

» 二分法 / 二进制 (Dichotomy / Binary)

Problems:

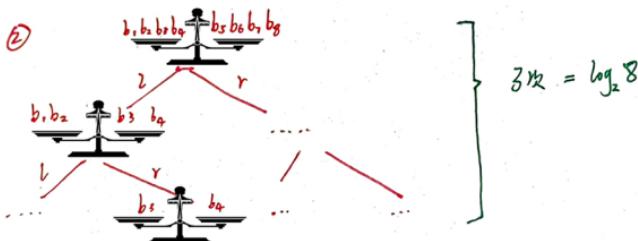
① 1000瓶水 $W = \{w_1, \dots, w_{1000}\}$

$\exists w \in W$ 有毒 (一瓶) 如果 rat 喝了会死, 要求找出哪瓶有毒的 rat 最少:

② 8个球 $B = \{b_1, \dots, b_8\}$ 外观相同.

$\exists b \in B$ 与其余7个一样的球相比较. 要求用天平最少次数可找到.

Ideas: 二分法求解:



$$\textcircled{2} \because 10 > \log_2 1000 > 9$$



$$w_i = B_9 B_8 B_7 B_6 B_5 B_4 B_3 B_2 B_1 B_0$$

$y_9 \quad y_8 \quad y_7 \quad y_6 \quad y_5 \quad y_4 \quad y_3 \quad y_2 \quad y_1 \quad y_0$

Algorithm: For w_i in W :

For B_j in w_i :

If $B_j == 1$: Drink(y_j, w_i);

Return $y_9 \dots y_0$;



ID (核酸检测) Covid-Test:



P_1



P_2



P_{1000}

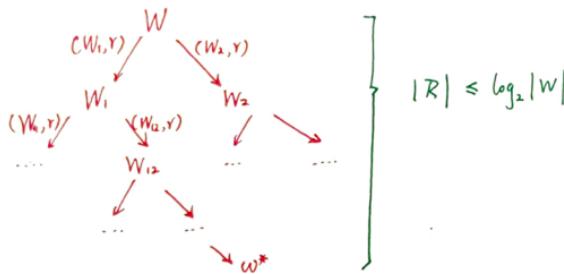
T1

T2

T10

$$P_i = B_9 \dots B_0$$

①



► 二进制数计算

$$\text{② } N = 11$$

$$\begin{array}{r} 11 \\ \text{---} \\ 1 \mid 1 \\ \text{---} \\ 1 \mid 0 \\ \text{---} \\ 0 \mid 10 \end{array} \quad N = (1011)_2$$

$$\begin{array}{r} 11 \\ \text{---} \\ 3 \mid 1 \\ \text{---} \\ 1 \mid 0 \end{array}$$

$$S_{10} = \{0, 1, 2, \dots, 9\}$$

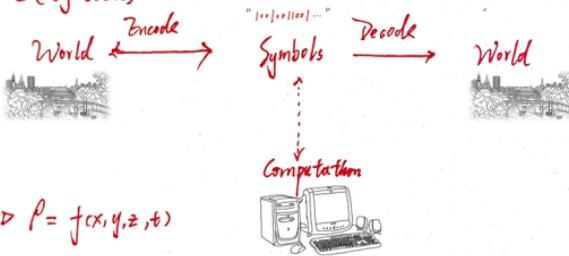
$$N = (13)_8$$

$$\begin{aligned} S_2 &= \{0, 1\} \\ S_8 &= \{0, 1, \dots, 7\} \end{aligned}$$

$$S_{16} = \{0, 1, \dots, 9, A, B, C, D, E, F\}$$

$$N = (8)_{16}$$

► 符号化 (Symbolic)



$$\text{③ } P = f(x, y, z, t)$$

$$\text{④ } P \vee \neg q \rightarrow r$$

$$\text{⑤ } 1000100111001101001\dots$$

$P = \text{"Sunny"}$

$q = \text{"Rain"}$

$r = \text{"Good for Hiking"}$

i) 并且: $P \wedge q$

ii) 或: $P \vee q$

iii) 非: $\neg q$

iv) 异或: $P \oplus q = (\neg P \wedge q) \vee (P \wedge \neg q)$

⑥ 双向“依赖关系”;

