Plasticity

- Spatial working memory: storing information in a continuum of attractors
- Memory in a noisy environment
- Short term plasticity: Tsodyks-Markram formalism and application to an ODR task

Spatial working memory and short term plasticity

- The models we have considered before are characterized by
  - Discrete attractors
  - Extremely non-linear neuron dynamics
  - Both features favor the performance of the system as we have well defined attraction basins

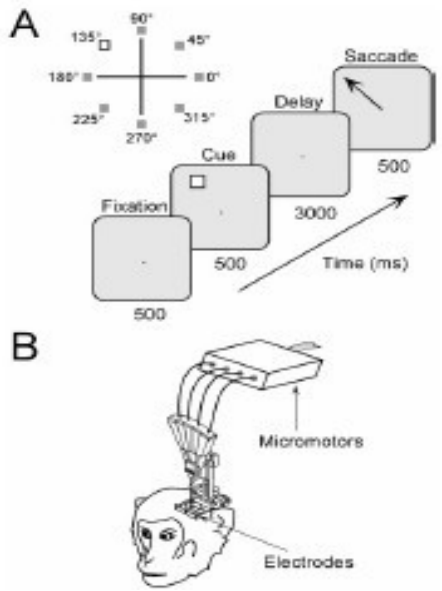
- What would be the most difficult case?
  - Continuous attractors
  - Quasi-linear dynamics

Spatial working memory and short term plasticity

- Continuous attractors
  - This will happen if we want to remember the value of a continuous variable (space, frequency, wavelength, etc)

- The non-linearity depends on
  - The intrinsic neuron dynamics
  - The level of noise: more noise will tend to linearize the effective f-I curve of the neurons

Persistent Activity During an Oculomotor Delayed Response Task



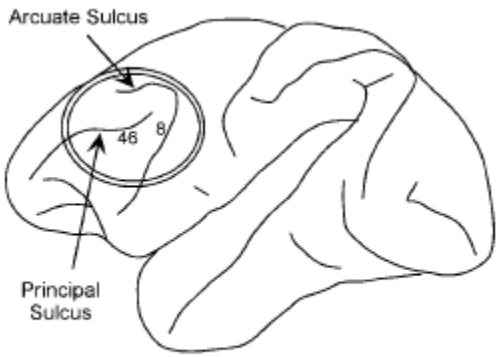


Figure 1. Location of electrophysiological recording in dorsolateral prefrontal cortex, centered on area *46* and the frontal eye fields (area *8*). This region included the caudal half of the principal sulcus and cortex lining the arcuate sulcus.

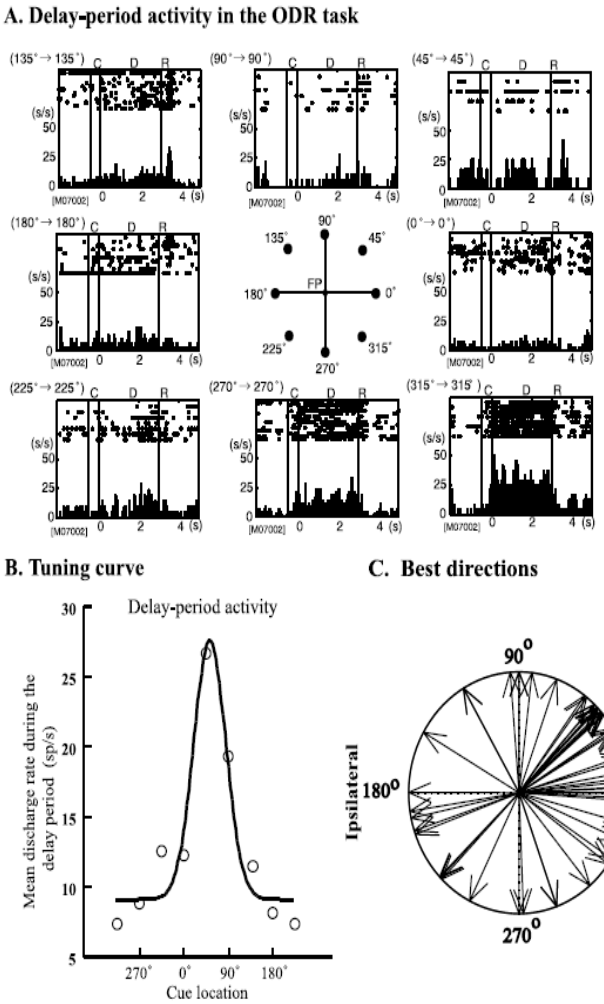
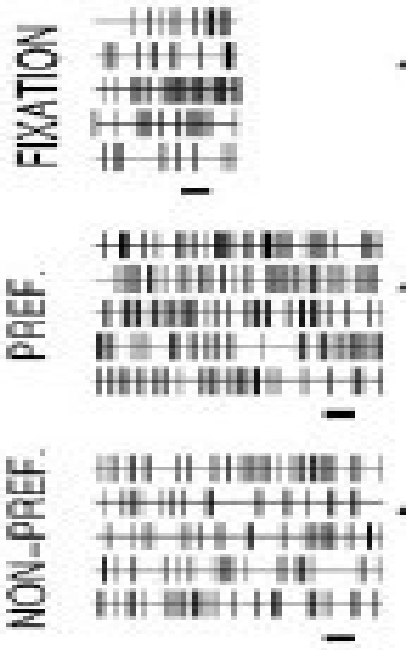
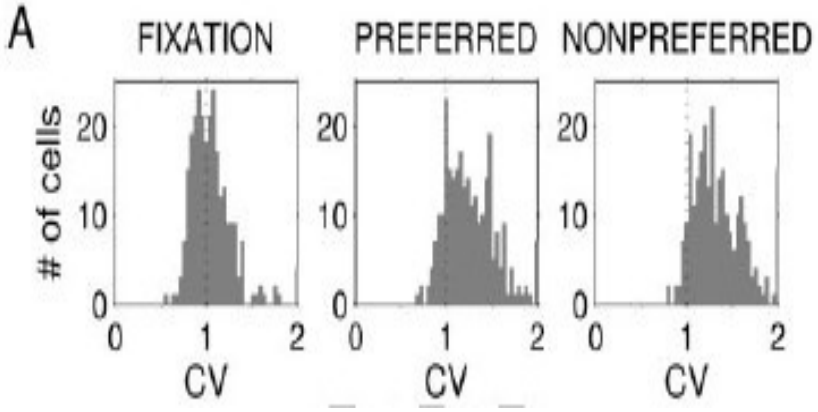


Fig. 2. Characteristics of delay-period activity observed in DLPFC neurons. (A) An example of directional delay-period activity. (B) An example of a tuning curve of directional delay-period activity. (C) Polar distribution of the best directions of delay-period activity. A majority of the best directions were directed toward the contralateral visual field.

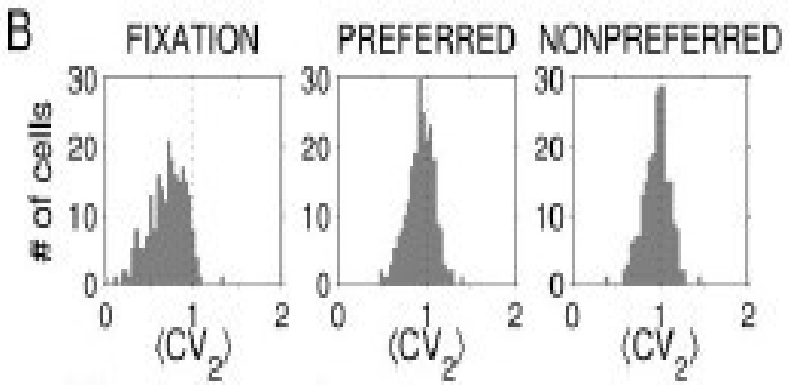
Funahashi, 2006

Persistent Activity During the Delay Period is Highly Variable





$CV=[<ISI^2>-<ISI>^2]^{1/2}/<ISI>$

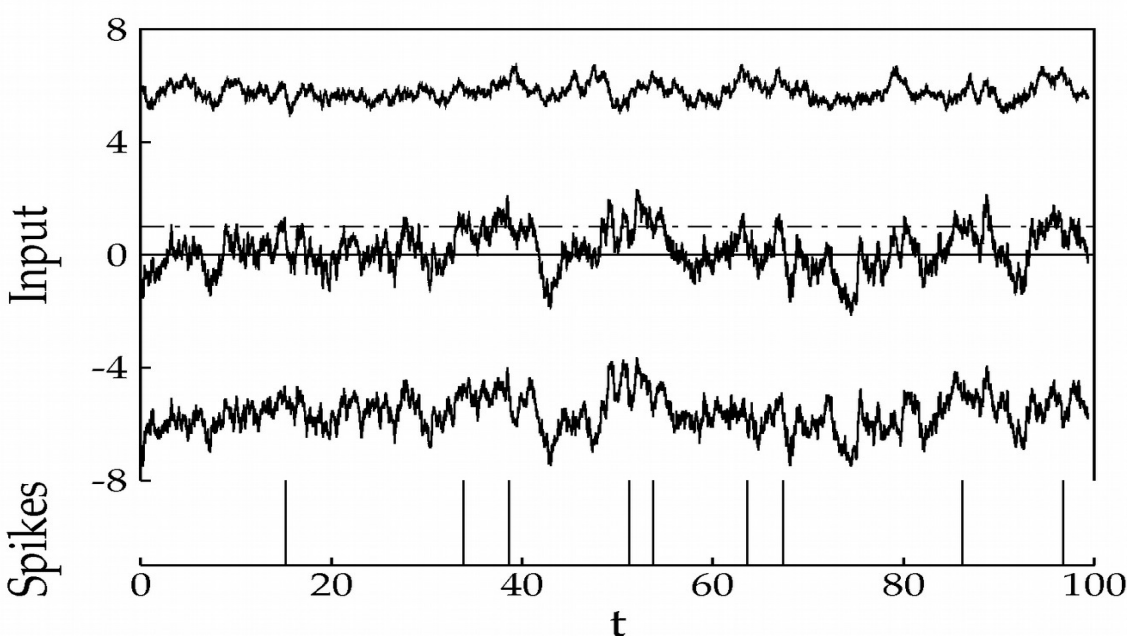


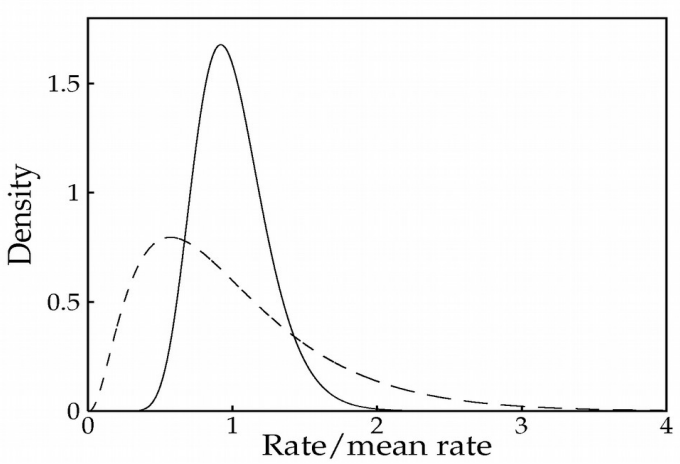
Compte et al, 2003

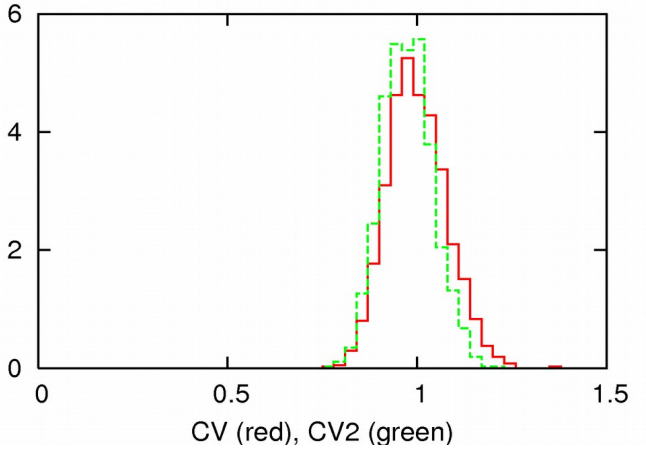
$<CV_2>=2<|ISI_{n+1}-ISI_n|/[ISI_{n+1}+ISI_n]>$

Balance of excitation and inhibition explains irregular activity...

