Analysis of Overtaking Rules in Multi-Lane Freeways: Based on State Transition Approach and Cellular Automaton

Luowei Zhou*, Wanjia Zhu, Xiuyu Wang and Meilin Zhu

***Abstract*—The keep-right-except-to-pass rule is widely implemented in multi-lane freeways all around the world, but it may not be an optimum one. In this paper, we proposed a State Transition Approach to analyze the "keep right" rule's performance theoretically. To test the theoretical results, we build a simulation model based on Improved Cellular Automaton, taking the safe distance effect and velocity difference effect into account. In order to seek a better traffic rule, we develop 3 new rules based on the old rule. After comparing them with the old rule in terms of the five evaluation criteria we think of, we arrive at our conclusion: in normal countries, the best traffic rule is the one which forbids cars from overtaking to achieve the best safety performance and highest traffic flow. In countries where the terrain is complex and the speed limits have to be adjusted frequently, the best rule is the one in which all existing lanes are identical, and drivers are allowed to drive on any lane as they can.**

***Index Terms*—keep-right-except-to-pass rule, state transition, cellular automaton, safe distance, velocity difference.**

I. Introduction

Recent changes in economics and technology have enabled an ever increasing households to own their private cars, but at the same time, have posed more pressure on highway capacity. It is necessary for the government to implement proper traffic rules which can maximize the traffic flow while ensuring safety. The most popular traffic rule for the time being is the keep-right-except-to-pass rule, which states that all drivers should drive in the right-most lane unless they are passing another vehicle, and when overtaking, drivers should move one lane to the left to pass the vehicle ahead and return to the right lane as soon as possible. This rule is widely implemented in most countries in the world, including the U.S and China. In Great Britain and some other countries, this rule is adjusted with a simple change of orientation, and we can call it the keep-left-except-to-pass rule.

However, is the "keep right" rule optimum? Or, is there any alternative traffic rule superior to this one in terms of traffic flow, safety, and other important factors? This is one main issue we want to deal with in this paper.

Recently, traffic flow research has attracted much attention from research community. Existing approaches can be divided into two categories: macroscopic model and microscopic model, and the former one mainly concentrates on collective behavior while the other one focuses on the individual behavior, such as lane-changing, between each vehicle.

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Macroscopic models [1][2], usually based on a continuum approach, have been successfully applied to single-lane freeway. When it comes to multi-lane situation, actually, these models turn out to be less efficient and sometimes highly complex.

As for microscopic models, the cellular automaton (CA) model has received increasing scientific researches since the 1990s [3]. In1992, Nagel and Schreckenberg [4] pointed out the first cellular automaton—NaSch Model—to simulate the single-lane freeway traffic modelling problem. Later, other approaches [5][6][11] dealing with the single-lane problem were proposed, such as the Optimal Velocity (OV) model and the Fukui-Ishibashi (FI) model. However, in real traffic, freeways are always multi-lane and vehicles can overtake each other by means of lane-changing behavior. Thus in 1996, Rickert [7] extended the single-lane NaSch Model to two-lane traffic condition with the help of specific lane-changing rules.

In regard to multi-lane CA models [8][9][10], lane-changing rules play significant roles in the modelling process. Lane-changing rules can be divided into asymmetric and symmetric ones according to the function of each lane. If the function of each lane is homogenous, the lane-changing rules are regarded as symmetric, vice versa. There exists some researches [12][13] of asymmetric lane-changing rules already. Also, current researches on modeling overtaking behavior pay some attention to the effects of age, gender, environment, turn signal effect or users aggressive overtaking maneuvers [14][15][16][17]. However, to the best of our knowledge, we are the first one to take the keep-right-except-to-pass rule into account and analyse overtaking performances on the basis of safe distance effect and velocity difference effect.

We propose a state transition approach to reproduce the traffic condition in multi-lane freeways and we also propose an improved cellular automaton model to simulate the asymmetric lane-changing rule. Considering that we have a better understanding of safe distance [24] as well as the relation between velocity differences and overtaking probability, we are firmly convinced that our models have good performance in reproducing the traffic condition in multi-lane freeways.

For the rest of our paper, we will first describe in detail the old keep-right-except-to-pass rule, together with the three new rules we think of. Then we set up five criteria to evaluate traffic rules. Next, we look into the keep-right-except-to-pass rule. We use a state transition approach to study its performance in light traffic. Heavy traffic situation is also considered. Afterwards, a improved cellular automaton model is built to analyse the Safe Distance Effect and the Velocity Difference Effect as well as verify the results given by the theoretical model. Finally, we compare our three new rules together with the old rule in terms of the five evaluation criteria and get some conclusions.

II. Problem Description

A. Overall System

As stated above, we are concerned with the overtaking behavior in freeways. To characterize the system, we have some general assumptions. First, for simplicity, we only consider a four-lane freeway (two in each direction), with a center dividing strip between the opposing traffic flows. Second, there are no stop lights or intersections to interrupt the flow of traffic, there are no no sharp turns. As to vehicles, we assume that there is only one type of vehicle called "car" in the freeway. Cars enter the freeway in a Poisson manner.

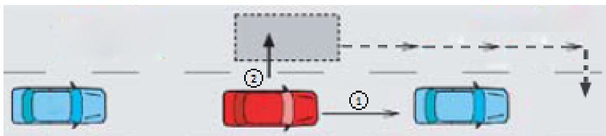


Fig. 1. Keep-right-except-to-pass rule.

