Question 7

a) 26^8

b) $A_{26}^8 = 26 \times 25 \times \cdots \times 19 = P(26, 8)$

c) 26^7

d) $A_{25}^7 = 25 \times 24 \times \cdots \times 19 = P(25, 7)$

Question 8

a) 0

b) $5! = 120$

c) $A_6^5 = 720 = P(6, 5)$

d) $A_7^5 = 2520 = P(7, 5)$

Question 9

a)   Assume set A has 100 elements,
   then $|A| = 100$,
   therefore A has 2^{100} subsets in total.
   The answer is $2^{100} - C(100, 0) - C(100, 1)$,
   which is $2^{100} - 101$

Question 10   (consider the row is sequential)

a)   1). Pick a place for bride $C(6, 1)$
   2). Pick 5 people from the remain 9 people
                                                $C(9, 5)$
   3). arrange the order for these 5 people $P(5, 5)$
   4) $C(6, 1) \times C(9, 5) \times P(5, 5) = 90720$

b).   1). Pick 2 places for bride & groom $C(6, 2)$
   2). Arrange the order for them $P(2, 2)$
   3). Pick 4 people from the remainder and order them
                   C_8^4 $C(8, 4) \times P(4, 4)$ sort

4). $C(6,2) \times P(2,2) \times C(8,4) \times P(4,4) = 50400$

C). 1). Pick one of the couple $C(2,1)$

2). Pick a 5 people and sort $C(8,5) \times P(6,6)$

3). $C(2,1) \times C(8,5) \times P(6,6) = \cancel{17440} 80640$

(consider the row is not sequential)

a) $C(9,5) = 126$

b) $C(8,4) = 70$

c) $C(2,1) \times C(8,\cancel{4}5) = 112$

Question 11

a) Pick 3 places for 1s $C(12,3) = 220$

b) Zero 1s $C(12,0) = 1$

one 1s $C(12,1) = 12$

two 1s $C(12,2) = 66$

Three 1s $C(12,3) = 220$

Total $1 + 12 + 66 + 220 = 299$

c) All possible strings 2^{12}

At least 3 1s $2^{12} - 1 - 12 - 66 = 4017$

Question 12 (consider the length is different

a) only 2 letters 1

3 letters $C_6^1 A_2^2 = C(6,1) \times P(2,2)$

4 letters $C_6^2 A_3^3 = C(6,2) \times P(3,3)$

⋮

8 letters $C_6^6 A_7^7 = C(6,6) \times P(7,7)$

Total $1 + C(6,1)P(2,2) + C(6,2) \times P(3,3) + \cdots + C(6,6) \times P(7,7) = 11743$

b) Assume E and D are one to another,

2 letters	$1 \times P(2,2)$
3 letters	$C(6,1) \times P(2,2) \times P(2,2)$
4 letters	$C(6,2) \times P(3,3) \times P(2,2)$
⋮	
7 letters	$C(6,6) \times P(7,7) \times P(2,2)$
Total	23486

While all possible ways of permutations are,

$$P(8,1) + P(8,2) + P(8,3) + \cdots P(8,8)$$
$$= 109600$$

Therefore the answer is $109600 - 23486 = 86114$

~~c) Assume the permutation doesn't contain CD nor DE.~~
~~1 letter   P(8,1)~~
~~2 letters   P(8,2)---2~~
~~3 letters~~

c) contain the string DE : same as a)
                                   11743

contain the string CD : 11743
contain CD and DE, which is CDE ;

3 letters	1
4 letters	$C(5,1) P(2,2)$
5 letters	$C(5,2) P(3,3)$
⋮	
8 letters	$C(5,5) A_6^6$
Total	1631

contain either CD or DE $11743 + 11743 - 1631 = 21855$

(consider the length is the same)

a). $P(7,7) = 5040$

b). DE : 5040

 ED : 5040

 Total : ~~A0~~ $P(8,8)$

The answer is $P(8,8) - 5040 - 5040 = 30240$

c) CD : 5040

 DE : 5040

 CDE : $P(6,6) = 720$

The answer is $5040 \times 2 - 720 = 9360$