NYU, Tandon School of Engineering
Bridge to CS — Spring 2018

Homework #1
Due by Friday 4/20 11:55pm

Submission instructions:

1. You should submit your homework in the NYU Classes system.
2. For this assignment you should turn in a '.pdf' file with your answers. Name your file 'YourNetID_hw1.pdf'
3. For questions 1-3, make sure to include the conversion calculations, not just the final answer.

Question 1:

A. Convert the following numbers to their decimal representation:
1. $10011011_2 = 1\cdot2^0 + 1\cdot2^1 + 0\cdot2^2 + 1\cdot2^3 + 1\cdot2^4 + 0\cdot2^5 + 0\cdot2^6 + 1\cdot2^7 = 155_{10}$
2. $456_7 = 6\cdot7^0 + 5\cdot7^1 + 4\cdot7^2 = 237_{10}$
3. $38A_{16} = 10\cdot16^0 + 8\cdot16^1 + 3\cdot16^2 = 906_{10}$
4. $2214_5 = 4\cdot5^0 + 1\cdot5^1 + 2\cdot5^2 + 2\cdot5^3 = 309_{10}$

B. Convert the following numbers to their binary representation:
1. $69_{10} = 64 + 4 + 1 = 2^6 + 2^2 + 2^0 = 1000101_2$
2. $485_{10} = 128 + 256 + 64 + 32 + 4 + 1 = 2^0 + 2^2 + 2^5 + 2^6 + 2^7 + 2^8 = 111100101$
3. $6D1A_{16} = (6)_2(D)_2(1)_2(A)_2 = (0110)_2(1101)_2(0001)_2(1010)_2$
$= 110\,1101\,0001\,1010_2$

C. Convert the following numbers to their hexadecimal representation:
1. $1101011_2 = 0110\ 1011 = 6B_{16}$
2. $895_{10} = 3\times256 + 7\times16 + 15 = 3\times16^2 + 7\times16^1 + 15\times16^0 = 37F_{16}$

Question 2:

Solve the following, do all calculation in the given base:
1. $7566_8 + 4515_8 = 14303_8$
2. $10110011_2 + 1101_2 = 11000000_2$
3. $7A66_{16} + 45C5_{16} = C02B_{16}$
4. $3022_5 - 2433_5 = 44_5$

$$1.\quad \begin{array}{r} 7566_8 \\ +\ 4515_8 \\ \hline 14303_8 \end{array} \qquad 2.\quad \begin{array}{r} 10110011_2 \\ +\ 1101_2 \\ \hline 11000000_2 \end{array} \qquad 3.\quad \begin{array}{r} 7A66_{16} \\ +\ 45C5_{16} \\ \hline C02B_{16} \end{array} \qquad 4.\quad \begin{array}{r} 3022_5 \\ -\ 2433_5 \\ \hline 0044_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation:

1. $124_{10} =$ $64+32+16+8+4 = 2^6+2^5+2^4+2^3+2^2 = (01111100)_{8 \text{ bit 2's}}$
2. $-124_{10} =$ ~~10000~~ 10000100 8 bit 2's comp
3. $109_{10} =$ $64+32+8+4+1 = 2^6+2^5+2^3+2^2+2^0 =$ 01101101 8 bit 2's comp
4. $-79_{10} =$ $10000000_2 - 01001111_2 = 10110001$ 8 bit 2's comp

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation:

1. $00011110_{8 \text{ bit 2's comp}} =$ $+(2+4+8+16) = 30_{10}$
2. $11100110_{8 \text{ bit 2's comp}} =$ $-(00011010)_2 = -26_{10}$
3. $00101101_{8 \text{ bit 2's comp}} =$ $+(1+4+8+32) = 45_{10}$
4. $10011110_{8 \text{ bit 2's comp}} =$ $-(01100010)_2 = -98_{10}$

Question 4:

A. For each of the following sets, determine whether 2 is a member of that set.

T **a)** $\{x \in \mathbf{R} | \ x \text{ is an integer greater than } 1\}$ T **b)** $\{2, \{2\}\}$

F **c)** $\{x \in \mathbf{R} | \ x \text{ is the square of an integer}\}$ F **d)** $\{\{2\}, \{\{2\}\}\}$

F **e)** $\{\{2\}, \{2, \{2\}\}\}$ F **f)** $\{\{\{2\}\}\}$

B. Determine whether each of these statements is true or false.

T **a)** $x \in \{x\}$ T **b)** $\{x\} \subseteq \{x\}$ F **c)** $\{x\} \in \{x\}$

T **d)** $\{x\} \in \{\{x\}\}$ T **e)** $\varnothing \subseteq \{x\}$ F **f)** $\varnothing \in \{x\}$

C. Find two sets A and B such that $A \in B$ and $A \subseteq B$. $A = \{a\}$ $B = \{a, \{a\}\}$

D. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

The first set = A
i. the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi. $B \subseteq A$

The second set = B
ii. the set of people who speak English, the set of people who speak Chinese A and B has no relavanc

iii. the set of flying squirrels, the set of living creatures that can fly $A \subseteq B$

