Analytical Geometry and Linear Algebra I, Lab 10

Conic sections (2nd order curve equation): Hyperbola

From general to canonical form

Tangent line to a curve

Questions for today

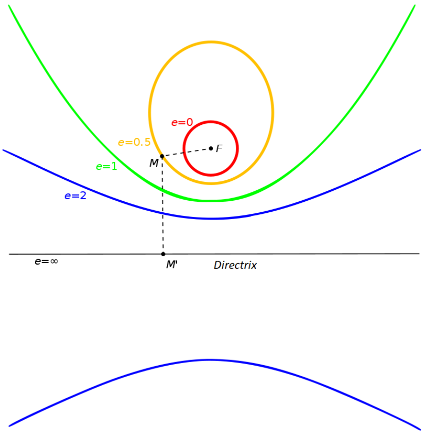
- How can I work with general form of 2nd order curve equation?
- How it relates with cone?
- What forms of equation do we have?

Some definitions, which can be helpful

Eccentricity, Directrix

Eccentricity is a measure of how much a conic section deviates from being circular.

It is a constant ration between distance from focal to point on the curve and from the point on the curve to **directrix**.



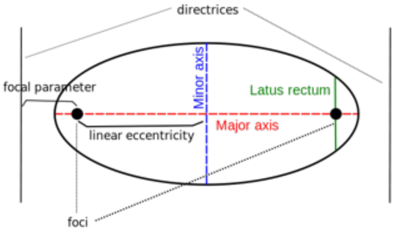
Some definitions, which can be helpful

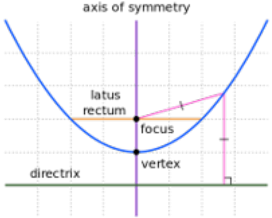
Linear eccentricity, Latus Rectrum, Focal parameter

The **linear eccentricity** is the distance between the center and the focus (or one of the two foci).

The **latus rectum** is the chord parallel to the directrix and passing through the focus (or one of the two foci).

The **focal parameter** is the distance from the focus (or one of the two foci) to the directrix.

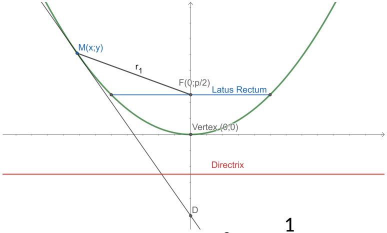




Parabola

Forms:

- **Canonical** $2py = x^2$
- **General** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where either $A = 0$ or $C = 0$, not both
- **Parametric** $\begin{cases} x = \sqrt{2}pt \\ y = pt^2 \end{cases}$



Parabola $y = x^2$, $p = \frac{1}{2}$

Properties:

- **Vertex** $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- **Center** Not defined
- **Eccentricity** $ecc = 1$
- **Linear Eccentricity** Not defined
- **Foci** $F = \begin{pmatrix} 0 \\ \frac{p}{2} \end{pmatrix}$

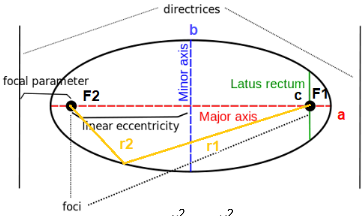
- **Latus Rectum** (length of chord) $2p$
- **Focal parameter** p
- **Discriminant** $\mathfrak{D} = B^2 - 4AC = 0$
- **Directrix eq.** $y = -\frac{p}{2}$

- **Tangent eq.** $yy_{tangent} = p(x + x_{tangent})$
- $r = |\overline{FM}| = \sqrt{(x - \frac{p}{2})^2 + y^2}$
- $\triangle MFD$ is isosceles, where MD – tangent to M

Ellipse

Forms:

- **Canonical** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- **General** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $AC > 0$
- **Parametric** $\begin{cases} x = a\,cos(\alpha) \\ y = b\,sin(\alpha) \end{cases}$



Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Properties:

- **Vertex** $\begin{pmatrix} \pm a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ \pm b \end{pmatrix}$
- **Center** $(0; 0)$
- **Eccentricity** $0 \leq ecc < 1$, $ecc = \sqrt{1 - \frac{b^2}{a^2}}$
- **Linear Eccentricity** $c = \sqrt{a^2 - b^2}$

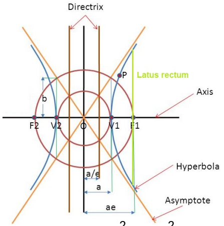
- **Foci** $F = \begin{pmatrix} \pm c \\ 0 \end{pmatrix} = \begin{pmatrix} \pm(ecc\ a) \\ 0 \end{pmatrix}$
- **Latus Rectum** (length of chord) $\frac{2b^2}{a}$
- **Focal parameter** $\frac{b^2}{\sqrt{a^2 - b^2}}$
- **Discriminant** $\mathfrak{D} = B^2 - 4AC < 0$
- **Directrix eq.** $x = \pm \frac{a}{ecc}$

- **Tangent eq.** $\frac{x_{tangent}x}{a^2} + \frac{y_{tangent}y}{b^2} = 1$
- $r_1 + r_2 = 2a$
- $r_{1,2} = |\overline{F_{1,2}M}| = \sqrt{(x \pm c)^2 + y^2}$
- $\frac{r_1}{d_1} = ecc$

Hyperbola

Forms:

- **Canonical** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- **General** $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $AC < 0$
- **Parametric** $\begin{cases} x = \dfrac{a}{cos(\alpha)} \\ y = b\, tg(\alpha) \end{cases}$



Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Properties:

- **Vertex** $\begin{pmatrix} \pm a \\ 0 \end{pmatrix}$
- **Center** $(0; 0)$
- **Eccentricity** $ecc > 1$,
  $ecc = \sqrt{1 + \frac{b^2}{a^2}}$
- **Linear Eccentricity**
  $c = \sqrt{a^2 + b^2}$

- **Foci** $F = \begin{pmatrix} \pm c \\ 0 \end{pmatrix} = \begin{pmatrix} \pm(ecc\ a) \\ 0 \end{pmatrix}$
- **Latus Rectum** (length of chord)
  $\frac{2b^2}{a}$
- **Focal parameter** $\frac{b^2}{\sqrt{a^2+b^2}}$
- **Discriminant** $\mathfrak{D} = B^2 - 4AC > 0$
- **Directrix eq.** $x = \pm \frac{a}{ecc}$

- **Asymptots eq.** $y = \pm \frac{b}{a} x$
- **Tangent eq.**
  $\frac{x_{tangent} x}{a^2} - \frac{y_{tangent} y}{b^2} = 1$
- $r = |\overline{F_{closest} M}| = |x\ ecc - a|$
- $\frac{r_1}{d_1} = ecc$

Task 1

Find the centre and eccentricity of the hyperbola $9x^2 - 4y^2 + 18x + 16y - 43 = 0$

Task 1

Answer

$$9\,(x^2 + 2x) - 4\,(y^2 - 4y) - 43 = 0$$
$$9\,(x + 1)^2 - 9 - 4(y - 2)^2 + 19 - 43 = 0$$
$$9\,(x + 1)^2 - 4\,(y - 2)^2 = 36$$

Hence, centre is $(-1, 2)$, $a^2 = 4$ and $b^2 = 9$.

$$\frac{(x+1)^2}{4} - \frac{(y-2)^2}{9} = 1$$

$$b^2 = a^2\,(e^2 - 1) \text{ gives } e = \frac{\sqrt{13}}{2}.$$

From general to canonical form

Special case: when $B = 0$

$Ax^2 + Cy^2 + 2Dx + 2Ey + F = 0$ — General form.