The most popular traffic rule for the time being is the keep-right-except-to-pass rule, which states that all drivers should drive in the right-most lane unless they are passing another vehicle, and when overtaking, drivers should move one lane to the left to pass the vehicle ahead and return to the right lane as soon as possible. Our first task is to analyze the performance of the keep-right-except-to-pass rule under various traffic conditions, which can be simplified to just light and heavy traffic conditions. Before start, we shall mention the definition of "light" and "heavy" traffic here. In common sense, when the number of vehicles in the freeway is very huge, we call the situation as "heavy traffic". But what does "huge" mean? This definition is rather vague and not appropriate for research use. In this paper, we state "light" and "heavy" in this way: if a car which changes lanes to overtake cannot find a place to return to its initial lane within certain time range, the traffic is heavy. Otherwise the traffic is light. In section **4**, we will use the variable λ (arrival rate) to illustrate. The closer λ is to 0.5, the heavier the traffic is.

B. Description of The New Rules

Intuitively, we know that if all the cars in the freeway obey the keep-right-except-to-pass rule, they will mostly travel in the right lane so that the traffic flow will be small and the freeway will be inefficient. Moreover, when the traffic is heavy, many cars in the left lane have no chance to return to the previous lane, so the "keep right" rule is broken even if drivers obey it. We will use theoretical analyses to prove these conjectures in section **3**. But here we shall think of three new rules based on our common sense in advance as alternatives to the old rule. In the following sections, we will illustrate that those new rules are more efficient than the old one in one specific aspect or another.

New Rule 1

Cars with different level of speed travel in the different lanes. The car drive in the faster lane can pass another vehicle by move one lane to the slower lane, pass, and return to their former travel lane. We divide the acceptable speed range to several intervals, and cars are required to drive in the lane according to their travel speed. Left lanes have higher speed requirement and right lanes have lower speed requirement. So the left-most lane have the highest speed requirement while the right-most lane have the lowest one. The car in the left lane can pass another vehicle by move one lane to the right, pass, and return to their former travel lane.

For a four-lane freeway (two for each direction), the rule requires cars with higher speed to drive in the left lane and cars with lower speed to drive in the right lane. The car in the left lane with higher speed can overtake another vehicle by occupying the right lane for a while and then return to their former travel lane. For example, when the speed limit is 100km/h, the speed requirement for left lane is 50-100km/h while that for right lane is 0-50km/h. And a faster car in the left lane can overtake another car by occupying the right lane for a while.

New Rule 2

Cars with different level of speed travel in the different lanes. Overtaking is forbidden in this rule and all cars travel along their own lane. We divide the acceptable speed range to several intervals, and cars are required to drive in the lane according to their travel speed.

As for a a four-lane freeway (two for each direction), the rule requires cars with higher speed to drive in the left lane and cars with lower speed to drive in the right lane. For example, if the speed limit is 100 km/h, the speed requirement for left lane is 50-100 km/h while that for right lane is 0-50 km/h.

New Rule 3

This is a rule that allows drivers to drive on all lanes and there are no difference between lanes. Cars should travel in their own lane unless they are passing another vehicle, in which case they move one lane to the another, pass, and return to their former travel lane.

In addition to the above descriptions, given that drivers would leave the freeway through entrances or leave temporarily to rest and service area, to make the second and third rules more practical, we recommend to build up buffer areas in freeways, allowing vehicles in left lane to change lane to right lane so as to leave the freeway. Generally speaking, buffer areas make up a small part of the whole freeway, thus, we neglect the effect of buffer areas on the overall performance. Further researches can be taken to study the influence of buffer areas.

III. Model

A. Traffic Rule Evaluation

We must set up several basic criteria to analyze the performance of the keep-right-except-to-pass rule. These evaluation criteria include both static ones (such as safety) and comparative static ones (such as performance in extreme conditions). Also, the criteria should be easily calculated from available data. We choose the following five evaluation criteria:

Traffic flow: The number of cars passing an observing point per unit of time. Here we set its unit as "vehicles per second".

Danger index: The average number of lane changes for each car in our assumed freeway. It measures safety. Overtaking is risky because the car behind may crash into the car ahead when changing lanes. The more frequently a car changes lanes, the more likely an accident may take place. In contrast, when a car travels in a fixed lane, it can adjust speed instantly when encountering another car ahead, so it is not possible that an crashing accident took place. For simplicity, we assume that accidents will happen only when the car changes lanes. So we can use the number of lane changes per car as a measure of safety. Its unit is "times per vehicle".

Average traffic speed: The average speed of all cars passing an observing point. Its unit is "kilometer per hour". It measures how fast cars are under a specific traffic rule.

USL effect: It stands for "Under-posted Speed Limit effect". Celeris paribus, if the traffic flow in a situation where the speed limit is too low is a, and the traffic flow in a situation where the speed limit is moderate is b, then the USL effect equals the ratio of a to b. It measures the performance of a traffic rule in extreme conditions. If the traffic flow decreases too sharply when speed limit is too low or too high, we do not regard the traffic rule as a good one.

OSL effect: Accordingly, it stands for "Over-posted Speed Limit effect". Its definition and function are similar to those of the USL effect.

Then how to evaluate a specific traffic rule? Since the traffic flow and the average traffic speed both measure the efficiency and capacity of freeways, the larger they are, the better the rule performs. For the danger index, we hope it to be as small as possible so as to decrease the probability of an accident. USL and OSL effects measure the rule's role in extreme conditions. What we want is that the traffic flow of the freeway was still relatively high when the speed limit was too low or too high. So these two effects of a good traffic rule should also be large.

B. Performance In Light Traffic

In light traffic condition, there are not many cars, so the distance between neighboring cars is big. Consequently, it is comparatively easy to change lanes, overtake and return without worrying about being collided or finding nowhere to return. In fact, the traffic state at time t can be computed from the traffic state at time 1 through multiple iterations. This is similar to the Markov model, but the difference lies in that the transition probability matrix is not constant. Following this thinking, we use a state transition approach to look for a steady state of traffic flow.

