Linear Regression Analysis HW2

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2019 年 3 月 22 日

1.

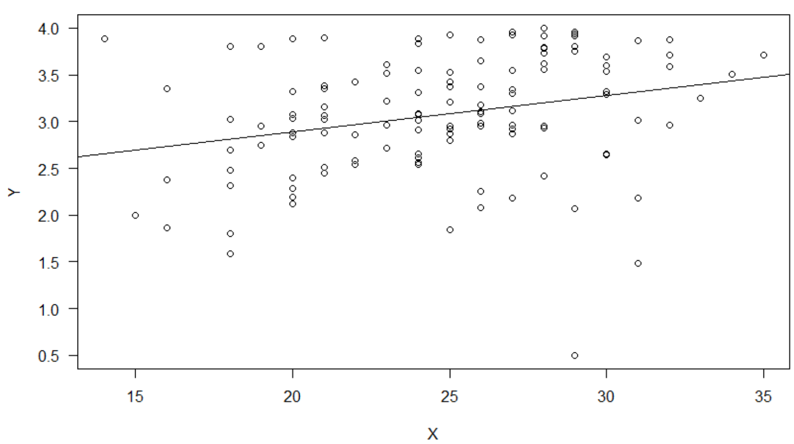
a.

$$\beta_0 = 2.11405, \beta_1 = 0.03883$$

The estimated regression function is

$$Y = 2.11405 + 0.03883X + \epsilon$$

b.



Surely it doesn't fit the data very well.

c.

With $X = 30$, we can obtain that $\hat{Y} = \beta_0 + \beta_1 X = 3.278863$

d.

Let δ be the change,

$$\hat{\delta} = \beta_1 = 0.03883$$

3.

a.

From $\hat{\beta_1} = argmin_{\beta_1} Q$, where $Q = \sum_{i=1}^{n}(Y_i - \beta_1 X_i)^2$, we obtain:

$$\frac{\partial Q}{\partial \beta_1} = -2\sum_{i=1}^{n} X_i(Y_i - \beta_1 X_i) = 0$$

It can be solved:

$$\hat{\beta_1} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2}$$

b.

$$\because \epsilon_i = Y_i - \beta_1 X_i \ i.i.d. \sim N(0, \sigma^2)$$

$$\therefore L(\beta_1 | X_1, \cdots, X_n, Y_1, \cdots, Y_n) = (2\pi\sigma^2)^{-n/2} exp(-\frac{\sum_{i=1}^{n} \epsilon_i^2}{2\sigma^2})$$

$$= (2\pi\sigma^2)^{-n/2} exp(-\frac{\sum_{i=1}^{n} (Y_i - \beta_1 X_i)^2}{2\sigma^2})$$

$$\therefore logL = -\frac{n}{2} log(2\pi\sigma^2) - \frac{\sum_{i=1}^{n} (Y_i - \beta_1 X_i)^2}{2\sigma^2}$$

From $\frac{\partial logL}{\partial \beta_1} = 0$ It can be solved:

$\hat{\beta_1} = \frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2}$, the same as the LSE.

c.

$$
\begin{aligned}
\because E\hat{\beta}_1 &= \frac{1}{\sum_{i=1}^n X_i^2} E \sum_{i=1}^n X_i Y_i \\
&= \frac{1}{\sum_{i=1}^n X_i^2} E(\beta_1 X_i^2 + X_i \epsilon_i) \\
&= \beta_1 + \frac{\sum_{i=1}^n X_i E\epsilon_i}{\sum_{i=1}^n X_i^2} \\
&= \beta_1
\end{aligned}
$$

$\therefore \hat{\beta}_1$ is unbiased.

4.

a.

Denote X_i as the i-th recovery time, $s^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2$.
Consider X_i $i.i.d. \sim N(\mu, \sigma^2)$,$\bar{X} \sim N(\mu, \sigma^2/n)$

$$H_0 : \mu = 123.7 \leftrightarrow H_1 : \mu > 123.7$$

$$\text{Let } T = \frac{\sqrt{n}(\bar{X} - 123.7)}{s} \sim t_{n-1}$$

From $P(T > t_{n-1;0.1}) = 10\%$ we can decide to reject H_0 if $T > t_{n-1;0.1}$.
Because $T = 2.0687 > t_{6;0.1}$, we don't reject H_0

b.

The same as a, because $T = 2.0687 > t_{6;0.05}$

5.

From what we have proved we can obtain:

$$\begin{aligned} b_0 &= \bar{Y} - b_1\bar{X} \\ &= \bar{Y} - \beta_1\bar{X} - \frac{\sum(X_i - \bar{X})\epsilon_i}{\sum(X_i - \bar{X})^2}\bar{X} \\ &= \beta_0 + \frac{\sum \epsilon_i}{n} - \frac{\sum(X_i - \bar{X})\epsilon_i}{\sum(X_i - \bar{X})^2}\bar{X} \end{aligned}$$

$$\because E(b_0) = E(\bar{Y} - \beta_1\bar{X}) + 0 = E(\frac{\sum(\beta_0 + \epsilon_i)}{n}) = \beta_0$$

$\therefore b_0$ is unbiased.

$$\because Var(b_0) = \sum(\frac{1}{n} - \frac{\bar{X}\sum(X_i - X)}{\sum(X_i - X)^2})^2\sigma^2$$

6.

$$\begin{aligned} \because Q &= \sum(Y_i - b_0 - b_1X_i)^2 \\ &= \sum((\beta_0 - b_0) + (\beta_1 - b_1)X_i)^2 \\ &= \sum_i(\sum_j \epsilon_j(\frac{1}{n} + \frac{(X_j - \bar{X})(X_i - \bar{X})}{\sum_k(X_k - \bar{X})^2}))^2 \end{aligned}$$

$$\therefore E(Q) = (n-2)(E\epsilon_i^2 - (E\epsilon)^2) = (n-2)Var(\epsilon) = (n-2)\sigma^2$$