Example of transformation from general to canonical form:

$$16x^2 + 25y^2 - 32x + 50y - 359 = 0 \Rightarrow$$

$$(16x^2 - 32x) + (25y^2 + 50y) - 359 = 0 \Rightarrow$$

$$16(x^2 - 2x) + 25(y^2 + 2y) = 359 \Rightarrow$$

$$16(x^2 - 2x + 1) + 25(y^2 + 2y + 1) = 350 + 16 + 25 \Rightarrow$$

$$16(x - 1)^2 + 25(y + 1)^2 = 400 \Rightarrow$$

$$\frac{(x-1)^2}{25} + \frac{(y+1)^2}{16} = 1$$

From general to canonical form

General case: classical method

Algorithm

1. Find angle of rotation, using $(C - A)sin(2\alpha) + Bcos(2\alpha) = 0$
2. Using roration matrix, write down a transformation from $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix}$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x'cos(\alpha) & -y'sin(\alpha) \\ x'sin(\alpha) & y'cos(\alpha) \end{bmatrix}$
3. Substitute it to original equation and simplify to a canoncial equation with shifting (for instance, $\frac{x' + x_{shift}}{2} + \frac{y' + y_{shift}}{8} = 1$)
4. Change the variables again ($\begin{bmatrix} x' \\ y' \end{bmatrix} \rightarrow \begin{bmatrix} x'' \\ y'' \end{bmatrix}$; $\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' + x_{shift} \\ y' + y_{shift} \end{bmatrix}$). It gives you a pure canoncical form.
5. Write a system which shows $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x'' \\ y'' \end{bmatrix}$.

From general to canonical form

Classical method: case study (1)

$xy = -2 \quad (1)$ a, b, c, e, directrix eq., asymptots -?

1) Classical method

$A=0,\ B=1,\ C=0$

$\underbrace{(A-C)}_{0}\sin 2\alpha + B\cos 2\alpha = 0 \Rightarrow \cos 2\alpha = 0 \Rightarrow 2\alpha = 90^\circ$

$\alpha = 45^\circ$

$$\begin{cases} x = \frac{1}{\sqrt2}x' - \frac{1}{\sqrt2}y' = \frac{1}{\sqrt2}(x'-y') \\ y = \frac{1}{\sqrt2}x' + \frac{1}{\sqrt2}y' = \frac{1}{\sqrt2}(x'+y') \end{cases} \quad (2)$$

$(2) \to (1)$

$\frac{1}{\sqrt2}\cdot\frac{1}{\sqrt2}(x'-y')(x'+y') = -2$

$(x'^2 - y'^2) = -4$

$\frac{y'^2}{4} - \frac{x'^2}{4} = 1$ – Canonical form

$a=2,\ b=2,\ e=\sqrt{1+1}=\sqrt2$

Directrix eq. in new basis $y' = \pm\frac{a}{e} = \pm\sqrt2 \quad (3)$

Asymptots eq. in new basis $x' = \pm y' \quad (4)$

Let's find a transformation $x(x',y') \to x'(x,y)$, using (2)

$$\begin{cases} x = \frac{x'-y'}{\sqrt2} \\ y = \frac{x'+y'}{\sqrt2} \end{cases} \Bigg| \cdot\sqrt2 \qquad \begin{cases} \sqrt2 x = x'-y' \\ \sqrt2 y = x'+y' \end{cases} \Big| \text{"2"+"1"},\ \text{"2"-"1"}$$

$$\begin{cases} \frac{x+y}{\sqrt2} = x' \\ \frac{y-x}{\sqrt2} = y' \end{cases} \quad (5)$$

Directrix eq. in old basis $(5) \to (3)$

$y - x = \pm 2$

Asymptots eq. in old basis $(5) \to (4)$

$x = 0,\ y = 0$

From general to canonical form

General case: Other way

Problem

In some cases it's quite tough to convert from general form to canonical using classical method from the class (bad numbers).

