

9. (1)

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

$$= \sqrt{\frac{1284 - 6 \times 14.33^2}{5}}$$

$$= \sqrt{10.38} = 3.22$$

∴ σ 之點估計為 3.22

(2) $1 - \alpha = 0.90, \frac{\alpha}{2} = 0.05, n-1 = 5$

$$\chi_{\frac{\alpha}{2}}^2 (n-1) = \chi_{0.05}^2 (5) = 11.07$$

$$\chi_{1-\frac{\alpha}{2}}^2 (n-1) = \chi_{0.95}^2 (5) = 1.15$$

∴ σ 之 90% 信賴區間為

$$\left[\sqrt{\frac{5 \times 10.38}{\chi_{0.05}^2 (5)}}, \sqrt{\frac{5 \times 10.38}{\chi_{0.95}^2 (5)}} \right] = \left[\sqrt{\frac{51.9}{11.07}}, \sqrt{\frac{51.9}{1.15}} \right] = (2.17, 6.72)$$

20. μ_1 為投資組合一獲利率之平均數, μ_2 為投資組合二獲利率之平均數, $n_1 = 9, \bar{x} = 9.67$
 $S_1 = 9.27, n_2 = 9, \bar{y} = 6.78, S_2 = 21.15$

(1) 因為 $\sigma_1^2 \neq \sigma_2^2, U = \frac{\frac{9.27^2}{9} + \frac{21.15^2}{9}}{\frac{9.27^2}{9} + \frac{21.15^2}{9}} = 10.96 \neq 11$

∴ $\mu_1 - \mu_2$ 之 95% 信賴區間為

$$(\bar{x} - \bar{y}) \pm t_{\frac{\alpha}{2}} (U) \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = (9.67 - 6.78) \pm t_{0.025} (11) \sqrt{\frac{9.27^2}{9} + \frac{21.15^2}{9}}$$

$$= 0.89 \pm 2.201 \times 7.70 = 0.89 \pm 16.95, \text{ 即 } (-16.06, 17.84)$$

(2) $1 - \alpha = 0.90, \chi_{\frac{\alpha}{2}}^2 (n_1 - 1) = \chi_{0.05}^2 (8) = 15.51, \chi_{1-\frac{\alpha}{2}}^2 (n_1 - 1) = \chi_{0.95}^2 (8) = 2.73$

∴ σ_1 之 90% 信賴區間為

$$\left[\sqrt{\frac{8 \times 9.27^2}{\chi_{0.05}^2 (8)}}, \sqrt{\frac{8 \times 9.27^2}{\chi_{0.95}^2 (8)}} \right] = \left[\sqrt{\frac{687.46}{15.51}}, \sqrt{\frac{687.46}{2.73}} \right] = (6.66, 15.87)$$

(3) $1 - \alpha = 0.90, F_{\frac{\alpha}{2}} (n_1 - 1, n_2 - 1) = F_{0.05} (8, 8) = 3.44, F_{1-\frac{\alpha}{2}} (n_1 - 1, n_2 - 1) = F_{0.95} (8, 8)$
 $= \frac{1}{F_{0.05} (8, 8)} = 0.29, \therefore \frac{\sigma_1^2}{\sigma_2^2}$ 之 90% 信賴區間為

$$\left[\frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}} (n_1 - 1, n_2 - 1)}, \frac{S_1^2}{S_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}} (n_1 - 1, n_2 - 1)} \right] = \left[\frac{9.27^2}{21.15^2} \times \frac{1}{3.44}, \frac{9.27^2}{21.15^2} \times \frac{1}{0.29} \right] = (0.06, 0.66)$$

助教出的習題

$$3, n=10, \bar{x}=13.63, s=6.05, n-1=9, 1-\alpha=0.98, \frac{\alpha}{2}=0.01$$

$$\bar{x} \pm t_{\frac{\alpha}{2}}(n-1) \frac{s}{\sqrt{n}} = 13.63 \pm t_{0.01}(9) \frac{6.05}{\sqrt{10}}$$

$$= 13.63 \pm 2.821 \times 1.91$$

$$= 13.63 \pm 5.39 \quad (8.24, 19.02)$$

$$4, \quad (1) \quad n=1200, \hat{p}=0.33, 1-\alpha=0.98$$

$$0.33 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.33 \pm 2.327 \times \sqrt{\frac{0.33 \times 0.67}{1200}} = 0.33 \pm 0.3$$

$$= (0.30, 0.36)$$

$$(2) \quad n=820, x=650, \hat{p}=\frac{650}{820}=0.79$$

$$1-\alpha=0.95, \frac{\alpha}{2}=0.025$$

$$0.79 \pm 1.96 \times \sqrt{\frac{0.79 \times 0.21}{820}}$$

$$= 0.79 \pm 1.96 \times 0.014$$

$$= 0.79 \pm 0.03$$

$$= (0.76, 0.82)$$

$$14, \quad (1) \quad n=15, \bar{x}=1.73, s=0.8, 1-\alpha=0.95, t_{\frac{\alpha}{2}}(n-1) = t_{0.025}(14) = 2.145$$

$$1.73 \pm t_{0.025}(14) \frac{0.8}{\sqrt{15}} = 1.73 \pm 2.145 \frac{0.8}{\sqrt{15}}$$

$$= 1.73 \pm 0.44$$

$$= (1.29, 2.17)$$

$$(2) \quad 1.73 \pm t_{0.10}(14) \frac{0.8}{\sqrt{15}}$$

$$= 1.73 \pm 1.345 \frac{0.8}{\sqrt{15}}$$

$$= 1.73 \pm 0.28$$

$$= (1.45, 2.01)$$