

**Homework #1**  
**Due by Friday 4/20 11:55pm**

**Submission instructions:**

1. You should submit your homework in the NYU Classes system.
2. For this assignment you should turn in a '.pdf' file with your answers. Name your file 'YourNetID\_hw1.pdf'
3. For questions 1-3, make sure to include the conversion calculations, not just the final answer.

**Question 1:**

A. Convert the following numbers to their decimal representation:

1.  $10011011_2 =$
2.  $456_7 =$
3.  $38A_{16} =$
4.  $2214_5 =$

B. Convert the following numbers to their binary representation:

1.  $69_{10} =$
2.  $485_{10} =$
3.  $6D1A_{16} =$

C. Convert the following numbers to their hexadecimal representation:

1.  $1101011_2 =$
2.  $895_{10} =$

**Question 2:**

Solve the following, do all calculation in the given base:

1.  $7566_8 + 4515_8 =$
2.  $10110011_2 + 1101_2 =$
3.  $7A66_{16} + 45C5_{16} =$
4.  $3022_5 - 2433_5 =$

### **Question 3:**

A. Convert the following numbers to their 8-bits two's complement representation:

1.  $124_{10} =$
2.  $-124_{10} =$
3.  $109_{10} =$
4.  $-79_{10} =$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation:

1.  $00011110_{8 \text{ bit 2's comp}} =$
2.  $11100110_{8 \text{ bit 2's comp}} =$
3.  $00101101_{8 \text{ bit 2's comp}} =$
4.  $10011110_{8 \text{ bit 2's comp}} =$

### **Question 4:**

A. For each of the following sets, determine whether 2 is a member of that set.

- |   |                                  |
|---|----------------------------------|
| <b>a)</b> $\{x \in \mathbf{R} \mid x \text{ is an integer greater than } 1\}$ | <b>b)</b> $\{2, \{2\}\}$         |
| <b>c)</b> $\{x \in \mathbf{R} \mid x \text{ is the square of an integer}\}$   | <b>d)</b> $\{\{2\}, \{\{2\}\}\}$ |
| <b>e)</b> $\{\{2\}, \{2, \{2\}\}\}$   | <b>f)</b> $\{\{\{2\}\}\}$        |

B. Determine whether each of these statements is true or false.

- |                                 |                                       |                                 |
|---------------------------------|---------------------------------------|---------------------------------|
| <b>a)</b> $x \in \{x\}$         | <b>b)</b> $\{x\} \subseteq \{x\}$     | <b>c)</b> $\{x\} \in \{x\}$     |
| <b>d)</b> $\{x\} \in \{\{x\}\}$ | <b>e)</b> $\emptyset \subseteq \{x\}$ | <b>f)</b> $\emptyset \in \{x\}$ |

C. Find two sets  $A$  and  $B$  such that  $A \in B$  and  $A \subseteq B$ .

D. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

- i. the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi.
- ii. the set of people who speak English, the set of people who speak Chinese
- iii. the set of flying squirrels, the set of living creatures that can fly

### **Question 5:**

Let  $A = \{a, b, c, d, e\}$  and  $B = \{a, b, c, d, e, f, g, h\}$ . Find:

- a)  $A \cup B$
- b)  $A \cap B$
- c)  $A - B$
- d)  $B - A$

**Question 6:**

Draw the Venn diagrams for each of these combinations of the sets  $A$ ,  $B$ , and  $C$ .

- a)  $A \cap (B - C)$
- b)  $(A \cap B) \cup (A \cap C)$
- c)  $(A \cap \overline{B}) \cup (A \cap \overline{C})$

**Question 7:**

Let  $A$ ,  $B$ , and  $C$  be sets. Use a membership table to show that:

- a)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- b)  $(B - A) \cup (C - A) = (B \cup C) - A$
- c)  $\overline{(A \cap B \cap C)} = (\overline{A} \cup \overline{B} \cup \overline{C})$
- d)  $(A - C) \cap (C - B) = \emptyset$
- e)  $(A - B) - C \subseteq (A - C)$

**Question 8:**

Let  $A$ ,  $B$ , and  $C$  be sets. Use a set identities to show that:

- a)  $A - B = \overline{B} - \overline{A}$
- b)  $(A \cap B) \cup (A \cap \overline{B}) = A$
- c)  $A - (B - C) = (A - B) \cup (A - \overline{C})$

**Question 9:**

Can you conclude that  $A=B$ , if  $A$ ,  $B$ , and  $C$  are sets, such that:

- a)  $A \cup C = B \cup C$ ?
- b)  $A \cap C = B \cap C$ ?
- c)  $(A \cup C = B \cup C) \text{ and } (A \cap C = B \cap C)$ ?

**Question 10:**

Find  $\bigcup_{i=1}^{\infty} A_i$  and  $\bigcap_{i=1}^{\infty} A_i$ , if for every positive integer  $i$ :

- a)  $A_i = \{-i, -i + 1, \dots, -1, 0, 1, \dots, i - 1, i\}$
- b)  $A_i = \{-i, i\}$
- c)  $A_i = [-i, i]$ , that is, the set of real numbers  $x$  with  $-i \leq x \leq i$
- d)  $A_i = [-i, \infty)$ , that is, the set of real numbers  $x$  with  $x \geq -i$