Learning to Check LTL Satisfiability and to Generate Traces via Differentiable Trace Checking

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Content

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Motivation

Linear temporal logic (LTL) satisfiability checking

- *e.g.*, input: $\square(r \rightarrow \diamond g)$, output: SAT
- important applications, *e.g.*, model checking[6], goal-conflict analysis[5,18], and business process[21]
- PSPACE-complete[28]

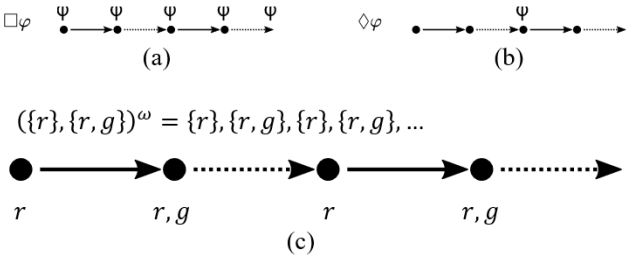

Figure 1: An example of LTL.

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- *logical approaches*: *e.g.*, based on logical reasoning mechanisms, such as model checking[23,24], tableau[1,11,27,29], temporal resolution[8,26], anti-chain[30], and Boolean satisfiability (SAT) problem[12–17]
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Learning-based approaches

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- Is it enough to answer the SAT or UNSAT enough?

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LTL satisfiability checking and trace generation (*SAT-and-GET*)

- *e.g.*, input: $p_1 \mathcal{U} \bigcirc p_2$, output: SAT because $(\{p_1, p_2\}, \{p_1\})^\omega \models p_1 \mathcal{U} \bigcirc p_2$

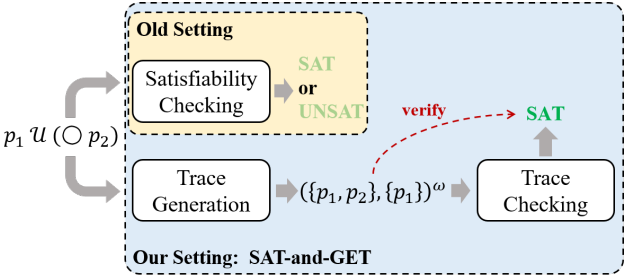

Figure 1: Overview of SAT-and-GET

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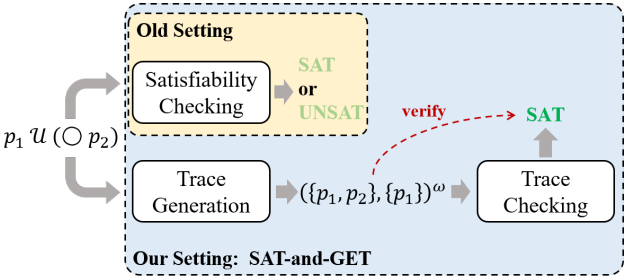

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- bridging the gap between the *continuous domain* and the *discrete domain*

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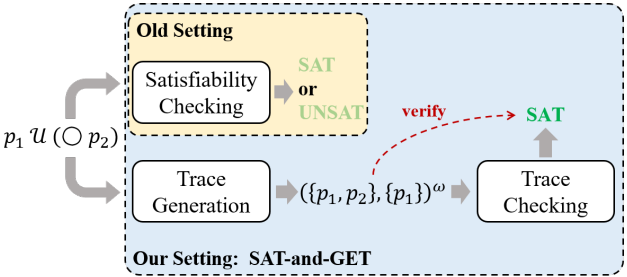

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- bridging the gap between the *continuous domain* and the *discrete domain*
- *supervision confusion* from multiple satisfiable traces

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- an *LTL encoding method* to parameterize a neural network and theoretically prove that its inference process (*neural trace checking*) is able to simulate LTL trace checking
- a novel approach *jointly* trains a neural network for LTL satisfiability checking and a neural network for trace generation guided by neural trace checking

Content

LTL Encoding

Definition 1 (LTL Encoding)

