Analytical Geometry and Linear Algebra I, Class #10

Innopolis University, October 2022

1. Determine types of curves given by the following equations[1]. For each of the curves, find its canonical coordinate system (i.e. indicate the coordinates of origin and new basis vectors in the initial coordinate system) and its canonical equation.

   (a) $9x^2 - 16y^2 - 6x + 8y - 144 = 0$;

   (b) $9x^2 + 4y^2 + 6x - 4y - 2 = 0$;

   (c) $12x^2 - 12x - 32y - 29 = 0$;

   (d) $xy + 2x + y = 0$;

   (e) $5x^2 + 12xy + 10y^2 - 6x + 4y - 1 = 0$;

   (f) $8x^2 + 34xy + 8y^2 + 18x - 18y - 17 = 0$;

   (g) $25x^2 - 30xy + 9y^2 + 68x + 19 = 0$;

   (h) $x^2 + 2xy + y^2 - 5x - 5y + 4 = 0$.

2. Prove that a curve given by $34x^2 + 24xy + 41y^2 - 44x + 58y + 1 = 0$ is an ellipse. Find the major and minor axes of this ellipse, its eccentricity, coordinates of its center and foci. Find the equations of axes and directrices of this ellipse.

3. Prove that a curve given by $7x^2 + 48xy - 7y^2 - 62x - 34y + 98 = 0$ is a hyperbola. Find the eccentricity of this hyperbola, coordinates of its center and foci. Find the equations of axes, asymptotes and directrices of this hyperbola.

4. Prove that a curve given by $x^2 + 2xy + y^2 + x = 0$ is a parabola. Find the coordinates of its vertex and focus. Find the equations of axis and directrix of this parabola.

5. Find the equations of lines tangent to curve $6xy + 8y^2 - 12x - 26y + 11 = 0$ that are

   (a) parallel to line $6x + 17y - 4 = 0$;

   (b) perpendicular to line $41x - 24y + 3 = 0$;

   (c) parallel to line $y = 2$.

6. Find the equations of lines tangent to curve $5x^2 + 6xy + 5y^2 - 16x - 16y - 16 = 0$ that pass through

   (a) point $(3; 3)$;

   (b) point $(0; -\frac{4}{5})$;

   (c) point $(0; 1)$.

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[1]In all problems in this section the coordinate system is considered to be Cartesian.