In addition to the general assumptions mentioned above, we consider here the following situation: all the cars enter the system through the right lane. According to speed, they are divided into 3 discrete states: State 1, State 2 and State 3. The speed of a car in State i is different from that in State j if $i \neq j$. After overtaking the car ahead , the car at a higher speed will return to the right-most lane as soon as possible, and travel at its previous speed v_i, because the drivers may prefer to different speeds in the freeway. If a car chooses not to overtake, it will change to the same state with the preceding car, avoiding collision. For simplicity, we arbitrarily ignore the accelerating process of overtaking. In other words, the overtaking process is finished immediately.

1) Notations:

- v_i: The speed of cars in State i, $i = 1,2,3$. For $i > j$, we have $v_i < v_j$.
- v_4: The speed of cars in the left lane. Whatever the car's free speed is, if it changes lanes to the left, it has to travel at the speed of v_4. We assume $v_4 > max(v_1, v_2, v_3)$ for the convenience of overtaking.
- $\pi_k^{(t)}$: The proportion of cars in state k after t times of transitions.
- p_{ij}: The probability that a car in State i moves to State j. According to subsection **3.3**, $j \geq i$.

2) A State Transition Approach: According to FRESIM(the freeway model within the CORSIM software), the probability of overtaking when a car catches up with another in the right lane positively correlate with their relative speed. For convenience, let p_1 represent the overtaking probability of when State 1 meets State 2, that is, $p_1 = p(v_1 - v_2)$, where $p > 0$ is a proportional coefficient. Similarly, $p_2 = p(v_1 - v_3)$, $p_3 = p(v_2 - v_3)$.

Figure 2 illustrates how an overtaking takes place. The car behind follows a probability distribution to decide whether to follow or overtake. If it choose to follow, its speed must slow down to avoid collision.

Initially, the proportions of cars in State 1,2 and 3 are $\pi_1^{(1)}$, $\pi_2^{(1)}$, $\pi_3^{(1)}$, respectively. Using the Bayes theorem, we can get the probability of State 1 maintaining its state:

$$p_{11} = \frac{\pi_2^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_1 + \frac{\pi_3^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_2$$

Similarly, we can get

$$p_{12} = \frac{\pi_2^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} (1 - p_1)$$

$$p_{13} = \frac{\pi_3^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} (1 - p_2)$$

Putting these probabilities together, the transition probability matrix is

$$\boldsymbol{T}^{(1)} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ 0 & p_3 & 1 - p_3 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Given the initial proportion vector $\boldsymbol{\pi}^{(1)} = (\pi_1^{(1)}, \pi_2^{(1)}, \pi_3^{(1)})$ and the transition probability matrix $\boldsymbol{T}^{(1)}$, we can infer that if a car catches up with another in the system, the proportion vector will be updated like the following:

$$\boldsymbol{\pi}^{(2)} = (\pi_1^{(2)}, \pi_2^{(2)}, \pi_3^{(2)}) = \boldsymbol{\pi}^{(1)} \cdot \boldsymbol{T}^{(1)} \tag{2}$$

Plugging (1) into (2), then extracting the first component of the vector, yields

$$\pi_1^{(2)} = \left[\frac{\pi_2^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_1 + \frac{\pi_3^{(1)}}{\pi_2^{(1)} + \pi_3^{(1)}} p_2 \right] \cdot \pi_1^{(1)}$$

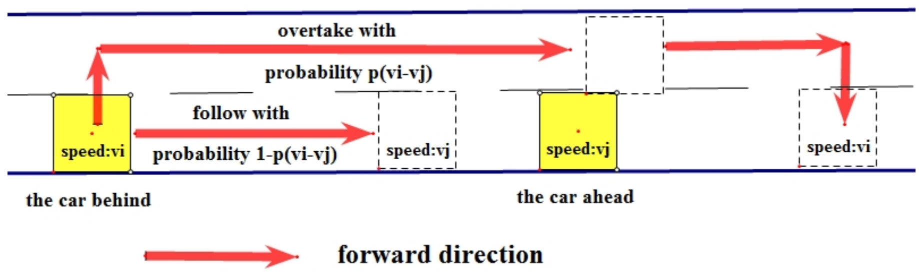


Fig. 2. The choice between following and overtaking($i < j$).

The transition probability matrix will also be updated because the proportion of cars in different states will change after one-time transition. The iteration formula is $\pi^{(n+1)} = \pi^{(n)} \cdot \boldsymbol{T}^{(n)}$. Repeating this process, we get

$$\pi_1^{(n+1)} = \left[\frac{\pi_2^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_1 + \frac{\pi_3^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_2\right] \cdot \pi_1^{(n)} \qquad (3)$$

Due to the property of simple average, we have

$$\frac{\pi_2^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_1 + \frac{\pi_3^{(n)}}{\pi_2^{(n)}+\pi_3^{(n)}}p_2 \leq max(p_1,p_2) < 1 \qquad (4)$$

So, plugging (4) into (3) and iterating for n times, we can get

$$0 \leq \pi_1^{(n+1)} \leq [max(p_1,p_2)]^n \cdot \pi_1^{(1)}$$

Taking the limit, then using the squeeze theorem, yields

$$\lim_{n\to\infty} \pi_1^{(n+1)} = 0$$

That means, after N times of transitions, the proportion of State 1 cars is zero if N approaches to infinity. Thus, $\pi^{(n)} = (0, \pi_2^{(n)}, \pi_3^{(n)})$, and the transition probability matrix between State 2 and State 3 is

$$\boldsymbol{T}_{23}^N = \begin{pmatrix} p_3 & 1-p_3 \\ 0 & 1 \end{pmatrix}$$

From $\boldsymbol{T}_{23}^N$, we can see that State 2 will transfer to State 3 with the probability of $1-p_3$, while State 3 can only maintain State 3 itself. Thus State 3 is an absorbing state. In other words, after $N+M$ times of transitions, all cars will be in State 3(the lowest speed) if M approaches to infinity, that is

$$\lim_{M,N\to\infty} \pi^{(N+M)} = (0,0,1)$$

3) A Numerical Test And Conclusion: We use a numerical method to simulate the iterating and matrix updating processes mentioned above, so as to test our model results.

