Application Note #39

KINEMETRICS

Transfer Function of Kinemetrics Instruments

S-Plane Representations

by
Dr. A. Trnkoczy & I.M. Standley

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Kinemetrics, Inc., 222 Vista Avenue, Pasadena, CA 91107 USA

Phone: [1] 626-795-2220 ■ Fax: [1] 626-795-0868

E-mail: services@kmi.com

Website: www.kinemetrics.com

ftp:\\ftp.kinemetrics.com

Transfer Functions of Kinemetrics Instruments

1. Introduction

The following report describes the s-plane transfer functions of Kinemetrics' instrumentation. Its purpose is to allow you to correct the instrument response where appropriate and contains the nominal transfer functions for most Kinemetrics sensors and gives zeros and pole for Kinemetrics recorders.

The nominal sensors' transfer functions are based on theoretical models of the electro-mechanical system or are determined empirically based on measurements For some instruments equations are also provided for using the instrument calibration parameters to determine a model with a better "fit" for the unit you are using.

For digitizers with analog anti-alias filters we have provided the nominal zeros and poles for the designs based on the nominal component values at room temperature. There will be small variations in actual field conditions due to both component tolerances and the effect of temperature changes on the components. However, for most applications these errors are insignificant.

Altus Digitizers use Digital Filtering for anti-alias protection that is implemented as Finite Impulse Response (FIR) Filters. These filters are identical in each unit, as they are not subject to component tolerances. However, be sure that you have the coefficients for the software version installed in the units. These coefficients are available for download from the Kinemetrics web site. In order to provide the very steep "brickwall" response of the Altus these filters have several hundred taps. Thus, this information is provided for those who wish to do complex post processing. However, we do not recommend trying to increase the frequency range of the instrument by "boosting" the amplitude at the FIR filter's corner.

Kinemetrics World Wide Web site is a good source for updated information on the Transfer functions of new instruments or on the current coefficients used in Altus instruments' Digital Filters.

2. FBA-11/FBA 23/FBA-23DH Strong Motion Accelerometers

2.1. Description of the Dynamic System

Kinemetrics FBA accelerometers are closed-loop, force-feedback sensors measuring the relative displacement of moving mass with respect to the sensor case (representing ground or structure motion). Their voltage output is proportional to the input acceleration in the frequency band from DC to 50 Hz (optionally to 100 Hz or 90 Hz for a 4 g unit). The sensor's properties are essentially equivalent to a second-order dynamic system with one pair of complex conjugate poles (p_1 and p_2). The sensors' transfer function (TF) depends almost entirely on the electronic components rather than on the mechanical components of the sensors. The influence on the Transfer Function of the physical mass, mechanical damping, spring elements and internal RC low-pass filter within the closed-loop path of the sensor are negligible for almost all applications. For more accurate transfer functions at high frequencies above 20 Hz, you can take into account the additional pole (p_3) of a passive, low-pass RC filter in the post-amplifier stage of the sensor electronics.

The FBA sensors are factory-calibrated as second-order systems. Nominal natural frequency is 50 Hz (or 100 Hz or 90 Hz for a 4 g unit) and nominal relative damping is 0.707. Natural frequency, relative damping, and sensor sensitivity at DC are factory-adjusted as close as practically possible to the nominal values and are noted in the calibration sheet of each sensor. In order to be more precise, you must take into account poles based on values from the calibration sheet rather than the nominal values. Sensors with different absolute sensitivity have the same second-order poles. However, the low-pass filter in the post-amplifier stage depends on the sensor’s full-scale sensitivity. Poles pertaining to mechanical elements and the internal low-pass filter in the trans-conductance amplifier also depend on the full-scale sensitivity of the sensors. However, they are insignificant in most cases and are not given in this document.

If you wish, you can determine the modification of the TF due to mechanical elements and the internal low-pass filter. A system identification program or MatLab functions can be used for this purpose.

2.2. Nominal Second-Order Transfer Function of the FBA accelerometers

The nominal second-order Transfer Function related to ground acceleration of the FBA having one pair of conjugated poles p_1 and p_2 is:

$$\frac{V(s)}{A(s)} = k_2 \cdot \frac{\omega_n^2}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2} = \frac{k_2 \cdot k_1}{(s - p_1) \cdot (s - p_2)} \qquad 1$$

where

ω_n is the sensor natural frequency in rad/s,

s is Laplace operator
ς is relative damping (dimensionless)
V(s) is Laplace transform of sensor output voltage in V
A(s) is Laplace transform of input acceleration either in m/s^2 or in g
k_2 is sensor absolute sensitivity at DC in Vs^2/m or V/g
p_1, p_2 is conjugate pair of poles
k_1 is $|p_1*p_2| = 9.844*10^4$

This can be solved as a quadratic to find the two conjugate complex poles:

$$p_1 = -\zeta \cdot \omega_n + j \cdot \omega_n \cdot \sqrt{1-\zeta^2} \qquad (2)$$

$$p_2 = -\zeta \cdot \omega_n - j \cdot \omega_n \cdot \sqrt{1-\zeta^2} \qquad (3)$$

These equations can then be used with the sensor absolute sensitivity at DC values below or, for better precision, those taken from the unit's calibration data to calculate these poles.

