Analytical Geometry and Linear Algebra I, HW #4

Innopolis University, September 2022

1 Inverse Matrix

1. Find inverse matrices for the following matrices:

   (a) $\begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$;

   (b) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}$;

   (c) $\begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$;

2. Solve matrix equations:

   (a) $X \begin{bmatrix} 2 & 2 & -1 \\ 2 & -1 & 2 \\ -1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 2 \\ 5 & 8 & -1 \end{bmatrix}$.

3. It is known that $A^2 + A + I = O$ (O is a zero matrix) for a square matrix A. Is it true that matrix A is invertible? If it is so, how can we find the inverse matrix?

2 Matrix Rank

1. Calculate the ranks of the following matrices:

   (a) $\begin{bmatrix} 13 & 16 & 16 \\ -5 & -7 & -6 \\ -6 & -8 & -7 \end{bmatrix}$;

3 Changing Basis and Coordinates

1. There are two different coordinate systems in space: O, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ and O', $\mathbf{e}'_1$, $\mathbf{e}'_2$, $\mathbf{e}'_3$. It is known that the old coordinates x, y, z are expressed through the new coordinates x', y', z' with the following formulas:

   $$x = x' + y' + z' - 1;\ y = -x' + z' + 3;\ z = -x' - y' - 2.$$

   (a) Find the transition matrix from the new basis to the old one and the transition matrix from the old basis to the new one.

(b) Find the coordinates of O, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ in the new coordinate system.

(c) Find the coordinates of O', $\mathbf{e}_1'$, $\mathbf{e}_2'$, $\mathbf{e}_3'$ in the old coordinate system.

2. Let us consider two coordinate systems in the plane: O, $\mathbf{e}_1$, $\mathbf{e}_2$ and O', $\mathbf{e}_1'$, $\mathbf{e}_2'$. Point O' has coordinates $(7; -2)$ in the old coordinate system, and vectors $\mathbf{e}_1'$, $\mathbf{e}_2'$ can be obtained from vectors $\mathbf{e}_1$, $\mathbf{e}_2$ by rotating them $60°$ (a) clockwise; (b) counterclockwise. Find the old coordinates of a point x, y given its new coordinates x', y'.