# End-to-end Learning of LTL$_f$ Formulae by Faithful LTL$_f$ Encoding

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## Motivation

**Problem.** Learning *linear temporal logic on finite traces* (LTL$_f$) formulae in *arbitrary form* to characterize the high-level behavior of a system from observation traces with *imperfect data*.

**Challenge.**

1. *huge search space* of the target formula in arbitrary form
2. *wrong search bias* resulting from imperfect data

## Contributions

1. In this paper, we propose an LTL$_f$ encoding method for faithfully bridging the neural network inference and the LTL$_f$ inference.
2. Based on this theoretical result, we propose an end-to-end approach, `TLTLf`, to learn LTL$_f$ formulae.
3. Results demonstrate that `TLTLf` still outperforms other approaches on imperfect data.

## Model Structure of `TLTLf`

**Definition 1** *Let $\mathbb{P}$ be a set of atomic propositions and $L \in \mathbb{N}$. The parameter set of `TLTLf` of size $L$ is defined as $\Gamma = \{(\Gamma_{\mathrm{r}})_{i,j} \in \mathbb{R} | 1 \leq i \leq L-2, i+2 \leq j \leq L\} \cup \{(\Gamma_{\mathrm{a}})_{i,j} \in \mathbb{R} | 1 \leq i \leq L, 1 \leq j \leq |\mathbb{P}|\} \cup \{(\Gamma_{\neg})_i, (\Gamma_{\wedge})_i, (\Gamma_{\mathsf{X}})_i, (\Gamma_{\mathsf{U}})_i, (\Gamma_{\mathrm{none}})_i \in \mathbb{R} | 1 \leq i \leq L\}$. For brevity, we also reuse $\Gamma$ to denote an assignment of the parameter set $\Gamma$ of `TLTLf`.*

**Definition 2** *An LTL$_f$ encoding of `TLTLf` of size $L$ is defined as $\theta = \{(\theta_{\mathrm{r}})_{i,j} \in \mathbb{R}^{(0,1)} | 1 \leq i \leq L-2, i+2 \leq j \leq L\} \cup \{(\theta_{\mathrm{a}})_{i,j} \in \mathbb{R}^{(0,1)} | 1 \leq i \leq L, 1 \leq j \leq |\mathbb{P}|\} \cup \{(\theta_{\neg})_i, (\theta_{\wedge})_i, (\theta_{\mathsf{X}})_i, (\theta_{\mathsf{U}})_i, (\theta_{\mathrm{none}})_i \in \mathbb{R}^{(0,1)} | 1 \leq i \leq L\}$, where $\mathbb{R}^{(0,1)}$ denotes the real value range from 0 to 1.*

An arbitrary parameter assignment of `TLTLf` can be converted to an LTL$_f$ encoding of `TLTLf` of the same size, as shown in the following equation.

$$\begin{aligned}&(\theta_{\mathrm{r}})_{i,j} = \frac{e^{(\Gamma_{\mathrm{r}})_{i,j}}}{(\eta_{\mathrm{r}})_i}, (\theta_{\mathrm{a}})_{i,j} = \frac{e^{(\Gamma_{\mathrm{a}})_{i,j}}}{(\eta_{\mathrm{op}})_i}, (\theta_{\neg})_i = \frac{e^{(\Gamma_{\neg})_i}}{(\eta_{\mathrm{op}})_i},\\ &(\theta_{\wedge})_i = \frac{e^{(\Gamma_{\wedge})_i}}{(\eta_{\mathrm{op}})_i}, (\theta_{\mathsf{X}})_i = \frac{e^{(\Gamma_{\mathsf{X}})_i}}{(\eta_{\mathrm{op}})_i}, (\theta_{\mathsf{U}})_i = \frac{e^{(\Gamma_{\mathsf{U}})_i}}{(\eta_{\mathrm{op}})_i}, (\theta_{\mathrm{none}})_i = \frac{e^{(\Gamma_{\mathrm{none}})_i}}{(\eta_{\mathrm{op}})_i},\end{aligned} \tag{1}$$

