# Improving Local Search Algorithms via Probabilistic Configuration Checking

Weilin Luo [1], Rongzhen Ye [1], Hai Wan [1,*], Shaowei Cai [2,*], Biqing Fang [1] Delong Zhang [1]

[1] School of Computer Science and Engineering, Sun Yat-sen University, Guangzhou, China
[2] State Key Laboratory of Computer Science, Institute of Software, Chinese Academy of Sciences, China

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# Content

# Definition of Problem

Combinatorial optimization problems (COPs), such as:

- **Minimum weight dominating set problem (MWDS)**. Given a vertex weighted graph, MWDS consists in identifying a subset of vertices $D \subseteq V$ with the smallest total weight such that all vertices are either in $D$ or adjacent to at least one vertex in $D$.
- **Maximum weight clique problem (MWCP)**. Given a vertex weighted graph, MWCP is to find a clique with the maximum total weight.

- Local search is a popular method for solving combinatorial optimization problems (COPs).
- Configuration checking (CC)[3] is an effective mechanism to alleviate the cycling problem for COPs, particularly those graph theoretic ones.

# Configuration checking (CC)

The main idea of the CC strategy is that if the configuration of a vertex remains unchanged since its last removal from the candidate solution, then it is forbidden to be added back into the candidate solution.

- MWDS: 2-level neighborhoods
- MWCP: 1-level neighborhoods

Neighborhoods with more than two levels can have an impact on a vertex and should be considered in the configuration.
Our work revolves around the following two questions.

- Is there a general way to define the configuration of a vertex?
- How does the new configuration lead to a further improvement of the CC strategy?

Table 1: Motivation example on BHOSLIB benchmark.

| Instances | CC²FS | | | CC²FS-H | | |
|---|---|---|---|---|---|---|
| | min | avg | time | min | avg | time (s) |
| frb30-15-1 | 212 | 214.2 | 2.59 | 212 | **212.0** | **0.11** |
| frb30-15-2 | 242 | 242.0 | 8.81 | 242 | 242.0 | **0.18** |
| frb30-15-3 | 175 | 175.0 | **0.01** | 175 | 175.0 | 0.03 |
| frb30-15-4 | 166 | 168.6 | 0.19 | 166 | **166.0** | **0.08** |
| frb30-15-5 | 160 | 160.0 | **0.01** | 160 | 160.0 | 0.02 |
| frb35-17-1 | 274 | 274.9 | 0.57 | 274 | **274.0** | **0.11** |
| frb35-17-2 | 208 | 208.3 | 67.44 | 208 | **208.0** | **0.29** |
| frb35-17-3 | 201 | 201.0 | 0.38 | 201 | 201.0 | **0.10** |
| frb35-17-4 | 286 | 286.8 | 0.73 | 286 | **286.0** | **0.45** |
| frb35-17-5 | 295 | 295.7 | 5.60 | 295 | **295.0** | **0.11** |
| frb40-19-1 | 262 | 262.0 | 1.99 | 262 | 262.0 | **0.14** |
| frb40-19-2 | 243 | 243.9 | 0.66 | 243 | **243.0** | **0.18** |
| frb40-19-3 | 250 | 251.8 | 102.43 | 250 | **250.0** | **3.88** |
| frb40-19-4 | 250 | 250.0 | 0.07 | **249** | **249.0** | 95.82 |
| frb40-19-5 | 272 | 282.3 | 0.31 | 272 | **277.6** | 82.50 |
| frb45-21-1 | 328 | 328.3 | 7.82 | 328 | **328.0** | **0.64** |
| frb45-21-2 | 259 | 261.7 | 46.82 | 259 | **259.1** | **1.39** |
| frb45-21-3 | 233 | 233.9 | 9.19 | 233 | **233.0** | **0.89** |
| frb45-21-4 | 399 | 399.2 | 22.67 | 399 | **399.0** | **0.95** |
| frb45-21-5 | 312 | 318.8 | 128.19 | 312 | **312.0** | **1.82** |
| frb50-23-1 | 261 | 264.7 | 90.71 | 261 | **261.0** | **8.97** |
| frb50-23-2 | 277 | 277.0 | **0.01** | 277 | 277.0 | 0.17 |
| frb50-23-3 | 299 | 301.9 | 1.21 | **281** | **292.2** | 31.13 |
| frb50-23-4 | 265 | 265.0 | 3.04 | 265 | 265.0 | **0.47** |
| frb50-23-5 | 410 | 418.8 | 128.09 | 410 | **411.1** | 158.96 |
| frb53-24-1 | 229 | 230.0 | 95.41 | 229 | **229.0** | **2.35** |
| frb53-24-2 | 298 | 298.0 | 1.04 | 298 | 298.0 | **0.38** |
| frb53-24-3 | 182 | 182.1 | 0.01 | 182 | **182.0** | 0.26 |
| frb53-24-4 | 189 | 189.0 | **0.02** | 189 | 189.0 | 0.27 |
| frb53-24-5 | 204 | 204.0 | **0.01** | 204 | 204.0 | 0.29 |
| frb56-25-1 | 229 | 229.2 | 0.32 | 229 | **229.0** | 0.41 |
| frb56-25-2 | 319 | 319.5 | 25.27 | 319 | **319.0** | **3.46** |
| frb56-25-3 | 336 | 338.7 | 30.21 | 336 | **336.0** | 38.80 |
| frb56-25-4 | 268 | 268.0 | **0.21** | 268 | 268.0 | 0.41 |
| frb56-25-5 | 425 | 427.6 | 103.13 | 425 | **425.0** | **1.83** |
| frb59-26-1 | 262 | 264.0 | 1.62 | 262 | **262.0** | 25.31 |
| frb59-26-2 | 383 | 391.1 | 97.04 | 383 | **383.0** | **5.64** |
| frb59-26-3 | 248 | 248.0 | 1.13 | **246** | **246.0** | 14.24 |
| frb59-26-4 | 248 | 248.9 | 47.47 | 248 | **248.0** | **0.62** |
| frb59-26-5 | 288 | 288.8 | 195.62 | 288 | **288.0** | **3.26** |
| frb100-40 | 350 | 350.0 | **2.04** | 350 | 350.0 | 9.70 |
| #win | 0 | 0 | 7 | **3** | **29** | **25** |