Let $\mathbb{P}$ be a set of atom propositions, ϕ an LTL formula over $\mathbb{P}$, $(V, E, v_1, \mathtt{lab}_V, \mathtt{lab}_E) = \mathtt{tree}(\phi)$, and $\langle v_1, \ldots, v_{|\phi|} \rangle = \mathtt{pretravel}(\mathtt{tree}(\phi))$. The *LTL encoding* η of ϕ is a 6-tuple

$$(\boldsymbol{\eta}_{right}, \boldsymbol{\eta}_{atom}, \boldsymbol{\eta}_{\neg}, \boldsymbol{\eta}_{\wedge}, \boldsymbol{\eta}_{\bigcirc}, \boldsymbol{\eta}_{\mathcal{U}}),$$

where $\boldsymbol{\eta}_{right} \in \{0,1\}^{|\phi|\times|\phi|}$, $\boldsymbol{\eta}_{atom} \in \{0,1\}^{|\phi|\times|\mathbb{P}|}$, and $\boldsymbol{\eta}_{\neg}, \boldsymbol{\eta}_{\wedge}, \boldsymbol{\eta}_{\bigcirc}, \boldsymbol{\eta}_{\mathcal{U}} \in \{0,1\}^{|\phi|}$; moreover, for all subscripts i and j, $(\boldsymbol{\eta}_{right})_{i,j} = 1$ if $\mathtt{lab}_E((v_i, v_j)) = R$ and $(\boldsymbol{\eta}_{right})_{i,j} = 0$ otherwise, $(\boldsymbol{\eta}_{atom})_{i,j} = 1$ if $\mathtt{lab}_V(v_i) = p_j$ and $(\boldsymbol{\eta}_{atom})_{i,j} = 0$ otherwise, $(\boldsymbol{\eta}_{\mathsf{o}})_i = 1$ if $\mathtt{lab}_V(v_i) = \mathsf{o}$ and $(\boldsymbol{\eta}_{\mathsf{o}})_i = 0$ otherwise, where $\mathsf{o} \in \{\neg, \wedge, \bigcirc, \mathcal{U}\}$.

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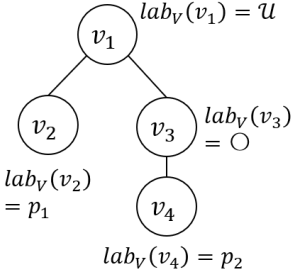
Let $\phi = p_1 \ \mathcal{U} \ \bigcirc p_2$ be an LTL formula. $\texttt{pretravel}(\texttt{tree}(\phi)) = \langle v_1, v_2, v_3, v_4 \rangle$. The LTL encoding η of ϕ is



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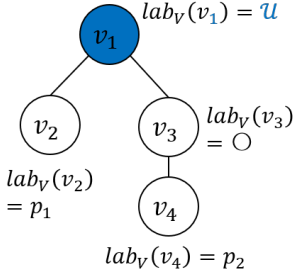

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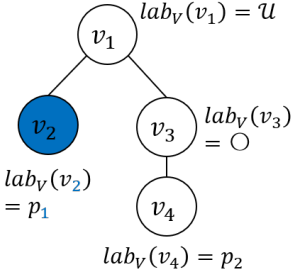

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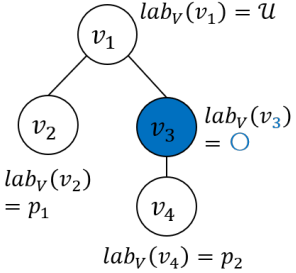

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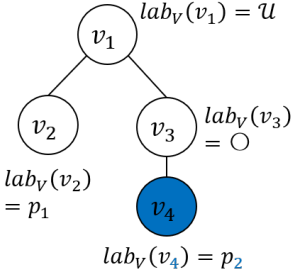