where

$$\begin{aligned}&(\eta_{\mathrm{r}})_i = \textstyle\sum_{j=i+2}^{L} e^{(\Gamma_{\mathrm{r}})_{i,j}} + \sum_{j=1}^{|\mathbb{P}|} e^{(\Gamma_{\mathrm{a}})_{i,j}} + e^{(\Gamma_{\mathrm{none}})_i} + e^{(\Gamma_{\neg})_i} + e^{(\Gamma_{\mathsf{X}})_i},\\ &(\eta_{\mathrm{op}})_i = \textstyle\sum_{j=1}^{|\mathbb{P}|} e^{(\Gamma_{\mathrm{a}})_{i,j}} + e^{(\Gamma_{\mathrm{none}})_i} + e^{(\Gamma_{\neg})_i} + e^{(\Gamma_{\wedge})_i} + e^{(\Gamma_{\mathsf{X}})_i} + e^{(\Gamma_{\mathsf{U}})_i}.\end{aligned} \tag{2}$$

**Example 1** *Let $\mathbb{P} = \{p_1, p_2\}$ and $\theta$ be an LTL$_f$ encoding of `TLTLf` of size 3 where $(\theta_{\neg})_1 = 0.8, (\theta_{\mathsf{X}})_1 = 0.3, (\theta_{\mathrm{a}})_{2,1} = \theta_{\mathrm{none}})_3 = 1$ and other parameters are assigned 0. The LTL$_f$ formula that $\theta$ represents is the most likely to be $\neg p_1$ while it may also be $\mathsf{X} p_1$ since $(\theta_{\mathsf{X}})_1 = 0.3$.*

## Inference of `TLTLf`

**Definition 3** *Let $\mathbb{P}$ be a set of atomic propositions and $\Gamma$ a parameter assignment of `TLTLf` of size $L \in \mathbb{N}$. $\theta$ is constructed from $\Gamma$ by Equation (1). Given a trace $\pi = s_0, s_1, ..., s_n$ over $\mathbb{P}$, `TLTLf` computes satisfaction vectors $x_i \in \mathbb{R}^L$ (where $0 \leq i \leq n$) defined as follows:*

$$\begin{aligned}(x_i)_j = \sigma(&\textstyle\sum_{k=1}^{|\mathbb{P}|} (\theta_{\mathrm{a}})_{j,k} I(p_k \in s_i) + (\theta_{\mathsf{X}})_j (x_{i+1})_{j+1}\\ &+ (\theta_{\neg})_j \sigma(1 - (x_i)_{j+1}) + (\theta_{\wedge})_j \sigma((x_i)_{j+1} + (r_i)_j - 1)\\ &+ (\theta_{\mathsf{U}})_j ((r_i)_j + \sigma((x_i)_{j+1} + (x_{i+1})_j - 1))),\end{aligned} \tag{3}$$

*where*

$$(r_i)_j = \textstyle\sum_{k=L}^{j+2} (\theta_{\mathrm{r}})_{j,k} (x_i)_k, \sigma(x) = \min(1, \max(0, x)), \tag{4}$$

*and $(x_i)_{L+1} = 0$, $(x_{n+1})_k = 0$ for all $1 \leq k \leq L+1$, and $I(C)$ returns 1 if $C$ is satisfied or 0 otherwise. By $\mathrm{ESat}(\theta, \pi)$ we denote the satisfaction relation between $\theta$ and $\pi$. Finally `TLTLf` outputs $\mathrm{ESat}(\theta, \pi)$ as $(x_0)_1$.*

## The Faithful Subclass of LTL$_f$ Encoding

**Definition 4** *Let $\theta$ be an LTL$_f$ encoding of `TLTLf` of size $L$. $\theta$ is said to be faithful if it satisfies the following conditions:*

