

例 6.4 (課本例題)

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$$E(x) = \mu, V(x) = \sigma^2 = E(x^2) - \mu^2$$

$$\text{則 } E(\bar{x}) = \mu, V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \frac{n-1}{n} \sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^n x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2$$

因此, $\hat{\theta}_2 = \sum_{i=1}^n (x_i - \bar{x})^2 / (n-1)$ 為母體變異數 σ^2 之無偏估計量, 而 $\hat{\theta}_1 = \sum_{i=1}^n (x_i - \bar{x})^2 / n$ 為 σ^2 之有偏估計量

CH4

6. (1) 是, 離散的

$$(2) X \sim B(10, 0.5), P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.623 = 0.377$$

$$(3) P(X \leq 4) = 0.377$$

$$34. P(X = x) = \frac{e^{-k} \cdot k^x}{x!}$$

$$P(X = k) = \sum_{x=0}^k \frac{M^x}{x!} e^{-M}$$

令 X 為 1 個月內發生無預警停駛次數
則 $X \sim P(0.5)$

$$(1) P(X=0) = \frac{e^{-0.5} \cdot 0.5^0}{0!} = e^{-0.5} = 0.6065$$

21 個月 $\rightarrow 1$ 次

17 個月 $\rightarrow 0.5$ 次

$$(2) P(X \geq 1) = 1 - P(X=0) = 1 - 0.6065 = 0.3935$$

35. 令 X 為 100 呎寬, 30 呎長的破璃氣球非瑕疵個數, 則 $X \sim P(3)$

$$(1) P(X=0) = \frac{e^{-3} \cdot 3^0}{0!} = 0.0498$$

$$(2) P(X=2) = \frac{e^{-3} \cdot 3^2}{2!} = 0.4232 - 0.1991 = 0.224$$

$$P(X=2) = P(X \leq 2) - P(X \leq 1)$$