# Related Work

The variants of CC:

- Redefinition of configuration (What we focus).
  - 1-level neighborhoods: [1,2,8].
  - 2-level neighborhoods: [7,9].
- Multi-value CC.
- Changes of CC rules.

Two CC-based algorithms for MWDS and MWCP:

- MWDS: CC$^2$FS [9]
- MWCP: LSCC [8]

# Content

# Preliminaries

- An undirected graph $G = (V, E)$ consists of a vertex set $V$ and an edge set $E \subseteq V \times V$.
- For an edge $e = \{u, v\}$, vertices $u$ and $v$ are the endpoints of edge $e$.
- Two vertices are *neighboring vertices* if and only if they belong to one edge.
- For a vertex $v$, the set of its neighboring vertices is $N(v) = \{u \in V \mid \{u, v\} \in E\}$, and its *degree* is $deg(v) = |N(v)|$.
- The distance between two vertices $u$ and $v$, denoted by $dist(u, v)$, is the number of edges in the shortest path between them.

**Definition 1 ($i$-th-level neighborhoods)**

For a vertex $v$, we define its $i$-th-level neighborhoods as $N_i(v) = \{u \in V \mid dist(u, v) = i\}$, $i$-level neighborhoods as $N_{\leq i}(v) = \bigcup_{j=1}^{i} N_j(v)$, and its $i^+$-level neighborhoods as $N_{>i}(v) = \bigcup_{j=i+1} N_j(v)$, particularly, $N_1(v) = N_{\leq 1}(v) = N(v)$.

# PC

## Definition 2 (Probabilistic configuration (PC))

Given an undirected graph $G = (V, E)$ and a candidate solution $S$, the probabilistic configuration (PC) of a vertex $v \in V$ is a vector consisting of the states of all vertices in $(\bigcup_{i=1}^{2}(N_i(v))^{p_i}) \cup (N_{>2}(v))^{p_3}$, where $p_i \in [0, 1]$, $i = 1, 2, 3$, and $\{*\}^{p_i}$ is randomly chosen vertices from the set $\{*\}$ with the size of $\lceil |\{*\}| \cdot p_i \rceil$.