- N_E, N_I excitatory and inhibitory neurons
- K connections per neuron (average)
- $1 << K << N_E, N_I$
- Synaptic efficacy O(1/sqrt(K))
- Average excitatory input O(sqrt(K))
- Average inhibitory input O(sqrt(K))
- Net input and fluctuations O(1)
  (van Vreeswijk and Sompolinsky, 1996)







But this is problematic for having persistent states

- $f_E=F(h_E)$, $f_I=F(h_I)$
- f: firing rate; h: synaptic input; F: input-output transfer function

$$h_E=\mathrm{sqrt}(K)\{G_{EE} f_E - G_{EI} f_I + I_E\}$$
$$h_I=\mathrm{sqrt}(K)\{G_{IE} f_E - G_{II} f_I + I_I\}$$

- In the limit of very large K it must be

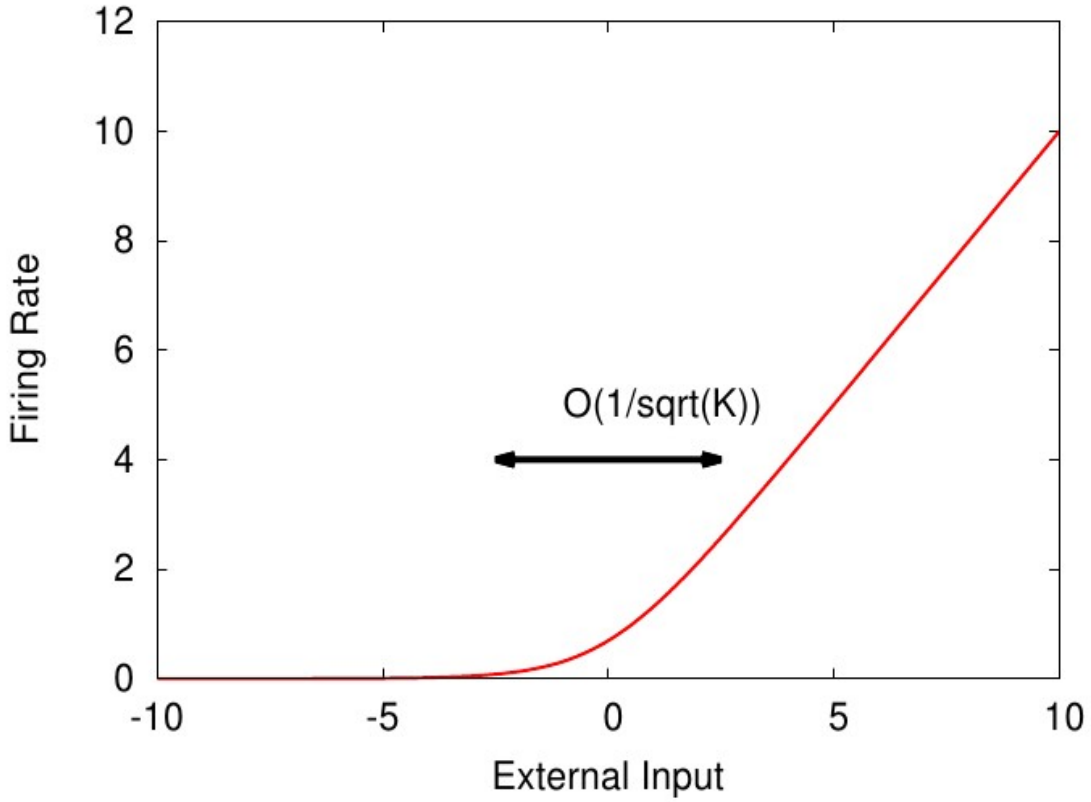
$$0=G_{EE} f_E - G_{EI} f_I + I_E$$
$$0=G_{IE} f_E - G_{II} f_I + I_I$$

- There is a LINEAR relation between the firing rates and the external inputs. Single neuron input-output transfer function (**F**) is irrelevant

- **Multistability is not possible between balanced states if synapses are linear**

But this is problematic for having persistent states

- On the other hand, for finite values of connectivity some non-linearity is still present
- Some models work in this regime (e.g. Renart et al, 2007)
- They require an increasingly precise tuning of parameters as K increases



How can we reconcile balanced states and persistent states in a robust way?

- A possible solution:

  Non Linearities in Synapses e.g. Short Term Synaptic Plasticity

Modeling short term synaptic plasticity

- The synaptic connection is characterized by its absolute amount of ‘resources,’ which can be partitioned into three states: effective, inactive, and recovered
- If all the resources are activated by a presynaptic spike, this would generate the maximal possible response defined here as the absolute synaptic efficacy
- Each presynaptic spike activates a certain fraction of resources available in the recovered state, which then quickly inactivates with a time constant of a few milliseconds and recovers with a time constant of about 1 sec.
- This model could reflect various possible biophysical mechanisms of synaptic depression, such as receptor desensitization or depletion of synaptic vesicles

(Tsodyks & Markram, 1997)

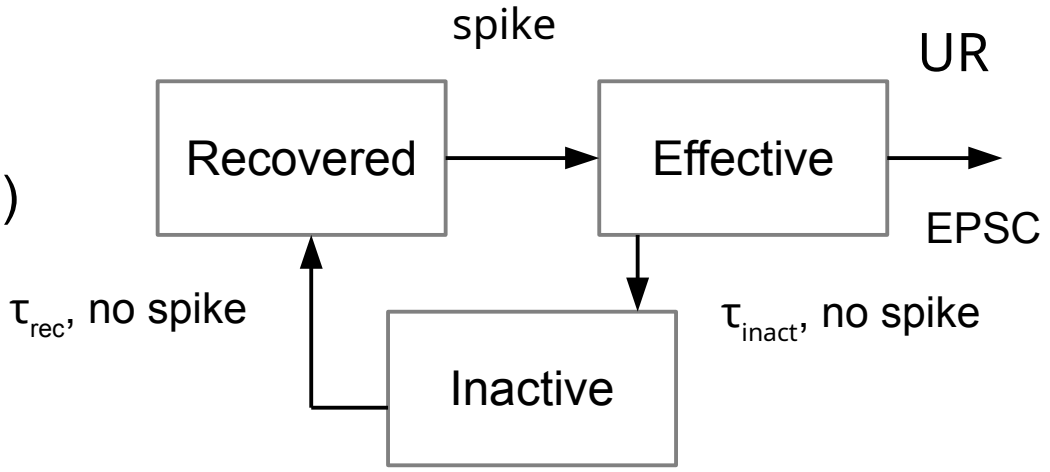
Modeling short term synaptic plasticity

- Kinetic equations for each on of the three states are

$$dR/dt = I/\tau_{rec} - U\,R\,\delta(t-t_{spike})$$

$$dE/dt = -E/\tau_{inact} + U\,R\,\delta(t-t_{spike})$$

$$dI/dt = -I/\tau_{rec} + E/\tau_{inact}$$



where U is the utilization of synaptic efficacy parameter

- The size of the excitatory postsynaptic current (EPSC) for a periodic spike train of frequency f is (in the limit of small τ_{inact}):

$$EPSC \approx 1/(f\,\tau_{rec})$$

Modeling short term synaptic plasticity

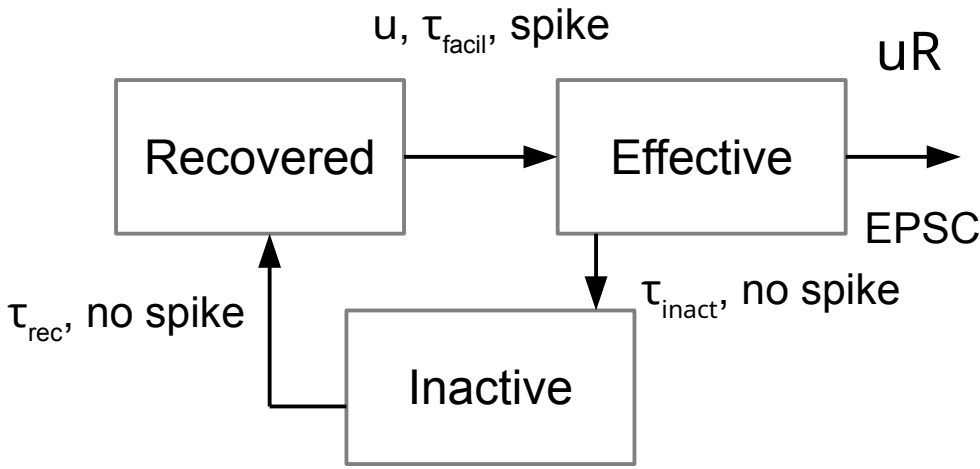
- In Markram, Wang & Tsodyks, 1998, facilitation is included in the model
- The utilization of synaptic efficacy is increased at each spike

$$dR/dt = I/\tau_{rec} - u\,R\,\delta(t-t_{spike})$$

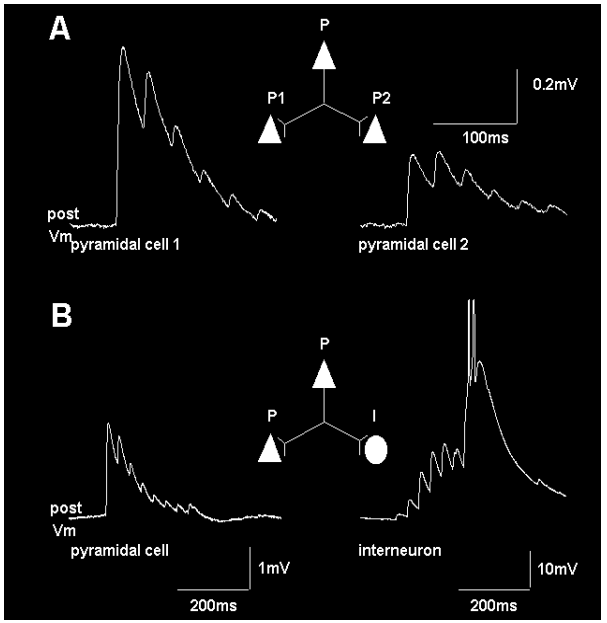
$$dE/dt = -E/\tau_{inact} + u\,R\,\delta(t-t_{spike})$$

$$dI/dt = -I/\tau_{rec} + E/\tau_{inact}$$

$$du/dt = (U-u)/\tau_{facil} + U(1-u)\,\delta(t-t_{spike})$$



Modeling short term synaptic plasticity



- 4 Key Synaptic Parameters (in the limit of small τ_{inact}):

- Absolute strength : A
- Probability of release : U
- Depression time constant: τ_{rec}
- Facilitation time constant: τ_{facil}

(Markram, Wang & Tsodyks, 1998;
Tsodyks, Pawelzik & Markram, 1998)

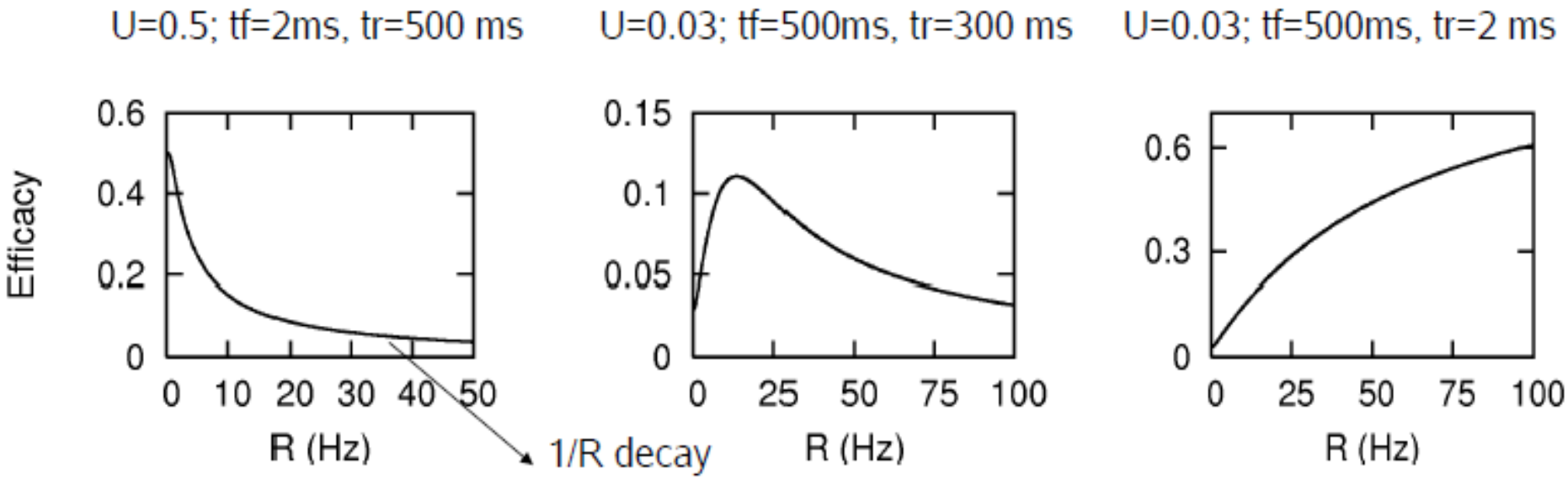
- Effective synaptic efficacy at the time of n+1th spike: A u_{n+1} R_{n+1}

$$u_{n+1} = u_n \exp\left(\frac{-\Delta t}{\tau_{\text{facil}}}\right) + U\left(1 - u_n \exp\left(\frac{-\Delta t}{\tau_{\text{facil}}}\right)\right)$$

$$R_{n+1} = R_n(1 - u_{n+1}) \exp\left(\frac{-\Delta t}{\tau_{\text{rec}}}\right) + 1 - \exp\left(\frac{-\Delta t}{\tau_{\text{rec}}}\right)$$

