Analytical Geometry and Linear Algebra 1

Introduction
Vector + Operations of vectors
Basis
Subspace

About me (1)

Bachelor in BMSTU

“CAD developer” track

ДИЧь / RAGE

(Древнерусские Игры и чай)

club leader

[Channel](#)

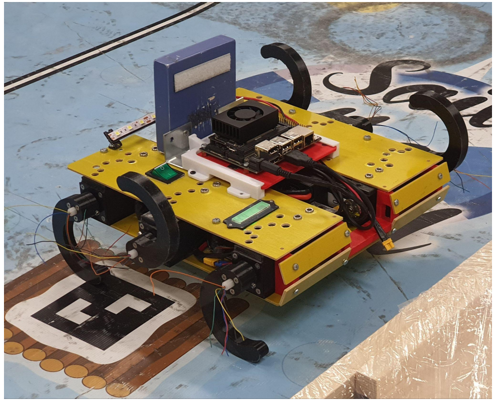
Finished PhD in Inno

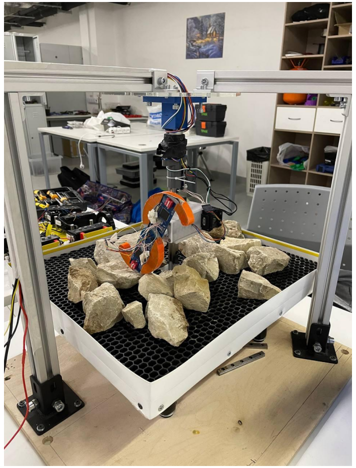
Research Fellow

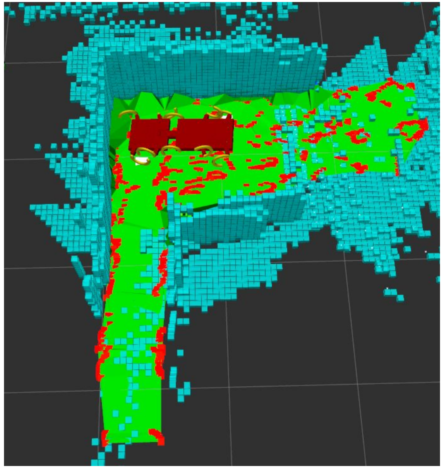
About me (2)

Research interest - Multilegged biomimetic robots

Dissertation title: Perception method development for a mobile multilegged robot in a cave environment