Note that the existing configurations are special cases of the PC.

Table 2: Configurations in the form of PC.

| Configuration | Probabilistic configuration form |
|---|---|
| $N_1(v)$ | $(N_1(v))^{1.0} \cup (N_2(v))^{0.0} \cup (N_{>2}(v))^{0.0}$ |
| $N_1(v) \cup N_2(v)$ | $\bigcup_{i=1}^{2}(N_i(v))^{1.0} \cup (N_{>2}(v))^{0.0}$ |

# PCC

**Algorithm 1** SelectConf($v$, $N_1(v)$, $N_2(v)$, $N_{>2}(v)$, $P$)

**Input**: a vertex $v$ and $N_1(v)$, $N_2(v)$, $N_{>2}(v)$, $P = \{p_1, p_2, p_3\}$ a set of probabilities.
**Output**: the configuration $Conf$.

1: $Q_1 \leftarrow$ randomly choose $\lceil |N_1(v)| \cdot p_1 \rceil$ vertices from $N_1(v)$;
2: $Q_2 \leftarrow$ randomly choose $\lceil |N_2(v)| \cdot p_2 \rceil$ vertices from $N_2(v)$;
3: $Q_3 \leftarrow$ randomly choose $\lceil |N_{>2}(v)| \cdot p_3 \rceil$ vertices from $N_{>2}(v)$;
4: $Conf(v) \leftarrow Q_1 \cup Q_2 \cup Q_3$;
5: **return** $Conf(v)$;

- Firstly, carry out a preprocessing to compute the 1st-, 2nd-, and $2^+$- level neighborhoods for each vertex, which can be quickly implemented with a breadth-first search algorithm with the time complexity of $O(|V| \cdot |E|)$, which only needs to be run once.
- In each search step, for the operated vertex $v$, select vertices from $N_1(v)$, $N_2(v)$ and $N_{>2}(v)$ to form its configuration (Algorithm 1).

# A PCC Strategy for MWDS

When removing a vertex $u$ from the candidate solution $S$

- $confCh[u] := 0$;
- calculate $Conf[u]$ according to Algorithm 1;
- for each vertex $w \in Conf[u]$, $confCh[w]$ is set to 1.

When adding a vertex $v$ to the candidate solution $S$

- calculate $Conf[v]$ according to Algorithm 1;
- for each vertex $w \in Conf[v]$, $confCh[w]$ is set to 1.

# A PCC Strategy for MWCP

When removing a vertex $u$ from the candidate solution $S$

- $confCh[u] := 0$;

When adding a vertex $v$ to the candidate solution $S$

- calculate $Conf[v]$ according to Algorithm 1;
- for each vertex $w \in Conf[v]$, $confCh[w]$ is set to 1.

When swapping a vertex $u$ from the candidate solution $S$ with a vertex $v$

- $confCh[u] := 0$;
- calculate $Conf[v]$ according to Algorithm 1;
- for each vertex $w \in Conf[v]$, $confCh[w]$ is set to 1.

# Complexity Analysis

- Given an undirected graph $G = (V, E)$, we use $\Delta(G)$ to denote $max\{|N(v)| \mid v \in V\}$.
- For the updating rules of the $1$-level neighborhoods configuration, the worst time complexity is $O(\Delta(G))$.
- As for the rules of the $2$-level neighborhoods configuration, the worst time complexity is $O(\Delta(G)^2)$.
- For PC, the worst time complexity is $O(|V|)$ ($p_i = 1.0$ for all $1 \leq i \leq 3$), which is the amount of the vertices in the graph. This could result in a rather time-consuming CC strategy. Fortunately, $p_i$ is not $1.0$ after a training process in most cases. As a result, the complexity of PCC is roughly comparable with CC.

# Automatic Configuration Process

here are three parameters, $p_1, p_2$, and $p_3$, in the definition of PC. In order to obtain a good configuration for a given COP, we use a configurator similar to the state-of-the-art automatic algorithm configurator – SMAC[4] to configure the probabilistic configuration.

