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Question 7

a. $f(n) = 5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3 \quad (n \geq 1)$

when $n \geq 1$, $2n^2 + 3n \geq 5$, thus

$5n^3 + 2n^2 + 3n \geq 5n^3$.

$f(n) \geq 5n^3$,

then $5n^3 \leq f(n) \leq 10n^3$

$5g(n) \leq f(n) \leq 10g(n)$ (when $n \geq 1$).

therefore, $f(n) = \theta(n^3)$

b. $f(n) = \sqrt{7n^2 + 2n - 8}$

$= \sqrt{7n^2 + 2n - 24 + 16}$

$= \sqrt{(n+2)(7n-12) + 16} \geq \sqrt{(n+2)(7n-12)}$

let $7n - 12 \geq n + 2$

$6n \geq 14$

$n \geq \frac{7}{3} \Rightarrow n \geq 3$.

thus when $n \geq 3$, $f(n) \geq \sqrt{(n+2)(7n-12)} \geq \sqrt{(n+2)^2} = n+2 \geq n$

$f(n) = \sqrt{7n^2 + 2n - 8} = \sqrt{(n+1)(7n-5) - 3} \leq \sqrt{(n+1)(7n-5)}$

let $7n - 5 \geq n + 1$

$6n \geq 6$

$n \geq 1$.

thus when $n \geq 1$, $f(n) \leq \sqrt{(n+1)(7n-5)} \leq \sqrt{(7n-5)^2} = 7n - 5 \leq 7n$

In conclusion, when $n \geq 3$, $n \leq f(n) \leq 7n$,

$g(n) \leq f(n) \leq 7g(n)$

therefore, $f(n) = \theta(n)$

Question 8

$P(E) = (\frac{3}{6})^6 = \frac{1}{64}$

Question 9

All outcomes that first bit is ~~a~~ 1 is {1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111}.

All outcomes that given first bit is a 1 and contains at least two consecutive 0s is {1000, 1001, 1100}.

$P(E) = \frac{3}{8}$

Question 10

a) Assume event E represents exactly 3 boys and 2 girls, then $P(E) = P(boy)^3 \cdot P(girl)^2$

$= C(3,1) \times 0.51^3 \times (1-0.51)^2 \approx \cancel{0.37}\ 0.96$

b) Assume event E represents at least one boy and F represents no boy, then

$P(E) = 1 - P(F) = 1 - (1-0.51)^5 \approx 0.972 \approx 0.97$

c) $P(\text{at least one girl}) = 1 - 0.51^5 \approx 0.965 \approx 0.97$

d) $P(\text{all children of the same sex}) = P(\text{all boys}) + P(\text{all girls}) = 0.51^5 + (1-0.51)^5 \approx 0.06$

e) $P(E) = P(\text{first boy}) + P(\text{at least 2 girls}) - P(\text{first boy and at least 2 girls})$

$= 0.51 + 1 - (0.51)^5 - 0.51^4 \times (1-0.51) \times C(4,1)$

$- 0.51 + [1 - 0.51^4 - 0.51^3 \times (1-0.51) \times C(4,1)]$

≈ 0.967

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Question 11

a) $P(\text{boy}) = P(\text{girl}) = 1/2 = 0.5$

$P(\text{no boy}) = P(\text{all girls}) = 0.5^5 = \frac{1}{32}$

b) $P(\text{boy}) = 0.51$

$P(\text{no boy}) = (1-0.51)^5 \approx 0.028$

c) $P(\text{no boy}) = [1-(0.51-\frac{1}{100})] \times [1-(0.51-\frac{2}{100})] \times$

$[1-(0.51-\frac{3}{100})] \times [1-(0.51-\frac{4}{100})] \times$

$[1-(0.51-\frac{5}{100})]$

≈ 0.038

Question 12

a) $P(\text{no failures}) = p^n$

b) $P(\text{at least 1 failure}) = 1-p^n$

c) $P(\text{at most 1 failure}) = p^n + C(n,1) \times (1-p) \times p^{n-1}$

$= p^n + n(1-p)p^{n-1}$

d) $P(\text{at least two failures}) = 1-p^n - C(n,1) \times (1-p) \times p^{n-1}$

$= 1-p^n - n(1-p)p^{n-1}$