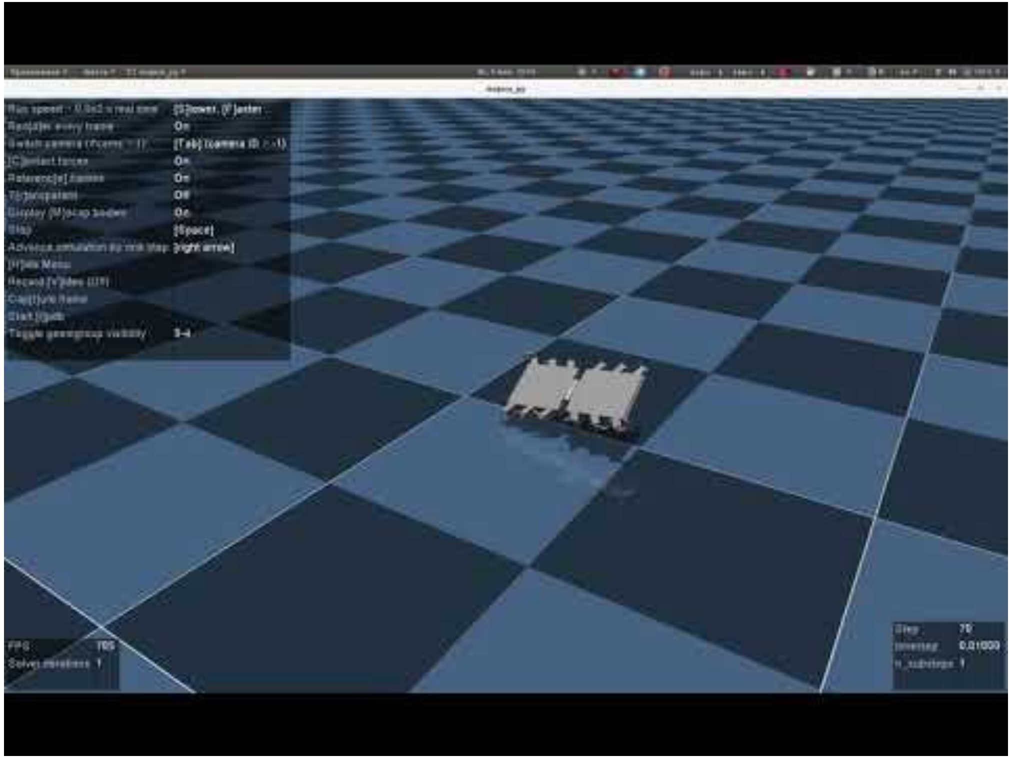


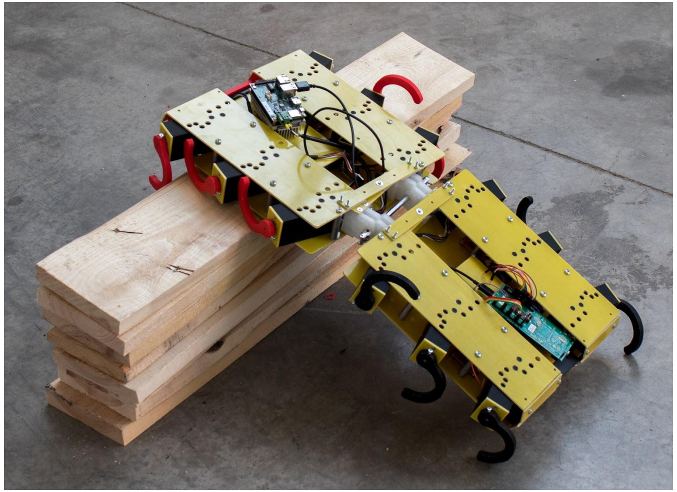




Where did I use Linear Algebra

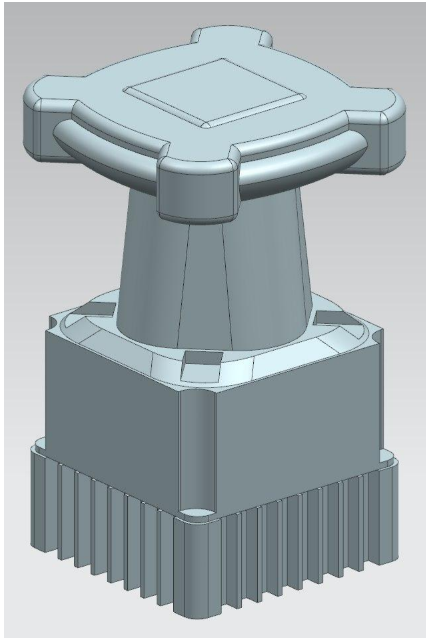
Where did I use linear algebra (1): Research