# Content

# Setting

Benchmarks:

- MWDS
  - T1 [6]
  - T2 [6]
  - UDG [6]
  - BHOSLIB [10]
  - DIMACS [5]
- MWCP
  - BHOSLIB [10]
  - DIMACS [5]

Competitors:

- MWDS: $CC^2FS$ [9], $CC^2FS$-P(ours, one PC for each COP), $CC^2FS$-SP(ours, one PC for each benchmark)
- MWCP: LSCC [8], LSCC-P(ours, one PC for each COP), LSCC-SP(ours, one PC for each benchmark)

# Comparison PCC with CC

Table 3: Comparison results of CC$^2$FS and CC$^2$FS-P on 7 benchmarks. We also report the sum of difference ('sum of diff') between CC$^2$FS-P and CC$^2$FS in 'min' and 'avg'. The smaller the difference, the better for CC$^2$FS-P.

| | Benchmarks | #inst. | CC$^2$FS #w.m. | CC$^2$FS #w.a. | CC$^2$FS-P #w.m. | CC$^2$FS-P #w.a. | sum of diff min | sum of diff avg |
|---|---|---|---|---|---|---|---|---|
| train | T1 | 5 | 0 | 0 | **5** | **5** | **-167** | **-167.0** |
| | T2 | 5 | 0 | 0 | **3** | **3** | **-49** | **-49.0** |
| | UDG | 4 | **4** | **4** | 0 | 0 | 30 | 30.0 |
| | BHOSLIB | 4 | 0 | 0 | **1** | **4** | **-1** | **-8.4** |
| | DIMACS | 8 | 0 | 0 | 0 | 0 | 0 | 0.0 |
| test | T1 | 525 | 10 | 10 | **152** | **152** | **-2129** | **-2129.0** |
| | T2 | 525 | 0 | 0 | **57** | **57** | **-697** | **-697.0** |
| | UDG | 116 | **81** | **81** | 16 | 16 | 541 | 541.0 |
| | BHOSLIB | 37 | 0 | 0 | **6** | **26** | **-52** | **-105.1** |
| | DIMACS | 29 | 0 | 0 | **2** | **9** | **-4** | **-27.2** |
| Total | | 1258 | 95 | 95 | **242** | **272** | **-2528** | **-2611.7** |

- CC$^2$FS-P is the better solver on these benchmarks except the UDG benchmark.
- Particularly, CC$^2$FS-P achieves much better minimum and average weights than those of CC$^2$FS on instances.
- CC$^2$FS-P performs worse even in the training set with respect to the UDG benchmark. Therefore, we argue that it results from the adverse bias generated from training data from other benchmarks.

# Comparison PCC with CC

Table 4: Experimental results of LSCC and LSCC-P on DIMACS and BHOSLIB benchmarks. We report the sum of difference between LSCC-P and LSCC in 'min' and 'avg'. The larger the difference, the better for LSCC-P.

| | Benchmarks | #inst. | LSCC #w.m. | LSCC #w.a. | LSCC-P #w.m. | LSCC-P #w.a. | sum of diff min | sum of diff avg |
|---|---|---|---|---|---|---|---|---|
| train | BHOSLIB | 4 | 1 | 0 | **2** | **4** | **13** | **67** |
| train | DIMACS | 8 | 0 | 1 | **3** | **3** | **124** | **166.8** |
| test | BHOSLIB | 37 | 2 | 3 | **14** | **26** | **315** | **462.7** |
| test | DIMACS | 29 | **2** | 2 | 0 | 2 | -19 | **52.9** |
| Total | | 78 | 5 | 6 | **19** | **35** | **433** | **749.4** |

- LSCC-P achieves better performance than LSCC in BHOSLIB benchmark.
- LSCC-P achieves comparable performance with LSCC in DIMACS benchmark.