We might as well set p_1=0.35, p_2=0.45, p_3=0.29.From our notations, $\pi_k^{(t)}$ is the proportion of cars in state k after t times of transitions, and we set their initial values as following: $\pi_1^{(1)}$=0.2, $\pi_2^{(1)}$=0.5, $\pi_3^{(1)}$=0.3. Then we use Matlab to get the following simulation figure:

From **Figure 3** we can clearly see that the proportion vector equals to (0, 0, 1) after 6 times of transitions. This simulation result is consistent with our model. Later, we will employ a simulation method to test this result again in subsection **3.5**.

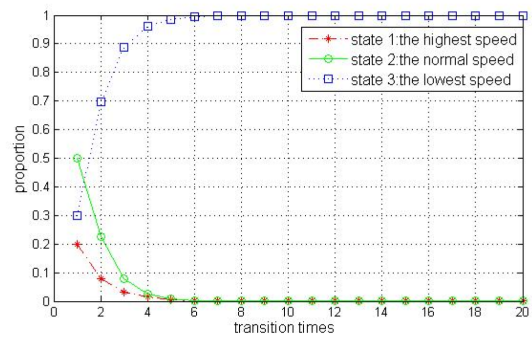


Fig. 3. Proportions of cars in different states after several transitions.

According to our above analysis, we can confidently conclude that if all the cars in the freeway obey the keep-right-except-to-pass rule, they will all travel in the right lane at a relatively low speed in the long term, while at the same time, the left lane is comparatively empty. So, the traffic flow, one of our evaluation criteria, under the “keep right” rule is small. In order to make better use of our freeways, a more flexible rule is needed which enable drivers to drive in the left lane under certain conditions.

This conclusion only applies to a “light traffic” situation, where cars can overtake and then return to the right lane easily. When the traffic is heavy, all is a different story. Unfortunately, because all cars will travel at the same low speed, traffic in the right lane is doomed to become heavy in the long run. This means that after a certain time point, a car changing lane to the left for overtaking will find nowhere to return! We will look into the “keep right” rule’s performance in heavy traffic in the next subsection.

C. Performance In Heavy Traffic

The model we have developed in light traffic cannot be applied to the situation where traffic is heavy. Since the car which changes lanes for overtaking can always find opportunities to go back to the right lane in light traffic, a transition probability matrix can be calculated to describe the whole system. However, in heavy traffic, a car overtaking another cannot always find a place to return to the right lane, and therefore has to stay in the left lane for a long enough time. As a result, cars which have a high speed and change lanes for overtaking will “pile up” in the left lane. That is to say, in the long run, cars in the right lane are all in State 3, traveling at the speed of v_3, while cars in the left lane are all traveling at a higher speed of v_4. Consequently, there will be no state transitions in heavy traffic, and we cannot get the transition probability matrix. This is the first reason why the state transition approach cannot be applied to heavy traffic.

Another important factor we should take into consideration is the safe distance. Safe distance is the minimum following distance between two neighboring cars to avoid collision. According to Wikipedia and New York State Department of Motor Vehicles, there exists a rule od thumb called “two-second rule” to calculate this safe distance, which states that a driver should ideally stay at least two seconds behind any vehicle that is directly in front of the driver’s vehicle. So if the speed of a car is v_3, then the driver should make sure that the distance between his car and the ahead one is no less than $2v_3$.

In light traffic, safe distance is ignored because there are few cars in the freeway. The distance between neighboring cars is big. However, in heavy traffic, we should consider the safe distance factor because cars in the right lane travel in a long queue. So it is not appropriate to reuse the analyzing method in light traffic. **Figure 4** shows how the heavy traffic system works.

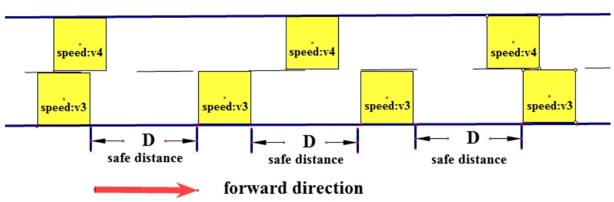


Fig. 4. Road condition when traffic is heavy.

From the analysis above, we can see that there also exists a steady state in heavy traffic situation. In the steady state, cars in the right lane are all at a speed of v_3, while cars in the left lane are all at a speed of v_4. This violates the “keep right” rule in that the left lane is used for driving. Moreover, there will be no transitions between these two states.

Finally, we can get two conclusions about the “keep right” rule in heavy traffic. First, the rule will be broken even if drivers obey it. Second, the danger index is the lowest when traffic is heavy because there is no overtaking.

IV. Simulation

We model the movement of each car separately and use a discrete time simulation to simulate the driving process. Given a set of traffic rules (for example, the keep-right-except-to-pass rule), the simulator can produce a prediction of the movement following the rules. We still assume here that the safe distance for overtaking is proportioned to the traffic speed, obeying the “two-second rule” stated above.

A. Notations

- T: The total time in the simulator.
- S: The length of the simulated freeway, with the unit of cells.
- λ: Arrival rate, number of cars arriving per second.
- L: Length of cell, length of cars.
- D: Safe distance, the safe distance for overtaking.
- d: safe-distance coefficient, determine the relation between the traffic speed v_n and safe distance D, that is, $D = d \cdot v_n$. As we have discussed the safe distance in the assumption, we set d to be 2.
- v_{max}: the maximum velocity of cars.
- p_1: Low-speed overtaking factor, determine the probability of overtaking when two vehicles have slight speed difference. We assume it to be 0.3.
- p_2: High-speed overtaking factor, determine the probability of overtaking when two vehicles have large speed difference. We set it to be 0.7.
- p_a: Acceleration factor, determine the probability of acceleration when cars meet certain conditions. We assume it to be 0.3.
- p_r: Randomization factor, determine the probability of the randomization process. We set it to be 0.7.