Multilegged robot StriRus

Where did I use linear algebra (2): CAD



Solid Modeling

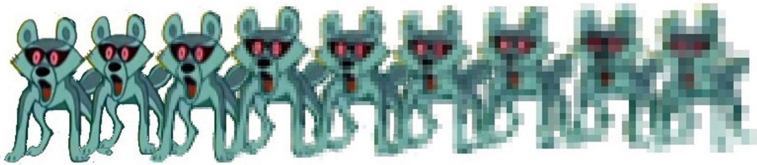
Surface Modeling

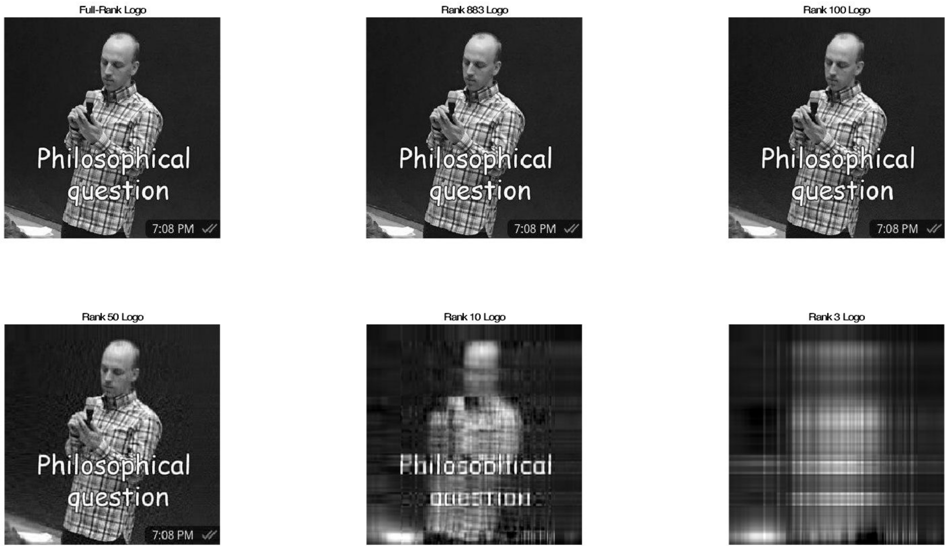
Image compression

Task: We want to compress our image for reducing the size

Solution: We can represent our picture as a matrix.

Next step is using SVD for reducing matrix rank.





More details in matlab code (pdf)

Computer vision

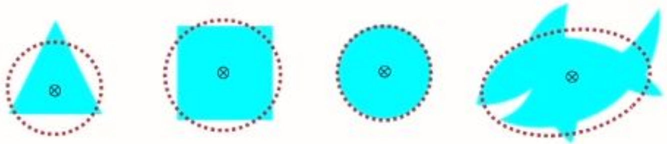
Task: we want to know the orientation of the object

Needed terms: [Centroid](), [Image moments]()

Solution: use equivalent ellipse method. We consider an ellipse, witch:

- *centred at the object's centroid*
- *same moments of inertia* about centroid.

Afterwards, we *find an ellipse* using *eigenvalues and eigenvectors*



Equivalent ellipse for several objects

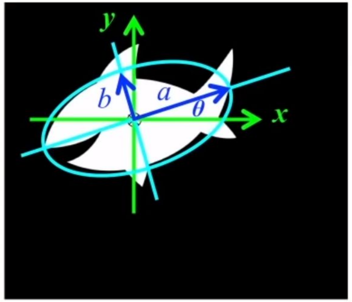
An ellipse with the inertia matrix

$$\mathbf{J} = \begin{pmatrix} \mu_{20} & \mu 11 \\ \mu 11 & \mu_{02} \end{pmatrix}$$

has radii

$$a = 2\sqrt{\frac{\lambda_1}{m_{00}}}, \quad b = 2\sqrt{\frac{\lambda_2}{m_{00}}}$$

