随想随感

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1 chapter 1

Lemma 1.1. *If a rectangle is the almost disjoint union of finitely many other rectangles, say* $R = \bigcup_{k=1}^{N} R_k$*, then*

$$|R| = \sum_{k=1}^{N} |R_k|$$

If $R, R_1, ..., R_N$ *are rectangles, and* $R \in \bigcup_{k=1}^{N} R_k$ *,then*

$$|R| \leq \sum_{k=1}^{N} |R_k|$$

Remark 1.2. 有限个几乎不交的并集，求面积可以直接相加；有交就小于等于。

theorem 1.3. *Every open subset* $\mathcal{O}$ *of* R *can be writen uniquely as acountable union of disjoint open intervals*

Remark 1.4. 任何开区间可以被分解为可数个不交的开区间的并集

theorem 1.5. *Every open subset* $\mathcal{O}$ *of* $\mathbb{R}^d$ *,*$d \geq 1$*, can be written as acountable union of almost disjoint closed cubes*

Remark 1.6. 用小立方体*(d*维*)*去不断逼近

definition 1.7 (Cantor Set)**.** 非空，有界，闭集，完全不连通，不可数集，零测

definition 1.8 (Exterior Measure)**.** *if* E *is any subset of* $\mathbb{R}^d$*, the exterior measure of* E *is*

$$m_*(E) = \inf \sum_{j=1}^{\infty} |Q_j|$$

consedering any arbitrary covering of a closed cube $Q \in \bigcup_{j=1}^{\infty} Q_j$($Q_j$ is also closed cube), note that it suffices to prove that

$$|Q| \leq \sum_{j=1}^{\infty} |Q_j|$$

稍微放大Q_j, 得到 open cube S_j, 之后可以利用有限覆盖定理, 得到

$$|Q| \leq (1+\epsilon) \sum_{j=1}^{M} |Q_j| \leq (1+\epsilon) \sum_{j=1}^{\infty} |Q_j|$$

So

$$|Q| \leq m_*(Q)$$

对于 closed cube Q, $m_*(Q) \leq |Q|$显然, $|Q| \leq m_*(Q)$ 需要稍微放大Q_j成 open cube S_j, 之后用有限覆盖定理

对于 open cube Q, 取 closure of Q, 那么$m_*(Q) \leq |Q|$也同样显然，$|Q| \leq m_*(Q)$ 需要取任意被Q覆盖的 open cube Q_0, 而任何对于Q的立方体开覆盖必然覆盖Q_0, 所以下确界 $m_*(Q_) \geq |Q_0|$, 又可以取充分接近, 可得$|Q| \leq m_*(Q)$