Homework 2

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4.19

(a)

$$(X_1 - \mu)'\Sigma^{-1}(X_1 - \mu) \sim \chi^2_6$$

(b)

Denote $c_i = \frac{1}{n},\ i = 1, 2, \cdots, n$

Then

$$\bar{X} = \sum_{i=1}^{n} c_i X_i \sim N_6(\mu, \frac{1}{n}\Sigma)$$

$$\sqrt{n}(\bar{X} - \mu) \sim N_6(0, \Sigma)$$

(c)

$$(n-1)S = \sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})' \sim W_6(\Sigma, n-1)$$

4.20

Assume that $B \in R^{p\times 6}$

Denote

$$Y_i = BX_i \sim N_p(B\mu, B\Sigma B')$$

Then

$$\bar{Y} = \frac{1}{n}\sum Y_i = B \times \frac{1}{n}\sum X_i = B\bar{X}$$

$$B(19S)B' = \sum_{i=1}^{n}(BX_i - B\bar{X})(BX_i - B\bar{X})' = \sum_{i=1}^{n}(Y_i - \bar{Y})(Y_i - \bar{Y})' \sim W_6(B\Sigma B', 19)$$

(a)

$$B(19S)B' = \sum_{i=1}^{n}(Y_i - \bar{Y})(Y_i - \bar{Y})' \sim W_6(B\Sigma B', 19)$$

(b)

$$B(19S)B' = \sum_{i=1}^{n}(Y_i - \bar{Y})(Y_i - \bar{Y})' \sim W_6(B\Sigma B', 19)$$

4.22

(a)

According to the CLT,

$$\bar{X} \sim N_p(\mu, \frac{1}{n}\Sigma)$$

(b)

According to the CLT,

$$n(\bar{X} - \mu)'S^{-1}(\bar{X} - \mu) \sim \chi_p^2$$

5.1

(a)

$$T^2 = \sqrt{n}(\bar{X} - \mu)'S^{-1}\sqrt{n}(\bar{X} - \mu)$$

```
n=4
p=2
X <- matrix(c(2,8,6,8,12,9,9,10), nrow = n)
mu <- matrix(c(7,11), nrow=p)
T2=sqrt(n)*t(colMeans(X)-mu)%*%solve(cov(X))%*%(sqrt(n)*(colMeans(X)-mu))
T2
```

```
##          [,1]
## [1,] 13.63636
```

(b)

$$\frac{2}{2} \times \frac{T^2}{3} \sim F_{2,2}$$

(c)

From $\Pr(\frac{2}{2} \times \frac{T^2}{3} > q) = 1 - \alpha$

We can obtain q

```
alpha=0.05
q=qf(1-alpha,p,n-p)
q
```

```
## [1] 19
```

Because $\frac{2}{2} \times \frac{T^2}{3} < q$, we accept H_0

5.2

Before X is changed, T^2 is

```
n=3
p=2
X <- matrix(c(6,10,8,9,6,3), nrow = n)
mu <- matrix(c(9,5), nrow=p)
T2=sqrt(n)*t(colMeans(X)-mu)%*%solve(cov(X))%*%(sqrt(n)*(colMeans(X)-mu))
T2
```

```
##           [,1]
## [1,] 0.7777778
```

After X is changed, T^2 is

```
C <-matrix(c(1,1,-1,1), nrow = 2)
X <- X%*%t(C)
mu <- C%*%mu
T2=sqrt(n)*t(colMeans(X)-mu)%*%solve(cov(X))%*%(sqrt(n)*(colMeans(X)-mu))
T2
```

```
##           [,1]
## [1,] 0.7777778
```

5.5

$$H_0 : \mu' = [0.55, 0.60] \leftrightarrow H_1 : \mu' \neq [0.55, 0.60]$$

```
n=42
p=2
x_bar<-matrix(c(0.564,0.603), nrow = p)
mu<-matrix(c(0.55,0.60), nrow = p)
S<-matrix(c(.0144,.0117,.0117,.0146), nrow = p)
T2=n*t(x_bar-mu)%*%solve(S)%*%(x_bar-mu)
T2/(n-1)*(n-p)/p
```

```
##           [,1]
## [1,] 0.5609776
```

From $\Pr(\frac{40}{2} \times \frac{T^2}{41} > q) = 1 - \alpha$

We can obtain q

```
alpha=0.05
q=qf(1-alpha,p,n-p)
q
```

```
## [1] 3.231727
```

Because $\frac{40}{2} \times \frac{T^2}{41} < q$, we accept H_0

The result is consistent with Figure 5.1 because it is equivalent that we accept H_0 and μ falls into the confidence ellipse.

5.7

Simultaneous confidence intervals:

```
data=read.table("T5-1.DAT")
alpha=0.05
n = dim(data)[1]
p = dim(data)[2]
X=as.matrix(data)
s=cov(X)
x_bar=colMeans(X)
for (i in 1:3)
{
  print(c(x_bar[i]-sqrt(p*(n-1)/(n-p)*qf(1-alpha,p,n-p))*sqrt(s[i,i]/n),x_bar[i]+sqrt(p*(n-1)/(n-p)*qf(
}
```

```
##       V1       V1
## 3.397768 5.882232
##       V2       V2
## 35.05241 55.74759
##        V3        V3
##  8.570664 11.359336
```

Borferroni intervals:

```
for (i in 1:3)
{
  print(c(x_bar[i]-qt(1-alpha/(2*p),n-1)*sqrt(s[i,i]/n),x_bar[i]+qt(1-alpha/(2*p),n-1)*sqrt(s[i,i]/n)))
}
```

```
##       V1       V1
## 3.643952 5.636048
##       V2       V2
## 37.10308 53.69692
##        V3        V3
##  8.846992 11.083008
```

The Borferroni intervals are more precise than simultaneous confidence intervals.

5.8

The value of a:

```
n=42
p=2
x_bar<-matrix(c(0.564,0.603), nrow = p)
mu<-matrix(c(0.55,0.60), nrow = p)
S<-matrix(c(.0144,.0117,.0117,.0146), nrow = p)
a=solve(S)%*%(x_bar-mu)
a
```

```
##            [,1]
## [1,]  2.308112
## [2,] -1.644172
```

```
T2_value=n*(t(a)%*%(x_bar-mu))%*%(t(a)%*%(x_bar-mu))*solve(t(a)%*%S%*%a)
T2_value
```

```
##          [,1]
## [1,] 1.150004
```

It is equal to T^2 in Exercise 5.5.