The nominal Transfer Function related to ground acceleration of the FBA taking into account the post amplifier low-pass filter is:

$$\frac{V(s)}{A(s)} = \frac{k_2 \cdot k_1}{(s-p_1)\cdot(s-p_2)\cdot(s-p_3)}$$

where
s is Laplace operator
V(s) is Laplace transform of sensor output voltage in V
A(s) is Laplace transform of input acceleration either in m/s^2 or in g
k_2 is sensor absolute sensitivity at DC in Vs^2/m or V/g
p_1, p_2 is conjugate pair of poles
p_3 is the pole of the post amplifier
k_1 is $|p_1*p_2*p_3| = 1.48*10^8$

2.3. Absolute sensitivity at DC - k_2 -of FBA accelerometers

Kinemetrics FBA sensors are produced with 0.25, 0.5, 1, 2, and 4 g full-scale acceleration range and have +/- 2.5 V full-scale output voltage range. This corresponds to the following nominal absolute sensitivity at DC.

Full Scale Acceleration Range in g	Sensor absolute sensitivity at DC in Vs^2/m	Sensor absolute sensitivity at DC in V/g
0.25g	1.0197	10
0.5g	0.5099	5
1.0g	0.2549	2.5
2.0g	0.1275	1.25
4.0g	0.06373	0.625

The measured value of k_2 for each sensor is included on the calibration card, this can be used in the transfer function instead of the nominal sensitivities given above.

2.4. Accurate Relative Damping & Natural Frequency values

The Calibration Card for the FBA contains accurately measured Natural Frequency and relative damping values of each individual sensor. These values are obtained by precise measurements during factory calibration process and vary slightly from sensor to sensor and from nominal value. Thus, to most precisely determine the poles these values can be substituted into equations 2 and 3.

2.5. Nominal TF's poles and zeros for the FBA accelerometers

If we assume that the FBA's natural frequency is 50, 90, or 100 Hz, and the relative damping is precisely 0.707 we obtain the following nominal poles for the second-order representation for constant input acceleration:

FBA Natural Frequency /Hz	Pole P1		Pole P2	
	Real Component [rad/s]	Imaginary Component [rad/s]	Real Component [rad/s]	Imaginary Component [rad/s]
50	- 222.1	+ 222.1	- 222.1	- 222.1
90	- 399.8	+ 399.8	- 399.8	- 399.8
100	- 444.2	+ 444.2	- 444.2	- 444.2

We can get a further improvement in the accuracy of the model by considering the pole p_3 due to the low-pass RC filter in the post amplifier. This pole is dependent on the Natural Frequency and full scale acceleration range of the sensors.

FBA Natural Frequency [Hz]	FBA full scale acceleration range in [g]	Pole P3	
		Real Component [rad/s]	Imaginary Component [rad/s]
50, 100	0.25, 0.5, 1	- 1000	0
50, 90	2,4	- 1500	0

Now as an example of the full TF, a 50 Hz, 1g FBA would have the following nominal poles:

	FBA Natural Frequency [Hz]	Pole P1	Pole P2	Pole P3
Real Component [rad/s]	50	-222.1	- 222.1	-1000
Imaginary Component [rad/s]		222.1	-222.1	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	FBA Natural Frequency [Hz]	Zero Z1	Zero Z2
Real Component [rad/s]	All	0	0
Imaginary Component [rad/s]		0	0

3. EpiSensor Force Balance Accelerometers (ES-T, EpiDeck, ES-U2, ES-SB,ES-DH (HypoSensor))

3.1. Description of the Dynamic System

EpiSensor accelerometers are closed-loop, force-feedback sensors measuring the relative displacement of a moving mass with respect to the sensor case (representing ground or structure motion). The sensor’s transfer function (TF) depends almost entirely on the electronic components rather than on the mechanical components of the sensors. The influence on the transfer function of the mechanical mass, damping, spring elements and internal RC low-pass filter within the closed-loop path of the sensor are negligible for almost all applications. However, the sensor transfer function is more complex than in the case of FBA accelerometers and cannot be represented accurately as a second order dynamic system. We need four poles to adequately describe the EpiSensor's TF. This transfer function is identical for all models of the EpiSensor.