# Discussion About the Bias of Training

Table 5: Comparison results of $CC^2FS$ and $CC^2FS$-SP.

| | Benchmarks | #inst. | $CC^2FS$ | | $CC^2FS$-SP | | sum of diff | |
|---|---|---|---|---|---|---|---|---|
| | | | #w.m. | #w.a. | #w.m. | #w.a. | min | avg |
| train | T1 | 5 | 0 | 0 | **5** | **5** | **-184** | **-184.0** |
| | T2 | 5 | 0 | 0 | **3** | **3** | **-49** | **-49.0** |
| | UDG | 4 | 1 | 1 | **2** | **2** | **-8** | **-8.0** |
| | BHOSLIB | 4 | 0 | 0 | **1** | **4** | **-1** | **-8.4** |
| | DIMACS | 8 | 0 | 0 | 0 | 0 | 0 | 0.0 |
| test | T1 | 525 | 10 | 10 | **154** | **154** | **-2205** | **-2205.0** |
| | T2 | 525 | 0 | 0 | **58** | **58** | **-697** | **-697.0** |
| | UDG | 116 | **47** | **47** | 41 | 41 | **-14** | **-14.0** |
| | BHOSLIB | 37 | 0 | 0 | **6** | **26** | **-52** | **-109.1** |
| | DIMACS | 29 | 0 | 0 | **2** | **9** | **-4** | **-27.1** |
| Total | | 1258 | 58 | 58 | **272** | **302** | **-3214** | **-3301.6** |

Table 6: Comparison results of $CC^2FS$-P and $CC^2FS$-SP.

| | Benchmarks | #inst. | $CC^2FS$-P | | $CC^2FS$-SP | | sum of diff | |
|---|---|---|---|---|---|---|---|---|
| | | | #w.m. | #w.a. | #w.m. | #w.a. | min | avg |
| train | T1 | 5 | 0 | 0 | **2** | **2** | **-17** | **-17.0** |
| | T2 | 5 | 0 | 0 | 0 | 0 | 0 | 0.0 |
| | UDG | 4 | 0 | 0 | **4** | **4** | **-38** | **-38.0** |
| | BHOSLIB | 4 | 0 | 0 | 0 | 0 | 0 | 0.0 |
| | DIMACS | 8 | 0 | 0 | 0 | 0 | 0 | 0.0 |
| test | T1 | 525 | 17 | 17 | **36** | **36** | **-76** | **-76.0** |
| | T2 | 525 | 1 | 1 | 1 | 1 | 0 | 0.0 |
| | UDG | 116 | 22 | 22 | **82** | **82** | **-555** | **-555.0** |
| | BHOSLIB | 37 | 0 | 0 | 0 | **1** | 0 | **-4.0** |
| | DIMACS | 29 | 0 | **1** | 0 | 0 | 0 | 0.1 |
| Total | | 1258 | 40 | 41 | **125** | **126** | **-686** | **-689.9** |

- $CC^2FS$-SP achieves a better performance than $CC^2FS$ on three benchmarks.
- In UDG benchmark, $CC^2FS$-SP and $CC^2FS$ are comparable.

# Discussion About the Bias of Training

Table 7: Comparison results of LSCC and LSCC-SP.

| | Benchmarks | #inst. | LSCC #w.m. | LSCC #w.a. | LSCC-SP #w.m. | LSCC-SP #w.a. | sum of diff max | sum of diff avg |
|---|---|---|---|---|---|---|---|---|
| train | BHOSLIB | 4 | 1 | 0 | **2** | **4** | **8** | **69.4** |
| train | DIMACS | 8 | 0 | 1 | **3** | **3** | **124** | **166.8** |
| test | BHOSLIB | 37 | 2 | 2 | **14** | **27** | **371** | **502.3** |
| test | DIMACS | 29 | **2** | 2 | 0 | 2 | -19 | **52.9** |
| Total | | 78 | 5 | 5 | **19** | **36** | **484** | **791.4** |

Table 8: Comparison results of LSCC-P and LSCC-SP.

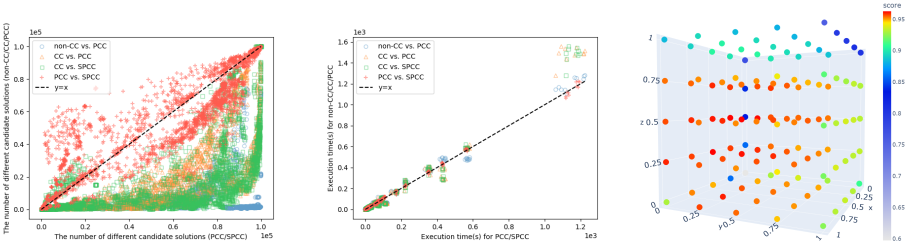
| | Benchmarks | #inst. | LSCC-P #w.m. | LSCC-P #w.a. | LSCC-SP #w.m. | LSCC-SP #w.a. | sum of diff max | sum of diff avg |
|---|---|---|---|---|---|---|---|---|
| train | BHOSLIB | 4 | 1 | 2 | **2** | 2 | -5 | **2.4** |
| train | DIMACS | 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| test | BHOSLIB | 37 | **9** | 8 | 6 | **15** | **56** | **39.6** |
| test | DIMACS | 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| Total | | 78 | **10** | 10 | 8 | **17** | **51** | **42** |

