Essentials of Analytical Geometry and Linear Algebra I, Class #2

Innopolis University, September 2022

1 Dot Product

1. Find $|\mathbf{a}|^2 - 2\sqrt{3}\mathbf{a}\cdot\mathbf{b} - 7|\mathbf{b}|^2$ given that $|\mathbf{a}| = 4$, $|\mathbf{b}| = 1$, $\angle(\mathbf{a}, \mathbf{b}) = 150°$.
2. Find the angle[1] between $\mathbf{a}=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}-5\\1\\-1\end{bmatrix}$.
3. Prove that vectors $\mathbf{b}(\mathbf{a}\cdot\mathbf{c})-\mathbf{c}(\mathbf{a}\cdot\mathbf{b})$ and $\mathbf{a}$ are perpendicular to each other.
4. All three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ have length of 3 and $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$. Find $\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{c}+\mathbf{c}\cdot\mathbf{a}$
5. Find an angle between $\mathbf{a}$ and $\mathbf{b}$ if:
   (a) $\mathbf{a}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$, $\mathbf{b}=\begin{bmatrix}-2\\2\\-2\end{bmatrix}$;
   (b) $\mathbf{a}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$, $\mathbf{b}=\begin{bmatrix}5\\1\\1\end{bmatrix}$;
   (c) $\mathbf{a}=\begin{bmatrix}3\\1\\-2\end{bmatrix}$, $\mathbf{b}=\begin{bmatrix}-2\\2\\-2\end{bmatrix}$.
6. There are two vectors on some basis $\mathbf{a}=\begin{bmatrix}x\\1-x\end{bmatrix}$, $\mathbf{b}=\begin{bmatrix}x^2-2x\\x^2-2x+1\end{bmatrix}$. It is needed to find x, when:
   (a) vectors are collinear;
   (b) they have the same direction.
7. There are two vectors $\mathbf{a}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}5\\1\\1\end{bmatrix}$. Length of $\mathbf{c}$ is equal to 1 and the vector is perpendicular to $\mathbf{a}$. The angle between $\mathbf{b}$ and $\mathbf{c}$ is $arccos(\sqrt{\frac{2}{27}})$. Find the coordinates of $\mathbf{c}$. How many solutions the task have?

[1] If not stated otherwise, the coordinate system in this section is supposed to be Cartesian.

2 Cross Product

1. Find the cross product[2] of:

   (a) vectors $\mathbf{a}=\begin{bmatrix}3\\-2\\1\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}2\\-5\\-3\end{bmatrix}$;

   (b) vectors $\mathbf{a}=\begin{bmatrix}3\\-2\\1\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}-18\\12\\-6\end{bmatrix}$.

2. Simplify the expressions:

   (a) $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b})$;

   (b) $(3\mathbf{a}-\mathbf{b}-\frac{1}{3}\mathbf{c})\times(2\mathbf{a}+\frac{3}{2}\mathbf{b}-3\mathbf{c})$.

[2] If not stated otherwise, the coordinate system in this assignment is supposed to be Cartesian.