1. $\forall \gamma \in \theta : \gamma = 0 \vee \gamma = 1$.
2. $\forall i \in [1, L] : (\theta_{\mathrm{none}})_i + \sum_{j=1}^{|\mathbb{P}|} (\theta_{\mathrm{a}})_{i,j} + (\theta_{\neg})_i + (\theta_{\wedge})_i + (\theta_{\mathsf{X}})_i + (\theta_{\mathsf{U}})_i = 1$.
3. $\sum_{j=1}^{|\mathbb{P}|} (\theta_{\mathrm{a}})_{L,j} + (\theta_{\mathrm{none}})_L = 1 \wedge \forall i \in [1, L-1] : \sum_{j=i+2}^{L} (\theta_{\mathrm{r}})_{i,j} + (\theta_{\mathrm{none}})_i + \sum_{j=1}^{|\mathbb{P}|} (\theta_{\mathrm{a}})_{i,j} + (\theta_{\neg})_i + (\theta_{\mathsf{X}})_i = 1$.
4. $(\theta_{\mathrm{none}})_1 = 0 \wedge \forall i \in [2, L] : \sum_{j=1}^{i-2} (\theta_{\mathrm{r}})_{j,i} + (\theta_{\neg})_{i-1} + (\theta_{\wedge})_{i-1} + (\theta_{\mathsf{X}})_{i-1} + (\theta_{\mathsf{U}})_{i-1} + (\theta_{\mathrm{none}})_i = 1$.
5. $\forall i \in [1, L-1] : (\theta_{\mathrm{none}})_{i+1} \geq (\theta_{\mathrm{none}})_i$.
6. $\forall i \in [1, L), \forall j \in (i, L], \forall t \in (i, j), \forall t' \in (j, L] : (\theta_{\mathrm{r}})_{i,j} + (\theta_{\mathrm{r}})_{t,t'} \leq 1$.

**Example 2** *Consider the LTL$_f$ encoding $\theta$ of `TLTLf` given in Example 1 again. A faithful LTL$_f$ encoding closed to $\theta$ is $\hat{\theta}$, where $(\hat{\theta}_{\neg})_1 = 1, (\hat{\theta}_{\mathsf{X}})_1 = 0, (\hat{\theta}_{\mathrm{a}})_{2,1} = 1, (\hat{\theta}_{\mathrm{none}})_3 = 1$ and other parameters are assigned 0. The LTL$_f$ formula that $\hat{\theta}$ represents is unique, which is $\neg p_1$.*

Faithful LTL$_f$ encodings and the prefix forms of LTL$_f$ formulae have one-to-one correspondence. We can construct faithful encoding $\theta_{\phi(L)}$ for any LTL$_f$ formula $\phi$ of length $L$. The inference result of `TLTLf` using a faithful encoding is proved to be the same as the inference result of the corresponding LTL$_f$ formula.

**Theorem 1** *For any LTL$_f$ formula $\phi$ with $\mathrm{pretravel}(T(\phi)) = v_1, v_2, ..., v_L$, any $L' \geq L$ and any trace $\pi = s_0, s_1, ..., s_n$, it holds that $\mathrm{ESat}(\theta_{\phi(L')}, \pi) = 1$ if $\pi \models \phi$, or $\mathrm{ESat}(\theta_{\phi(L')}, \pi) = 0$ otherwise.*

## Learning LTL$_f$ Formulae by LTL$_f$ Encoding

**Framework.** The framework of `TLTLf` is summarized as follows.

1. First build `TLTLf` parameterized by an LTL$_f$ encoding and then train it to distinguish positive traces from negative traces.
2. Then extract the formula from `TLTLf`.