3.2. Nominal transfer function of the EpiSensor

We have determined a good empirical model of the system, which uses two pairs of conjugate poles to represent the transfer function of the sensor. If this transfer function is corrected for the sensor absolute sensitivity at DC, the amplitude agreement is within ±.0.5 dB over the bandwidth of the sensor. The phase agreement is within ± 2.5° in the 0 - 100 Hz band and within ± 5° over the full bandwidth of the instrument. This model can be represented as:

$$\frac{V(s)}{A(s)} = \frac{k1 * k2}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)}$$

where

s	is the Laplace operator,
p_1,p_2,p_3,p_4	are two pairs of conjugate poles
k_1	is $\lvert p_1 * p_2 * p_3 * p_4 \rvert = 2.46 \times 10^{13}$,
k_2	is absolute sensitivity of sensor at DC in V/g or Vs^2/m,
V (s)	is the Laplace transform of the output voltage in V,
A (s)	is the Laplace transform of the input acceleration in g or m/s^2.

3.3. Absolute sensitivity at DC - k_2 -of the EpiSensor

The EpiSensor can be configured to have different absolute sensitivities at DC. The Absolute sensitivity depends on, the sensor Full Scale Acceleration Range, its full-scale output voltage range, and its output configuration, single ended or differential. The following two tables give the absolute sensitivity at DC in V/g and Vs^2/m units for different sensor configurations.

Full Scale Acceleration Range in g	**EpiSensor nominal absolute sensitivity at DC - k_2 - in V/g**			
	Single-ended ± 2.5V output	**Single-ended ± 10V output**	**Differential ± 5V output**	**Differential ± 20V output**
1/4g	10	40	20	80
1/2g	5	20	10	40
1g	2.5	10	5	20
2g	1.25	5	2.5	10
4g	0.625	2.5	1.25	5

Full Scale Acceleration Range in g	**EpiSensor nominal absolute sensitivity at DC - k_2 - in Vs^2/m**			
	Single-ended ± 2.5V output	**Single-ended ± 10V output**	**Differential ± 5V output**	**Differential ± 20V output**
1/4g	1.0197	4.0789	2.0445	8.1577
1/2g	0.5099	2.0445	1.0197	4.0789
1g	0.2549	1.0197	0.5099	2.0445
2g	0.1275	0.5099	0.2549	1.0197
4g	0.06373	0.2549	0.1275	0.5099

3.4. Nominal TF's poles and zeros for the EpiSensor

Nominal poles and zeros of EpiSensor's transfer function are as follows:

	Pole P1	Pole P2	Pole P3	Pole P4
Real Component [rad/s]	-981	-981	-3290	-3290
Imaginary Component [rad/s]	1009	-1009	1263	-1263

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	FBA Natural Frequency [Hz]	Zero Z1	Zero Z2
Real Component [rad/s]	All	0	0
Imaginary Component [rad/s]		0	0

4. The SS-1 Short-Period Seismometer

4.1. Description of the Dynamic System

The Ranger SS-1 short-period seismometer is a passive electrodynamic seismic sensor. Its output above the natural frequency is essentially proportional to the ground velocity. It can be accurately represented by a second-order dynamic system. Its nominal natural frequency f_n is 1 Hz (ω_n = 6.283 rad/s). The sensor's damping “element” is electrodynamic in nature. The choice of the relative damping is left to you. The relative damping is determined by selection of an external damping resistor R_x. Normally; a relative damping of around 0.707 is used.

SS-1 seismometers are completely calibrated in our factory. Natural frequency and absolute sensitivity are adjusted as close as practically possible to nominal values. The open generator constant, G_o, of the main coil, the external damping resistor, R_x, for nominal relative damping of 0.707, and the internal coil resistance, R_c, are shown on the calibration sheet of each sensor. After selection of the relative damping and corresponding damping resistor, coil resistance allows calculation of the loaded generator constant, G_L.

Note: If you calibrate the SS-1 using the calibration coil the resultant transfer function will include the effect of the mutual induction between the calibration coil and the main coil. This becomes significant for frequencies above 5 Hz. This interaction can be removed by post-processing to account for the mutual inductance.

4.2. Absolute sensitivity or open and loaded generator constant of the SS-1 seismometer

The nominal open-generator constant G_0 of the SS-1 for standard coil resistance, Rc, of 5.000 ohms is

G_0 = 345 [Vs/m]

This is the sensor's absolute sensitivity to constant input velocity with nothing connected to the sensor output. For practical use, the loaded generator constant G_L must be calculated according to the equation below, where G_0 is the open generator constant, R_c is the coil resistance given in the calibration sheet, and R_x is the damping resistor you select.

$$G_L = G_0 \cdot \frac{R_x}{R_x + R_c}$$

Note that, in the case of a relatively low input impedance seismic recorder connected to the output of the seismometer, the resistance of the parallel combination of the physical damping resistor and the input impedance of the recorder must correspond to the selected external damping resistor Rx value.