B. Safe Distance Effect and Velocity Difference effect

Before introducing our improved CA model for overtaking rules, we will first analyse the safe distance effect and velocity effect about the highway traffic. They are both crucial for enhance the traffic flow, average traffic speed and safety in multi-lane highway.

Safe distance effect, that is, the safe distance requirement between each car has a essential influence on the performance of the multi-lane highway, such as the Danger index, Traffic flow. Intuitively, we know that the greater safe distances, the safer highways. Also, the traffic flow might decrease when the safe distance is greater. Previously, some researchers [19][20] theoretically considered the effect of safe distance requirement on the duration of overtaking and performance of highways. Nevertheless, their numerical tests are not powerful enough to reflect the overall effect of safe distance effect. In this paper, we evaluate the highway performance under different safe distance requirement and strive to find the relation between safe distance and our evaluation criteria.

In addition, generally speaking, cars with large velocity difference are more likely to overtake each other. Here we suppose that, in safe condition, if a car’s velocity and its preceding car’velocity meet the inequality $v_n \geq v_{n+1} + 2$ (the unit of v_n and v_{n+1} is $5m/s$), the following car has a probability of p_2 to overtake its preceding car; otherwise, the probability will decrease to p_1. However, lack of precise statistic, we cannot steadfastly prove that relation is perfect to reflect the real performance of multi-lane highways. More researches are still in great need to analyse the effect of more complex relation between velocity difference and overtaking possibility. Perhaps there is an approximate linear relation or non-linear relation between them. Here, we analyse the system’s overall performance under different p_1, p_2 and strive to take as many as occasions into account to reproduce the real situation to the fullest.

C. Improved CA model for Overtaking Rules

Since we simulate the traffic condition in a freeway, we might as well first describe the freeway and cars in our cellular automaton model. First, the freeway is a rectangular grid, with two rows and S columns. We only consider two lanes of the forward direction (see **Figure 5**). The first row of the grid is the left lane and the second row of the grid is the right lane. Each row has S square cells, the length of each side is L. The $2 \cdot S$ cells can be either empty or occupied by a car with a velocity $v = 0, 1, \ldots, v_{max}$. Cars are numbered $1, 2, \ldots, N$, and our interest is focused on the car numbered n, noted by a velocity v_n. We assume that L equals to 5 meters and v_{max} equals to 6. Also, we set the total time T in the simulator to be 20000 seconds and the length of simulated freeway to be 20000 cells, that is 100 kilometers.

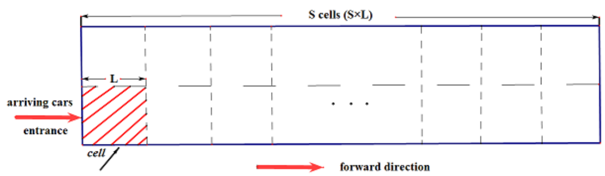


Fig. 5. Description of the simulated freeway.

Further explanations about the simulator are presented in **Figure 6**. An integer gap denotes the number of empty cells in front of a car with velocity v_n and location x_n, and gap_f represent the number of empty cells between a car and its following car in the other lane while gap_p represent the number of empty cells between a car and its preceding car in the other lane.

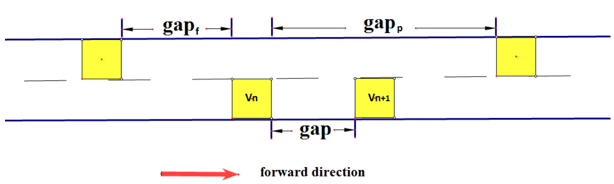


Fig. 6. Illustration of the gaps and velocities in the CA simulation

Before introducing our asymmetric lane-changing rules, that is, keep-right-except-to-pass rule, we briefly recall the definition of the Nagel-Schareckenberg (NaSch) model:

(i) Acceleration: If $(v_n \le gap - 1)$ Then $v_n \to min(v_{max}, v_n + 1)$;
(ii) Deceleration: If $(v_n \ge gap + 1)$ Then $v_n \to min(v_n, gap)$;
(iii) Randomization: $v_n \to max(v_n - 1, 0)$ with probability p;
(iv) Movement(position update): $x_n \to x_n + v_n$.

Next, when the popular keep-right-except-to-pass rule is taken into account, we set up an improved asymmetric lane-changing CA model-Overtaking Cellular Automaton(OCA) to deal with the specific overtaking rule. The new model can be expressed as:

(i) Acceleration: If $(gap \ge d \cdot v_n)$ then $v_n \to min(v_{max}, v_n + 1)$ with probability p_a;
(ii) Left→ right("Return" process, $gap < d \cdot v_n$):
If $(gap_f \ge d \cdot v_n \& gap_p \ge d \cdot v_n)$
Then car moves from left lane to right lane;
(iii) Right→ left("Go" process, $gap < d \cdot v_n$):
If $(gap_f \ge d \cdot v_n \& gap_p \ge d \cdot v_n)$
Then car moves from right lane to left lane with probability: $(p_1$, if $v_n < v_{n+1} + 2)$ or $(p_2$, if $v_n \ge v_{n+1} + 2)$;
(iv) Deceleration: If $(gap < d \cdot v_n \& v_n > v_{n+1})$ Then $v_n \to v_{n+1}$;
(v) Randomization: $v_n \to max(v_n - 1, 0)$ with probability p_r;
(vi) Movement(position update): $x_n \to x_n + v_n$.