Modeling short term synaptic plasticity

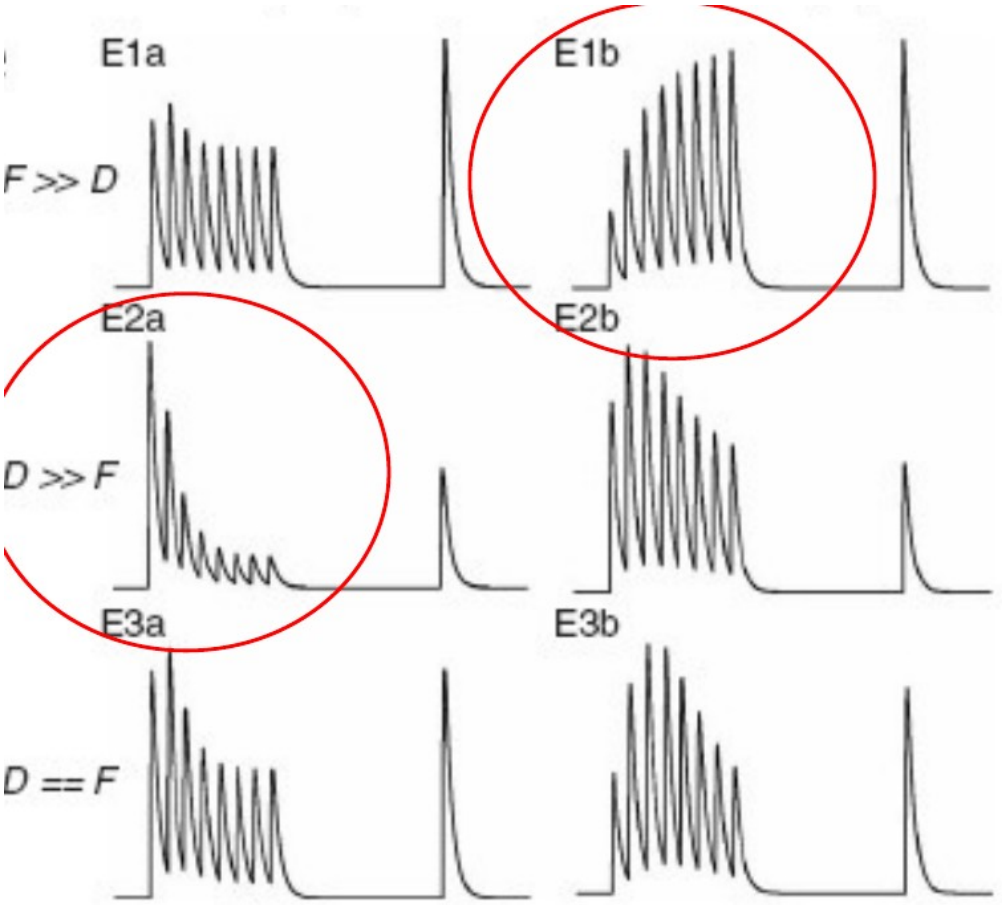
- $U, \tau_{rec}, \tau_{facil}$ control the relation between depression and facilitation.
- For periodic spike trains:



Modeling short term synaptic plasticity

- Diversity in the pre-frontal cortex
- $F=\tau_{facil}$
  $D=\tau_{rec}$

  Wang et al. (2006)

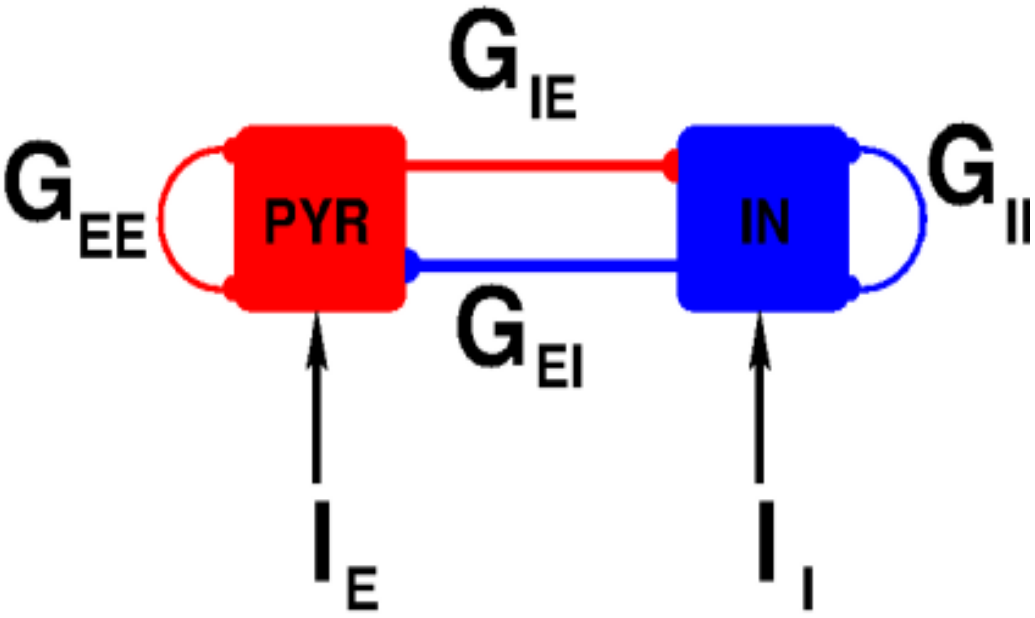


Modeling short term synaptic plasticity

- How will this system behave in a network
- Some kind of self-consistent calculation is necessary
- Mean field theory: Mongillo et al. PRL (2012)
- Two populations of LIF neurons with sparse connectivity (K connections per neuron K<<N)
- Coupling strengths proportional to $1/K^{1/2}$
- If we have plasticity only in the E→E connections the final result is that the firing rates can be evaluated by multiplying J_{EE} by p_R
- P_R can be evaluated self-consistently

Modeling short term synaptic plasticity

- Two population network with sparse connectivity and plasticity in the E->E connections



Modeling short term synaptic plasticity

- For instance

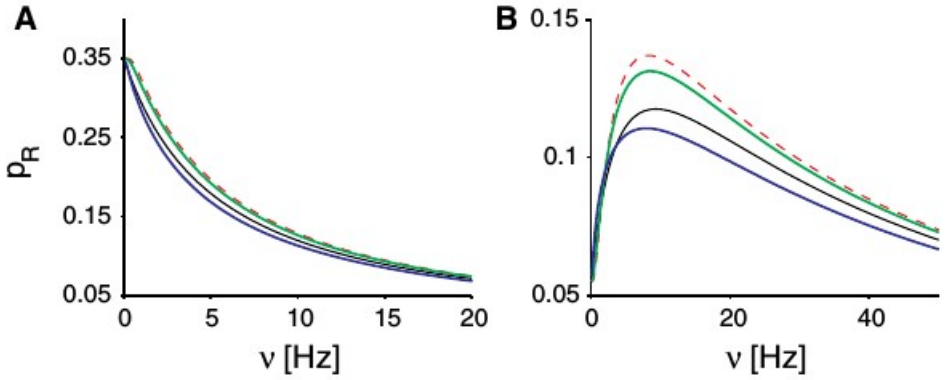


FIG. 1 (color online). The probability of release p_R vs the spiking rate ν for different ISI pdfs. From top to bottom: δ function (red dashed), inverse Gaussian with CV $= 0.5$ (green), exponential (black), inverse Gaussian with CV $= 1.5$ (blue). (a) Depressing transmission: p_R decreases with increasing ν. Parameters: $U = 0.35$, $\tau_F = 150$ ms, $\tau_D = 700$ ms. (b) Facilitating transmission: p_R first increases, then decreases with increasing ν. Parameters: $U = 0.055$, $\tau_F = 650$ ms, $\tau_D = 200$ ms.

- This generates additional NON-LINEARITY

Modeling short term synaptic plasticity

- Introducing a firing rate dependence in the E->E connections, the balance equations become:

$$0 = G_{EE}\, p_R(f_E)\, f_E - G_{EI}\, f_I + I_E$$
$$0 = G_{IE}\, f_E - G_{II}\, f_I + I_I$$

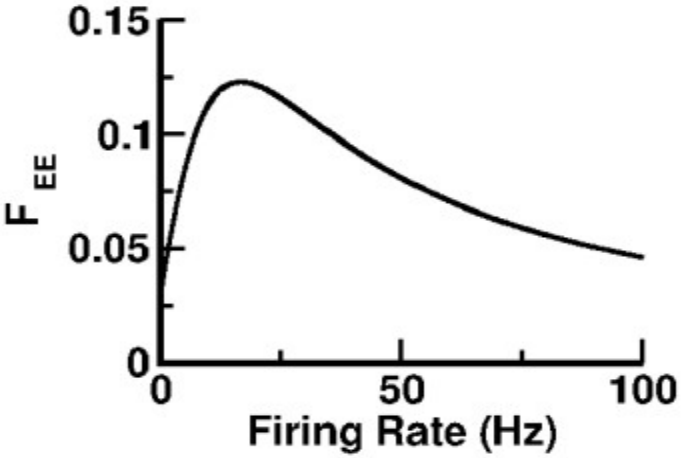
- Mean field equations for leaky integrate-and fire neurons (Mongillo et al, 2012)

- $p_R(f_E)$ depends very weakly on the statistics of the spike trains

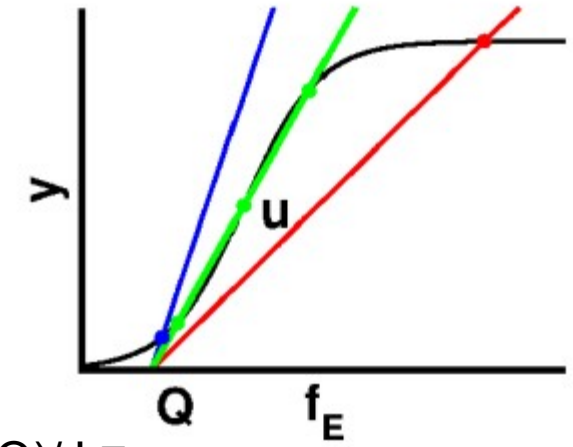
Short term synaptic plasticity allows bistability in unstructured network

- The input-output transfer at population level are non-linear
- This allows to have bistability:

Synapse with facilitation and depression



For a given value of the coupling two stable solutions



$f_E F_{EE}(f_E)=(f_E-Q)/J \equiv y$

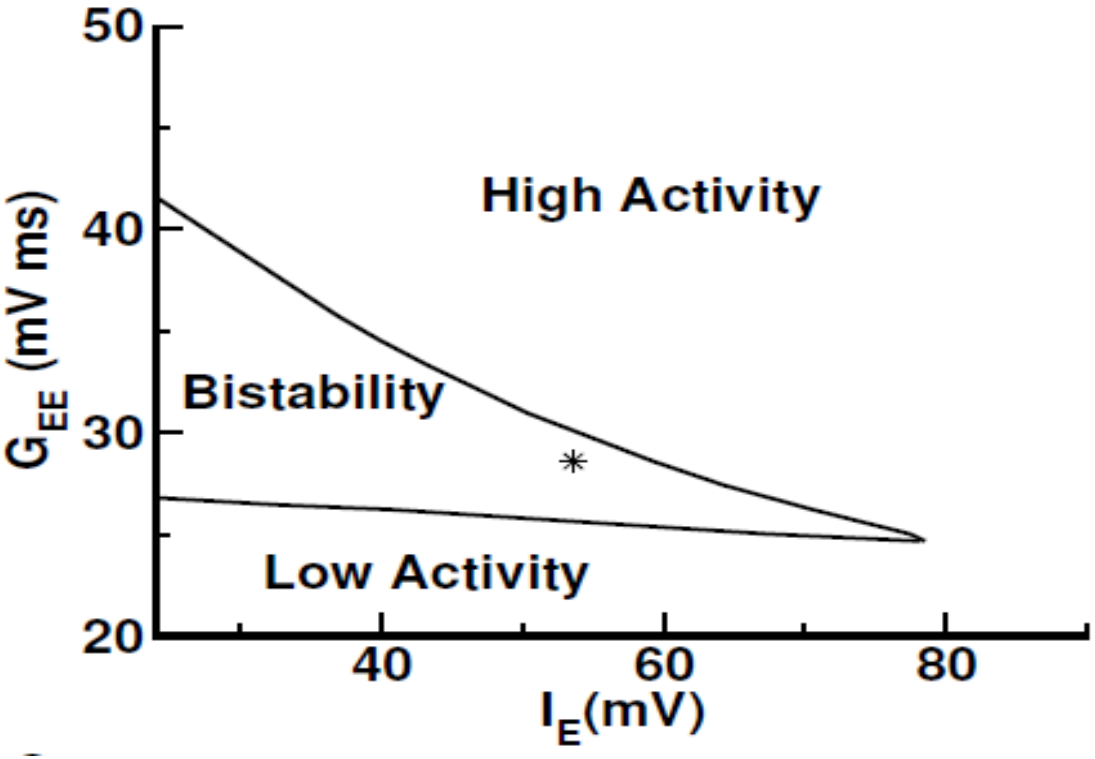
$Q=(G_{II} I_E/G_{EI} - I_I)/G_{IE}$

$J=G_{II} G_{EE}/(G_{EI} G_{IE})$

Short term synaptic plasticity allows bistability in unstructured network

- Bistability in a wide region of the parameter space:

Phase diagram



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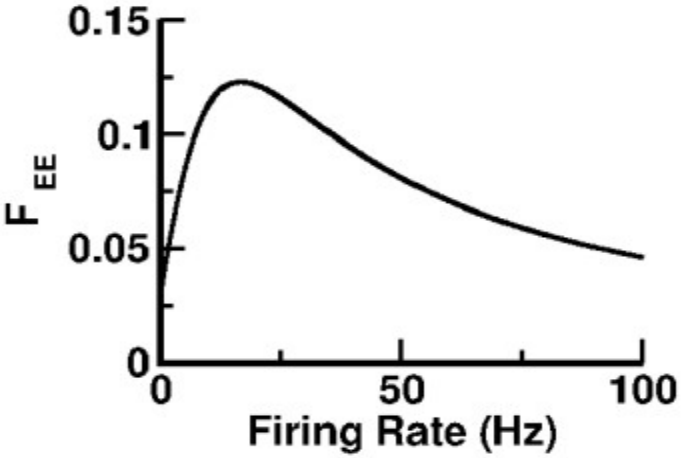

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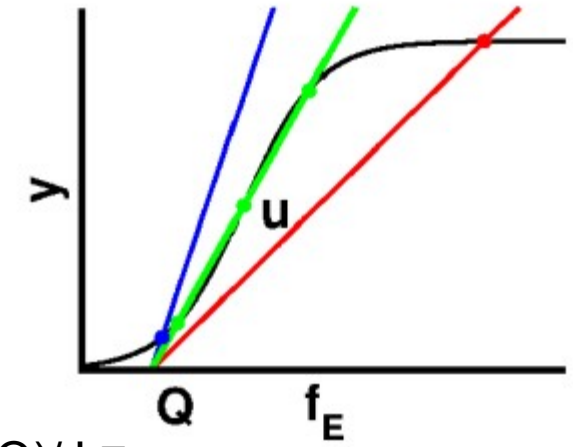


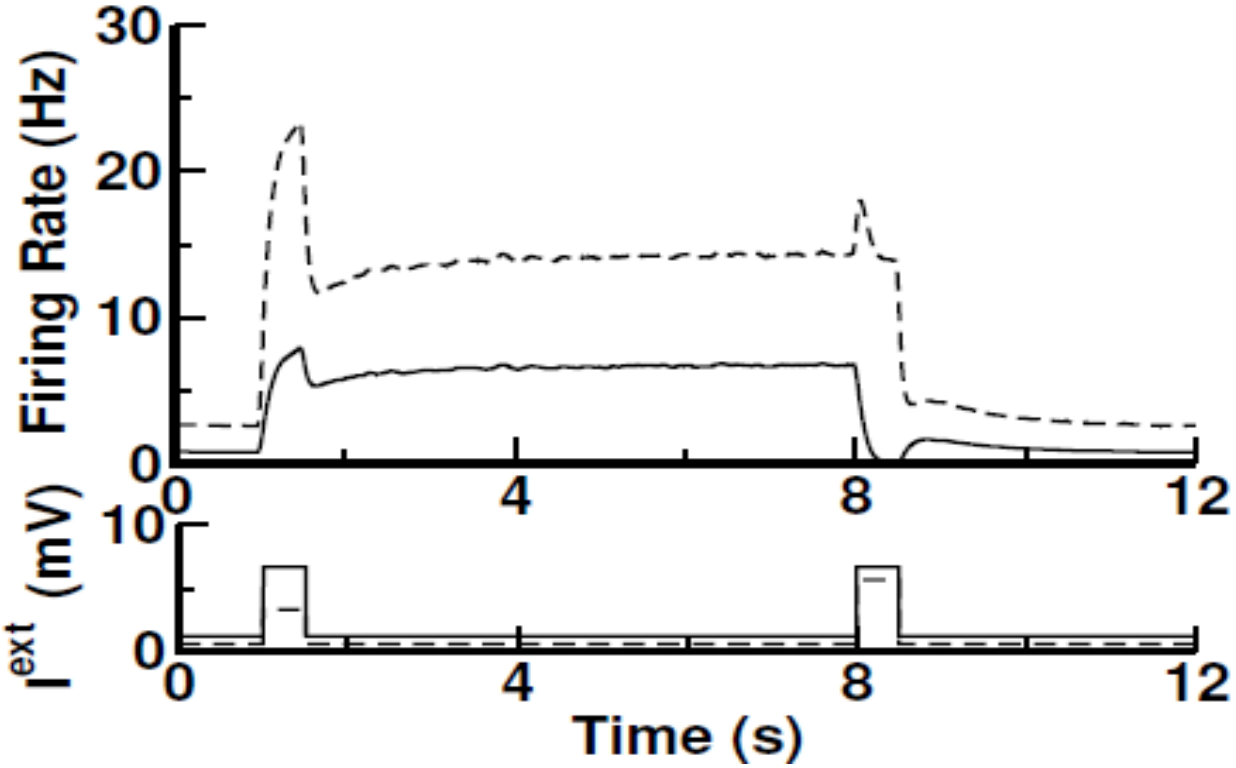
$f_E F_{EE}(f_E)=(f_E-Q)/J \equiv y$

$Q=(G_{II} I_E/G_{EI} - I_I)/G_{IE}$

$J=G_{II} G_{EE}/(G_{EI} G_{IE})$

Short term synaptic plasticity allows bistability in unstructured networks

- Switch between DOWN and UP states
- These two states are balanced and display high temporal variability (CV approx. 1)

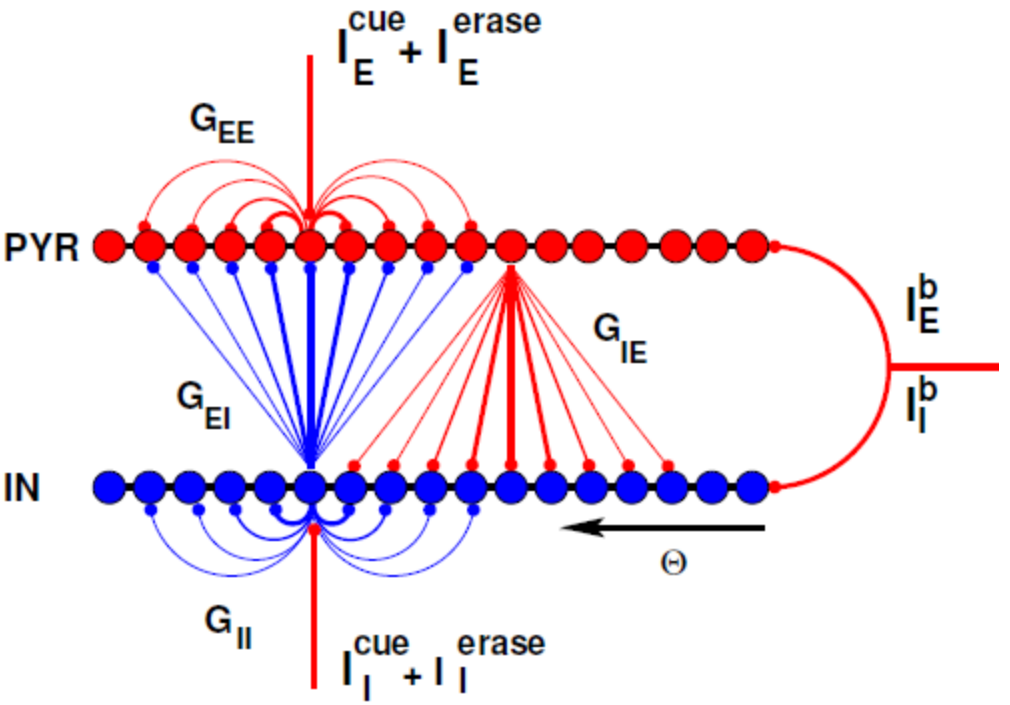


Short term synaptic plasticity allows multistability

- But is it possible to have a continuum of states?
  - Coexistence of a low-rate homogeneous state and an infinite number of high-rate spatially modulated states
  - That can be achieved with spatially modulated interactions: more probability of connections if the neurons are located at similar spatial locations
  - The specific state should be chosen by a cue-dependent transient stimulus (Hansel & Mato ,2012)

The Model for the ODR task

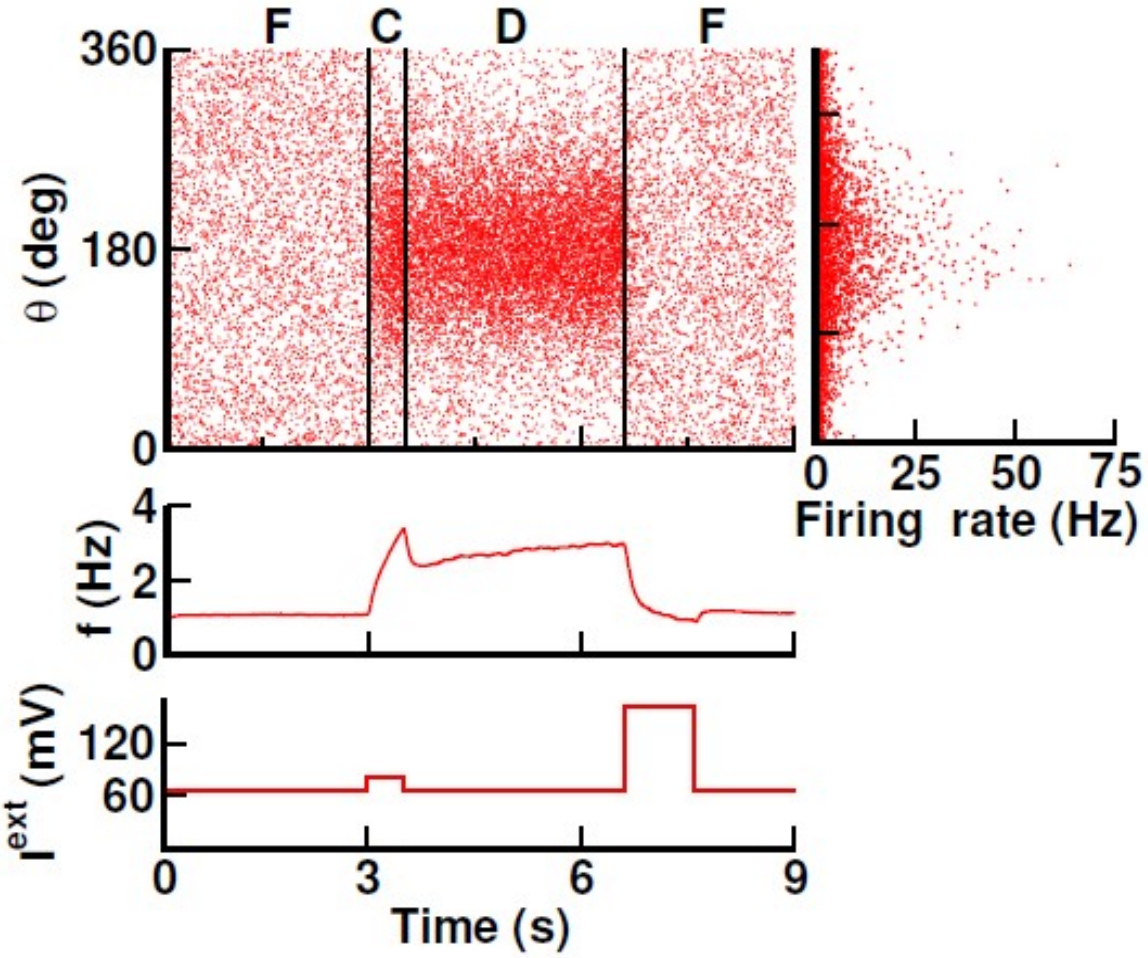
- N_E excitatory (PYR) and N_I inhibitory neurons (IN)
- Ring: architecture: probability of connections depends on the distance along the ring.
- Average number of connections per neuron: K
- Interactions: exponential time dependence.
  Excitation:
  mixture of AMPA and NMDA
  Inhibition: $GABA_A$
- E->E interactions are facilitating
- External input is time dependent to simulate the task



