# Answer Keys to Assignment 1

**Question 1.2** T, M, L, R are strategies that survive iterated elimination of strictly dominated strategies.

(M, L) and (T, R) are pure-strategy Nash equilibria.

**Question 1.6** Suppose that $(q_1^*, q_2^*)$ constitutes a pure-strategy Nash equilibrium.

If $q_1^* > 0$, $q_2^* > 0$, then by first-order conditions,

$$\begin{aligned} q_1^* &= \frac{a - 2c_1 + c_2}{3}; \\ q_2^* &= \frac{a - 2c_2 + c_1}{3}. \end{aligned}$$

This is the case when $0 < c_i < a/2$.

However, if $2c_2 > a + c_1$, $q_2^*$ cannot be positive. Thus, $q_2^* = 0$, and by firm 1's best response function:

$$q_1^* = \frac{a - c_1}{2}.$$

As a final step, we compute firm 2's profit function given $q_1^*$:

$$\begin{aligned} \pi_2 &= (a - q_1^* - q_2)q_2 - c_2 q_2; \\ \frac{\partial \pi_2}{\partial q_2} &= a - c_2 - q_1^* - 2q_2 \\ &= \frac{1}{2}(a + c_1 - 2c_2) - 2q_2. \end{aligned}$$

Since $\partial \pi_2 / \partial q_2 < 0$ for any $q_2 > 0$, it is optimal for firm 2 to choose $q_2^* = 0$ given $q_1^*$. Hence, $((a - c_1)/2, 0)$ constitutes a Nash equilibrium.

**Question 1.8**

a). $x_1^* = x_2^* = 0.5$ is the unique pure-strategy Nash equilibrium.

b). Suppose that $(x_1^*, x_2^*, x_3^*)$ constitutes a pure-strategy Nash equilibrium.

**Case 1**: $x_1^* = x_2^* = x_3^* = 0.5$. Each candidate gets 1/3 of the votes. However, any of them can deviate to 0.4 and get a higher number of votes.

**Case 2**: $x_1^* = x_2^* = x_3^* \neq 0.5$. Each candidate gets 1/3 of the votes. However, any of them can deviate to 0.5 and get more than 1/2 of the votes.

**Case 3**: $x_1^* = x_2^* \neq x_3^*$. In this case, all candidates want to move as close to 0.5 as possible, but that would lead to case 1, a contradiction.

**Case 4**: $x_1^*, x_2^*, x_3^*$ are all different from each other. This case is similar to case 3.

Therefore, there is no pure-strategy Nash equilibrium of the game.

**Question 1.13** (Apply to Firm 2, Apply to Firm 1) and (Apply to Firm 1, Apply to Firm 2) are two pure-strategy Nash equilibria.

Let $(q, 1-q)$ denote worker 1's mixed strategy in which he/she plays "Apply to Firm 1" with probability $q$, and $(r, 1-r)$ denote worker 2's mixed strategy in which he/she plays "Apply to Firm 1" with probability $r$.

If $((q^*, 1-q^*), (r^*, 1-r^*))$ constitutes a mixed-strategy Nash equilibrium, then by Proposition 2.4, given worker 2's strategy, worker 1's expected payoff from choosing "Apply to Firm 1" and "Apply to Firm 2" should be equal. That is,

$$r^* \cdot \frac{1}{2}\omega_1 + (1-r^*) \cdot \omega_1 = r^* \cdot \omega_2 + (1-r^*) \cdot \frac{1}{2}\omega_2. \tag{1}$$

Similarly,

$$q^* \cdot \frac{1}{2}\omega_1 + (1-q^*) \cdot \omega_1 = q^* \cdot \omega_2 + (1-q^*) \cdot \frac{1}{2}\omega_2. \tag{2}$$

Combine (1) and (2), we have:

$$q^* = r^* = \frac{2\omega_1 - \omega_2}{\omega_1 + \omega_2}.$$

**Bonus question** There exists a (symmetric) mixed-strategy Nash equilibrium in which each firm chooses its price $p$ according to a cumulative distribution function $F(p)$ over $[c, +\infty)$. The proof goes as follows.

Given that firm 2 chooses its price $p_2$ according to $F(p)$, firm 1's expected profit from choosing $p_1$ is $p_1[1 - F(p_1)]$. By Proposition 2.4, firm 1 should find it indifferent between choosing any $p_1 \in [c, +\infty)$. Therefore, $p_1[1 - F(p_1)]$ should equal a constant, denoted by $k$. Hence,

$$F(p) = 1 - \frac{k}{p}.$$

Since $p < c$ leads to a negative profit, we have $F(c) = 0$, which implies $k = c$. Finally, when $p \to +\infty$, $F(p) \to 1$, so $F(p)$ is distributed over $[c, +\infty)$. Thus, the equilibrium price distribution is

$$F(p) = 1 - \frac{c}{p}.$$