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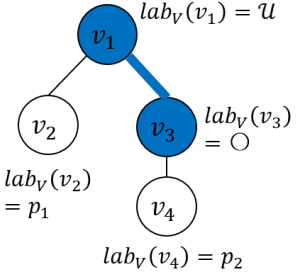

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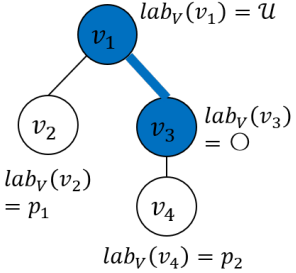

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Neural Modeling of Trace Checking

Definition 2 (Tensorized Trace)

Let $\mathbb{P}$ be a set of atom propositions and π a trace over $\mathbb{P}$. The *tensorized trace* of π is a 2-tuple $(\boldsymbol{s}, \boldsymbol{l})$ computed by Algorithm 2, where $\boldsymbol{s} \in \mathbb{R}_{[0,1]}^{|\pi| \times |\mathbb{P}|}$ and $\boldsymbol{l} \in \mathbb{R}_{[0,1]}^{|\pi|}$, and where $\mathbb{R}_{[0,1]}$ denotes the real value range from 0 to 1.

Algorithm 2: TENSORIZE

Input: A trace π over a set of atomic propositions $\mathbb{P}$.
Output: The tensorized trace $(\boldsymbol{s}, \boldsymbol{l})$ of π.

```
s ← 0 where s ∈ {0,1}^{|π|×|ℙ|}, l ← 0 where l ∈ {0,1}^{|π|}
for each state π[i] of π do
    for each atomic proposition p_j ∈ ℙ do
        if p_j ∈ π[i] then
            (s)_{i,j} ← 1
    if i is loop-start time then
        (l)_i ← 1
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for *each state* $\pi[i]$ *of* π **do**
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$$\boldsymbol{s} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{l} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

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- NTCNet is parameterized by $\theta = \eta$.
- The satisfaction vectors $\boldsymbol{x}_i \in \mathbb{R}^{|\phi|}$ where $i \in [1, |\pi|]$ are initialized by $\mathbf{0}$.
- $(\boldsymbol{x}_i)_j$ is computed as follows:

$$
\begin{aligned}
(\boldsymbol{x}_i)_j = \sigma(\quad & (\boldsymbol{p}_i)_j + \\
& (\boldsymbol{\theta}_\neg)_j (1 - (\boldsymbol{x}_i)_{j+1}) + \\
& (\boldsymbol{\theta}_\wedge)_j \sigma((\boldsymbol{x}_i)_{j+1} + (\boldsymbol{r}_i)_j - 1) + \\
& (\boldsymbol{\theta}_\bigcirc)_j (\boldsymbol{x}_{i+1})_{j+1} + \\
& (\boldsymbol{\theta}_\mathcal{U})_j (\boldsymbol{u}_i)_j^{(|\pi|)} \quad), \\
(\boldsymbol{x}_{|\pi|+1})_{j+1} = \quad & \textstyle\sum_{k=1}^{|\pi|} (\boldsymbol{l})_k (\boldsymbol{x}_k)_{j+1},
\end{aligned}
\tag{1}
$$

Table 1: Converting logical operations to soften forms.

formal logic	$a \wedge b$	$a \vee b$	$\neg a$
soften logic	$\sigma(a + b - 1)$	$\sigma(a + b)$	$1 - a$

$$
(\boldsymbol{p}_i)_j = \sum_{k=1}^{|\mathbb{P}|} (\boldsymbol{\theta}_{atom})_{j,k} (\boldsymbol{s})_{i,k}. \tag{2}
$$

$$
(\boldsymbol{r}_i)_j = \sum_{k=j+2}^{|\pi|} (\boldsymbol{\theta}_{right})_{j,k} (\boldsymbol{x}_i)_k. \tag{3}
$$

$$
\begin{aligned}
