Analytical Geometry and Linear Algebra I, Class #7

Innopolis University, October 2021

1. Find the centroid of the triangle formed by the lines given by the equations $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$. And draw the picture.
2. Find the condition that one of the lines given by $ax^2 + 2hxy + by^2 = 0$ may be perpendicular to one of the lines given by $a_1x^2 + 2h_1xy + by^2 = 0$. And draw the picture.
3. Find the equation of the plane passing through the point $(2, -3, 4)$ and parallel to the plane $2x - 5y - 7z + 15 = 0$. And draw the picture.
4. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$. And draw the picture.
5. Find the ratio in which the line joining the points $(2, -1, 4)$ and $(6, 2, 4)$ is divided by the plane $x + 2y + 3z + 5 = 0$. And draw the picture.
6. Find the equation of the plane which passes through the intersection of the planes $2x + 3y + 10z - 8 = 0, 2x - 3y + 7z - 2 = 0$ and is perpendicular to the plane $3x - 2y + 4z - 5 = 0$. And draw the picture.
7. Find the equations of the line passing through the point $(1, 2, 3)$ and perpendicular to the planes $x - 2y - z + 5 = 0$ and $x + y + 3z + 6 = 0$. And draw the picture.
8. Find the perpendicular distance from the point $(1, 3, -1)$ to the line $\frac{x-13}{5} = \frac{y+8}{-8} = \frac{z-31}{1}$. And draw the picture.
9. Find the distance of the points $(2, 3, -5), (3, 4, 7)$ from the plane $x + 2y - 2z = 9$ and prove that these points lie on the opposite sides of the plane. And draw the picture.
10. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$. And draw the picture.
11. Draw the following three cases and find how much solutions does the systems have.

    (a) $\begin{cases} 2x + 3y - z = 1, \\ x - 2y + z = 2, \\ x + y + z = 1 \end{cases}$

    (b) $\begin{cases} 2x - 3y + z = 3, \\ x - 2y + 2z = 2, \\ x - y - z = 1 \end{cases}$

    (c) $\begin{cases} 2x - 3y + z = 3, \\ x - 2y + 2z = 0, \\ x - y - z = 1 \end{cases}$