AGLA 1. TEST 1. **VARIANT 1.** 15 points, 60 minutes

Full name:	Group:

Task:	1	2	3	4	5	6	Total
Score:							

1. (2 points) For each of the following statements mark it as True or False. Justify each answer.

(a) If matrix B is produced by interchanging two columns of matrix A, then $\det(B) = -\det(A)$. Explain your answer in 2×2 case.	
(b) For any square matrix A there exists exactly one inverse matrix.	

2. (2 points) Decompose the vector $\mathbf{p} = (1, 2, 3)$ into components parallel and perpendicular to the vector $\mathbf{q} = (1, -2, 2)$.

3. (2 points)

   (a) Find the matrix product AB if $A = \begin{bmatrix} x & -2 & -1 \\ 4 & 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -5 & 1 \\ 1 & -3 \\ 2 & x \end{bmatrix}$

   (b) Find the largest possible value of determinant (AB).

4. (3 points) Point A has coordinates (5; -1; 8) in the old coordinate system. Find its coordinates in the new coordinate system obtained from the initial one by transferring the origin to point N that has coordinates (33; -1; 2) in the old coordinate system.

5. (3 points) Subspace S of $\mathbb{R}^3$ is formed by linear combination of vectors v_1 and v_2. Find a vector v that is orthogonal to S, if $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

6. (3 points) Let

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{1}$$

   Find All **Natural** numbers ($k \in \mathbb{N}$) where: $A^k = A^{-1}$ (Also you need to check if A is invertible), Note that $A^k = \underbrace{A.A \ldots A}_{k\ times}$

End of Test 1

AGLA 1. TEST 1. **VARIANT 2.** 15 points, 60 minutes

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	9	Total
Score:										

1. (2 points) For each of the following statements mark it as True or False. Justify each answer.

(a) For any matrices A, B, C: $(ABC)^\top = C^\top B^\top A^\top$	
(b) Inverse matrix (A^{-1}) always exists.	

2. (2 points) Decompose the vector $\mathbf{p} = (2, 4, 6)$ into components parallel and perpendicular to the vector $\mathbf{q} = (1, 2, -2)$.

3. (2 points)

   (a) Find the matrix product AB if $A = \begin{bmatrix} 4 & -2 & 1 \\ 2 & x & -5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ x & -3 \end{bmatrix}$

   (b) Find the largest possible value of determinant (AB).

4. (3 points) Point A has coordinates (-3; 29; -1) in the old coordinate system. Find its coordinates in the new coordinate system obtained from the initial one by transferring the origin to point N that has coordinates (1; -1; 2) in the old coordinate system.

5. (3 points) Subspace S of $\mathbb{R}^3$ is formed by linear combination of vectors v_1 and v_2. Find a vector v that is orthogonal to S, if $v_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}$

6. (3 points) Let

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 \\ 1/5 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix} \tag{2}$$

   Find All **Natural** numbers ($k \in \mathbb{N}$) where: $A^k = A^{-1}$ (Also you need to check if A is invertible), Note that $A^k = \underbrace{A.A\ldots A}_{k\ times}$

End of Test 1

AGLA 1. TEST 1. **VARIANT 3.** 15 points, 60 minutes

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (2 points) For each of the following statements mark it as True or False. Justify each answer.

(a) By definition, a matrix C is orthogonal iff $C^{-1} = C$.	
(b) The determinant of a matrix always defined.	

2. (2 points) Decompose the vector $\mathbf{p} = (3, 6, 9)$ into components parallel and perpendicular to the vector $\mathbf{q} = (1, -2, 2)$.

3. (2 points)

   (a) Find the matrix product AB if $A = \begin{bmatrix} 2 & x & 5 \\ 4 & 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & x \\ -3 & 2 \\ -1 & 2 \end{bmatrix}$

   (b) Find the largest possible value of determinant (AB).

4. (3 points) Point A has coordinates (2; -4; 6) in the old coordinate system. Find its coordinates in the new coordinate system obtained from the initial one by transferring the origin to point N that has coordinates (7; 1; -4) in the old coordinate system.

5. (3 points) Subspace S of $\mathbb{R}^3$ is formed by linear combination of vectors v_1 and v_2. Find a vector v that is orthogonal to S, if $v_1 = \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$

6. (3 points) Let

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \tag{3}$$

   Find All **Natural** numbers ($k \in \mathbb{N}$) where: $A^k = A^{-1}$ (Also you need to check if A is invertible), Note that $A^k = \underbrace{A.A \ldots A}_{k \ times}$

End of Test 1

AGLA 1. TEST 1. **VARIANT 4.** 15 points, 60 minutes

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (2 points) For each of the following statements mark it as True or False. Justify each answer.

(a) The determinant of a transition matrix (between two bases) can be any real number.	
(b) Any straight line is a subspace of R^2	

2. (2 points) Decompose the vector $\mathbf{p} = (2, 4, 6)$ into components parallel and perpendicular to the vector $\mathbf{q} = (1/3, -2/3, 2/3)$.

3. (2 points)

   (a) Find the matrix product AB if $A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & x \end{bmatrix}$, $B = \begin{bmatrix} 5 & -1 \\ x & 2 \\ -3 & -1 \end{bmatrix}$

   (b) Find the largest possible value of determinant (AB).

4. (3 points) Point A has coordinates (1; -1; -19) in the old coordinate system. Find its coordinates in the new coordinate system obtained from the initial one by transferring the origin to point N that has coordinates (4; -5; 2) in the old coordinate system.

5. (3 points) Subspace S of $\mathbb{R}^3$ is formed by linear combination of vectors v_1 and v_2. Find a vector v that is orthogonal to S, if $v_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

6. (3 points) Let

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1/7 \\ 0 & -7 & 0 \end{bmatrix} \tag{4}$$

   Find All **Natural** numbers ($k \in \mathbb{N}$) where: $A^k = A^{-1}$ (Also you need to check if A is invertible), Note that $A^k = \underbrace{A.A \dots A}_{k\ times}$ ...

End of Test 1