(\boldsymbol{u}_i)_j^{(t)} = & \begin{cases} (\boldsymbol{r}_i)_j, \\ \sigma((\boldsymbol{x}_i)_{j+1} + (\boldsymbol{u}_{i+1})_j^{(t-1)} - 1) + \\ (\boldsymbol{u}_i)_j^{(1)}), \end{cases} \\
(\boldsymbol{u}_{|\pi|+1})_j^{(t)} = & \textstyle\sum_{k=1}^{|\pi|} (\boldsymbol{l})_k (\boldsymbol{u}_k)_j^{(t)}.
\end{aligned}
\tag{4}
$$

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- $\texttt{NTCNet}((\boldsymbol{s}, \boldsymbol{l}) | \eta) = (\boldsymbol{x}_1)_1$ denotes the result of *neural trace checking* of the satisfaction relation between π and ϕ.

Neural Modeling of Trace Checking

Theorem 1 (Correctness of Neural Trace Checking)

Let ϕ be an LTL formula and η its LTL encoding. For any trace π, $\texttt{NTCNet}(\textsc{tensorize}(\pi)|\eta) = 1$ if and only if $\pi \models \phi$.

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- Lemma 2 shows that $(\boldsymbol{u}_i)_j^{(t)} = 1$, if there exists a time i' where $0 \leq i' - i < t$ such that $\pi_{i'} \models \phi_k$ and $\pi_{i''} \models \phi_{j+1}$ for all $i'' \in [i, i')$.

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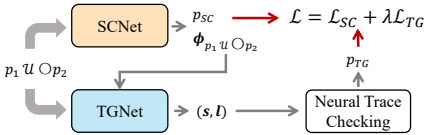
- Lemma 3 shows that $\boldsymbol{u}$ is monotonically increasing with time points.
- NTCNet is *differentiable*.

Overview of VSCNet

Train VSCNet

- SCNet extracts the formula representation ϕ and predicts the satisfiable probability p_{SC}.
- TGNet exploits ϕ to generate a tensorized trace $(\boldsymbol{s}, \boldsymbol{l})$.
- jointly training of SCNet and TGNet with NTCNet:

$$\mathcal{L} = \mathcal{L}_{SC} + \lambda \mathcal{L}_{TG} \qquad (5)$$



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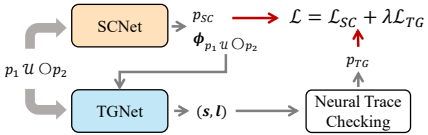

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Apply VSCNet to solve SAT-and-GET

Algorithm 3: Solving the SAT-and-GET problem

Input: An LTL formula ϕ, and two threshold hyperparameters $\beta_{\top}$ for finding tensorized traces and β_{SAT} for determining the satisfiability of ϕ.

Output: Whether ϕ is satisfiable or not and a satisfiable trace π of ϕ if it is satisfiable.

$(p_{SC}, \boldsymbol{\phi}) \leftarrow \texttt{SCNet}(\phi)$
$(\boldsymbol{s}, \boldsymbol{l}) \leftarrow \texttt{TGNet}(\boldsymbol{\phi})$
$\pi \leftarrow \textsc{round}((\boldsymbol{s}, \boldsymbol{l}), \beta_{\top})$
if $\pi \models \phi$ *or* $p_{SC} > \beta_{SAT}$ **then**
  return *SAT*, π
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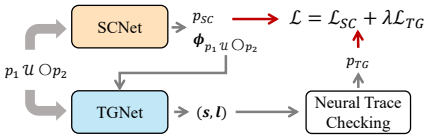

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1. obtain the p_{SC} and a *soften* tensorized trace $(\boldsymbol{s}, \boldsymbol{l})$

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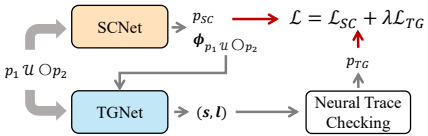

