# Homework 5

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## Prob-1

### a.

by PC:

```
library("stats")
R=matrix(c(1,0.83,0.78,0.83,1,0.67,0.78,0.67,1), 3)
respc <- princomp(covmat=R,cor = T,scores = T)
res1 <- respc$loadings %*% diag(respc$sdev)
L1 <- res1[,1]
Phi1 <- diag(diag(R - L1 %*% t(L1)))
L1
```

```
## [1] -0.9514150 -0.9098748 -0.8881720
```

```
Phi1
```

```
##            [,1]      [,2]      [,3]
## [1,] 0.09480941 0.0000000 0.0000000
## [2,] 0.00000000 0.1721278 0.0000000
## [3,] 0.00000000 0.0000000 0.2111505
```

by MLE:

```
res2 <- factanal(covmat=R,factors=1,rotation = 'varimax')
L2 <- res2$loadings[1:3]
Phi2 <- diag(res2$uniquenesses)
L2
```

```
## [1] 0.9829892 0.8443632 0.7934980
```

```
Phi2
```

```
##            [,1]      [,2]      [,3]
## [1,] 0.03373219 0.0000000 0.0000000
## [2,] 0.00000000 0.2870513 0.0000000
## [3,] 0.00000000 0.0000000 0.3703609
```

### b.

Prefer PC, because the specific variances in PC are approximately less than those in MLE.

### c.

The specific variances:

```
res2$uniquenesses
```

```
## [1] 0.03373219 0.28705126 0.37036094
```

The communalities:

```
L2^2
```

```
## [1] 0.9662678 0.7129492 0.6296390
```

The proportion of variance explained by each factor:

```
L2^2/3
```

```
## [1] 0.3220893 0.2376497 0.2098797
```

The residual matrix:

```
R - L2 %*% t(L2) - Phi2
```

```
##               [,1]          [,2]          [,3]
## [1,] -1.146924e-08  8.060989e-08  4.532572e-08
## [2,]  8.060989e-08 -4.526785e-07 -4.749103e-07
## [3,]  4.532572e-08 -4.749103e-07  3.552657e-08
```

## Prob-2 9.19

```
data=read.table("T9-12.DAT")
p=7
Z=scale(data, center = TRUE, scale = TRUE)
```

**a.**

```
res2 <- factanal(Z,2,scores='regression')
res2
```

```
##
## Call:
## factanal(x = Z, factors = 2, scores = "regression")
##
## Uniquenesses:
##    V1    V2    V3    V4    V5    V6    V7
## 0.069 0.070 0.123 0.005 0.474 0.614 0.029
##
## Loadings:
##    Factor1 Factor2
## V1 0.852   0.452
## V2 0.868   0.419
## V3 0.717   0.602
## V4 0.148   0.987
## V5 0.501   0.525
## V6 0.619
## V7 0.946   0.277
##
##                Factor1 Factor2
## SS loadings      3.545   2.071
## Proportion Var   0.506   0.296
## Cumulative Var   0.506   0.802
##
```

```
## Test of the hypothesis that 2 factors are sufficient.
## The chi square statistic is 117.2 on 8 degrees of freedom.
## The p-value is 1.25e-21
```

```
res3 <- factanal(Z,3,scores='regression')
res3
```

```
##
## Call:
## factanal(x = Z, factors = 3, scores = "regression")
##
## Uniquenesses:
##    V1    V2    V3    V4    V5    V6    V7
## 0.039 0.034 0.088 0.005 0.447 0.005 0.038
##
## Loadings:
##    Factor1 Factor2 Factor3
## V1 0.793   0.374   0.438
## V2 0.911   0.317   0.185
## V3 0.651   0.544   0.438
## V4 0.255   0.964
## V5 0.542   0.465   0.207
## V6 0.299           0.950
## V7 0.917   0.180   0.298
##
##                Factor1 Factor2 Factor3
## SS loadings      3.175   1.718   1.453
## Proportion Var   0.454   0.245   0.208
## Cumulative Var   0.454   0.699   0.906
##
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 62.18 on 3 degrees of freedom.
## The p-value is 2.01e-13
```