4.3. Nominal transfer function of the SS-1 seismometer

The second order transfer function related to constant ground velocity of the SS-1 seismometer is:

$$\frac{V(s)}{v(s)} = G_L \cdot \frac{s^2}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2} = G_L \cdot \frac{(s - z_1) \cdot (s - z_2)}{(s - p_1) \cdot (s - p_2)}$$

where

s	is the Laplace operator,
ω_n	is the sensor natural frequency in rad/s,
ς	is relative damping (dimensionless)
p_1,p_2	is a pair of conjugate poles
z_1, z_2	are two zeros at coordinate origin
G_L	is absolute sensitivity of sensor to constant ground velocity or loaded generator constant in Vs/m
V (s)	is the Laplace transform of the output voltage in V,
v (s)	is the Laplace transform of the input velocity in m/s.

4.4. Nominal TF's poles and zeros for the SS-1 seismometer

Nominal zeros and poles of the transfer function reduced to the constant input velocity are:

	Pole P1	Pole P2	Zero Z1	Zero Z2
Real part [rad/s]	- 4.44	- 4.44	0	0
Imaginary part [rad/s]	+ 4.44	- 4.44	0	0

You may use equations 2 and 3, and data from the sensor's calibration sheet for a more precise pole representation.

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), one zero at the coordinate origin of the s plane must be added to the above transfer function.

	Zero Z3
Real part [rad/s]	0
Imaginary part [rad/s]	0

5. WR1 Wide Band Seismometer

5.1. Description of the Dynamic System

The WR-1 wide-band seismometer is mechanically similar to the SS-1 seismometer; however, its mass position is controlled by electronic circuitry in a closed-loop, force-feedback manner. Its closed-loop TF has the properties of a second-order system with a natural frequency at 20 Hz and relative damping close to a nominal value 0.707. This corner frequency is determined almost entirely by the electronic components. The sensor has three outputs:

- DC-coupled acceleration output. Acceleration-proportional output with frequency pass-band from DC to 20 Hz
- AC-coupled acceleration output. Acceleration-proportional output with frequency pass-band from 0.05 Hz to 20 Hz (optionally from 0.02 Hz to 20 Hz), and
- Velocity output. Velocity-proportional output with frequency pass-band from 0.05 Hz to 20 Hz (optionally from 0.02 Hz to 20 Hz.)

The sensor is composed of two stages. The first one is a force-feedback loop that measures relative displacement of moving mass with respect to the case of the sensor with a capacitive displacement transducer. The output of this stage is proportional to the input acceleration (poles p_1 and p_2). The second stage is a passive, single-pole, RC low-pass filter (pole p_3) that is followed by a post-amplifier that also contains an additional single-pole, low-pass filter (pole p_4).The output of this amplifier is the DC-coupled acceleration-proportional output of the WR-1.

This signal is then fed through a passive, single-pole, high-pass filter (pole p_5 and zero z_1) and an active signal follower. This stage produces the band-pass limited, AC-coupled, acceleration-proportional output of the sensor.

This signal is then fed through an active integrator, whose output is proportional to ground velocity in the frequency range from 0.05 Hz (optionally from 0.02 Hz) to 20 Hz. The integrator's electronics has two poles (poles p_6 and p_7) and one zero (zero z_2). This zero and the higher pole (p_6) lay at approximately the same frequency, thus this stage effectively integrates all input signals above the lower pole (pole p_7), which is equivalent to the low frequency corner of the sensor.

5.2. Nominal transfer function of the WR-1 seismometer at Acceleration-Proportional DC-Coupled Output

The nominal Transfer Function related to ground acceleration of the WR-1 seismometer at DC acceleration proportional output is:

$$\frac{V(s)}{A(s)} = \frac{k_1 \cdot G_0}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)}$$

where

s	is the Laplace operator,
p_1,p_2	is a pair of complex conjugate poles
p_3,p_4	are poles of the two low-pass filters in the electronics
k_1	is $\lvert p_1*p_2*p_3*p_4 \rvert = 1.624*10^{10}$,
G_0	is generator constant (absolute sensitivity)at DC in V/g or Vs^2/m,
V (s)	is the Laplace transform of the output voltage in V,
A (s)	is the Laplace transform of the input acceleration in g or m/s^2.

5.3. Absolute sensitivity or generator constant at DC - G0 -of the WR-1 seismometer at acceleration-proportional DC-coupled output

The DC-coupled acceleration-proportional output of the WR-1 has the following nominal generator constant at DC:

G_0 = 25.49 [Vs^2/m] or 250 [V/g]

The precise value for each sensor is given on its calibration sheet.

Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant for each application.

5.4. Nominal TF's poles and zeros for the WR-1 seismometer at acceleration-proportional DC-coupled output

The poles of the nominal transfer function for DC-coupled acceleration-proportional output are:

	Pole P1	Pole P2	Pole P3	Pole P4
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z3	Zero Z4
Real Component [rad/s]	0	0
Imaginary Component [rad/s]	0	0

5.5. Nominal transfer function of the WR-1 seismometer at Acceleration-Proportional AC-Coupled Output

The nominal Transfer Function related to ground acceleration of the WR-1 seismometer at AC acceleration proportional output is:

$$\frac{V(s)}{A(s)} = \frac{(s-z_1)\cdot G_0}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)\cdot(s-p_5)}\cdot\frac{1}{k_1}$$

where

s	is the Laplace operator,
p_1,p_2	is a pair of complex conjugate poles
p_3,p_4	are poles of the two low-pass filters
p_5, z_1	pole and zero of the high-pass filter
G_0	is the generator constant (absolute sensitivity) at f_G=1Hz (ω_G=6.283 rad/s) in V/g or Vs^2/m,
V (s)	is the Laplace transform of the output voltage in V,
A (s)	is the Laplace transform of the input acceleration in g or m/s^2

and

$$k_1 = \left| \frac{(j\omega_G - z_1)}{(j\omega_G - p_1)(j\omega_G - p_2)(j\omega_G - p_3)(j\omega_G - p_4)\cdot(j\omega_G - p_5)} \right|$$

For nominal poles and zeros and 0.05 Hz low frequency corner of the WR-1, k1 equals:

$k1 = 6.148 \cdot 10^{-11}$

For nominal poles and zeros and 0.02 Hz low frequency corner of the WR-1, k1 equals:

$k1 = 6.155 \cdot 10^{-11}$

5.6. Absolute sensitivity or generator constant at 1 Hz - G0 -of the WR-1 seismometer at Acceleration-Proportional AC-Coupled Output

The WR-1 seismometer has the following nominal generator constant at the AC-coupled acceleration-proportional output and at f_G = 1 Hz (ω_G = 6.283 rad/s):

G_0 = 25.49 [Vs^2/m] or 250 [V/g]

Note that this generator constant is not related to DC conditions as before but it is valid at a certain frequency f_G.

The precise value for each sensor is shown on its calibration sheet. Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant for each application.

5.7. Nominal TF's poles and zeros for the WR-1 seismometer at Acceleration-Proportional AC-Coupled Output

Zeros and poles of nominal transfer function for AC-coupled acceleration output are as follows:

Standard WR-1 with a 0.05 Hz low frequency corner:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Zero Z1
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.314	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0

The WR-1 with a 0.02 Hz low frequency corner option:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Zero Z1
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.126	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z3	Zero Z4
Real Component [rad/s]	0	0
Imaginary Component [rad/s]	0	0

5.8. Nominal transfer function of the WR-1 seismometer at velocity proportional output

The nominal Transfer Function related to ground velocity of the WR-1 seismometer at AC velocity proportional output is:

$$\frac{V(s)}{v(s)} = \frac{(s-z_1)\cdot(s-z_2)\cdot(s-z_3)\cdot G_0}{(s-p_1)(s-p_2)(s-p_3)(s-p_4)\cdot(s-p_5)\cdot(s-p_6)\cdot(s-p_7)} \cdot \frac{1}{k_1}$$

where

s	is the Laplace operator,
$p_1 \ldots p_7$	are poles of the system
$z_1 \ldots z_3$	are zeros of the system
G_0	is generator constant (absolute sensitivity) at f_G=1Hz (ω_G=6.283 rad/s) in V/g or Vs^2/m,
V (s)	is the Laplace transform of the output voltage in V,
v (s)	is the Laplace transform of the input velocity in m/s

and

$$k_1 = \left| \frac{(j\omega_G - z_1)\cdot(j\omega_G - z_2)\cdot(j\omega_G - z_3)}{(j\omega_G - p_1)(j\omega_G - p_2)(j\omega_G - p_3)(j\omega_G - p_4)\cdot(j\omega_G - p_5)\cdot(j\omega_G - p_6)\cdot(j\omega_G - p_7)} \right|$$

For nominal poles and zeros and 0.05 Hz low frequency corner of the WR-1, k1 equals:

$k1 = 6.543 \cdot 10^{-11}$

For nominal poles and zeros and 0.02 Hz low frequency corner of the WR-1, k1 equals:

$k1 = 6.550 \cdot 10^{-11}$

5.9. Absolute sensitivity or generator constant at f_G - G_0 - of the WR-1 seismometer at velocity-proportional output

The WR-1 seismometer has the following nominal generator constant at the velocity proportional output and f_G = 1 Hz (ω_G = 6.283 rad/s):

G_0 = 160 [Vs/m]

Note that this generator constant is not related to DC conditions as in the case of both DC coupled outputs but it is valid at a certain frequency f_G.