Solution

$f = Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

can be converted into canonical form in new variables $\tilde{x}, \tilde{y}$

by the equation: $\dfrac{\tilde{x}^2}{-S/(\lambda_1^2\lambda_2)} + \dfrac{\tilde{y}^2}{-S/(\lambda_1\lambda_2^2)} = 1$, where

- S is determinant of $\begin{pmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{pmatrix}$ matrix
- $\lambda_{1,2}$ are the eigenvalues of $\begin{pmatrix} A & B/2 \\ B/2 & C \end{pmatrix}$ matrix.

  It can be found using this equation:
  $\lambda^2 - (A + C)\lambda + (AC - (B/2)^2) = 0$

Using *canonical form*, we can find $a,\ b,\ c,\ ecc$ for the curve, but not *coordinate dependent properties (like vertex, directrix eq.)*.

For this purpose, we need to find **angle** and **shift**.

Angle: $(C - A)\sin(2\alpha) + B\cos(2\alpha) = 0; \rightarrow \alpha = \ldots$

Shift (center of the curve)

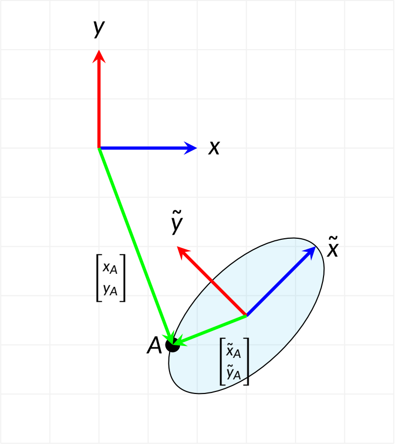
$\begin{cases} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{cases} \rightarrow$ 2 equations of line $\rightarrow$ solve system $(x_c; y_c)$

Using this transformation, we can find original coordinates, knowing the new one

$$\begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & x_c \\ \sin(-\alpha) & \cos(-\alpha) & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_A \\ \tilde{y}_A \\ 1 \end{bmatrix}$$

From general to canonical form

General case: Other way (2)



From general to canonical form

Other method: case study

$xy = -2 \ (1)$ a, b, c, e, directrix eq., asymptots -?

2) Other method

$$0x^2 + xy + 0y^2 + 0x + 0y + 2 = 0 \ (2)$$

$A=0 \quad B=1 \quad C=0 \quad D=0 \quad E=0 \quad F=2$

$$\frac{x'^2}{-S \cdot \frac{1}{\lambda_1^2 \lambda_2}} + \frac{y'^2}{-S\frac{1}{\lambda_1 \lambda_2^2}} = 1$$

$$S = \det\begin{pmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{pmatrix} = \det\begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} = -\frac{1}{2}(1) = -1/2$$

$$eig\begin{pmatrix} A & B/2 \\ B/2 & C \end{pmatrix} = \begin{vmatrix} A-\lambda & B/2 \\ B/2 & C-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - (A+C)\lambda + (AC - (B/2)^2) = 0 \Rightarrow \lambda^2 = \frac{1}{4} \Rightarrow$$

$$\Rightarrow \lambda_{1,2} = \pm\frac{1}{2}$$

$$\frac{x'^2}{\frac{1}{2} \cdot \frac{1}{\frac{1}{4}\cdot(-\frac{1}{2})}} + \frac{y'^2}{\frac{1}{2} \cdot \frac{1}{\frac{1}{4}\cdot\frac{1}{2}}} = 1 \Rightarrow \frac{y'^2}{4} - \frac{x'^2}{4} = 1 \ (3)$$

$a=2,\ b=2,\ e=\sqrt{1+1} = \sqrt{2}$

Directrix eq. in new basis $y' = \pm\frac{a}{e} = \pm\sqrt{2} \ (4)$

Asymptots eq. in new basis $x' = \pm y' \ (5)$

For finding coordinate dependent data (equations like (4), (5)), we need to find angle of rotation and a shift.