where $\lambda_1 > \lambda_2$ are the eigenvalues of **J**



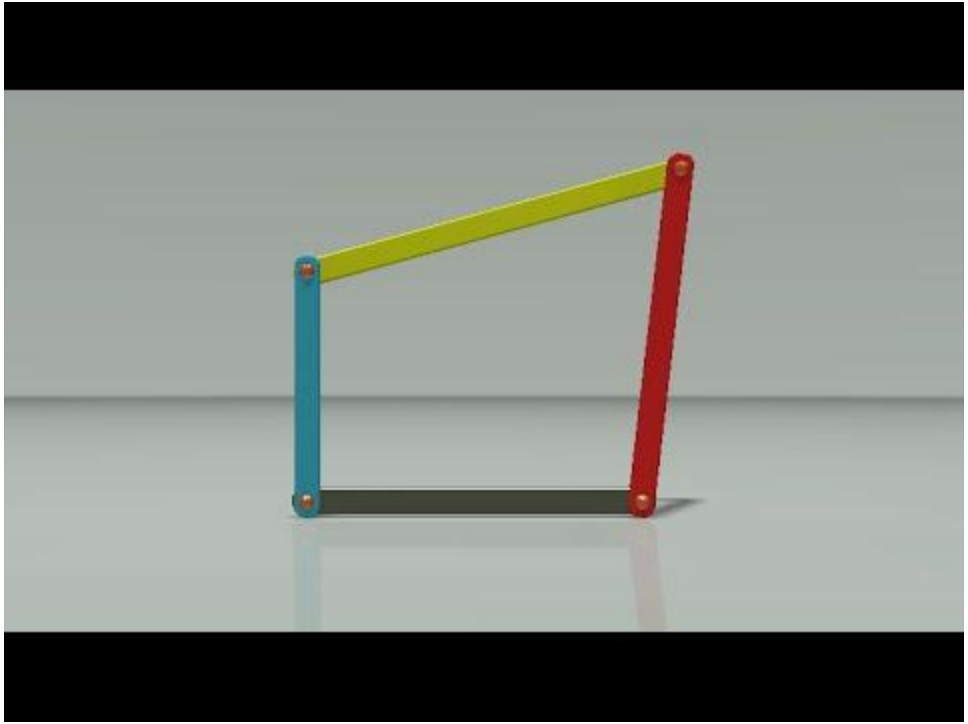
major axis length a
minor axis length b

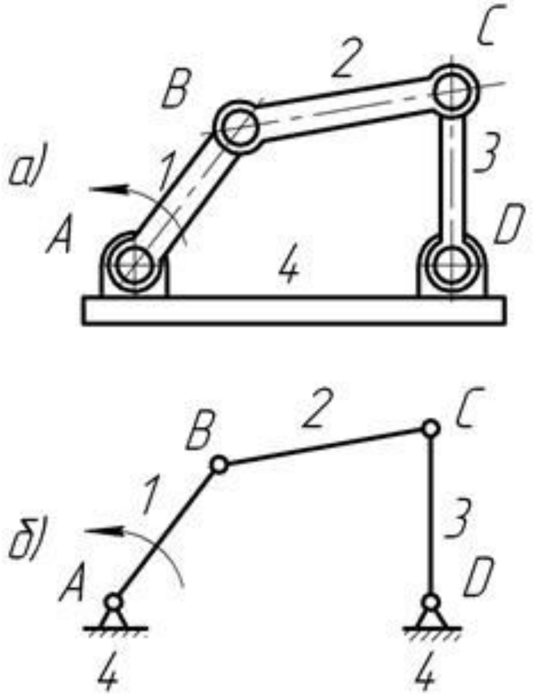
Orientation is $\theta = \tan^{-1}\frac{v_y}{v_x}$

Where v is the eigenvector corresponding to the largest eigenvalue

Feature extraction masterclass video

Where did I use linear algebra (3): Mechanics





Short summary

1. I used to use the course knowledge in my research career
2. It's not a useless course and I'll try to show it (practical cases) (not only from robotics)
3. I will give you an intuition of some basics

Agreements among the group (for now)

Pretest

Vector

Span

Linear combination

Linear independence

Length

Subspace

Vector

Column vectors. Examples

$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ — we will use **this notation**! We represent vectors as **columns**!

Basis

Basis in $\mathbb{R}^2$

A **set** of vectors is a *basis* of $\mathbb{R}^2$ if it spans $\mathbb{R}^2$ and this set is **linearly independent**.

Standard basis in $\mathbb{R}^2$

$\{\hat{\mathbf{i}}, \hat{\mathbf{j}}\} = \{(1,0),(0,1)\}$ is a basis of $\mathbb{R}^2$. They are the standard basis in $\mathbb{R}^2$.

Standard basis in $\mathbb{R}^3$

$\{\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\} = \{(1,0,0),(0,1,0),(0,0,1)\}$ is a basis of $\mathbb{R}^2$. They are the standard (canonical) basis in $\mathbb{R}^3$.

Linear Independence

Linearly independent vectors in $\mathbb{R}^2$

Two vectors $\mathbf{a}$ and $\mathbf{b}$ are *linearly independent* if for $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1\mathbf{a} + \alpha_2\mathbf{b} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = 0$.

Linearly independent vectors in $\mathbb{R}^3$

Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are *linearly independent* if for $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$, $\alpha_1\mathbf{a} + \alpha_2\mathbf{b} + \alpha_3\mathbf{c} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = \alpha_3 = 0$.

Span

Span

Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_n}\} \subset V$.

$$span(S) \equiv \left\{ \mathbf{w} \in V : \mathbf{w} = \sum_{k=1}^{n} c_k \mathbf{v_k}, \quad \forall c_k \in \mathbb{R} \right\}$$

In words, $W = span(S)$ is the set of all (possible) linear combinations of the vectors $\mathbf{v_1}, \mathbf{v_2}, \dots, \mathbf{v_n}$.
Note that W is a subspace of V.