The precise value for each sensor is shown on its calibration sheet. Since the WR-1 is an active sensor, there is no need for a damping resistor or for calculation of the loaded generator constant.

5.10. Nominal TF's poles and zeros for the WR-1 seismometer at velocity-proportional output

Zeros and poles of the nominal transfer function of velocity-proportional output are as follows:

Standard WR-1 with a 0.05 low frequency corner:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Zero Z1	Zero Z2	Zero Z3
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.314	- 6.45	- 0.30	0	- 7.25	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0	0	0	0	0

The WR-1 with a 0.02 low frequency corner option:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Zero Z1	Zero Z2	Zero Z3
Real part [rad/s]	- 88.8	- 88.8	- 1000	- 1030	- 0.126	- 6.45	- 0.098	0	-7.25	0
Imaginary part [rad/s]	+ 88.8	- 88.8	0	0	0	0	0	0	0	0

Note: To get the TF representation reduced to a constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), one zero at the coordinate origin of the s-plane must be added to the above transfer functions.

	Zero Z4
Real Component [rad/s]	0
Imaginary Component [rad/s]	0

6. The SSA-1 and SSA-2 Digital Accelerometers

The digital SSA-1 and SSA-2 accelerographs have standard 50 Hz FBA sensors built-in. All zeros and poles of the FBA are also active in the SSA-1's transfer function. In addition there is an analog two-pole Butterworth anti-aliasing filter at 50 Hz built into both accelerographs. Thus, the overall TF of an SSA-1 or SSA-2 with 50 Hz natural frequency and 2 g full-scale acceleration range-internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5
Real part [rad/s]	- 222.1	- 222.1	- 1500	- 222.1	- 222.1
Imaginary part [rad/s]	+ 222.1	- 222.1	0	+ 222.1	- 222.1

The overall TF of an SSA-1 or SSA-2 with 50 Hz natural frequency and 0.25, 0.5, or 1 g full-scale acceleration range internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5
Real part [rad/s]	- 222.1	- 222.1	- 1000	- 222.1	- 222.1
Imaginary part [rad/s]	+ 222.1	- 222.1	0	+ 222.1	- 222.1

For accelerographs with external sensors you must add pole p4 and p5 to the zeros and poles of the actual sensor to get the full transfer function of the recording system.

Note: To get the TF representation reduced to constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s-plane must be added to the above transfer function.

	Zero Z1	Zero Z2
Real part rad/s]	0	0
Imaginary Part [rad/s]	0	0

7. SSA-16 Digital Accelerometer

The SSA-16 digital accelerograph has standard FBA-11 sensors built-in. All zeros and poles of the FBA are also active in the SSA-16's transfer function. In addition there is an analog 6-pole Butterworth anti-aliasing filter at 50 Hz built into the SSA-16 accelerograph. Thus the overall TF of the SSA-16 with 50 Hz natural frequency and 2 g full-scale acceleration range internal accelerometer has the following zeros and poles:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6	Pole P7	Pole P8	Pole P9
Re. part [rad/s]	- 222.1	- 222.1	- 1500	- 81.3	- 81.3	- 222.1	- 222.1	- 303.5	- 303.5
Im. part [rad/s]	+ 222.1	- 222.1	0	+ 303.5	- 303.5	+ 222.1	- 222.1	+ 81.3	- 81.3

If another type of accelerometer is installed in the SSA-16, take into account its transfer function and add poles from P4 to P9 from the above table.

Note: To get the TF representation reduced to constant input displacement compatible with the standardized GSE PAZ calibration header (CAL1 format), two zeros at the coordinate origin of the s plane

	Zero Z1	Zero Z2
Real Part rad/s]	0	0
Imaginary Part [rad/s]	0	0

must be added to the above transfer function.

8. SSR-1 Digital Recorder

The SSR-1 digital recorder uses analog, six-pole anti-aliasing filters. Their corner frequency depends on the options you chose. The following options for corner frequencies are available: 1, 2.5, 5 10, 15, 25, 50, 125, 250 Hz. Standard values are 50, 15 and 5 Hz. You may order either Butterworth or Bessel filters. Three filters can be built into the recorder at the same time, and they are software selectable.

The SSR-1 recorder also has a built-in, first-order RC high-pass filter at a 0.01 Hz corner frequency. It can be switched on or off through software.

The following table shows the poles of standard corner frequencies and Butterworth SSR-1's anti-aliasing filters.

