Question 7

a. $P(X=1) = \frac{26-5}{26} = \frac{21}{26}$

$P(X=3) = \frac{5}{26}$

b. $EV(X) = 1 \times \frac{21}{26} + 3 \times \frac{5}{26} = \frac{18}{13} \approx 1.585$

c. $V(X) = \frac{21}{26} \cdot (1 - \frac{18}{13})^2 + \frac{5}{26} \cdot (3 - \frac{18}{13})^2 = \frac{105}{169} \approx 0.621$

Question 8

a. $P(X=2) = \frac{21}{26} \times \frac{21}{26} = \frac{441}{676}$

$P(X=4) = 1 - \frac{441}{676} - \frac{25}{676} = \frac{105}{338}$

$P(X=6) = \frac{5}{26} \times \frac{5}{26} = \frac{25}{676}$

b. (1) $E(X) = 2 \cdot \frac{441}{676} + 4 \times \frac{105}{338} + 6 \cdot \frac{25}{676} = \frac{36}{13} \approx 2.769$

(2) $E(X) = E(X_1) + E(X_2) = \frac{18}{13} + \frac{18}{13} = \frac{36}{13} \approx 2.769$

Question 9

a. For a single dice, $P(X=3) = 2 \times P(X=1) = 2 \times P(X=2) = \cdots 2 \times P(X=2)$

thus $5 \cdot P(4) + 2 \cdot P(4) = 1$,

$P(X=1) = \frac{1}{7}$, $P(X=3) = \frac{2}{7}$

Let X be the sum of two dice, then

~~$P(X=2) =$~~ $E(X) = E(X_1) + E(X_2)$

~~$P(X=$~~ $E(X) = 2 \cdot E(X_1)$

while $E(X_1) = 1 \cdot \frac{1}{7} + 2 \cdot \frac{1}{7} + 3 \cdot \frac{2}{7} + 4 \cdot \frac{1}{7} + 5 \cdot \frac{1}{7} + 6 \cdot \frac{1}{7} = \frac{24}{7}$

~~therefore~~ $E(X) = 2 \cdot \frac{24}{7} = \frac{48}{7} \approx 6.857$

b. The expected value of single fair dice is,

$E(X_1) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$

Let X be the sum of 3 fair dice, then

$E(X) = E(X_1 + X_1 + X_1) = E(X_1 + X_1) + E(X_1) = E(X_1) + E(X_1) + E(X_1)$

$= 3 \cdot E(X_1) = 10.5$