1 - \frac{1}{1+\exp\{l_{t,i}\}}$

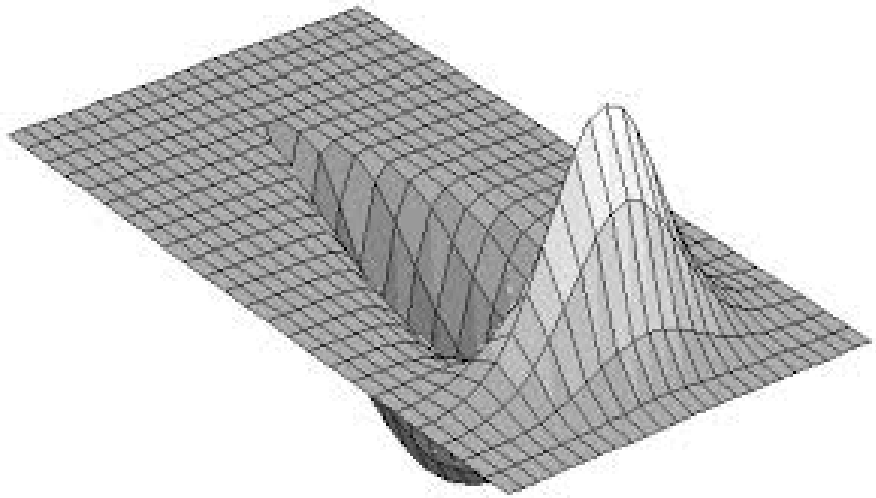
$inverse\_sensor\_model(m_i, x_t, z_t) = \log \frac{p(m_i|z_t,x_t)}{1-p(m_i|z_t,x_t)}$

# Sensor model

Simple version



All details:



# Occupancy grid and sensor fusion

## Question

How to create a map using simultanously various sensors?

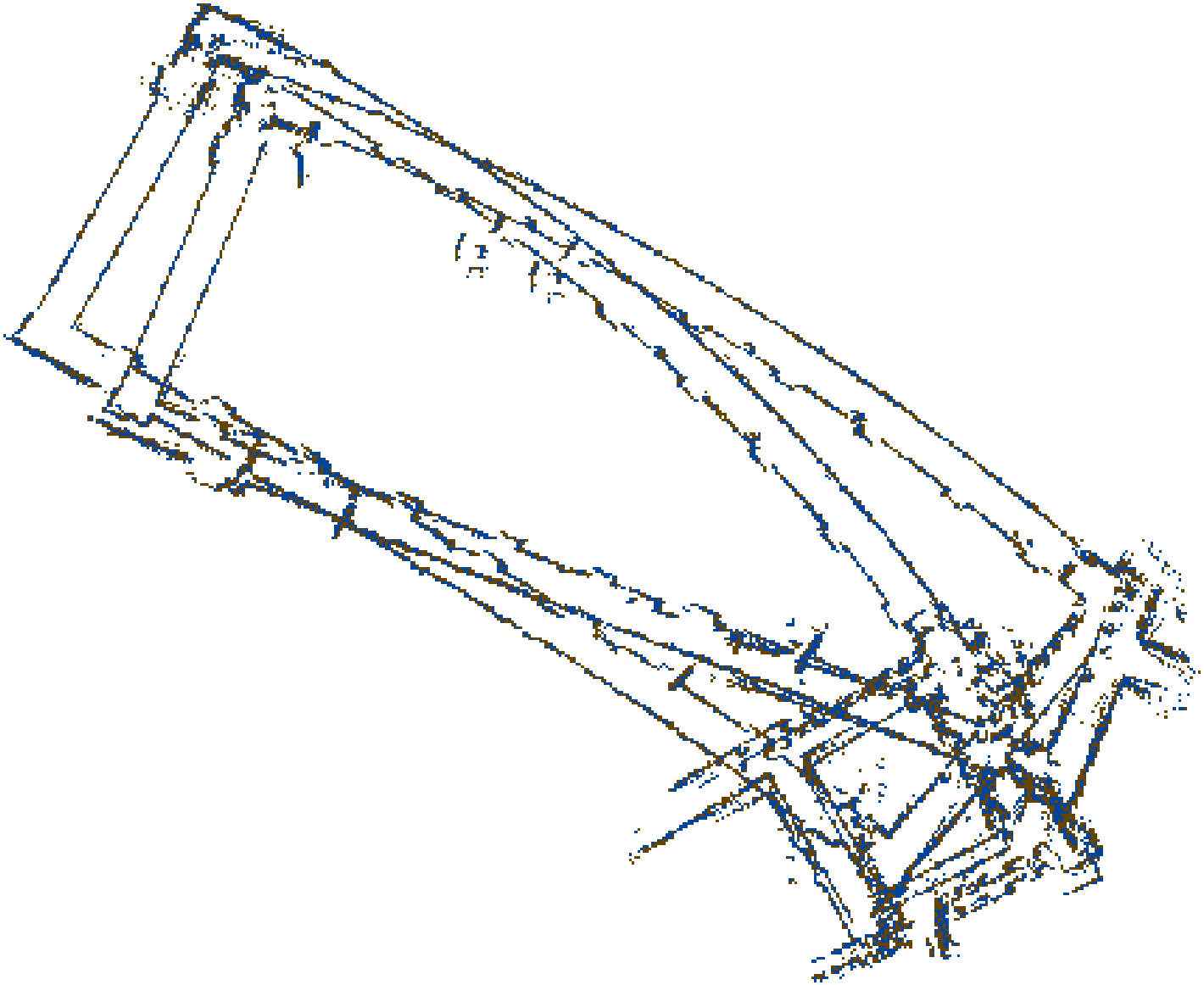
1. (simple) Run the algorithm consecutively for each sensor and the same map.
   Problem: If sensors detect various obstacles, map may be unstable
2. (typical) Create and update separate maps for each sensor type, when a map is needed – combine the sensor maps with chosen method, for example, if measurements are independent

$$p(m_i) = 1 - \Pi_k(1 - p(m_i^k)),$$

or using the most pessimistic hypothesis

$$p(m_i) = \max_k p(m_i^k).$$

# Consequences of odometric error in cycles



[0]Thrun. Integrating topological and metric maps...