Angle

$$(A-C)\sin(2\alpha) + B\cos(2\alpha) = 0 \Rightarrow 2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$$

$A-C = 0$

Shift

Partial derivative

$$\frac{\partial(2)}{\partial x} = 0\cdot 2x + 1\cdot 1\cdot y + 0 + 0\cdot 1 + 0 + 0 = 0 \Rightarrow y_c = 0$$

$$\frac{\partial(2)}{\partial(y)} = 0 + 1\cdot 1\cdot x + 0\cdot 2\cdot y + 0 + 0\cdot 1 + 0 = 0 \Rightarrow x_c = 0$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} c(-\alpha) & -s(-\alpha) & x_c \\ s(-\alpha) & c(-\alpha) & y_c \\ 0 & 0 & 1 \end{bmatrix}}_{H} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

If youneed to find just coordinates (like foci), put it here

If you need to find an equation -> $H^{-1}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \ (6)$

Put it $(6) \to (4)$

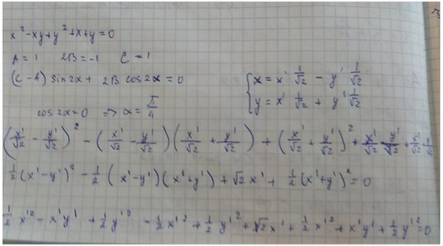
$(6) \to (5)$

Task 2

Transform $x^2 - xy + y^2 + x + y = 0$ into pure canoncial form

Task 2

Answer



$x^2 - xy + y^2 + x + y = 0$

$A = 1 \quad 2B = -1 \quad C = 1$

$(C - A)\sin 2\alpha + 2B\cos 2\alpha = 0$

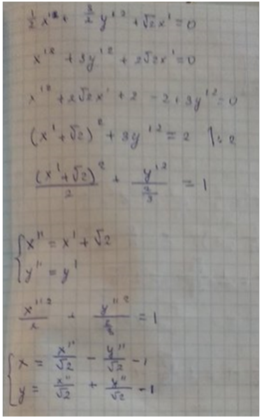
$\cos 2\alpha = 0 \Rightarrow \alpha = \frac{\pi}{4}$

$\begin{cases} x = x'\frac{1}{\sqrt2} - y'\frac{1}{\sqrt2} \\ y = x'\frac{1}{\sqrt2} + y'\frac{1}{\sqrt2} \end{cases}$

$\left(\frac{x'}{\sqrt2} - \frac{y'}{\sqrt2}\right)^2 - \left(\frac{x'}{\sqrt2} - \frac{y'}{\sqrt2}\right)\left(\frac{x'}{\sqrt2} + \frac{y'}{\sqrt2}\right) + \left(\frac{x'}{\sqrt2} + \frac{y'}{\sqrt2}\right)^2 + \frac{x'}{\sqrt2} - \frac{y'}{\sqrt2} + \frac{x'}{\sqrt2} + \frac{y'}{\sqrt2}$

$\frac12(x'-y')^2 - \frac12(x'-y')(x'+y') + \sqrt2 x' + \frac12(x'+y')^2 = 0$

$\frac12 x'^2 - x'y' + \frac12 y'^2 - \frac12 x'^2 + \frac12 y'^2 + \sqrt2 x' + \frac12 x'^2 + x'y' + \frac12 y'^2 = 0$



$\frac12 x'^2 + \frac32 y'^2 + \sqrt2 x' = 0$

$x'^2 + 3y'^2 + 2\sqrt2 x' = 0$

$x'^2 + 2\sqrt2 x' + 2 - 2 + 3y'^2 = 0$

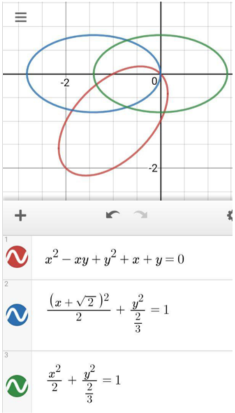
$(x' + \sqrt2)^2 + 3y'^2 = 2 \quad | : 2$

