Date

Question 9

Let set A be consist of any 5 integers.

Let set B be all possible remainders when an integer is divided by 4, then $B = \{0, 1, 2, 3, 4\}$, $|B| = 4$

Assume there is no repetitive element in A, then $|A| = 5$.

According to Pigeonhole principle, there must be two elements in set A mapping to set B.

Assume there are repetitive elements in A, such as $a_1 = a_2$, then their remainders when divided by 4 are equal.

Question 10

Let set A be the network consisting of 6 computers.

Let set B be all possible results of ~~and~~ number of other computers connected,

then $B = \{1, 2, 3, 4, 5\}$, $|B| = 5$

since $|A| = 6$,

according to pidgeonhole principle, there are at least two computers that are directed to the same number of other computers.

Question 11

Let set A be the 51 numbers selected from $\{1, 2, \cdots, 100\}$,

then $|A| = 51$,

Note that for each element from $[1, 50]$, there is an element from $[51, 100]$ that their sum equals to 101,

the corresponding relation can be represented as

$B = \{\{1, 100\}, \{2, 99\}, \cdots \{50, 51\}\}$, $|B| = 50$.

Assume that ~~50 elemen~~ there is only one element in A is in range [1, 50] and all others are 51 ~ 100, there's always an element in B that shows a corresponding integer that makes their sum equal to 101.

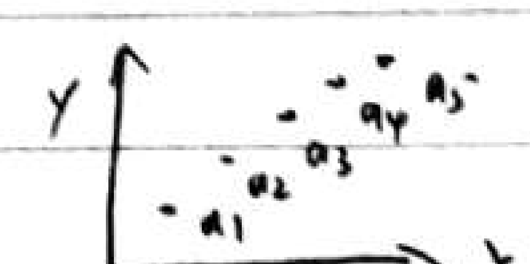
Question 12

Let set A be the 51 houses. $|A| = 51$.

Let set B be all non-consecutive even integers from 1000 to 1099, then $B = \{1000, 1002, 1004, \cdots, 1098\}$

$|B| = 50$.

Assume that all houses with even-number-address are in A, then the only one left cannot be an even number without repetition, thus there is always one ~~even~~ odd number in A. Therefore, there is always an even number beside that odd number, which means these two ~~house~~ addresses are consecutive.



Question 13

~~Let~~ Let set A be all possible pair of points that their mid points are integer coordinates,

$A = \{\{a_1, a_3\}, \{a_2, a_4\}, \{a_3, a_5\}, \{a_1, a_5\}\}$

Let set B be all possible integer midpoints,

$B = \{a_2, a_3, a_4\}$

Since, $|A| = 4$, $|B| = 3$

then for ~~every~~ a specific element in B, there exists ~~at least~~ two elements in A mapping to it,

which is to say, for every possible integer midpoint, there is at least one pair of points that can form a line that fits the given condition.

eg. for mid point a_3 (3, 3), there are two corresponding lines, ~~some~~ which is (a_1, a_5) and (a_2, a_4).