## Result Analysis

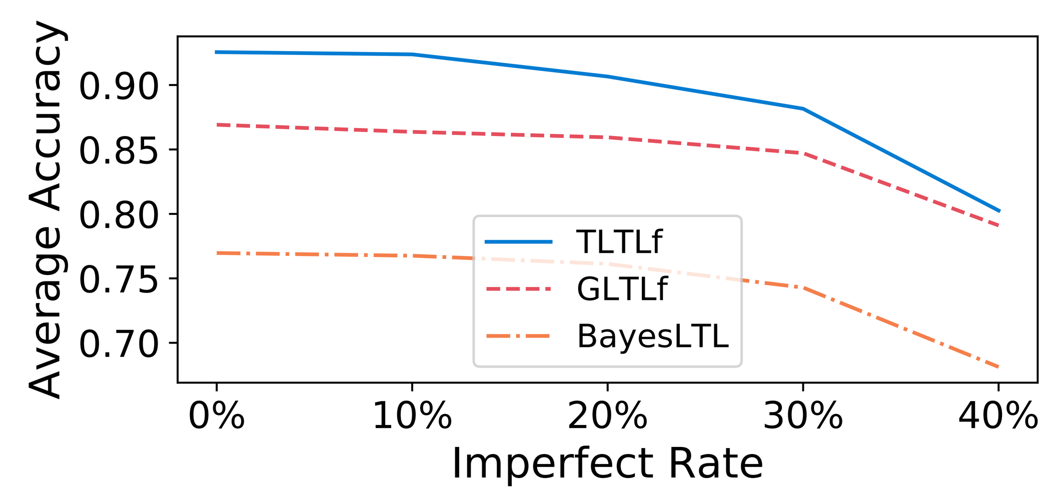
**Competitor.** Competitors. We compared TLTLf with four SOTA approaches, including C.&M.(Camacho and McIlraith 2019), BayesLTL (Kim et al. 2019), MaxSAT-DT (Gaglione et al. 2021) and GLTLf (Luo et al. 2022). C.&M. cannot learn from imperfect data but can learn arbitrary formulae. BayesLTL can learn from imperfect data but cannot learn arbitrary formulae. MaxSAT-DT, GLTLf and our proposed TLTLf can learn arbitrary formulae from imperfect data.

**Comparisons across datasets.**

| | $k_f = 3$ | | | $k_f = 6$ | | | $k_f = 9$ | | | $k_f = 12$ | | | $k_f = 15$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Acc(%) | F$_1$(%) | $N_s$ | Acc(%) | F$_1$(%) | $N_s$ | Acc(%) | F$_1$(%) | $N_s$ | Acc(%) | F$_1$(%) | $N_s$ | Acc(%) | F$_1$(%) | $N_s$ |
| `MaxSAT-DT` | 100 | 100 | 49 | 100 | 100 | 19 | 100 | 100 | 8 | 100 | 100 | 5 | 100 | 100 | 5 |
| `C.&M.` | 99.77 | 99.77 | 50 | 97.93 | 96.79 | 47 | 97.14 | 95.55 | 35 | 95.10 | 91.91 | 20 | 93.74 | 87.49 | 8 |
| `BayesLTL` | 85.19 | 85.96 | 50 | 77.94 | 76.78 | 50 | 74.08 | 75.73 | 50 | 72.77 | 73.47 | 50 | 74.85 | 77.32 | 50 |
| `GLTLf` | 94.34 | 94.27 | 50 | 90.09 | 90.37 | 50 | 84.02 | 83.29 | 50 | 83.08 | 83.21 | 50 | 83.07 | 83.50 | 50 |
| `TLTLf` | 98.00 | 97.92 | 50 | 95.38 | 95.47 | 50 | 91.90 | 91.33 | 50 | 89.59 | 88.98 | 50 | 90.40 | 90.22 | 50 |

- `TLTLf` significantly surpasses `BayesLTL` and `GLTLf`.
- Although `MaxSAT-DT` and `C.&M.` are in the lead, they cannot solve long formulae.

**Comparisons on Imperfect Data.**




- `TLTLf` performs better then other approaches on imperfect data.
- `TLTLf` can handle long formulae with higher performance compared to other approaches.

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