算法分析与设计基础作业1

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a.

$\forall f(n) = \Theta(n^2)$

$\exists c_1, c_2, n_0 > 0, s.t. \forall n > n_0, 0 \le c_1 n^2 \le f(n) \le c_2 n^2$

$\therefore c_1 n^2 \le 2n + f(n) \le c_2 n^2 + 2n$

取$c_3 > c_2, n_1 = max\{n_0, \dfrac{2}{c_3 - c_2}\}$

$\because \exists c_1, c_3, n_1 > 0, s.t. \forall n > n_1, 0 \le c_1 n^2 \le 2n + f(n) \le c_2 n^2 + 2n \le c_3 n^2$

$\therefore 2n + f(n) = \Theta(n^2)$

$\forall f(n) = \Theta(n^2)$

$\exists c_1, c_2, n_0 > 0, s.t. \forall n > n_0, 0 \le c_1 n^2 \le f(n) \le c_2 n^2$

$\therefore c_1 n^2 - 2n \le f(n) - 2n \le c_2 n^2$

取$c_3 < c_1, n_1 = max\{n_0, \dfrac{2}{c_1 - c_3}\}$

$\because \exists c_2, c_3, n_1 > 0, s.t. \forall n > n_1, 0 \le c_3 n^2 \le c_1 n^2 - 2n \le f(n) - 2n \le c_2 n^2$

$\therefore f(n) = \Theta(n^2) + 2n$

综上可证：$\Theta(n^2) + 2n = \Theta(n^2)$

b.

$$\forall f(n) \in \Theta(g(n)),$$

$$\because \exists c_1, c_2, n_0 > 0, s.t. \forall n > n_0, 0 \le c_1 g(n) \le f(n) \le c_2 g(n)$$

$$\therefore \lim_{n \to \infty} \frac{f(n)}{g(n)} \ne 0$$

$\therefore f(n) \notin o(g(n))$

即$\Theta(g(n)) \cap o(g(n)) = \emptyset$

c.

欲证明原命题，只需证明

$\exists f(n), g(n),$满足$f(n) \in O(g(n)), f(n) \notin \Theta(g(n)) \cup o(g(n))$

取$g(n) = n^3, f(n) = \begin{cases} n^3, & \text{n为奇数} \\ n^2, & \text{n为偶数} \end{cases}$

满足这个条件，所以原命题成立

d.

若$f(n) \geq g(n)$

则$f(n) = max\{f(n), g(n)\}$

$\therefore \forall n > 0, \frac{1}{2}(f(n) + g(n)) \leq f(n) \leq f(n) + g(n)$

$\therefore f(n) = \Theta(f(n) + g(n))$

若$f(n) < g(n)$

则$g(n) = max\{f(n), g(n)\}$

$\therefore \forall n > 0, \frac{1}{2}(f(n) + g(n)) \leq g(n) \leq f(n) + g(n)$

$\therefore g(n) = \Theta(f(n) + g(n))$

综上可证：$max\{f(n), g(n)\} = \Theta(f(n) + g(n))$

3-3

(a) 排列如下：

$2^{2^{n+1}}$

2^{2^n}

$(n+1)!$

$n!$

e^n

$n \cdot 2^n$
2^n
$(\frac{3}{2})^n$
$(lgn)^{lgn}$
n^{lglgn}
$(lgn)!$
n^3
4^{lgn}
n^2
$nlgn$
$lg(n!)$
n
2^{lgn}
$(\sqrt{2})^{lgn}$
$2^{\sqrt{2lgn}}$
lg^2n
lnn
$\sqrt{lgn}$
$lnlnn$
2^{lg^*n}
lg^*n
$lg^*(lgn)$
$lg(lg^*n)$
$n^{1/lgn}$
1
划分的等价类为：
$\{2^{2^{n+1}}\}$
$\{2^{2^n}\}$
$\{(n+1)!\}$
$\{n!\}$
$\{e^n\}$
$\{n \cdot 2^n\}$
$\{2^n\}$

$\{(\frac{3}{2})^n\}$

$\{(lgn)^{lgn}, n^{lglgn}\}$

$\{(lgn)!\}$

$\{n^3\}$

$\{4^{lgn}, n^2\}$

$\{nlgn, lg(n!)\}$

$\{n, 2^{lgn}\}$

$\{(\sqrt{2})^{lgn}\}$

$\{2^{\sqrt{2lgn}}\}$

$\{lg^2 n\}$

$\{lnn\}$

$\{\sqrt{lgn}\}$

$\{lnlnn\}$

$2^{lg^* n}$

$\{lg^* n\}$

$\{lg^*(lgn)\}$

$\{lg(lg^* n)\}$

$\{n^{1/lgn}, 1\}$

(b)

$$f(n) = \begin{cases} 0, & n\text{为奇数} \\ 2^{2^{n+2}}, & n\text{为偶数} \end{cases}$$