- the PC specially trained for each benchmark can improve the performance.
- Surprisingly, in DIMACS benchmark, the performance of LSCC-SP and LSCC-P are the same because they share the same PC.

# Performance on Reducing the Cycling Problem

first $10^5$ candidate solutions.

Figure 1: (a) Comparison of the number of different candidate solutions of non-CC, CC, PCC, SPCC. (b) Comparison of CPU time. (c) The scores of CC$^2$FS-P with different configurations on MWDS benchmark. The X-axis, Y-axis, and Z-axis represent $p_1, p_2$, and $p_3$ respectively.




(a) (b) (c)

- The result shows that PCC does not have much more time consumption to improve the exploration in local search and potentially alleviates the cycling problem.
- It is necessary to tune for an optimal PC for a given COP.

# Content

# Conclusion and Future Work

Conclusion:

1. We observe that neighborhoods with different levels should have different contributions to alleviate the cycling problem.
2. We expand the configuration with probability, i.e., probabilistic configuration (PC), to capture the contributions of vertices at different levels, resulting in the probabilistic configuration checking (PCC) strategy.
3. Our experimental results confirm that the PC can improve existing local search algorithms in two classic COPs, i.e., MWDS and MWCP, due to alleviating the cycling problem.

Future work:

1. making the method suitable for massive graphs.
2. extending our approach to other COPs.

# References I

[1] Cai, S.; and Su, K. 2013. Local search for Boolean Satisfiability with configuration checking and subscore. *Artificial Intelligence*, 204: 75–98.

[2] Cai, S.; Su, K.; Luo, C.; and Sattar, A. 2013. NuMVC: An efficient local search algorithm for minimum vertex cover. *Journal of Artificial Intelligence Research*, 46: 687–716.

[3] Cai, S.; Su, K.; and Sattar, A. 2011. Local search with edge weighting and configuration checking heuristics for minimum vertex cover. *Artificial Intelligence*, 175(9-10): 1672–1696.

[4] Hutter, F.; Hoos, H. H.; and Leyton-Brown, K. 2011. Sequential model-based optimization for general algorithm configuration. In *LION*, 507–523.

[5] Johnson, D. S.; and Trick, M. A. 1996. *Cliques, coloring, and satisfiability: second DIMACS implementation challenge, October 11-13, 1993*, volume 26. American Mathematical Soc.

[6] Jovanovic, R.; Tuba, M.; and Simian, D. 2010. Ant colony optimization applied to minimum weight dominating set problem. In *Proceedings of the 12th WSEAS international conference on Automatic control, modelling & simulation, (ACMOS-2010)*, 322–326.

[7] Wang, Y.; Cai, S.; Chen, J.; and Yin, M. 2018. A Fast Local Search Algorithm for Minimum Weight Dominating Set Problem on Massive Graphs. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, (IJCAI-2018)*, 1514–1522.

[8] Wang, Y.; Cai, S.; and Yin, M. 2016. Two efficient local search algorithms for maximum weight clique problem. In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, (AAAI-2016)*, 805–811.

[9] Wang, Y.; Cai, S.; and Yin, M. 2017. Local Search for Minimum Weight Dominating Set with Two-Level Configuration Checking and Frequency Based Scoring Function. *Journal of Artificial Intelligence Research*, 58: 267–295. ArXiv: 1702.04594.

[10] Wu, Q.; Hao, J.-K.; and Glover, F. 2012. Multi-neighborhood tabu search for the maximum weight clique problem. *Annals of Operations Research*, 196(1): 611–634.

# Thank you for your listening!