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Apply VSCNet to solve SAT-and-GET

1. obtain the p_{SC} and a *soften* tensorized trace $(\boldsymbol{s}, \boldsymbol{l})$
2. round probabilistic $(\boldsymbol{s}, \boldsymbol{l})$ to π

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$\pi[i] \leftarrow \emptyset$ for each $(\boldsymbol{s})_i$
for *each* $(\boldsymbol{s})_{i,j} \in \boldsymbol{s}$ **do**
  if $(\boldsymbol{s})_{i,j} > \beta_{\mathsf{T}}$ **then**
    $\pi[i] \leftarrow \pi[i] \cup \{p_j\}$
$k \leftarrow \arg\max_i((\boldsymbol{l})_i)$
$\pi \leftarrow \pi[1], \ldots, (\pi[k], \ldots, \pi[m])^{\omega}$

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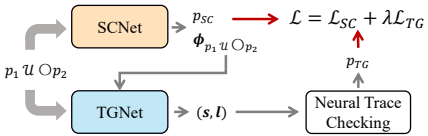

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Apply VSCNet to solve SAT-and-GET

1. obtain the p_{SC} and a *soften* tensorized trace $(\boldsymbol{s}, \boldsymbol{l})$
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3. check whether $\pi \models \phi$ by a classical trace checking algorithm [22]

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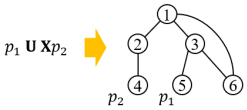
SCNet and TGNet



- construct the one-step unfolded graph (OSUG) [20]

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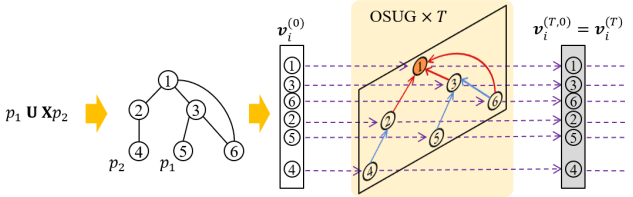
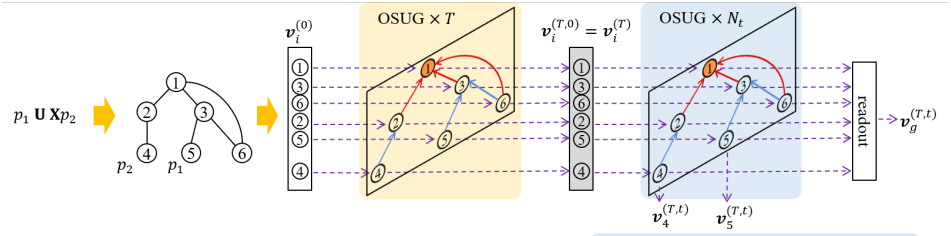


$$\boldsymbol{v}_g^{(T)} = \text{MEAN}(\boldsymbol{v}_1^{(T)}, \ldots, \boldsymbol{v}_6^{(T)})$$
$$p_{SC} = \text{MLP}(\boldsymbol{v}_g^{(T)})$$

- construct the one-step unfolded graph (OSUG) [20]
- compute the graph embedding $\boldsymbol{v}_g^{(T)} \in \mathbb{R}^{d_h}$ and vertex embeddings $\boldsymbol{v}_i^{(T)} \in \mathbb{R}^{d_h}$ using a 10-layer OSUG-based extractor

SCNet and TGNet



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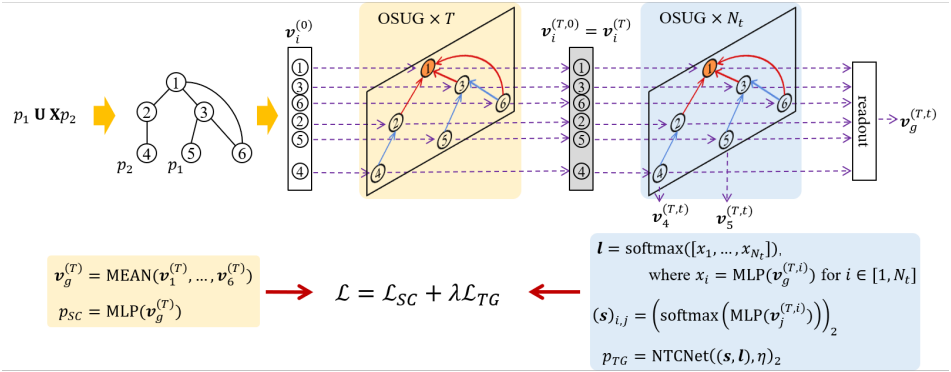

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Highlights of VSCNet

VSCNet ensures the soundness in $1/2$ of cases.

- Given an LTL formula ϕ, SCNet returns a satisfiability conclusion and TGNet returns a trace π.
- $\pi \models \phi$ and SCNet answers that ϕ is *satisfiable*: π confirms the answer of SCNet.
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VSCNet are able to check the satisfiability of an LTL formula in polynomial time.

- The inference time of both SCNet and TGNet is polynomial in terms of the size of the input formula.
- Although the inference time does not include the training time, model training can be done offline and once.

Content

Competitors and Datasets

Competitors: a random approach, two logic-based approaches, and seven neural network-based approaches