There are two stages of overtaking process, the "Go" process (see Figure 7) and "Return" process (see Figure 8). In the "Go" process, if a car meets certain conditions, it moves from the right lane to the left lane, pass the car ahead; in the "Return" process, if a car meets certain conditions, it returns to right lane and finish the whole overtaking process. To express it more clearly, we called the car which is in front of another car "the car ahead", and the car which is behind another car "the car behind". We have several steps to describe each of the process.

The "Go" process
Step 1: For the car in the right lane, judge the value of each cell within safe distance D ahead of the car. If there are cars within the safe distance, go to Step 2;
Step 2: When the speed of the car behind larger than the speed of the car ahead, judge the value of each cell within safe distance ahead and behind in the left lane, as we showed in the **Figure 7**. So that the driver can make sure it is safe to overtake other cars. If there are positive integers within the safe distance, the car behind decelerates to the same speed as the car ahead, otherwise, the car behind judge the speed difference of them. If the speed difference lower than L m/s, go to Step 3, otherwise, go to the Step 4;

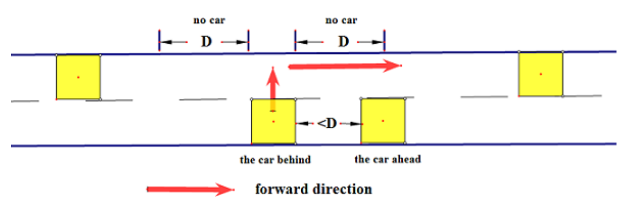


Fig. 7. The "Go" process.

Step 3: the car behind has a possibility of p_1 to overtake, the car change from the right lane to the left lane;
Step 4: the car behind has a possibility of p_2 to overtake, the car change from the right lane to the left lane, obviously, p_2 is larger than p_1.
The "Return" process
Step 1: For a car in the left lane, firstly, it judges the value of each cell within safe distance ahead and behind in the right lane. If there is no car, the car return to right lane, and the "Return" process is finished (see Figure 8), otherwise, go to the Step 2;

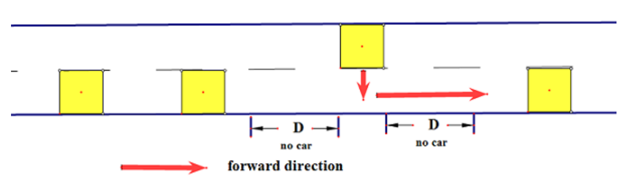


Fig. 8. The "Return" process.

Step 2: The car judge the value of each cell within safe distance ahead of the car. If there are cars within the safe distance and have a lower speed, the car behind decelerates to the same speed as the car ahead.

D. Analysis of Safe Distance Effect and Velocity Difference Effect

Our simulator can reproduce the real environment in freeways and calculate the five evaluation criteria as long as the arrival rate is provided. For example, results are listed below in **Table I** when $\lambda = 0.25$ (vehicles per second) or $\lambda = 0.4$ (vehicles per second). Thus we can analyze the relation between these evaluation criteria and arrival rate (λ) by creating plots of these criteria separately. **Figure 9** to **Figure 10** shows results by running simulating program. As we mention before, greater arrival rate indicates a greater volume of traffic, so the pictures can indicate the role of the old rule in different traffic conditions.

Arrival rate λ	0.25	0.4
Danger index	1.3607	0.9135
Average traffic speed	3.2367	3.2497
Traffic flow	0.2136	0.3316
OSL effect	1.1140	1.1224
USL effect	0.9689	0.9856

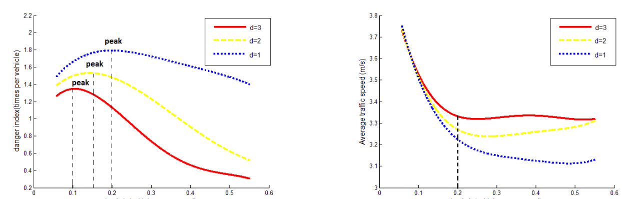
TABLE I
SIMULATION RESULTS.

Generally speaking, as can be seen in **Figure 9(a)** and **Figure 10(a)**, the danger index firstly increases with the increasing of arriving cars. Then the danger index reaches a peak value. After the peak value, the danger index drops and the traffic condition appeals to be more safe. Actually, in the real world, drivers are more safe when the traffic is too light or too heavy compared to the normal volume. For one thing, there is no need to overtake when the freeway has few cars; for another, it is difficult to overtake when the traffic is full of cars. Also, as is shown in **Figure 9(b)** and **Figure 10(b)**,there exists an approximate reverse correlation between average traffic speed and λ. Since greater arrival rate indicates a greater volume of traffic, more cars mean slower driving.

More importantly, through the relation between Danger index and arrival rate, average traffic speed and arrival rate, we can draw some conclusion about the Safe Distance Effect and Velocity Difference Effect. With the increasing of safe-distance coefficient(d), the Danger index decreases rapidly. Meanwhile, the peak value point moves towards the right. For instance, $\lambda = 0.1$ is the peak value point when $d = 1$, while the peak value point becomes greater as $\lambda = 0.2$ when $d = 3$. However, the overtaking probability under different velocity difference cannot affect the peak value point, as is shown in **Figure 10(a)**. In this condition, despite the increasing of Danger index, the peak value point remain constantly as $\lambda = 0.15$. Thus, drivers' desire to overtaking does not necessarily affect the department of transportation's strategy to keep the freeways in safe condition. Efforts should be taken to make the real-time traffic flow much higher than $\lambda = 0.15$ to decrease the Danger index.