50 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 81.3	- 81.3	-222.1	-222.1	-303.5	-303.5
Imaginary part [rad/s]	+ 303.5	- 303.5	+ 222.1	- 222.1	+ 81.3	- 81.3

15 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 24.4	- 24.4	- 66.6	- 66.6	- 91.0	- 91.0
Imaginary part [rad/s]	+ 91.0	- 91.0	+ 66.6	- 66.6	+ 24.4	- 24.4

5 Hz Butterworth antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	- 8.13	- 8.13	- 22.2	- 22.2	- 30.4	- 30.4
Imaginary part [rad/s]	+ 30.4	- 30.4	+ 22.2	- 22.2	+ 8.13	- 8.13

The general case for a Butterworth filter with corner frequency, f_C [Hz]:

$$p_1[rad/s] = -2\pi \cdot f[Hz](0.259 + j0.966)$$

$$p_2[rad/s] = -2\pi \cdot f[Hz](0.259 - j0.966)$$

$$p_3[rad/s] = -2\pi \cdot f[Hz](0.707 + j0.707)$$

$$p_4[rad/s] = -2\pi \cdot f[Hz](0.707 - j0.707)$$

$$p_5[rad/s] = -2\pi \cdot f[Hz](0.966 + j0.259)$$

$$p_6[rad/s] = -2\pi \cdot f[Hz](0.966 - j0.259)$$

Zeros and poles of standard corner frequencies and Bessel anti-aliasing filters are shown below:

50 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-169.2	-169.2	-251.2	-251.2	-285.7	-285.7
Imaginary part [rad/s]	302.1	-302.1	176.6	-176.6	58.3	-58.3

15 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-50.8	-50.8	-75.4	-75.4	-85.7	-85.7
Imaginary part [rad/s]	90.6	-90.6	53.0	-53.0	17.5	-17.5

5 Hz Bessel antialiasing filter:

	Pole P1	Pole P2	Pole P3	Pole P4	Pole P5	Pole P6
Real part [rad/s]	-16.9	-16.9	-25.1	-25.1	-28.6	-28.6
Imaginary part [rad/s]	30.2	-30.2	17.7	-17.7	5.83	-5.83

The zero and pole of the high-pass filter with the corner frequency at 0.01 [Hz] (only active if switched on) is shown below:

	Pole P7	Zero Z1
Real Part rad/s]	- 0.0628	0
Imaginary Part [rad/s]	0	0

Zeros and poles of the SSR-1 data recorder are simply added to the transfer function of the sensors used, no matter which kind of representation of the TF we deal with.

9. The Altus Family of Seismic Recorders

The Altus family of seismic recorders, the K2, Mt. Whitney, Makalu and Etna/EtnaSI use digital filters to provide anti-alias protection. These filters are implemented as two or three stage, multi-rate, Finite Impulse Response (FIR) filters. They offer extremely steep low pass amplitude response. The -3dB point is at 40% of the Sampling Frequency, while at the Nyquist Frequency (50% of the Sampling Frequency) the amplitude is -120dB. In all but the Etnas you can select either a causal or a non-causal version of the final filter depending on your application.

The first-stage (A) filters are non-causal FIR filters. The Etnas require that their first filter have sinc^3 compensation. A 2kHz data stream from the A/D converter is decimated by the appropriate factor to get an intermediate data stream at twice the final sampling rate. At final sample rates below 100 sps, this requires a second A filter. The final decimate by 2 brickwall filter is available with either causal or non-causal coefficients as the application requires. The Makalu has double precision coefficients. The table below lists the recorder, filter file name, number of coefficients and decimation for the anti-alias filters used in the Altus Recorder Family. All coefficients are contained in the accompanying .COE files.

Altus Recorder Anti-Alias Filters			
K2 and Mt. Whitney			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250 (47, /4)	---	FIRBNC (137, /2) FIRBC (111, /2)
200	FIRA200 (57, /5)	---	“”
100	FIRA100 (113, /10)	---	“”
50	FIRA100 (113, /10)	FIRA50 (59, /2)	“”
40	FIRA200 (57, /5)	FIRA200 (57, /5)	“”
20	FIRA100 (113, /10)	FIRA200 (57, /5)	“”
Makalu			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250DP (57, /4)		FIRBNCDP (173, /2 FIRBCDP (173, /2)
200	FIRA200DP (71, /5)	---	“”
100	FIRA100DP (143, /10)	---	“”
50	FIRA100DP (143, /10)	FIRA50DP (85, /2)	“”
	FIRA200DP (71, /5)	FIRA200DP (71, /5)	“”
	FIRA100DP (143, /10)	FIRA200DP (71, /10)	“”
Etna			
Final sps	First A Filter	Second A Filter	Final Brickwall
250	FIRA250SS3 (47, /4)	---	FIRBNC (137, /2)
200	FIRA200S3 (57, /5)	---	“”
100	FIRA100S3 (113, /10)	---	“”