Table 2: Overview of Competitors

Approach	SC	TG
random	✓	✓
Aalta [14,17]	✓	✓
nuXmv [3]	✓	✓
TreeNN-inv [19]	✓	×
TreeNN-MP [19]	✓	×
TreeNN-con [19]	✓	×
Transformer-SC [19]	✓	×
Transformer-TG [10]	×	✓
Transformer	✓	✓
OSUG [20]	✓	✓
VSCNet-T (our)	✓	✓
VSCNet-G (our)	✓	✓

Competitors and Datasets

Competitors: a random approach, two logic-based approaches, and seven neural network-based approaches
Datasets: *SPOT* and large-scale datasets[17,19]

- *LTL-as-LTL$_f$*[25]: some coming from industrial scenes
- *LTL$_f$-Specific*[4,12]: generated by common patterns
- *NASA-Boeing*[2,9]: real-world specifications
- *DECLARE*[7]: widely used in the business process management

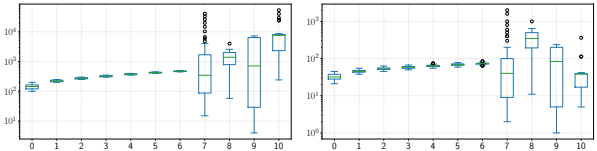

Figure 4: The boxplots of the formula size and the number of atomic propositions

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Setups

Training and testing

- train all neural networks on the training set of *SPOT*-$[100, 200)$ with hyperparameters optimized on the validation set of *SPOT*-$[100, 200)$
- *directly* evaluate all neural networks on the test set of *SPOT*-$[100, 200)$ to demonstrate their performance on in-distribution datasets
- *directly* evaluate all neural networks on the *SPOT* test sets with larger formulae and on the large-scale datasets to test the generalizability of neural networks on out-of-distribution datasets

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Metrics

- LTL satisfiability checking: accuracy (acc.), precision (pre.), recall (rec.), and F1 score
- semantic accuracy (sacc.)[10], where D is a set of satisfiable formulae and π is the trace generated by approaches:

$$\text{sacc.} = \frac{|\{\phi \in D | \pi \models \phi\}|}{|D|}, \tag{6}$$

Comparison on Synthetic Datasets

Table 3: Evaluation results on the test set of *SPOT*-$[100, 200)$

approach	acc.	pre.	rec.	F1	sacc.	time
random	50.01	50.01	49.28	49.64	51.88	0
Aalta	100.00	100.00	100.00	100.00	100.00	12,784,302
nuXmv	100.00	100.00	100.00	100.00	100.00	6,848
TreeNN-inv	93.27	97.75	88.58	92.94	-	434
TreeNN-MP	86.54	90.74	81.40	85.82	-	455
TreeNN-con	89.38	95.96	82.22	88.56	-	459
Transformer-SC	72.56	74.24	69.10	71.58	-	**104**
Transformer-TG	-	-	-	-	47.67	5,484
Transformer	59.30	62.79	45.64	52.86	45.09	10,601
OSUG	98.47	99.15	97.79	98.46	55.22	2,816
VSCNet-T (our)	93.45	97.47	89.22	93.16	71.86	364
VSCNet-G (our)	**99.15**	**99.47**	**98.83**	**99.15**	**91.01**	**156**

- On in-distribution datasets, compared with neural network-based approaches, VSCNet achieves SOTA performance on SAT-and-GET.

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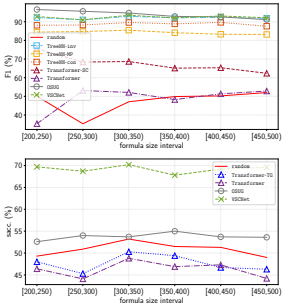

