Algebra-based Model for Concept Learning

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Abstract

Concept learning task requires a model to recognize the abstract concept in the given existing event. Previous models didn't make full use of dense inner connections of the concepts, which may limit their capability of concept recognization. To remedy this, we propose introducing the theory about abstract algebra to better utilize the computability and closure in the concept elements.

Due to the requirement of learning of many concepts and their connections from raw images, having a sufcient amount of diverse and easy-to-customize images with corresponding concepts is needed, then the 3D virtual scene generated tool Fuzzy World is proposed to satisfy the theoretical verification in this paper. While the concepts can be combined flexibly for generalization, the proposed approach is modality-agnostic and can be applied to many types of learning tasks.

Overview

Many advanced artificial intelligence tasks, such as generalizing from limited experience, abstract reasoning and planning, need to identify abstract concepts from entities. In this paper, we consider the problem of using queries to learn an unknown concept.

Examples of concepts include visual("green" or "triangle"), spatial("on left of","behind"), temporal ("slow", "after"), size ("bigger", "smaller") among many others. The first-order predicate logic is used to describe the connection between concepts and entities, With such a connection, the learning samples can be organized into queries as $r_i(o_1, o_2) = \text{predicate}(\text{object}_1, \text{object}_2)$, and then the dataset $R = \{r_i\}$ can be used to train the learning function. Specifically, the learning function is a mapping from the input of the unknown pattern to the representation of the specific concept, such as $f_0 : \mathcal{X} \rightarrow \mathcal{R}$, which gives the linguistic expression of the concept r_i contained in the image X_i, where the training sample is organized as $\mathcal{D} = \{(X_i, r_i)\}$.

It is worth noting the inner connection between similar concepts, specifically, this is some regular patterns about the computability and closure between concepts. Now consider a set of action concepts like this:

$$\{\text{run, walk, stand up, sit down, jump, static, slow down}\}$$

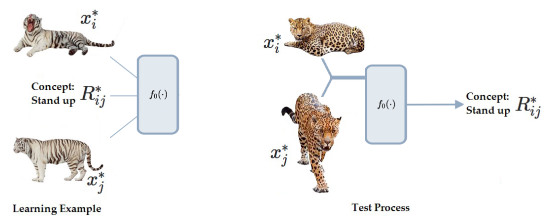

Figure 1: The example of concept learning task: notice the "inner computability" of the concept set.

Suppose we treat each action as a transformation and pre-limit some arithmetic rules, then operations on this set similar as multiplication can be defined, such as: run $\odot$ slow down $=$ walk,walk $\odot$ walk $=$ walk,$\cdots$, which means computability.And notice that the results of computation still belong to the set of action concepts, which means closure.

Such patterns have certain similarities with the properties of some algebraic structures in abstract algebra(i.e. groups, rings), such as closure and computability. Abstract algebra is an increasingly popular tool for machine learning, it facilitates learning methods like parallel training (Izbicki 2013).In this paper, we combine some properties of abstract algebra with neural networks to solve problems related to concept learning.

The benefits of introducing the nature of abstract algebra into the design of neural networks to solve concept learning problems can be stated as follows:

- **The combination of symbolism and neural connectionism**: There is an increasing trend in recent research work to combine the symbolism and neural connectionism (Tran and Garcez 2018), which can combine the powerful fitting ability of the neural network with the prior knowledge of the symbolic reasoning mechanism, and finally achieve more powerful logical reasoning ability.
- **Generalization**: The use of interoperable relationships between concepts increases the relevance of concepts, so models can achieve better generalization capabilities.
- **Few-shot leaning**: Since it can be promoted by introducing relationships between samples, more cross-mapping can be constructed, which means better samples utilization efficiency.

Dataset and Tasks

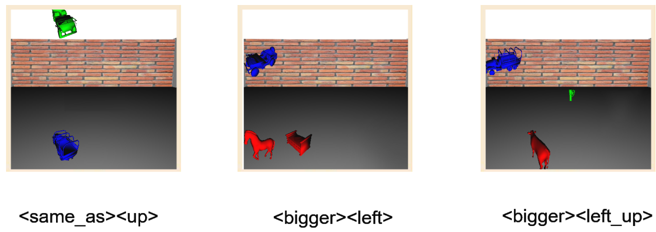

Figure 2: The dataset generated by Fuzzy World: formed by images and corresponding descriptions(visual pattern space $\mathcal{X}$ and the concept semantic space $\mathcal{R}$).

