Analytical Geometry and Linear Algebra I, Class #9

Innopolis University, October 2022

1. Find the foci, latus rectum, vertices and directrices of the following parabola: $y^2 + 4x - 2y + 3 = 0$.
2. Find the equations of the tangent and normal to the parabola $y^2 = 4(x-1)$ at $(5, 4)$.
3. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$ one of whose vertices is at the vertex of the parabola. Find its side.
4. Find the equation of the ellipse whose foci are $(4, 0)$ and $(-4, 0)$ and $e = 1/3$
5. Find the eccentricity, foci and the length of the latus rectum of the ellipse $9x^2 + 4y^2 = 36$
6. Find the equation of the normal to the ellipse $3x^2 + 2y^2 = 5$ at $(-1, 1)$.
7. The equation $25(x^2 - 6x + 9) + 16y^2 = 400$ represents an ellipse. Find the centre and foci of the ellipse. How should the axis be transformed so that the ellipse is represented by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$?
8. Find the locus of the poles with respect to the ellipse of the tangents to the parabola $y^2 = 4px$.
9. Find the equation of a parablola that has a line $x = 8$ for a directrix and point $F(7; 0)$ for a focus.
10. Find the eccentricity of an ellipse given that:
    (a) its major axis subtends an angle of 120° at the endpoints of its minor axis;
    (b) the segment between a focus and the farthest vertex subtends an angle of 90° at the endpoints of its minor axis.