Essentials of Analytical Geometry and Linear Algebra I, Class #1

Innopolis University, August 2022

1. Points $\mathbf{A}(3,-2)$ and $\mathbf{B}(1,4)$ are given. The $\mathbf{M}$ point is on the line $\mathbf{AB}$ in the way that $|\mathbf{AM}| = 3|\mathbf{AB}|$. Find coordinates of the $\mathbf{M}$ point, if:
1. The points $\mathbf{M}$ and $\mathbf{B}$ are from the same side from A.
2. The points $\mathbf{M}$ and $\mathbf{B}$ are from the different sides from A.

2. Check if the result of each of the following operations is a vector or not. Explain your answer.
1. $\mathbf{a} + \mathbf{b}$, if $\mathbf{a}$ and $\mathbf{b}$ are vectors
2. $\mathbf{a} - \mathbf{a}$, if $\mathbf{a}$ is a vector
3. $\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix}$
4. $\begin{bmatrix}2x + 15 - 4y\\ y - x\end{bmatrix}$, if x and y are integer numbers
5. $\begin{bmatrix}x + y\\ 2y + 122 - 3x\end{bmatrix} - \begin{bmatrix}x + y\\ 2y + 122 - 3x\end{bmatrix}$, if x and y are real numbers

3. Three vectors are given $\mathbf{a}\begin{bmatrix}1\\2\end{bmatrix}, \mathbf{b}\begin{bmatrix}-5\\-1\end{bmatrix}, \mathbf{c}\begin{bmatrix}-1\\3\end{bmatrix}$. Find the vectors $2\mathbf{a}+3\mathbf{b}-\mathbf{c}$ and $16\mathbf{a} + 5\mathbf{b} - 9\mathbf{c}$.

4. Check for each case if the following set of vectors is a basis or not. Explain your answer.
1. $\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix}$
2. $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}$
3. $\begin{bmatrix}1\\2\\3\end{bmatrix}$
4. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\3\end{bmatrix}$
5. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}$
6. $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}$

5. Check for each case if the following set of vectors is a subspace or not. Explain your answer.
1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0,\ y > 0$
5. Inner circle with the radius $r = 5$

6. Find the coordinates of the gravity center of a triangular plate **ABC** with vertices in points $\mathbf{A}(3,1), \mathbf{B}(6,3), \mathbf{C}(0,2)$.

7. Check for each case if the following set of vectors is a coplanar or not. Explain your answer.

1. $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\1\\5\end{bmatrix}, \begin{bmatrix}-1\\3\\2\end{bmatrix}$
2. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\4\end{bmatrix}, \begin{bmatrix}1\\3\\3\end{bmatrix}$
3. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}3\\3\\3\end{bmatrix}$

8. In the plane of the triangle **ABC** find the point **O** such that $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \mathbf{0}$. Are there such points outside of the triangle?
Note: **0** is a zero-vector.