**b.**

```
varimax(res2$loadings)
```

```
## $loadings
##
## Loadings:
##    Factor1 Factor2
## V1 0.852   0.452
## V2 0.869   0.419
## V3 0.717   0.602
## V4 0.148   0.986
## V5 0.501   0.525
## V6 0.619
## V7 0.946   0.276
##
##                Factor1 Factor2
## SS loadings      3.546   2.070
## Proportion Var   0.507   0.296
## Cumulative Var   0.507   0.802
```

```
##
## $rotmat
##              [,1]          [,2]
## [1,] 0.9999999583 -0.0002886942
## [2,] 0.0002886942  0.9999999583
```

```
varimax(res3$loadings)
```

```
## $loadings
##
## Loadings:
##    Factor1 Factor2 Factor3
## V1 0.794   0.374   0.438
## V2 0.911   0.317   0.185
## V3 0.651   0.544   0.438
## V4 0.255   0.964
## V5 0.542   0.465   0.207
## V6 0.299           0.950
## V7 0.917   0.180   0.297
##
##                Factor1 Factor2 Factor3
## SS loadings      3.175   1.718   1.452
## Proportion Var   0.454   0.245   0.207
## Cumulative Var   0.454   0.699   0.906
##
## $rotmat
##               [,1]          [,2]          [,3]
## [1,]  1.000000e+00 4.309125e-05 -0.0001185372
## [2,] -4.311812e-05 1.000000e+00 -0.0002267045
## [3,]  1.185274e-04 2.267096e-04  0.9999999673
```

**c.**

$m = 2$:

```
L2 <- res2$loadings[1:p]
# estimated communalities:
L2^2
```

```
## [1] 0.72615998 0.75417611 0.51442065 0.02178635 0.25075505 0.38276608
## [7] 0.89458243
```

```
# specific variances:
res2$uniquenesses
```

```
##         V1         V2         V3         V4         V5         V6
## 0.06919160 0.07038038 0.12330883 0.00500000 0.47358490 0.61363862
##         V7
## 0.02881701
```

```
# LL'+\psi:
L2 %*% t(L2) + diag(res2$uniquenesses)
```

```
##           [,1]      [,2]      [,3]       [,4]       [,5]       [,6]
## [1,] 0.7953516 0.7400355 0.6111888 0.12577909 0.42671804 0.52720908
## [2,] 0.7400355 0.8245565 0.6228674 0.12818248 0.43487178 0.53728301
```

```
## [3,] 0.6111888 0.6228674 0.6377295 0.10586478 0.35915676 0.44373728
## [4,] 0.1257791 0.1281825 0.1058648 0.02678635 0.07391237 0.09131855
## [5,] 0.4267180 0.4348718 0.3591568 0.07391237 0.72433995 0.30980724
## [6,] 0.5272091 0.5372830 0.4437373 0.09131855 0.30980724 0.99640470
## [7,] 0.8059838 0.8213846 0.6783743 0.13960548 0.47362545 0.58516306
##           [,7]
## [1,] 0.8059838
## [2,] 0.8213846
## [3,] 0.6783743
## [4,] 0.1396055
## [5,] 0.4736254
## [6,] 0.5851631
## [7,] 0.9233994
```

$m = 2$:

```
L3 <- res3$loadings[1:p]
# estimated communalities:
L3^2
```

```
## [1] 0.62960502 0.83080519 0.42421514 0.06504818 0.29380086 0.08948464
## [7] 0.84163641
```

```
# specific variances:
res3$uniquenesses
```

```
##         V1         V2         V3         V4         V5         V6
## 0.03857165 0.03448071 0.08812176 0.00500000 0.44662048 0.00500000
##         V7
## 0.03750980
```

```
# LL'+\psi:
L3 %*% t(L3) + diag(res3$uniquenesses)
```

```
##           [,1]      [,2]      [,3]       [,4]      [,5]       [,6]
## [1,] 0.6681767 0.7232421 0.5168056 0.20237258 0.4300913 0.23736044
## [2,] 0.7232421 0.8652859 0.5936667 0.23247014 0.4940559 0.27266152
## [3,] 0.5168056 0.5936667 0.5123369 0.16611569 0.3530365 0.19483516
## [4,] 0.2023726 0.2324701 0.1661157 0.07004818 0.1382433 0.07629425
## [5,] 0.4300913 0.4940559 0.3530365 0.13824331 0.7404213 0.16214396
## [6,] 0.2373604 0.2726615 0.1948352 0.07629425 0.1621440 0.09448464
## [7,] 0.7279413 0.8362033 0.5975240 0.23398060 0.4972660 0.27443311
##           [,7]
## [1,] 0.7279413
## [2,] 0.8362033
## [3,] 0.5975240
## [4,] 0.2339806
## [5,] 0.4972660
## [6,] 0.2744331
## [7,] 0.8791462
```