△ 演绎逻辑 (Predicate Logic)

$$\text{Father}(X, Y) \wedge \text{Mother}(Y, Z) \rightarrow \text{GrandPa}(X, Z)$$

▲ CSAT Problem 可满足性问题:

找到 $X_1 \dots X_m$ 的赋值使 $L = \{L_1 \dots L_n\}$ 全为真,

其中子句 $L_i \in L$ 不加 $L_i \notin X_1 \dots X_m$, 即 $\neg L_i = X_1 \vee \neg X_2 \vee X_3$

- 1) 全部的考试真题有如下命题：
- 有及格
 - 有人不及格
 - 全不及格

且 A, B, C 有且仅一个真，求 A, B, C 真值

$$L_1: A \oplus C$$

$$L_2: \neg B \wedge (A \oplus C)$$

$$L_3: \neg B \wedge \neg C$$

Idea: (Branch and Bounding) 分支定界法：



▷ (Bit-Computation) 位运算：

$$X = 1001$$

$$Y = 1110$$

$$S = \{ \Lambda, V, \neg, \oplus, \ll, \gg, \dots \}$$

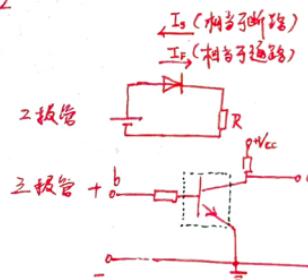
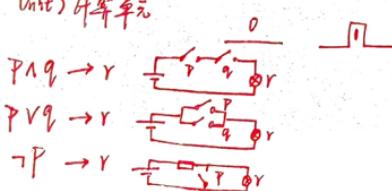
右移

$$X \gg n \leftrightarrow X \cdot 2^n$$

$$X \ll n \leftrightarrow X / 2^n$$

$$\triangleright X = (1001)_2 = 9 \quad X \gg 2 = (100100)_2 = 36 = 9 \cdot 2^2$$

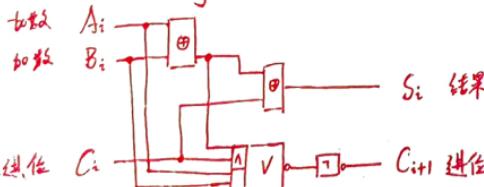
▷ (Computing Unit) 计算单元



$$\begin{aligned} b=1 &\rightarrow c=0 \\ b=0 &\rightarrow c=1 \end{aligned}$$

通电时 b⊕c 取反

▷ 总线 (Adding Unit)



上一位进位 Ci

$$\begin{array}{l} A_i \\ B_i \\ C_i \end{array} \equiv \boxed{+} \quad S_i \quad C_{i+1}$$

$$A + B = S$$

$$\begin{array}{r} A_1 \ A_2 \ \dots \ A_n \\ + \ B_1 \ B_2 \ \dots \ B_m \\ \hline S_{n+1} \ S_n \ \dots \ S_1 \end{array}$$

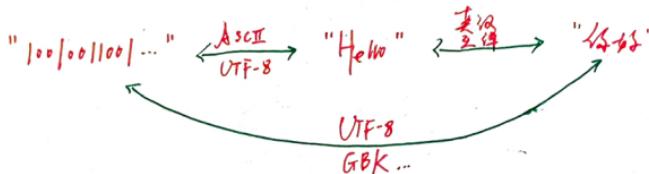


Q Hamming Code is " Partition " Satisfies

$$\Rightarrow \bigcup_i N_i = N \quad (\text{覆盖性})$$

$$\text{iii) } \left| \bigcap_i N_i \setminus \bigcup_j N_j \right| = 1 \quad (\text{连通性})$$

▷ (Encoding) 字符/因你编码.



► 因缘第28：RGB or Black White

(学技三 阵23)

000 1000
000 1000
001 0100
010 0000
100 0000

$$\left\{ \begin{array}{l} \text{RGB: } I \in \mathbb{N}^{mn \times 3} \\ \text{Black-white: } I \in \mathbb{N}^{mn} \end{array} \right.$$