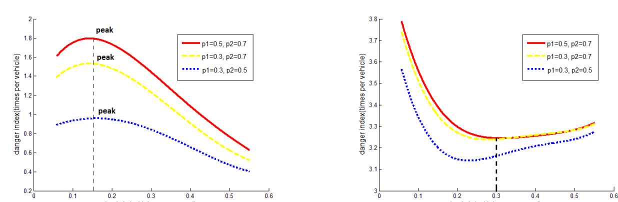
Also, from **Figure 9(a)**, we know that when safe distance requirement becomes larger, the danger index will drop more rapidly after the peak value point. For instance, given that $d = 2$, corresponding to the "2-second rule", when λ changes from 0.15 to 0.4, the Danger index decreases by 33%. The decreasing will reach 67% when safe-distance coefficient d changes to 3. In contrast, if safe distance requirement becomes smaller, the danger index will stay at a high level no matter how we control the traffic flow. Therefore, as for drivers, they should live up to the safe distance requirement to well guarantee their safety; as for departments of transportation, they should strive to coordinate the traffic flow to be far away from the peak value point.

In addition, as for Average traffic speed, both Safe Distance Effect and Velocity Difference Effect are contributing. On one hand, when the arrival rate is relatively low (below 0.2), the Velocity Difference Effect is responsible for the average speed differences. The increasing of p_1 and p_2, especially high-speed overtaking factor p_2, can lead to speed boosting. On the other hand, when the arrival rate is larger (above 0.3), driver's desire to overtaking has fewer influence on average speed. Instead, the Safe Distance Effect becomes really useful and indicates a increasing of Average traffic speed accompanying with the increasing of arrival rate. Considering that in normal traffic condition, the arrival rateλ is closed to 0.3, thus keeping in appropriate safe distance also worth it to pursue a higher Average traffic speed.



(a) Danger index (b) Average traffic speed

Fig. 9. Safe Distance Effect



(a) Danger index (b) Average traffic speed

Fig. 10. Velocity Difference Effect

Here we want to mention the relationship between our simulation results and our model in section **3**. In fact, the simulation results justify the model. Firstly, as for the light condition, we have mentioned a state transition approach to analyse the problem. Then we draw the conclusion that if all the cars in the freeway obey the keep-right-except-to-pass rule, they will all travel in the right lane at a relatively low speed in the long term, while at the same time, the left lane is comparatively empty. In the simulation, we set the arrival rate λ to be 0.25, and we can clearly see from the **Figure 11** that cars in the right lane are much more than cars in the left lane, so the theoretical model is proved.

Secondly, as for the heavy condition, we have concluded that cars in the right lane are all at a same speed while cars in the left lane are all at another speed, moreover, they will be no transitions between these two states. To verify this conclusion, we assume the arrival rate λ to be 0.5, the result is shown in **Figure 12**. From the picture, we notice that many cars in the left lane have no chance to return to the previous lane, so the "keep right" rule is broken even if drivers obey it.

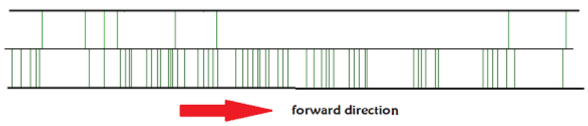


Fig. 11. A printscreen of simulation in light traffic, green lines represent cars.

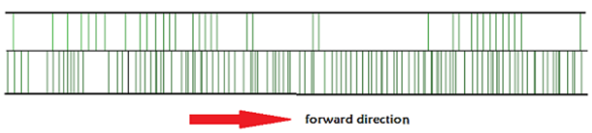
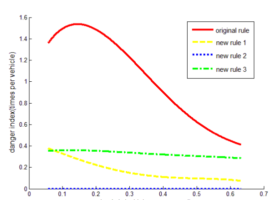


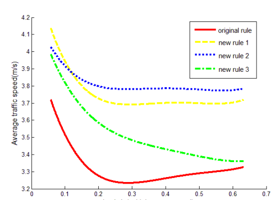
Fig. 12. A printscreen of simulation in heavy traffic, green lines represent cars.

E. Comparison Between Rules

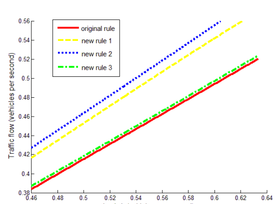
After better understanding of the Safe Distance Effect and Velocity Different effect, we use the improved cellular automaton model to simulate the respective traffic conditions under each of the new rules (mentioned in section **2.2**). The results are shown in **Figure 13**



(a) Danger index vs λ of all rules



(b) Average traffic speed vs λ of all rules



(c) Traffic flow vs λ of all rules

Fig. 13. Relation between traffic criteria and arrival rate(λ)

Of course, the value of each evaluation criterion is dependent on the value of λ (arrival rate). Since $\lambda < 0.2$ means light traffic and $\lambda > 0.5$ means heavy traffic, specifically, we set $\lambda = 0.3$ to calculate. In this way, we can compare the rules' performance in moderate traffic (below 45 vehicles per kilometer). Through running programmes in Matlab, we calculate the five evaluation criteria of each traffic rule. We show the results in **Table II**.

As we have pointed out before, for danger index, the smaller it is, the safer and better the traffic rule is. While for other four evaluation criteria, the larger, the better.

Although we have got the exact values of the five evaluation criteria for each new rule, it is hard to directly compare the rules because a rule may perform well in one aspect while bad in another aspect, and the units of different criteria are also different. Another reason is about the danger index. It negatively relates to the performance of a rule while other criteria show positive relations. In order to effectively compare these new traffic rules and choose the best one, we have to normalize all the criteria, and then convert values to an interval between 0 and 1, from worst to best. We call these normalized criteria as "evaluation indexes" (EI).