Question 5:

Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

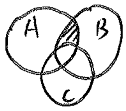
a) $A \cup B$ $= \{a, b, c, d, e, f, g, h\}$

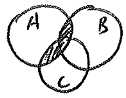
b) $A \cap B$ $= \{a, b, c, d, e\}$

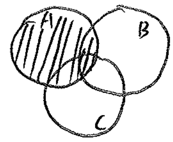
c) $A - B$ $= \varnothing$

d) $B - A$ $= \{f, g, h\}$

Question 6 :

a) 

b) 

c) 

Question 6:

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B - C)$

b) $(A \cap B) \cup (A \cap C)$

c) $(A \cap \overline{B}) \cup (A \cap \overline{C})$

Question 7:

Let A, B, and C be sets. Use a membership table to show that:

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $(B - A) \cup (C - A) = (B \cup C) - A$

c) $\overline{(A \cap B \cap C)} = (\overline{A} \cup \overline{B} \cup \overline{C})$

d) $(A - C) \cap (C - B) = \varnothing$

e) $(A - B) - C \subseteq (A - C)$

Answered on the following pages.

Question 8:

Let A, B, and C be sets. Use a set identities to show that:

a) $A - B = \overline{B} - \overline{A}$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

c) $A - (B - C) = (A - B) \cup (A - \overline{C})$

Question 8 :

a) $A - B = A \wedge \bar{B} = \bar{B} \wedge A = \bar{B} -$

b) $(A \wedge B) \vee (A \wedge \bar{B}) = [(A \wedge B) \vee A]$
$[(A \wedge B) \vee \bar{B}]$

$= A \wedge [(\bar{B} \vee A) \wedge (\bar{B} \vee B)]$

$= A \wedge [(\bar{B} \vee A) \wedge U$

$= A \wedge (\bar{B} \vee A)$

$= (A \wedge \bar{B}) \vee (A \wedge A$

$= A \vee (A \wedge \bar{B})$

$= A \vee (A - B)$

$\because A - B = \{x \mid x \in A$ and x

$\therefore A - B \subseteq A$

$\therefore A \vee (A - B) = A$

c) $A - (B - C)$

$= A \wedge \overline{B - C}$

$= A \wedge \overline{B \wedge \bar{C}}$

$= A \wedge (\overline{B} \vee \overline{\bar{C}})$

$= A \wedge (\bar{B} \vee C)$

$= (A \wedge \bar{B}) \vee (A \wedge$

$= (A - B) \vee (A - \bar{C}$

Question 9:

Can you conclude that $A=B$, if A, B, and C are sets, such that:

a) $A \cup C = B \cup C$? True

b) $A \cap C = B \cap C$? True

c) ($A \cup C = B \cup C$) *and* ($A \cap C = B \cap C$)? True

Question 10:

Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$, if for every positive integer i:

a) $A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$

b) $A_i = \{-i, i\}$

c) $A_i = [-i, i]$, that is, the set of real numbers x with $-i \le x \le i$

d) $A_i = [-i, \infty)$, that is, the set of real numbers x with $x \ge -i$

a) $\bigcup_{i=1}^{\infty} A_i = \{-i, -i+1, \cdots, -1, 0, 1, \cdots, i-1, i\}$

$\bigcap_{i=1}^{\infty} A_i = \{-1, 0, 1\}$

b) $\bigcup_{i=1}^{\infty} A_i = \{-i, -i+1, \cdots, -1, 1, \cdots, i-1, i\}$

$\bigcap_{i=1}^{\infty} A_i = \phi$

c) $\bigcup_{i=1}^{\infty} A_i = [-i, i]$

$\bigcap_{i=1}^{\infty} A_i = [-1, 1]$

d) $\bigcup_{i=1}^{\infty} A_i = [-i, \infty)$

$\bigcap_{i=1}^{\infty} A_i = [-1, \infty)$

a)

A	B	C	$B \wedge C$	$A \cup (B \wedge C)$	$A \cup B$	$A \vee C$	$(A \vee B) \wedge (A \vee C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

b)

A	B	C	$B-A$	$C-A$	$(B-A) \cup (C-A)$	$B \cup C$	$(B \cup C)-A$
1	1	1	0	0	0	1	0
1	1	0	0	0	0	1	0
1	0	1	0	0	0	1	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0

c)

A	B	C	$A \wedge B \wedge C$	$\overline{A \wedge B \wedge C}$	$\bar{A} \vee \bar{B} \vee \bar{C}$
1	1	1	1	0	0
1	1	0	0	1	1
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
0	0	0	0	1	1

$\boxed{\text{Question 7}}$

d)

A	B	C	A − C	C − B	(A−C) ∧ (C−B)
1	1	1	0	0	0
1	1	0	1	0	0
1	0	1	0	1	0
1	0	0	1	0	0
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	1	0
0	0	0	0	0	0

e)

A	B	C	A − B	(A − B) − C	A − C
1	1	1	0	0	0
1	1	0	0	0	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$(A - B) - C \subseteq (A - C)$ is true.