Figure 5: Evaluation results on other *SPOT* test sets.

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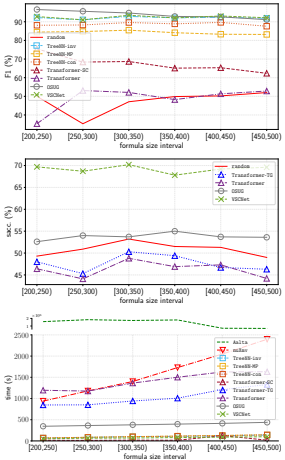
- VSCNet is significantly more efficient than logic-based SOTA approaches while achieving comparable performance.



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Comparison on Large-Scale Datasets

Table 4: Evaluation results on the large-scale datasets.

approach	*LTL-as-LTL$_f$*				*LTL$_f$-Specific*				*NASA-Boeing*				*DECLARE*			
	acc.	F1	sacc.	time	acc.	F1	sacc.	time	acc.	F1	sacc.	time	acc.	F1	sacc.	time
random	50.75	65.46	54.96	0	51.59	47.88	92.96	0	62.90	77.23	24.19	0	53.21	69.46	0.00	0
Aalta	100.00	100.00	100.00	122,682	100.00	100.00	100.00	36,027	100.00	100.00	100.00	3,601	100.00	100.00	100.00	61,220
nuXmv	100.00	100.00	100.00	6,804	100.00	100.00	100.00	766	100.00	100.00	100.00	41	100.00	100.00	100.00	3,743
TreeNN-inv	88.05	93.53	-	453	**98.53**	**98.37**	-	305	83.87	91.23	-	33	78.38	87.88	-	176
TreeNN-MP	82.18	90.03	-	480	96.29	95.98	-	234	70.97	83.02	-	25	95.50	97.70	-	129
TreeNN-con	84.61	91.10	-	475	98.35	98.11	-	230	51.61	68.09	-	25	0.00	0.00	-	129
Transformer-SC	72.99	82.64	-	**161**	83.12	75.45	-	**55**	71.05	83.08	-	**3**	0.00	0.00	-	**3**
OSUG	8.62	15.87	55.26	11,444	55.71	71.55	93.09	5,173	3.23	6.25	**46.77**	341	10.09	18.33	0.00	15,730
VSCNet-T (our)	88.35	**93.71**	**81.46**	368	90.76	90.56	83.80	177	**90.32**	**94.92**	**46.77**	21	95.41	97.65	0.00	94
VSCNet-G (our)	**88.85**	94.03	51.68	**91**	45.94	61.66	**93.36**	**25**	83.87	91.23	43.55	**3**	**100.00**	**100.00**	0.00	**6**

- All neural network-based approaches exhibit varying degrees of performance fluctuations on the large-scale datasets.

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- VSCNet is comparable with the logic-based approaches in LTL satisfiability checking.
- VSCNet is significantly more efficient than the logic-based approaches.

Ablation Study

Table 5: Ablation results of VSCNet-T and its variants on all *SPOT* test sets.

variants	$[100,200)$			$[200,250)$			$[250,300)$			$[300,350)$			$[350,400)$			$[400,450)$			$[450,500)$		
	acc.	F1	sacc.	acc.	F1	sacc.	acc.	F1	sacc.	acc.	F1	sacc.	acc.	F1	sacc.	acc.	F1	sacc.	acc.	F1	sacc.
VSCNet-T	93.45	93.16	**71.86**	93.10	92.78	**69.70**	91.60	91.05	**68.70**	93.80	93.47	**70.20**	92.55	92.08	**67.80**	**93.45**	**93.09**	**69.20**	**92.60**	**92.16**	**69.60**
- w/o SS	77.75	71.39	55.51	75.95	68.33	51.90	77.00	70.13	54.00	76.50	69.28	53.00	77.20	70.47	54.40	75.55	67.64	51.10	75.80	68.07	51.60
- w STS	93.66	**93.37**	54.53	**93.15**	**92.81**	51.30	**92.00**	**91.50**	52.10	**94.35**	**94.07**	53.60	**93.30**	**92.90**	52.30	93.35	92.97	51.10	92.45	91.97	48.60