Subspace

Definition

W is a <u>subspace</u> of V if

a)	$W \subset V$	(subset)
b)	$\mathbf{u}, \mathbf{v} \in W \Rightarrow \mathbf{u} + \mathbf{v} \in W$	(closure under addition)
c)	$\mathbf{u} \in W, \lambda \in \mathbb{R} \Rightarrow \lambda \mathbf{u} \in W$	(closure under scalar multiplication)

Task 1

Three vectors are given $\mathbf{a}\begin{bmatrix}1\\2\end{bmatrix}$, $\mathbf{b}\begin{bmatrix}-5\\-1\end{bmatrix}$, $\mathbf{c}\begin{bmatrix}-1\\3\end{bmatrix}$.

Find the vectors $2\mathbf{a}+3\mathbf{b}-\mathbf{c}$ and $16\mathbf{a}+5\mathbf{b}-9\mathbf{c}$.

Task 2

Check if the result of each of the following operations is a vector or not. Explain your answer.

1. $\mathbf{a} + \mathbf{b}$, if $\mathbf{a}$ and$\mathbf{b}$ are vectors
2. $\mathbf{a} - \mathbf{a}$, if $\mathbf{a}$ is a vector
3. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
4. $\begin{bmatrix} 2x + 15 - 4y \\ y - x \end{bmatrix}$, if x and y are integer numbers
5. $\begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix} - \begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix}$, i if x and y are real numbers

Task 3

Check for each case if the following set of vectors is a basis or not.
Explain your answer.

1. $\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\2\end{bmatrix}$

2. $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\2\end{bmatrix}$

3. $\begin{bmatrix}1\\2\\3\end{bmatrix}$

4. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\3\end{bmatrix}$

5. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}$

6. $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}$

Task 3: answers

Basis in $\mathbb{R}^2$

A **set** of vectors is a *basis* of $\mathbb{R}^2$ if it spans $\mathbb{R}^2$ and this set is **linearly independent**.

Check for each case if the following set of vectors is a basis or not.
Explain your answer.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ —
2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ —
4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$
5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ —
6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ —

Task 4

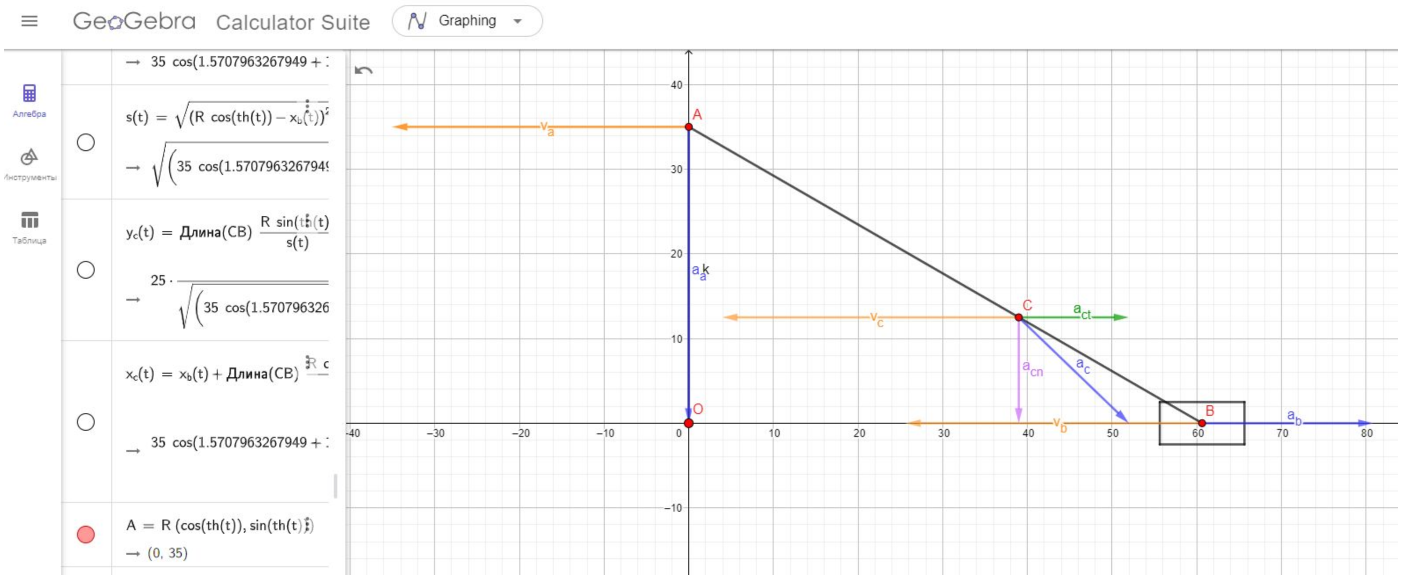
Points $\mathbf{A}(3, -2)$ and $\mathbf{B}(1,4)$ are given. The $\mathbf{M}$ point is on the line $\mathbf{AB}$ in the way that $|\mathbf{AM}| = 3|\mathbf{AB}|$. Find coordinates of the $\mathbf{M}$ point, if:

