Date

Question 5

a. Claim: For any positive integer n, 3 divide n^3+2n has no remainder.

I) Base : for $n=1$, $1^3+2\cdot 1=3$, as required.

II) Inductive step : We assume that $(n-1)^3+2(n-1)=3k$ $(k\in N)$, and we'll show that $n^3+2n=3t$ $(t\in N)$.

$k\geq 1$

$$(n-1)^3+2(n-1)=3k$$

$$n^3-1-3n^2+3n+2n-2=3k$$

$$n^3+5n-3-3n^2=3k$$

$$n^3+2n=3k+3+3n^2-3n$$

$$=3(k+1+n^2-n)$$

Let $t=k+1+n^2-n$, t is an integer, as requested

b. Claim: For any positive integer $n(n\geq 2)$ can be written as a product of primes.

I) Base: for $n=2$, $2=2\cdot 3^0\cdot 5^0\cdots$ we get the only prime factor 2 as requested.

II) Inductive Step: We assume that every k $(k<n)$ can be written as a product of primes, $k=2^{i_1}3^{i_2}5^{i_3}\ldots$, where i is a non-negative integer. we'll show that n can be written as $n=2^{i_1}3^{i_2}5^{i_3}\ldots$ as requested

We have 2 cases:

a) n is a prime number:

In this case, $n=n\cdot 1=n\cdot 2^0\cdot 3^0\cdot 5^0\ldots$, as required.

b) n is not a prime number:

In this case, n can be written as $a\cdot b$ (both a and b are not equal to n or 1), ~~as~~ given the definition of prime numbers. Then by the inductive hypothesis for $a < n$ and $b < n$, we get that $a = 2^{i_1} \cdot 3^{i_2} 5^{i_3} \ldots$ and $b = 2^{i_1'} \cdot 3^{i_2'} \cdot 5^{i_3'} \ldots$ Thus n can be written as $n = 2^{i_1 + i_1'} \cdot 3^{i_2 + i_2'} \cdot 5^{i_3 + i_3'} \ldots$, we get n written in the requested form.