$\frac{(x'+\sqrt2)^2}{2} + \frac{y'^2}{\frac23} = 1$

$\begin{cases} x'' = x' + \sqrt2 \\ y'' = y' \end{cases}$

$\frac{x''^2}{2} + \frac{y''^2}{\frac23} = 1$

$\begin{cases} x = \frac{x''}{\sqrt2} - \frac{y''}{\sqrt2} - 1 \\ y = \frac{x''}{\sqrt2} + \frac{y''}{\sqrt2} - 1 \end{cases}$



Task 3

Prove that a curve given by $7x^2 + 48xy - 7y^2 - 62x - 34y + 98 = 0$ is a hyperbola. Find the eccentricity of this hyperbola, coordinates of its center and foci. Find the equations of axes, asymptotes and directrices of this hyperbola.

Task 3

Answer

Eccentricity is $\sqrt{2}$;

center — $(1; 1)$;

foci — $F_1(-\frac{1}{5}; \frac{13}{5})$, $F_2(\frac{11}{5}; \frac{-3}{5})$;

real axis is $4x + 3y - 7 = 0$;

imaginary axis — $3x - 4y + 1 = 0$;

directrices are $3x - 4y - 4 = 0$ and $3x - 4y + 6 = 0$;

asymptotes — $x + 7y = 9$, $7x - y = 6$.

Task 4

Find the equation to the hyperbola that passes through $(2; 3)$ and has for its asymptotes the lines $4x + 3y - 7 = 0$ and $x - 2y = 1$

Task 4

Answer

The combined equation of the asymptotes is $(4x + 3y - 7)(x - 2y - 1) = 0$.
Hence, the equation of the hyperbola is $(4x + 3y - 7)(x - 2y - 1) + k = 0$.
This pass through (2,3).

$$(8+9-7)(2-6-1)+k=0$$
$$\therefore\ k=50$$

Hence, the equation of the hyperbola is

$$(4x+3y-7)(x-2y-1)+50=0$$
$$\text{(i.e.)}\quad 4x^2-5xy-6y^2-11x+11y+57=0$$

Task 5

Find the equations of lines tangent to curve $6xy + 8y^2 - 12x - 26y + 11 = 0$ that are

(a) parallel to line $6x + 17y - 4 = 0$;

(b) perpendicular to line $41x - 24y + 3 = 0$;

(c) parallel to line $y = 2$.

Task 5

Answer

$$6xy + 8y^2 - 12x - 26y + 11 = 0 \quad (1)$$

Parallel to the line $6x + 17y - 4 = 0 \quad (2)$

It means, that $\underline{\underline{k}}x + b = y$

$\rightarrow$ shoud be the same in tangent (1) and (2)

k in (2) $k = -\frac{6}{17}$

$$\frac{d(1)}{dx} = 6xy' + 16yy' - 12 - 26y' + 6y$$

$$y'(6x + 16y - 26) = 12 - 6y$$

$$y' = \frac{12 - 6y}{6x + 16y - 26} = k$$

$$\frac{12 - 6y}{6x + 16y - 26} = -\frac{6}{17} \Rightarrow y = 6x + 8 \quad (3)$$