The Model for the ODR task

- Leaky Integrate-and-Fire neurons
- N_E =64000, N_I =16000 (some simulations up to 256000)
- K=1600 (EE, IE) or 400 (EI,II) (some simulations up to 8000)
- t_{AMPA}= 3 ms, t_{NMDA}= 50 ms, t_{GABA}= 4 ms
- U=0.03, τ_r=250 ms, τ_f=450 ms (plasticity only in E->E connections)
- Probability of connections: Gaussian function σ=60 degrees (EE, II, EI) or 70 degrees (IE)

The network displays cue related multistability



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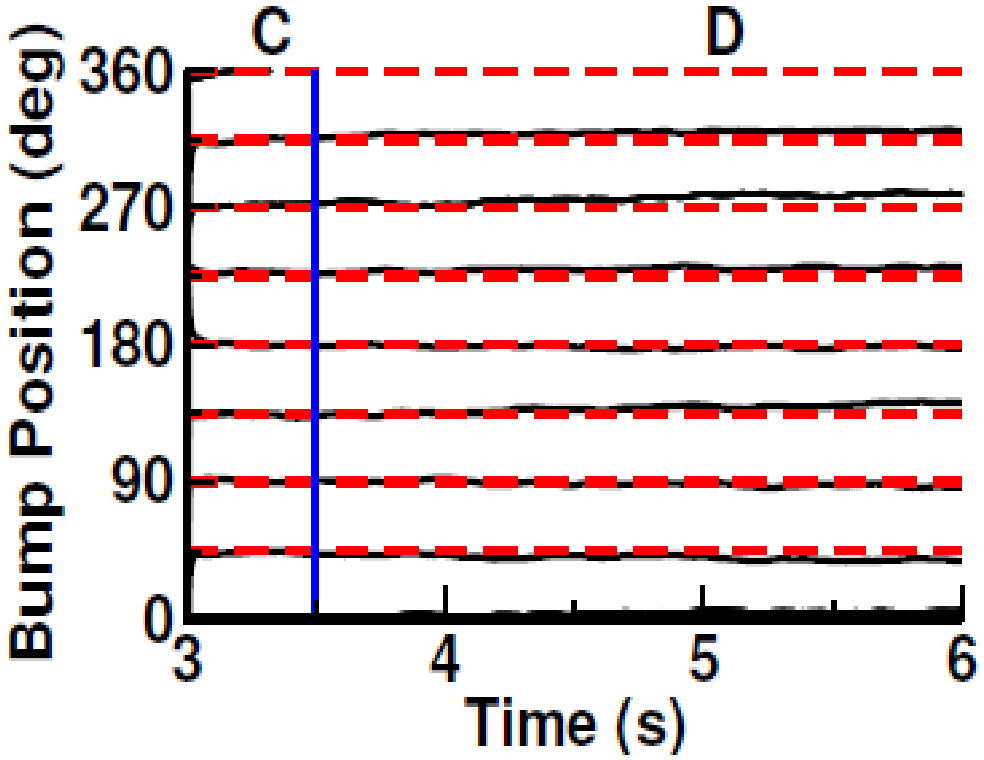
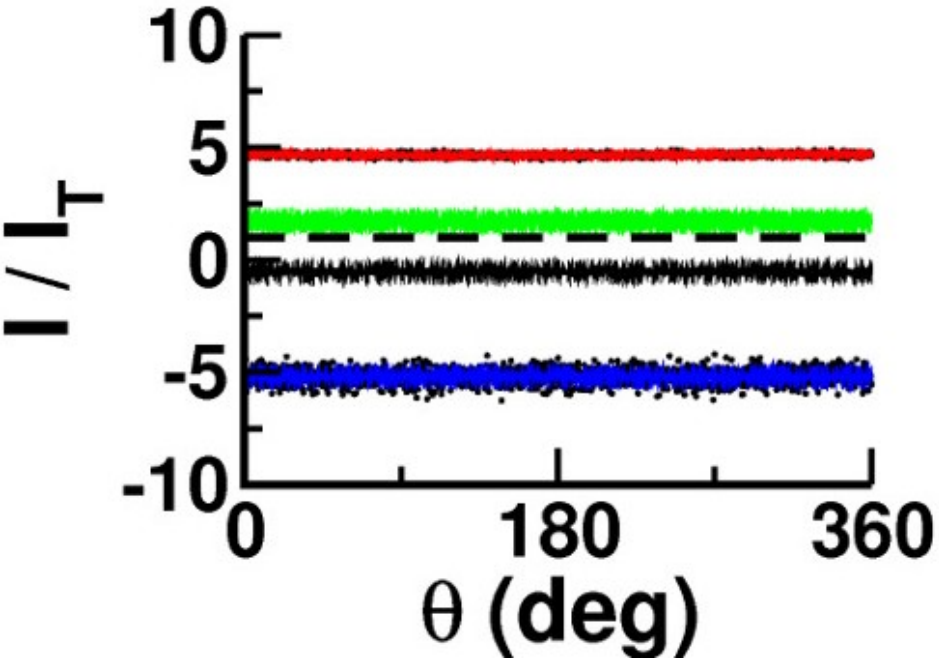
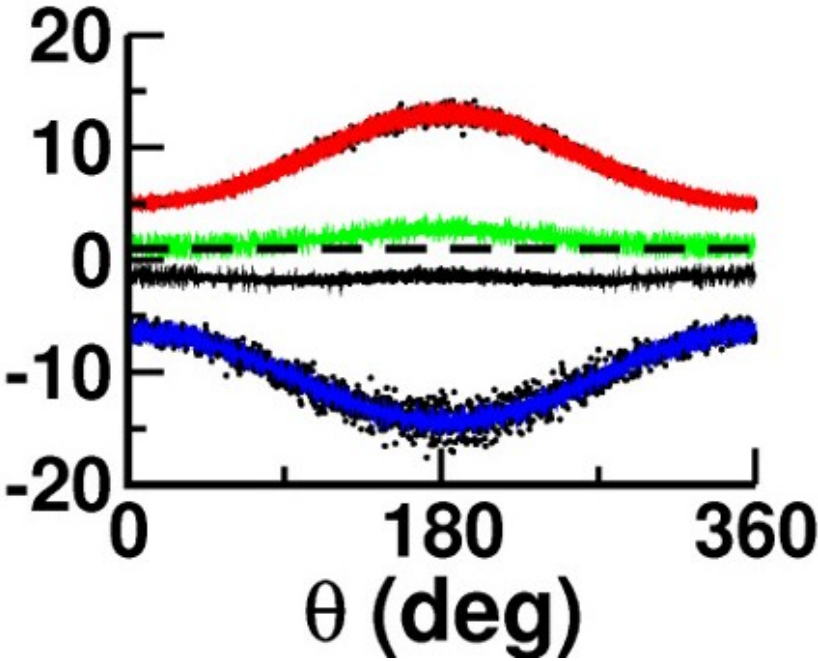


Both baseline and delay states are balanced

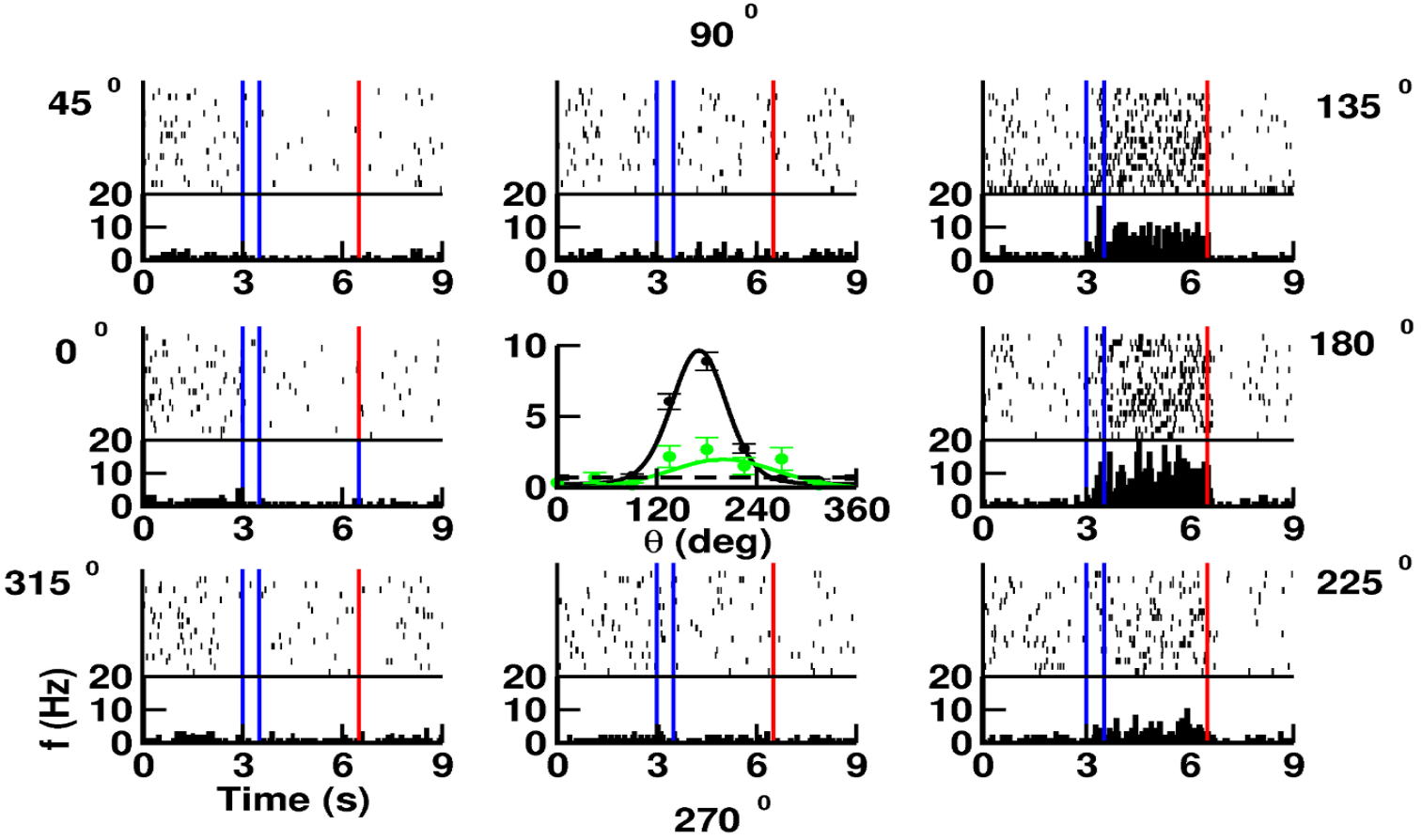
- Red: excitatory input
- Blue: inhibitory input
- Black: total input
- Green: total input + 1.5 SD