The abstract concept set with transformation relationships between the elements and the definition of the ring can be linked. In this paper, the following concept set is considered:

$$\{\text{left, up, bigger, right down, same size, same position}, \cdots\}$$

There are two kinds of operations between the elements in the definition of the ring: $+$ and $\times$(note that this is different from the operation between integers or real numbers), we can predefine the meaning of these two operations, in this paper we will regard the above concepts as "transform operators".Specifically, $+$ is defined as a comprehensive effect and $\times$ is defined as a mandatory conversion. Not strictly, the following examples can be given:

$$\begin{aligned}
&< \text{smaller} >< \text{left} > + < \text{same size} >< \text{up} > \\
&=< \text{smaller} >< \text{up left} > \\
&< \text{smaller} >< \text{left} > \times < \text{same size} >< \text{up} > \\
&=< \text{same size} >< \text{up} >
\end{aligned}$$

The samples generated by Fuzzy World is formed as (X_i, r_i),where X_i denotes the visual data and the r_i refers to the semantic data.The X_i is an image divided by size $d \times d$ and in each section can be placed with a meta-object and the algorithm can recognize the concept relationship between meta-objects with spatial parameters and generates semantic descriptive data r_i automatically.

Model

We can construct a concept learning model with division ring structure:Division Ring Network(DRN). A DRN(see the whole architecture of DRN at Figure.3) lets us apply the techniques above the division ring theory to create concept learning methods flexibly. Let X_i, X_j and r_i, r_j denote vision and semantic samples from dataset $\mathcal{D}$, notice that(using the recognizer $f_0(\cdot)$):

$$\begin{cases} +(r_i, r_j) = +(f_0(X_i), f_0(X_j)) = r' \\ \times(r_i, r_j) = \times(f_0(X_i), f_0(X_j)) = r'' \end{cases}$$

where the r', r'' denote the combination after the operation of $+(\cdot,\cdot), \times(\cdot,\cdot)$;And notice the following relations by the characteristic of homomorphism to $f_0(.)$:

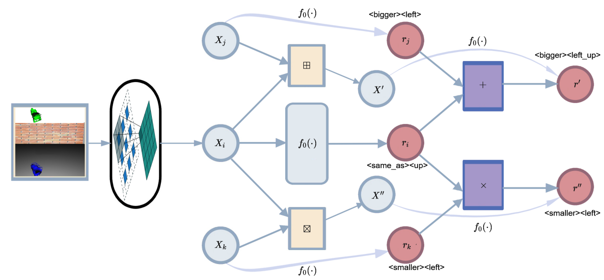

Figure 3: The architecture of the proposed model:The operations $\boxplus, +, \boxtimes, \times$ in the ring are all implemented by multi-layer neural network fitting.

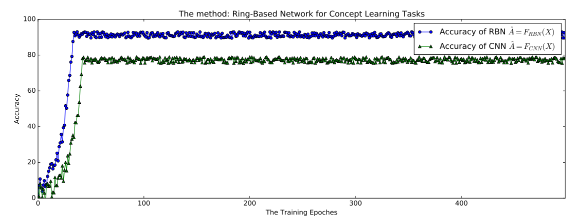

Figure 4: The proposed model is superior to the baseline in samples utilization efficiency, optimal accuracy and generalization.

$$\begin{cases} f_0(\boxplus(X_i, X_j)) = f_0(X_i \boxplus X_j) = +(f_0(X_i), f_0(X_j)) = r' \\ f_0(\boxtimes(X_i, X_j)) = f_0(X_i \boxtimes X_j) = \times(f_0(X_i), f_0(X_j)) = r'' \end{cases}$$

Combining the above two points, we can propose the model structure as shown in the Figure.3, in which $\boxplus, +, \boxtimes, \times$ are all implemented by multi-layer neural network fitting. Convolutional neural network(CNN) is used to extract visual features, We might as well make the extracted advanced visual feature matrix denoted by X_i, X_j.

Result

In this preliminary work, we proposed ring-based concept learning models for neural network design. Taking advantage of the properties of computability and closure in algebraic structure like ring, the neural network model DRN can optimize better and more efficiently, and achieve better learning results.The virtual $3D$ scene training tool Fuzzy World is proposed to meet the theoretical verification.

References

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