1. The points $\mathbf{M}$ and $\mathbf{B}$ are from the same side from $\mathbf{A}$.

2. The points $\mathbf{M}$ and $\mathbf{B}$ are from the different sides from $\mathbf{A}$.

Possible tool for simulation

[Geogebra]



Task 4 (1 of possible solutions)

- Find the distance between points $\mathbf{A}$ and $\mathbf{B}$
- Find the equation for the line $|\mathbf{AB}|$
- Find 2 points on the line with distance $3|\mathbf{AB}|$ from A.

Task 5 + 6

Find the coordinates of the gravity center of a triangular plate $\mathbf{ABC}$ with vertices in points $\mathbf{A}(3{,}1), \mathbf{B}(6{,}3), \mathbf{C}(0{,}2)$.

In the plane of the triangle $\mathbf{ABC}$ find the point $\mathbf{O}$ such that

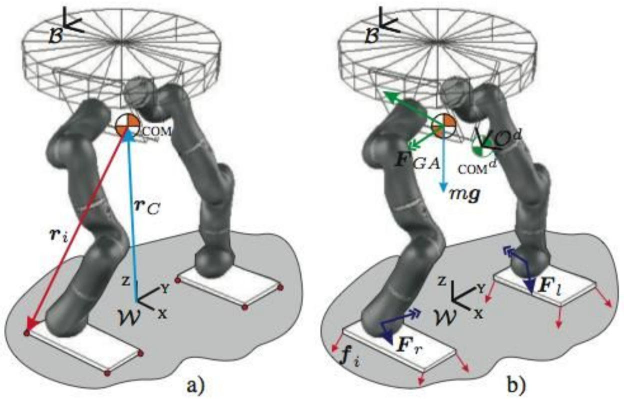
$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \mathbf{0}$. Are there such points outside of the triangle?

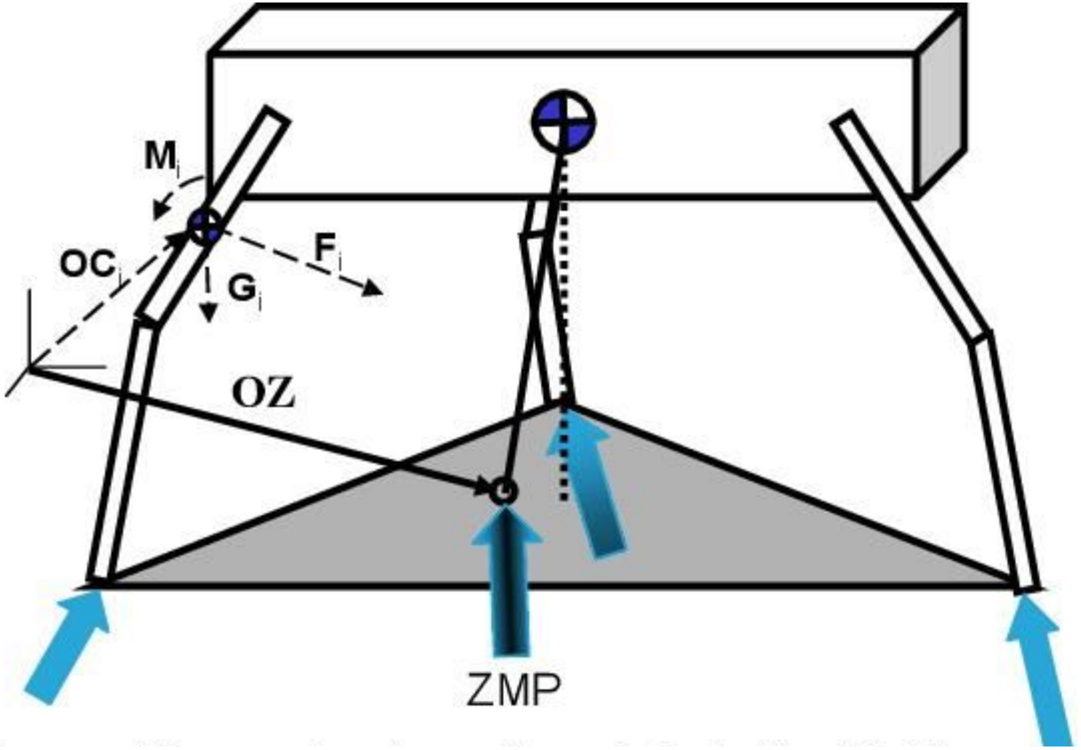
Note: $\mathbf{0}$ is a zero-vector.