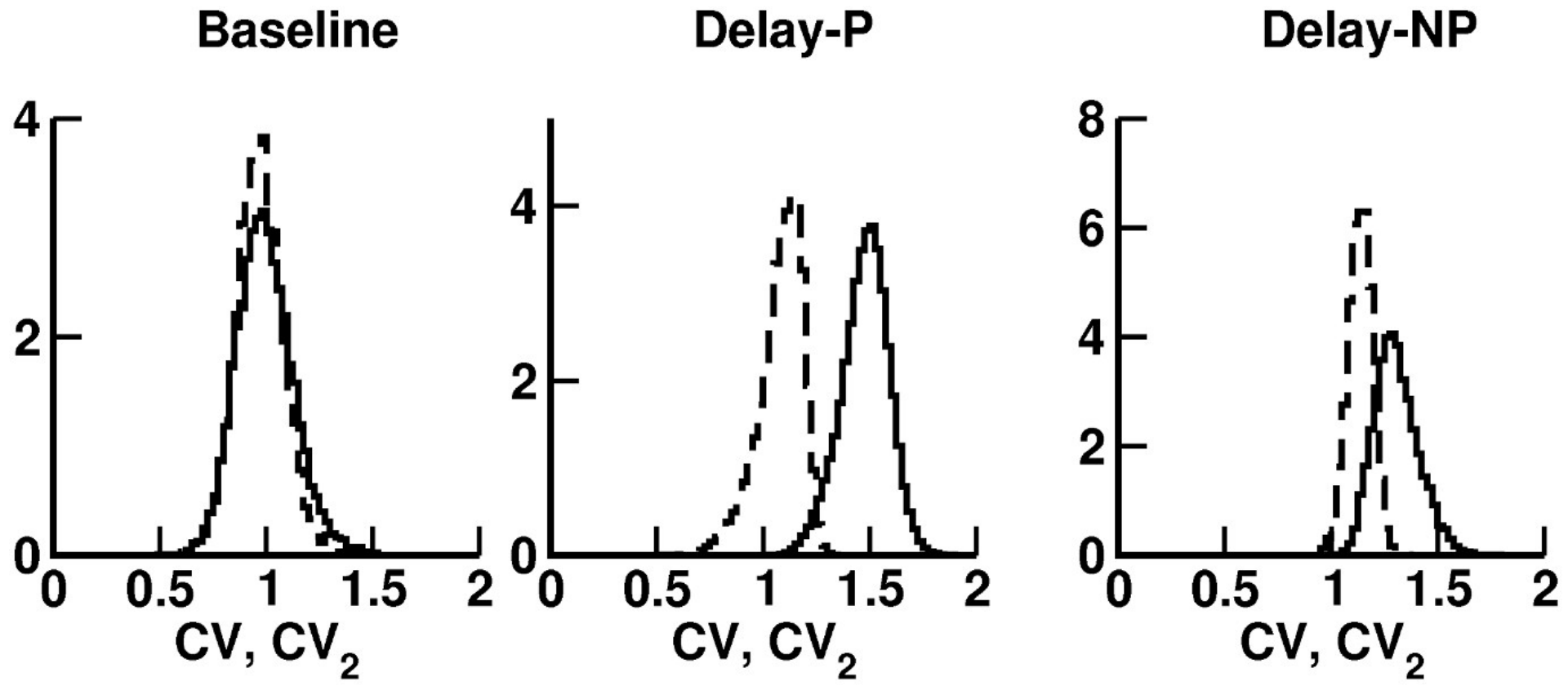
Both baseline and delay activities are highly irregular

- Black: tuning curve during delay
- Green: tuning curve during cue



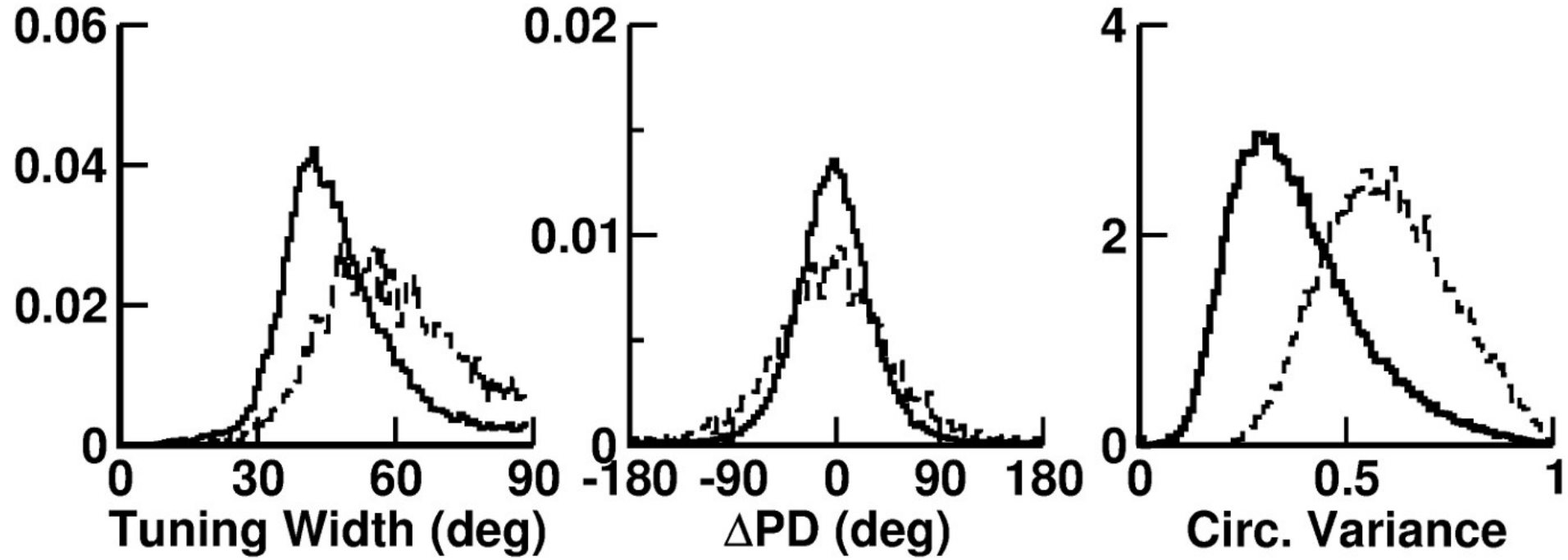
Both baseline and delay activities are highly irregular

- Solid line E pop
- Dashed line: I pop



Tuning properties are heterogeneous

- Solid line E pop
- Dashed line: I pop
- ΔPD: Preferred direction – position on the ring
- Circular Variance=1-c_1/c_0 , where c_n is the n^{th} Fourier component of the tuning curve



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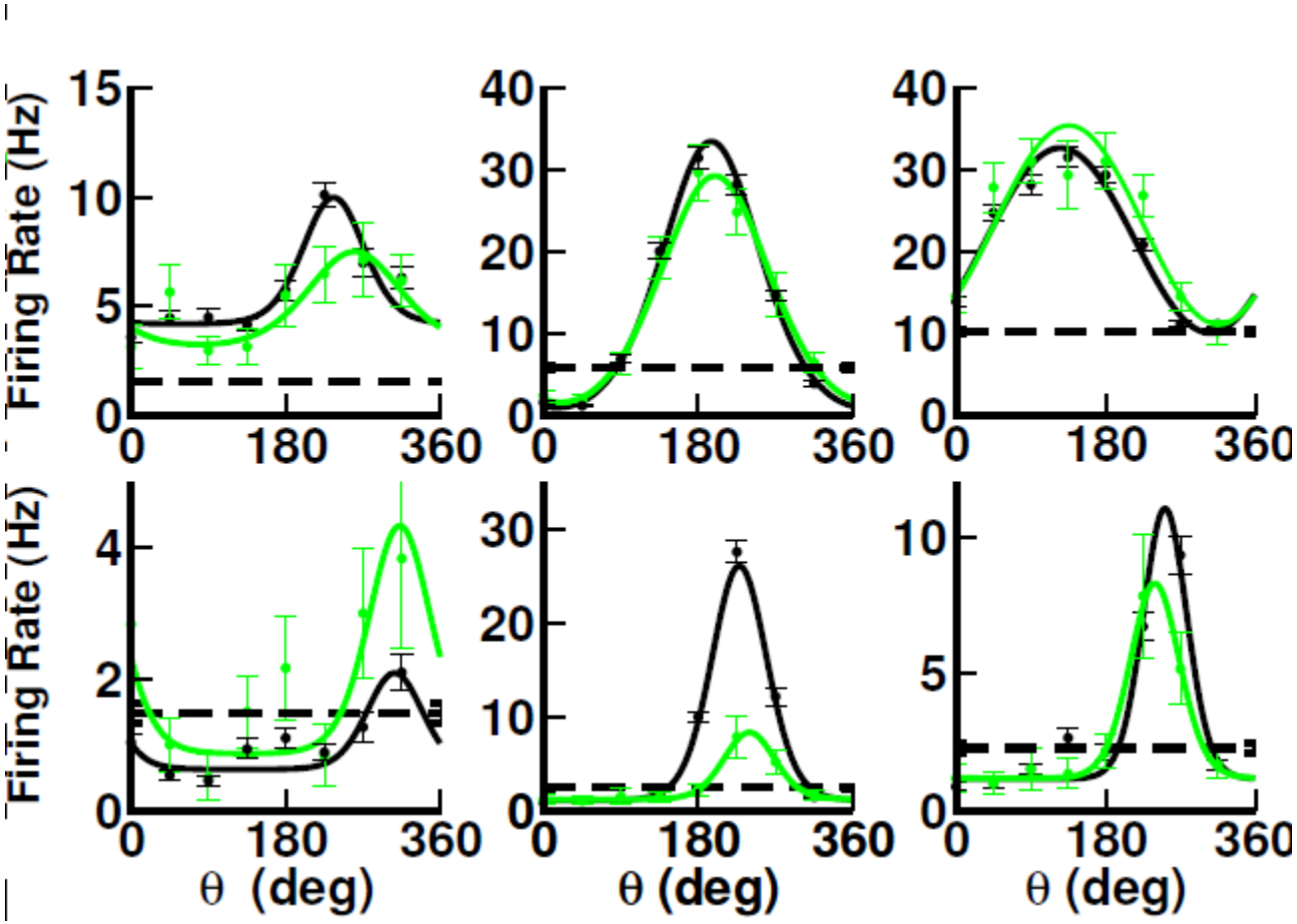
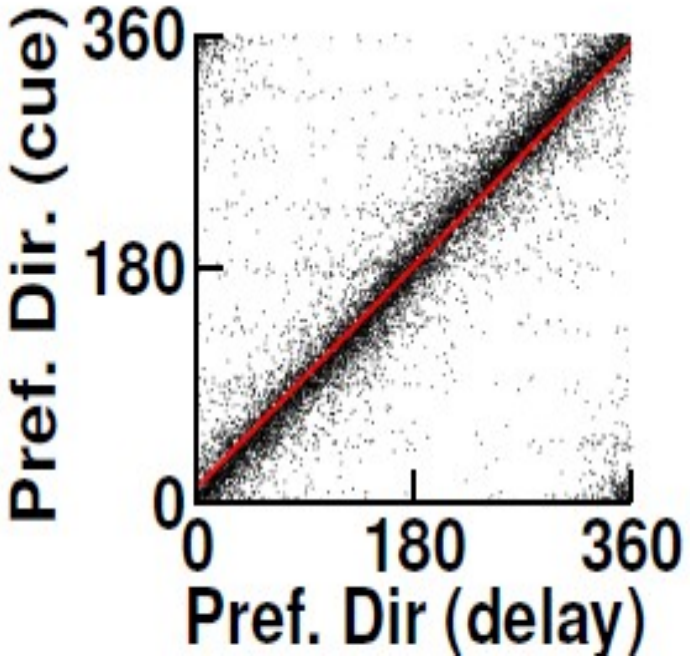
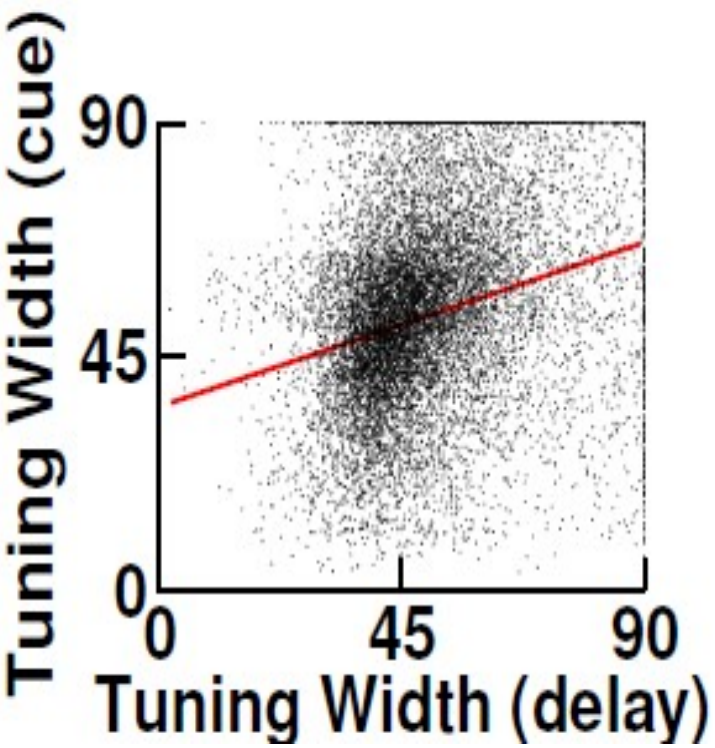