$(3) \Rightarrow (1)$

$$6x(6x+8) + 8(6x+8)^2 - 12x - 26(6x+8) + 11 = 0 \Rightarrow$$

$$x_1 = -\frac{7}{6}; \quad x_2 = \frac{-5}{6} \quad (4)$$

$$(4) \Rightarrow (3) \quad y_1 = 1 \quad y_2 = 3 \quad (5)$$

Parallel lines have the same "k" (slope), but different "b" - intercept

$$(5), (4) \rightarrow 6x + 17y + b = 0$$

$$b_1 = -10 \qquad 6x + 17y - 10 = 0$$

$$b_2 = -46 \Rightarrow 6x + 17y - 46 = 0$$

Task 5

Answer

(a) $6x + 17y - 10 = 0$ and $6x + 17y - 46 = 0$

(b) $24x + 41y - 22 = 0$ and $24x + 41y - 94 = 0$

(c) no solution

Task 6

Determine types of curves given by the following equations. For each of the curves, find its canonical coordinate system (i.e. indicate the coordinates of origin and new basis vectors in the initial coordinate system) and its canonical equation.

(a) $9x^2 - 16y^2 - 6x + 8y - 144 = 0$;

(b) $9x^2 + 4y^2 + 6x - 4y - 2 = 0$;

(c) $12x^2 - 12x - 32y - 29 = 0$;

(d) $xy + 2x + y = 0$;

(e) $5x^2 + 12xy + 10y^2 - 6x + 4y - 1 = 0$;

(f) $8x^2 + 34xy + 8y^2 + 18x - 18y - 17 = 0$;

(g) $25x^2 - 30xy + 9y^2 + 68x + 19 = 0$;

(h) $x^2 + 2xy + y^2 - 5x - 5y + 4 = 0$.

Task 6

Answer

(a) hyperbola $\frac{X^2}{16} - \frac{Y^2}{9} = 1$, $O'(\frac{1}{3};\ \frac{1}{4})$, $\mathbf{i}(1;\ 0)$, $\mathbf{j}(0;\ 1)$;

(b) ellipse $X^2 + \frac{Y^2}{4/9} = 1$, $O'(-\frac{1}{3};\ \frac{1}{2})$, $\mathbf{i}(0;\ 1)$, $\mathbf{j}(-1;\ 0)$;

(c) parabola $X^2 = \frac{8}{3}X$, $O'(\frac{1}{3}; -1)$, $\mathbf{i}(0;\ 1)$, $\mathbf{j}(-1;\ 0)$;

(d) hyperbola $\frac{X^2}{4} - \frac{Y^2}{4} = 1$, $O'(-1; -2)$, $\mathbf{i}(\frac{1}{\sqrt{2}};\ \frac{1}{\sqrt{2}})$, $\mathbf{j}(-\frac{1}{\sqrt{2}};\ \frac{1}{\sqrt{2}})$;

(e) ellipse $\frac{X^2}{1}4 + Y^2 = 1$, $O'(3; -2)$, $\mathbf{i}(\frac{3}{\sqrt{13}}; -\frac{2}{\sqrt{13}})$, $\mathbf{j}(\frac{2}{\sqrt{13}};\ \frac{3}{\sqrt{13}})$;

(f) hyperbola $\frac{X^2}{1/9} - \frac{Y^2}{1/25} = 1$, $O'(1; -1)$, $\mathbf{i}(\frac{1}{\sqrt{2}}; -\frac{1}{\sqrt{2}})$, $\mathbf{j}(\frac{1}{\sqrt{2}};\ \frac{1}{\sqrt{2}})$;

(g) parabola $Y^2 = \frac{6}{\sqrt{34}}$, $O'(-\frac{11}{17};\ \frac{10}{17})$, $\mathbf{i}(-\frac{3}{\sqrt{34}};\ -\frac{5}{\sqrt{34}})$, $\mathbf{j}(\frac{5}{\sqrt{34}}; -\frac{3}{\sqrt{34}})$;

(h) two parallel lines given by the equations $x + y = 4$ and $x + y = 1$ in initial coordinates.

Reference material

- Conic sections (slides, rus)
- How to find centre of conic (video, eng)
- Find the equation of major and minor axis of the given conic
- How to go from general to canonical form (mathprofi, rus)

Deserve “A” grade!

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Room 105 (Underground robotics lab)