10. The Q330 & Q330HR

The instrument response of the Quanterra Q330 and Q330HR are available for download by registered users at:

www.q330.com

11. The Rock/Rock+ Family of Seismic Recorders

The Rock® family of seismic recorders, Granite, Basalt, Obsidian, and others use digital filters to provide anti-alias protection. These filters are implemented as two to eight stage, multi-rate, Finite Impulse Response (FIR) filters. They offer extremely steep low pass amplitude response. The -3dB point is at 40% of the Sampling Frequency, while at the Nyquist Frequency (50% of the Sampling Frequency) the amplitude is -130dB.

The first-stage (A) filters have $sinc^5$ compensation. A 30K data stream from the A/D converter is decimated by the appropriate factors to get to an intermediate data stream at twice the final sampling rate. The final filter is a decimate by 2 brickwall filter, causal or non-causal. The table below lists the final sample rate, filter name, number of coefficients and decimation for the anti-alias filters used in the Rock Recorder Family. All coefficients are contained in the accompanying .COE files in App39F-coe.zip.

Rock Recorder Anti-Alias Filters

5000 sps (Obsidian only):
- coefA3_60 (30, /3)
- coefB2_60 (60, /2) or
  - coefB2C_60 (60, /2)

2000 sps:
- coefA5_50_s5c (59, /5)
- coefB3_80 (179, /3) or
  - coefB3C_80 (179, /3)

1000 sps:
- coefA5_50_s5c (59, /5)
- coefA3_50 (35, /3)
- coefB2_80_140 (133, /2) or
  - coefB2C_80_140 (133, /2)

500 sps:
- coefA5_50_s5c (59, /5)
- coefA3_50 (35, /3)
- coefA2_20 (85, /2)
- coefB2_80 (173, /2) or
  - coefB2C_80 (173, /2)

250 sps:
- coefA5_50_s5c (59, /5)
- coefA3_50 (35, /3)
- coefA2_20 (85, /2)
- coefA2_20 (85, /2)
- coefB2_80 (173, /2) or
  - coefB2C_80 (173, /2)

200 sps:
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

100 sps
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA2_20 (85, /2)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

50 sps:
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA4_50 (69, /4)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

20 sps:
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA2_20 (85, /2)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

10 sps:
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA4_50 (69, /4)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

1 sps:
coefA5_50_s5c (59, /5)
coefA3_50 (35, /3)
coefA2_20 (85, /2)
coefA4_50 (69, /4)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

12. The Etna2 Family of Accelerographs

The Etna2® family of accelerographs, Etna2, iCOBI3, and others use digital filters to provide anti-alias protection. These filters are implemented as two to seven stage, multi-rate, Finite Impulse Response (FIR) filters. They offer extremely steep low pass amplitude response. The -3dB point is at 40% of the Sampling Frequency, while at the Nyquist Frequency (50% of the Sampling Frequency) the amplitude is -130dB.

An 8K data stream from the A/D converter is decimated by the appropriate factors to get to an intermediate data stream at twice the final sampling rate. The final filter is a decimate by 2 brickwall filter, causal or non-causal. The table below lists the final sample rate, filter name, number of coefficients and decimation for the anti-alias filters used in the Etna2 Family. All coefficients are contained in the accompanying .COE files in App39F-coe.zip.

Etna2 Anti-Alias Filters

500 sps:
  coefA8_197 (197, /8)
  coefB2_80 (173, /2) or
    coefB2C_80 (173, /2)

250 sps:
  coefA8_197 (197, /8)
  coefA2_20 (85, /2)
  coefB2_80 (173, /2) or
    coefB2C_80 (173, /2)

200 sps:
  coefA8_197 (197, /8)
  coefB5_350 (350, /5)) or
    coefB5C_350 (350, /5)

100 sps
  coefA8_197 (197, /8)
  coefA5_50 (85, /5)
  coefB2_80 (173, /2)) or
    coefB2C_80 (173, /2)

50 sps:
  coefA8_197 (197, /8)
  coefA5_50 (85, /5)
  coefA2_20 (85, /2)
  coefB2_80 (173, /2)) or
    coefB2C_80 (173, /2)

20 sps:

coefA8_197 (197, /8)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

10 sps:

coefA8_197 (197, /8)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefA2_20 (85, /2)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)

1 sps:

coefA8_197 (197, /8)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefA5_50 (85, /5)
coefA2_20 (85, /2)
coefA2_20 (85, /2)
coefB2_80 (173, /2)) or
coefB2C_80 (173, /2)