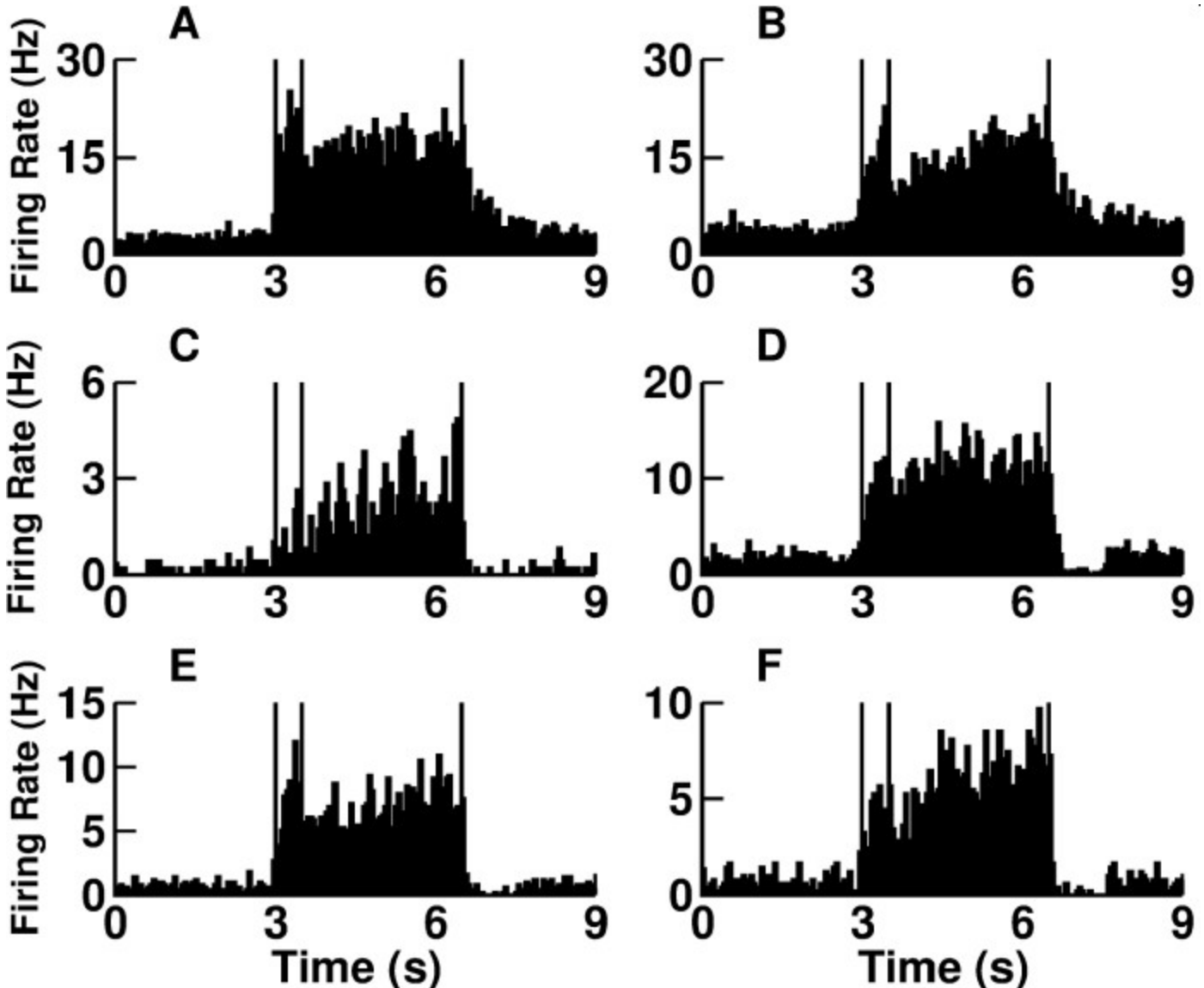
Tuning properties are heterogeneous

- Preferred directions are strongly correlated between cue and delay but tuning widths are not





Dynamics during cue period is also heterogeneous



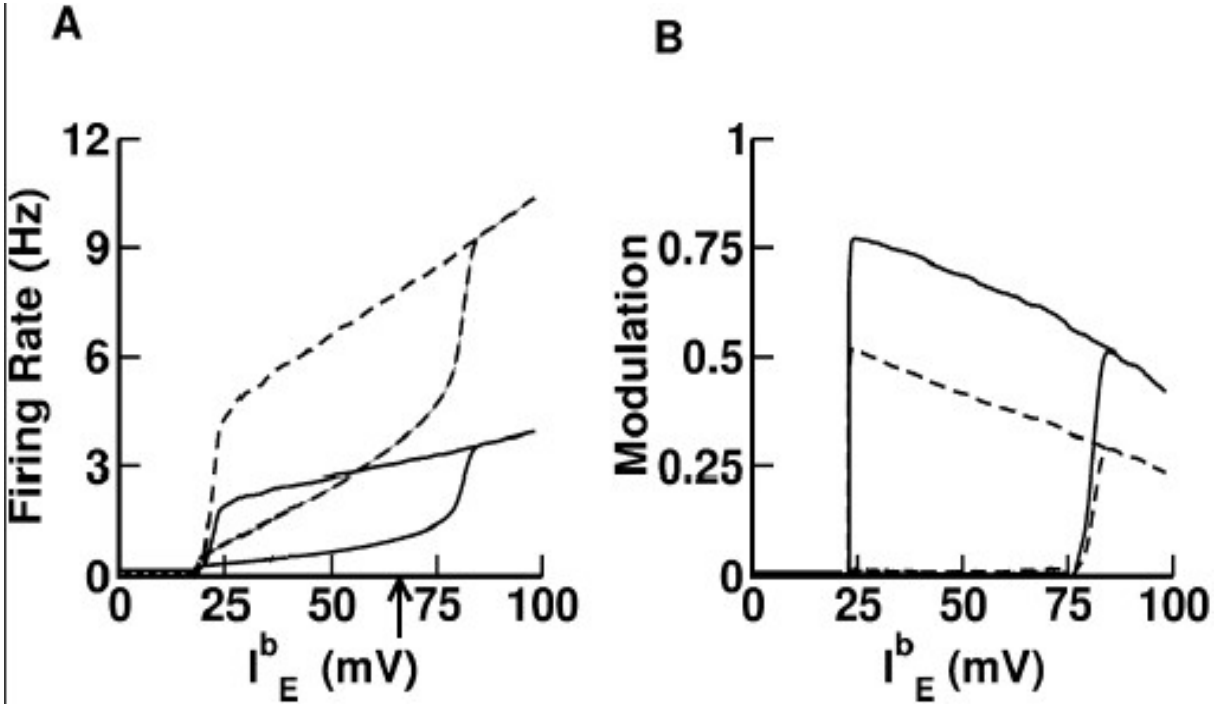
Multistability exists in a broad range of external inputs