The definition of these indexes are as follows. Let S_{ij} denote the simulation result of the rule $i (1 \le i \le 4)$ under evaluation criterion $j (1 \le j \le 5)$. And EI_{ij} denotes the evaluation index of the rule i under evaluation criterion j. When $j = 1,3,4,5$, the evaluation indexes are defined as

$$EI_{ij} = \frac{(S_{ij} - min_{1\le i\le 4} S_{ij})}{max_{1\le i\le 4} S_{ij} - min_{1\le i\le 4} S_{ij}}$$

These indexes are for the traffic flow, the average traffic speed, the USL effect and the OSL effect. For simplicity, we call them TFI, ATSI, USLI, OSLI respectively. For danger index (when $j = 2$), we call its EI as "safety index" (SI), the larger SI is, the safer the rule is.

$$SI = EI_{i2} = \frac{1}{1 + S_{i2}}$$

For all $1 \le i \le 4$, $1 \le j \le 5$, we have $0 \le EI_{ij} \le 1$.

The values of all evaluation indexes are listed below in **Table III**.

We use a radar chart of **Figure 14** to display our results.

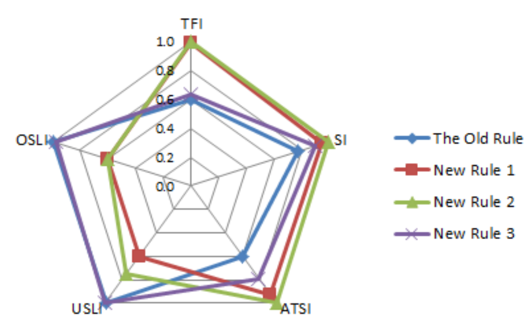


Fig. 14. A view of evaluation indexes for each traffic rule (mapping from (0,1) to (0.6,1)).

The traffic rule whose border is the most outside is the best. From this radar chart we can clearly see that the keep-right-except-to-pass rule is absolutely not optimum. What's more, to our surprise, the performance of New Rule 2 is the best on the whole, superior to other rules in 3 of our 5 individual evaluation

	the old rule	New Rule 1	New Rule 2	New Rule 3
Traffic Flow	0.2512	0.2745	0.2751	0.2530
Danger Index	1.2539	0.1440	0	0.3361
Average Traffic Speed	3.2457	3.6900	3.7824	3.5023
USL Effect	0.9695	0.8961	0.9242	0.9701
OSL Effect	1.1190	1.0379	1.0371	1.1142

TABLE II
EVALUATION OF THE NEW RULES.

	the old rule	New Rule 1	New Rule 2	New Rule 3
TFI	0	0.9749	1.0000	0.0753
SI	0.4437	0.8741	1.0000	0.7484
ATSI	0	0.8278	1.0000	0.4781
USLI	0.9919	0	0.3803	1.0000
OSLI	1.0000	0.0099	0	0.9418

TABLE III
VALUES OF THE FIVE NORMALIZED EVALUATION INDEXES.

indexes (larger traffic flow, higher safety index, large average traffic speed).

This superiority means that the relatively best performance can be achieved if overtaking is prohibited in the freeway. We can see the tradeoff between individual and collective interest here. Drivers who overtake to seek a higher speed indeed realize their individual interest because they may go to work or return home faster. But at the same time, traffic flow, average speed of all vehicles and safety may decrease, many resources are wasted and the collective interest cannot be realized. If we want to maximize the utility of the whole society, like to maximize the traffic flow, we must implement a rule prohibiting overtaking, which deteriorates people's individual interest.

We can also see the tradeoff between five evaluation criteria. A highly efficient traffic rule in normal conditions will not perform well in extreme conditions. New Rule 2 is of this kind. When the speed limit is moderate, traffic under this rule has the biggest flow and average speed, also very safe. But unfortunately, when the speed limit is too low or too high, this rule cannot maintain its good performance. New Rule 3 is also of this kind, but faces the opposite dilemma.

Then what should the government do in view of this tradeoff? We suggest that in a country where the terrain is complex and the speed limits have to be adjusted frequently, the government should take New Rule 3 for its stability in extreme conditions, or local traffic jam may emerge in different parts of the freeway with different speed limits. But in a country where the speed limits always remain stable (this is a normal country, we think), the government should take New Rule 2 to achieve the best safety performance and highest traffic flow.

V. CONCLUSION

To discuss the performance of traffic rules, we set up five evaluation criteria including both static and comparative static standards. Then we apply them to the keep-right-except-to-pass rule in four-lane freeways. First, we build a state transition model to analyze. In light traffic situation, we conclude that if all the cars in the freeway obey the "keep right" rule, they will mostly travel in the right lane at a relatively low speed. Consequently, the traffic flow will be small and the freeway is inefficient. In heavy traffic situation, however, many cars in the left lane have no chance to return to the previous lane, so the "keep right" rule is broken even if drivers subjectively obey it. Second, we create a simulation model to test the theoretical models, it turns out that the simulation results verify our former conclusions well.

Since the old "keep right" rule is not optimum, we come up with three new rules to compare with it. After calculation and comparison, we find that the comparatively best traffic rule completely forbids the overtaking behavior, suggesting that both lanes can be used as carriageways and all cars should drive in their own initial lanes. This new rule performs very well in terms of traffic flow, safety and average traffic speed. But "best" is not always the case. In countries where the terrain is complex and the speed limits have to be adjusted frequently, the best rule is the one which allows drivers to drive on all lanes, meaning that there are no difference between lanes. This new rule performs very stably in all conditions, including extreme conditions.

Our analysis is a combination of microscopic cellular automaton methods and macroscopic state transition methods, which makes it highly comprehensive and convincing. Also, we take account of the Safety Distance Effect, taking into account the "2-second rule" of highway traffic. What's more, we let the overtaking probability be affected by the speed spread of the two neighbouring cars, referred as Velocity Different Effect, which fits well with the reality. But we also overlook some aspects. Lack of empirical data, it is difficult for us to test our simulation results. More statistic data should be collected by relevant departments to test the theory. For future works, it is also better to take into account various types of vehicles like cars and trucks ([21][22][23] are good examples), different types of vehicles may lead to different results.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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