- w/o NTC & w STS	**93.74**	93.44	55.10	92.85	92.44	52.30	91.85	91.28	53.60	93.60	93.22	54.20	**93.30**	92.87	54.60	92.85	92.41	53.70	92.55	92.04	53.90

- VSCNet-T vs. VSCNet-T w STS:
  - The supervision of satisfiable traces is harmful to generating satisfiable traces.
- VSCNet-T vs. VSCNet-T w/o NTC & w STS:
  - NTCNet guides trace generation better than directly using a satisfiable trace for supervision.

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VSCNet-T	93.45	93.16	**71.86**	93.10	92.78	**69.70**	91.60	91.05	**68.70**	93.80	93.47	**70.20**	92.55	92.08	**67.80**	**93.45**	**93.09**	**69.20**	**92.60**	**92.16**	**69.60**
- w/o SS	77.75	71.39	55.51	75.95	68.33	51.90	77.00	70.13	54.00	76.50	69.28	53.00	77.20	70.47	54.40	75.55	67.64	51.10	75.80	68.07	51.60
- w STS	93.66	**93.37**	54.53	**93.15**	**92.81**	51.30	**92.00**	**91.50**	52.10	**94.35**	**94.07**	53.60	**93.30**	**92.90**	52.30	93.35	92.97	51.10	92.45	91.97	48.60
- w/o NTC & w STS	**93.74**	93.44	55.10	92.85	92.44	52.30	91.85	91.28	53.60	93.60	93.22	54.20	**93.30**	92.87	54.60	92.85	92.41	53.70	92.55	92.04	53.90

- VSCNet-T vs. VSCNet-T w STS:
  - The supervision of satisfiable traces is harmful to generating satisfiable traces.
- VSCNet-T vs. VSCNet-T w/o NTC & w STS:
  - NTCNet guides trace generation better than directly using a satisfiable trace for supervision.
- VSCNet-T vs. VSCNet-T w/o SS:
  - The joint learning of the module for trace generation and the module for satisfiability checking is mutually reinforcing.
- The ablation study on VSCNet-G exhibits similar results.

Content

Conclusion and Discussion

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1. Bridge between LTL trace checking and neural trace checking.
2. Design a joint approach of LTL satisfiability checking and trace generation, which can be realized by a neural network trainable with classical gradient descent.
3. Theoretically prove the correctness of the bridge and empirically confirmed the effectiveness of the joint approach.

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Discussion

1. Out-of-distribution datasets: plan to explore parameter-efficient fine-tuning to improve the generalizability in future work.
2. Fixed trace size: in practice, it may be sufficient to set N_t to a small constant for most cases and our approach also works for every LTL formula that has satisfiable traces whose sizes are smaller than N_t.
3. Verifiable unsatisfiable results: future work will also study how to explore neural network technology to generate unsatisfiable cores.

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