- Solid line E pop
- Dashed line: I pop

$m_E(\Theta,t) = m_0 + m_1 \cos(\Theta - \Theta_0) + \ldots$

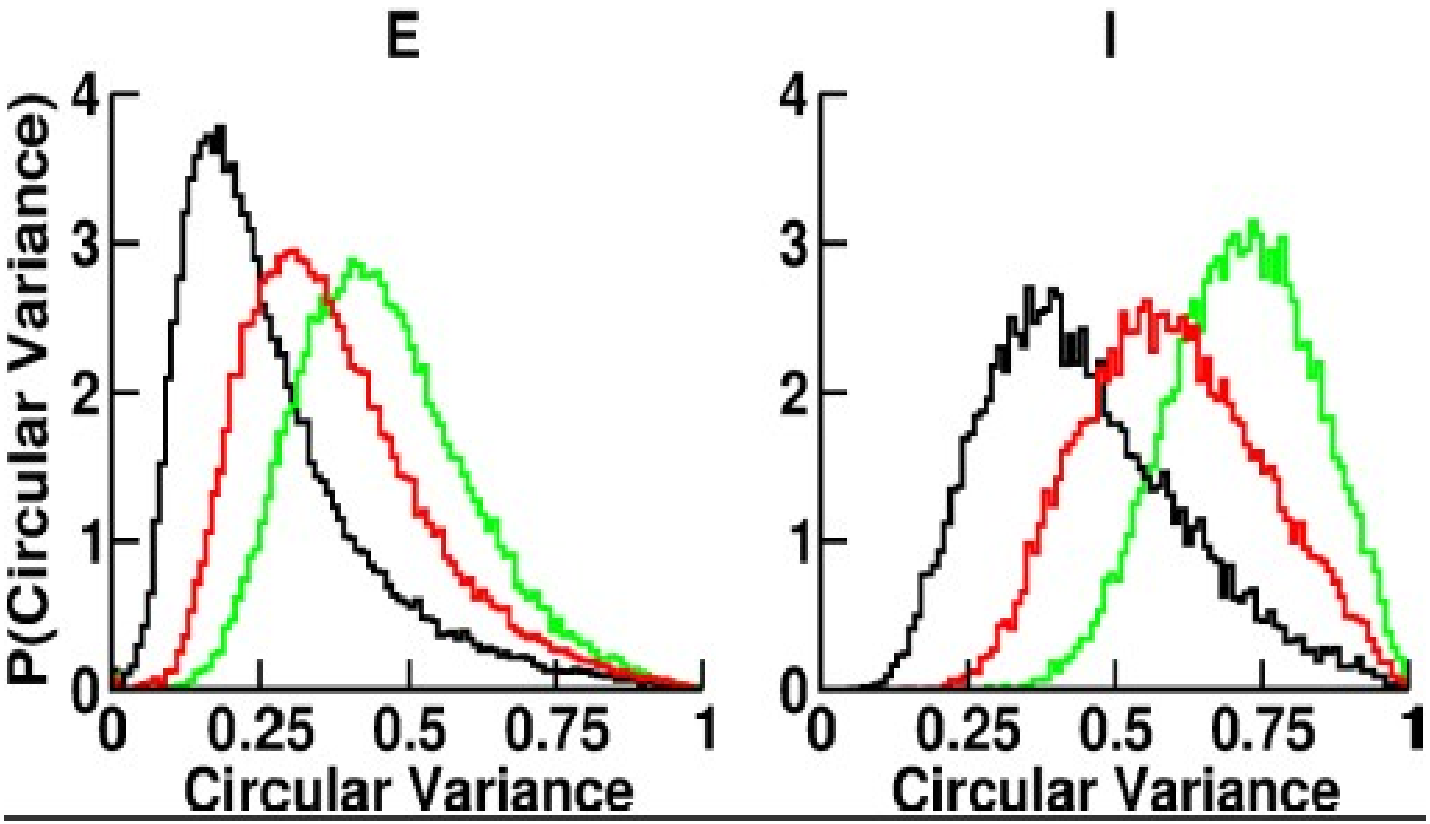
Population averaged = m_0

Normalized spatial modulation = m_1 / m_0

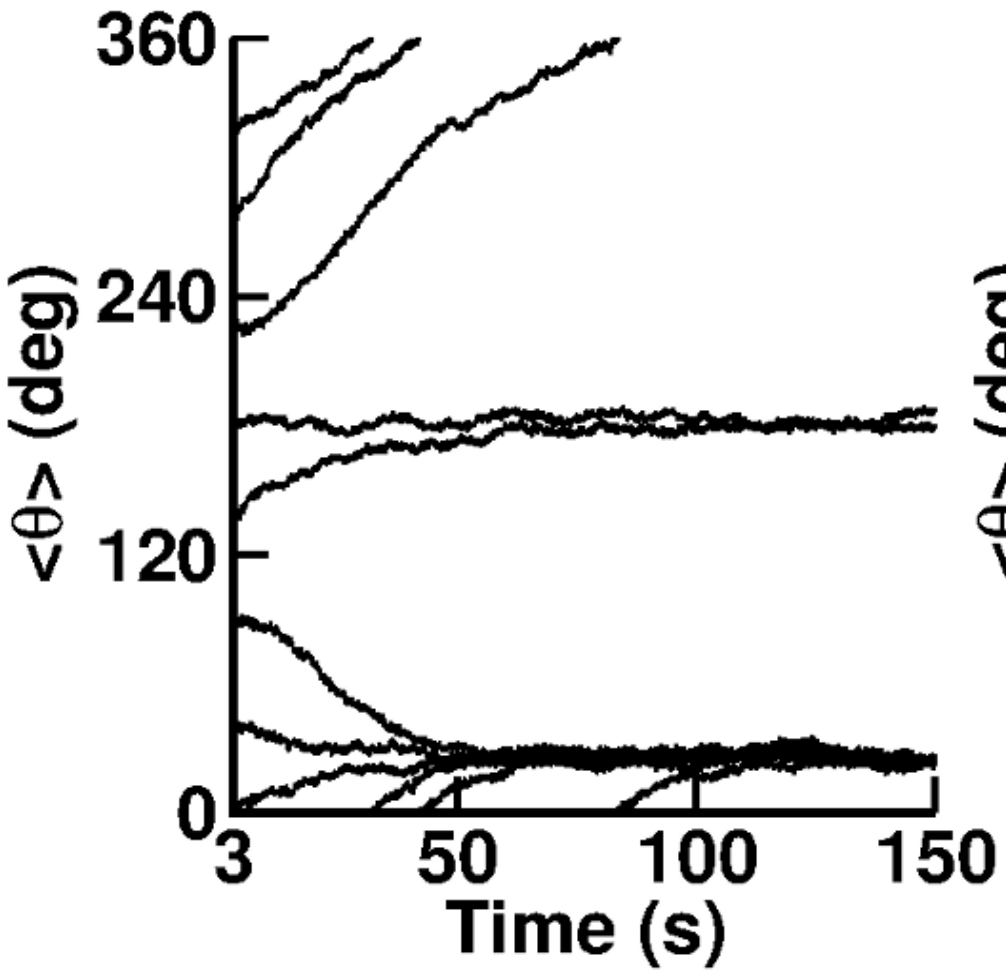


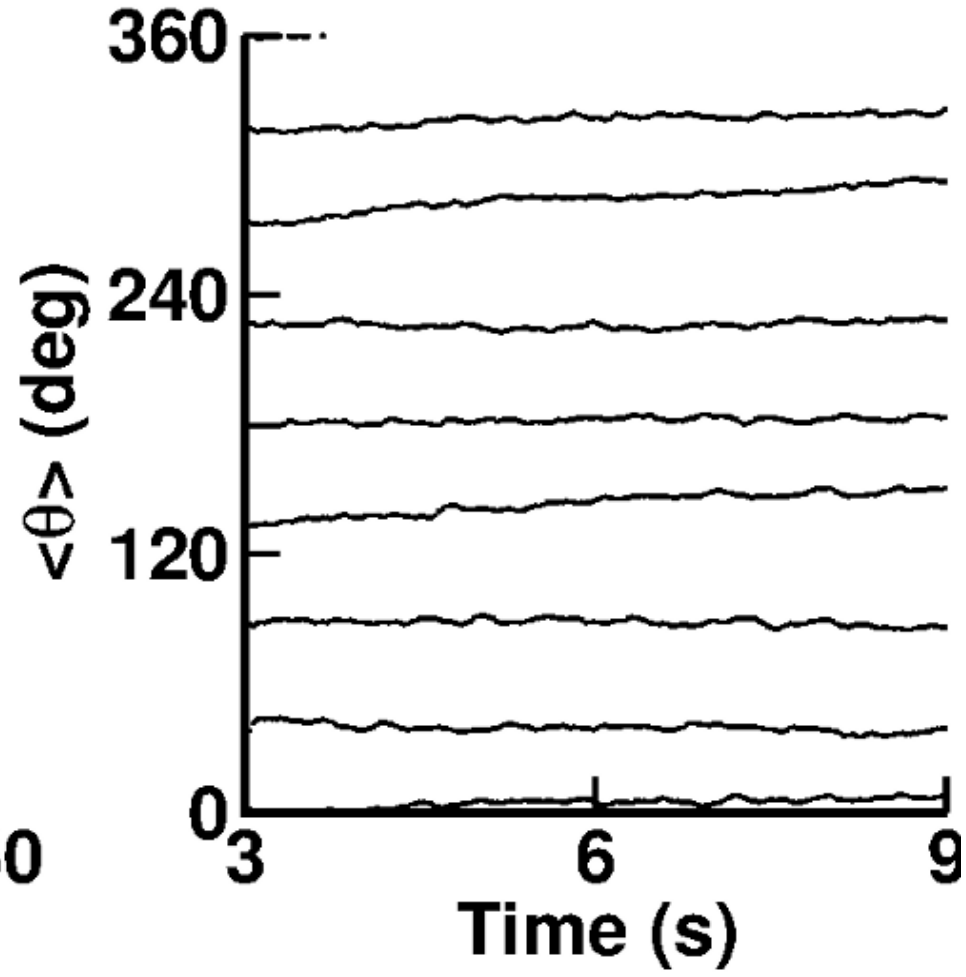
But the modulation depends on σ

- $\sigma_{II}=\sigma_{EI}$=40 deg (green)
- $\sigma_{II}=\sigma_{EI}$=60 deg (red)
- $\sigma_{II}=\sigma_{EI}$=70 deg (black)



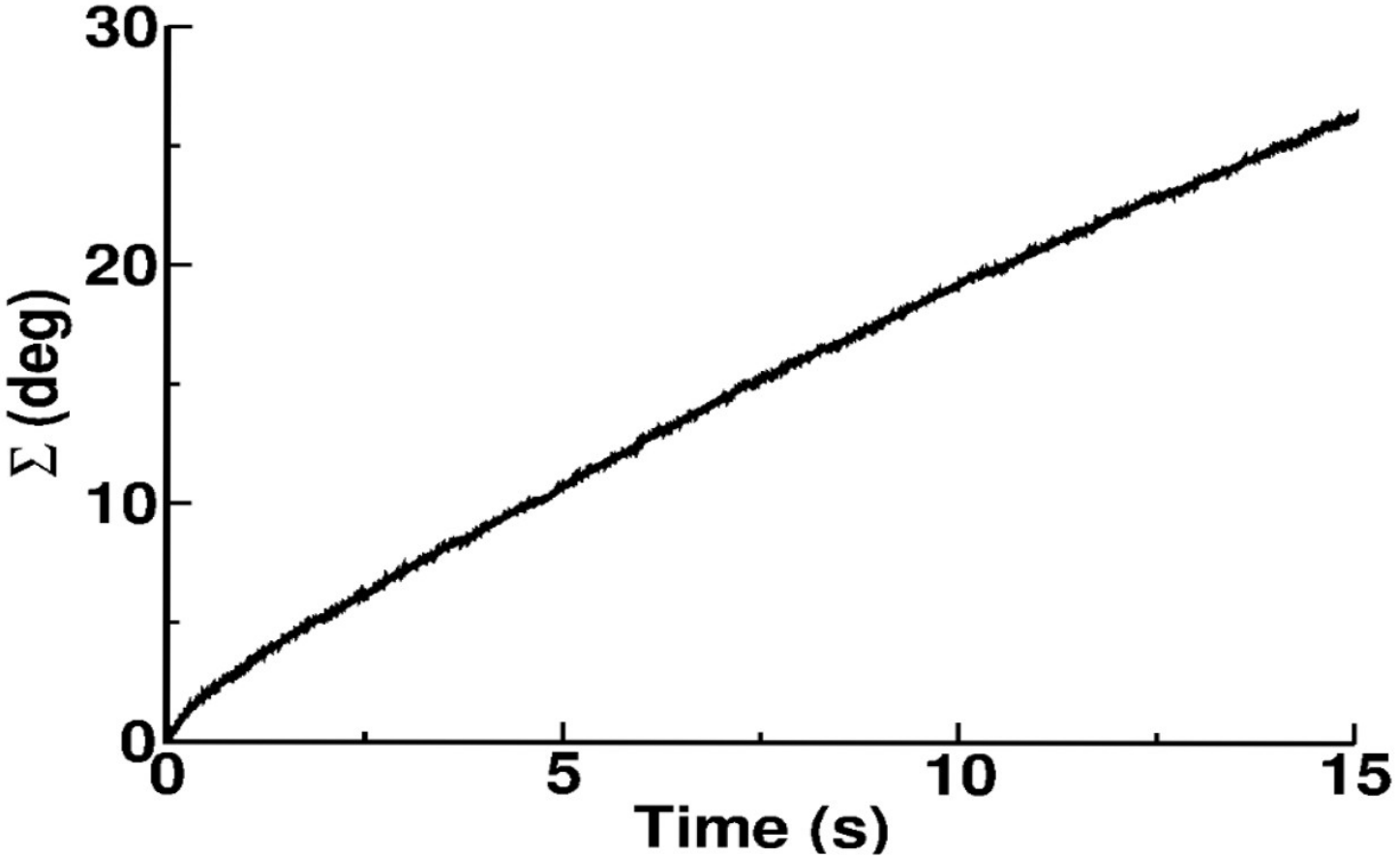
The bump drifts at very long times



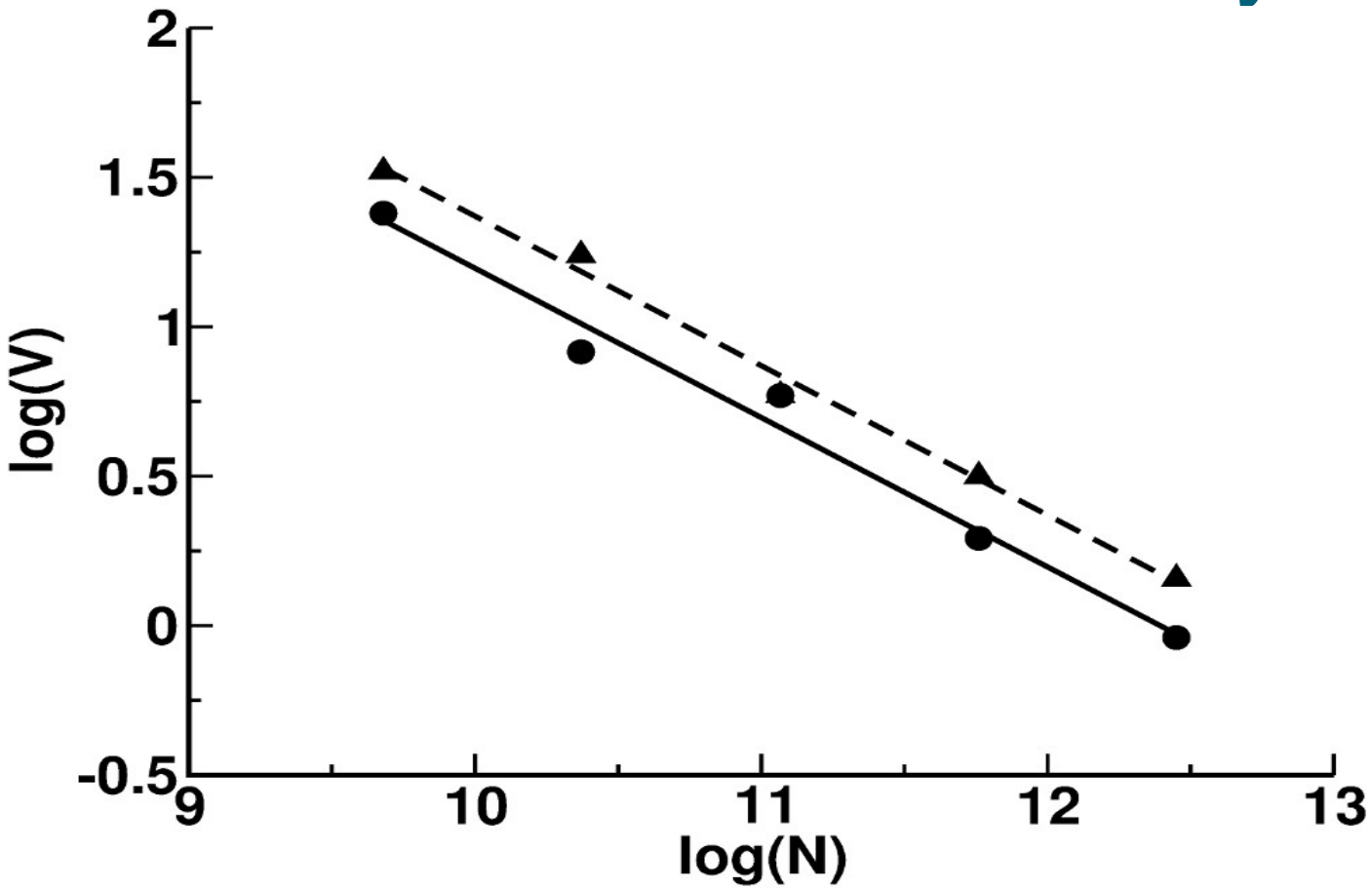


The velocity is constant at long times

- Σ: average quadratic displacement of the bump



Velocity decreases with the size of the network and the connectivity



- Solid line: K
- Dashed line: K/2

For plausible size and connectivity one can easily obtain a drift of 0.5− 1 degrees/sec, in line with experimental data (White et al. 1994, Ploner et al. 1998).

Summary of the properties of the model

- Balanced state in cue and delay periods
- Highly variable activity (CV approx. 1)
- Highly heterogeneous neuronal properties (tuning widths, preferred directions)
- Heterogeneous dynamics during the delay period
- Slowly drifting bump
- Robust with respect to changes in external input, values of couplings, spatial width of the probability of connections, number of connections, parameters of the synaptic plasticity

Next lecture

- Metaplasticity
- Reinforcement Learning