I prefer $m = 3$, because it represents more variances.

**e.**

```r
x<-c(110,98,105,15,18,12,35)
z=(x-colMeans(data))/diag(var(data))
# weighted least squares method:

# regression method:
res3$scores
```

```
##           Factor1      Factor2     Factor3
##  [1,] -0.78726929 -0.363904411 -0.49178225
##  [2,] -1.41588080 -0.738299159  0.20526714
##  [3,] -0.09897504 -0.800287410 -0.67912278
##  [4,] -0.45973504  0.579198613  0.82942096
##  [5,]  0.14454285 -0.368944907  0.68283151
##  [6,] -0.99920030 -0.069396094  0.53141949
##  [7,] -0.26295604 -0.497676143 -0.64396841
##  [8,]  0.93145104  1.485233300  1.79323841
##  [9,] -0.01057333 -0.334771034  1.21685451
## [10,]  1.08392076  0.429819603 -0.16518875
## [11,]  0.12553494  0.160310018  0.65721557
## [12,]  0.93188788 -0.522820190 -1.50954723
## [13,]  0.20143108  1.195807884  0.08550963
## [14,]  0.37065768 -0.944657367  0.16022974
## [15,]  0.55093674  0.345095427 -1.42983108
## [16,] -0.66693145 -0.881718882 -2.46907824
## [17,]  0.06428254 -0.081389611  0.20824113
## [18,]  0.57924865 -0.209944665  0.05173886
## [19,] -0.09731110 -1.620707009  1.30917499
## [20,] -0.75209831  1.700241690  0.35281847
## [21,] -0.98478814 -0.034194678 -1.43638766
## [22,]  1.53526711 -2.022205582 -0.15119779
## [23,] -1.03632052 -0.285794688 -1.40157270
## [24,]  0.65608225  0.022168913  0.49992946
## [25,]  0.84906750  1.023170892  0.37499359
## [26,] -0.04159567 -0.269686705 -1.70988431
## [27,]  0.84739427  0.497332373  0.41060632
## [28,]  1.94524042 -0.830504617 -0.34430645
## [29,] -1.85493684 -0.396483998  1.79149106
## [30,]  1.85018894 -1.054865716 -0.30205557
## [31,]  1.10158311  1.495891344 -0.69814394
## [32,] -1.82234490  0.951019523 -0.74734749
## [33,] -0.10625511 -1.075981934  0.30843018
## [34,] -1.31646940  0.009307396  0.62639479
## [35,]  1.18743012  1.479094235 -0.72582131
## [36,]  1.26493797 -1.206096153  1.35835209
## [37,] -1.33609607  2.141164202 -0.96099188
## [38,] -1.12873222  0.706023995  1.99469820
## [39,]  1.14975870  0.685063214  0.30269804
## [40,]  0.81849768  0.510790697  0.41633799
## [41,]  0.17179851 -0.641671662  1.17651063
## [42,] -0.13275072  0.546286828 -2.21743557
## [43,]  0.25052424  1.460631006 -0.43481486
## [44,] -1.11137698 -2.365417059 -0.27550507
## [45,] -0.37311549 -1.006156953  0.38546806
## [46,] -0.03590758  1.763405312  0.61627862
```

```
## [47,] -1.38072215 -0.751315731  0.67346741
## [48,] -1.74134468 -0.378166069 -0.69993562
## [49,]  0.15003089  0.669018806  0.62567745
## [50,]  1.19199128 -0.103016844 -0.15137535
```