Center of gravity VS Center of mass

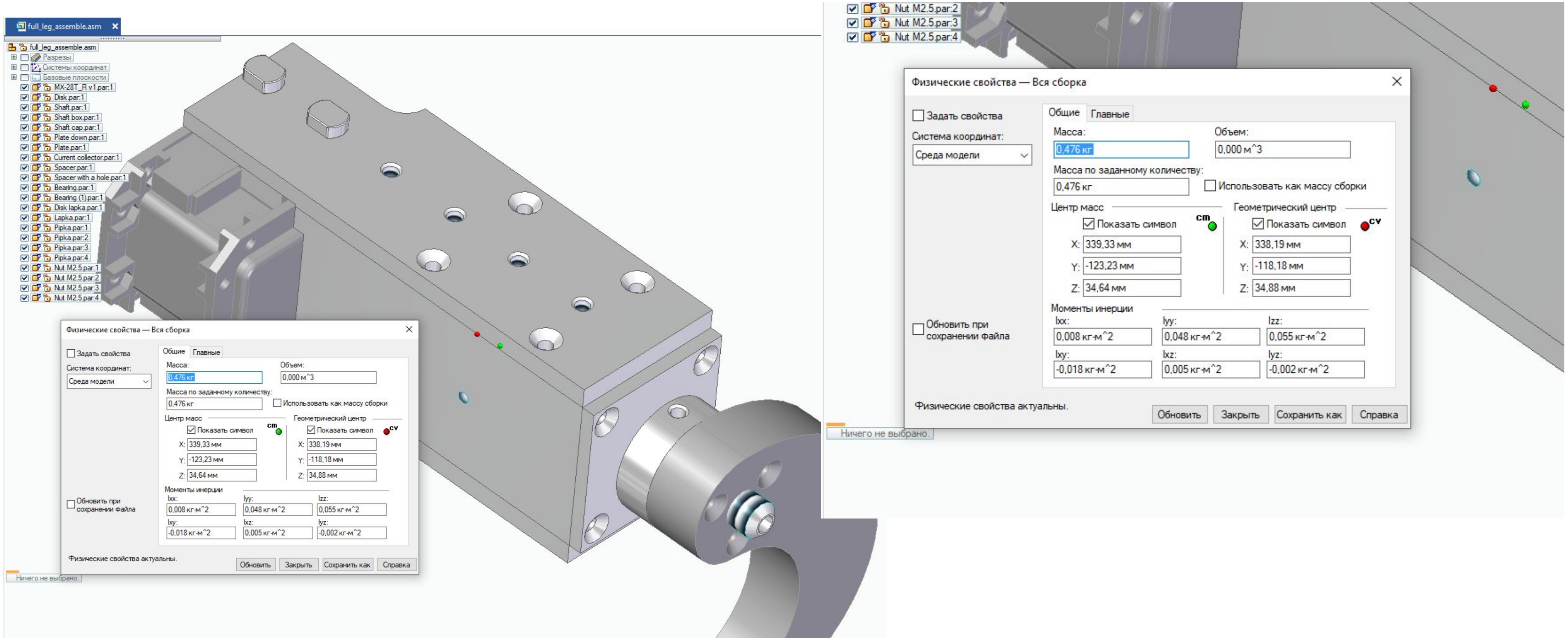
For classical mechanics - it’s the same. More info here

Where a center of mass can be used?

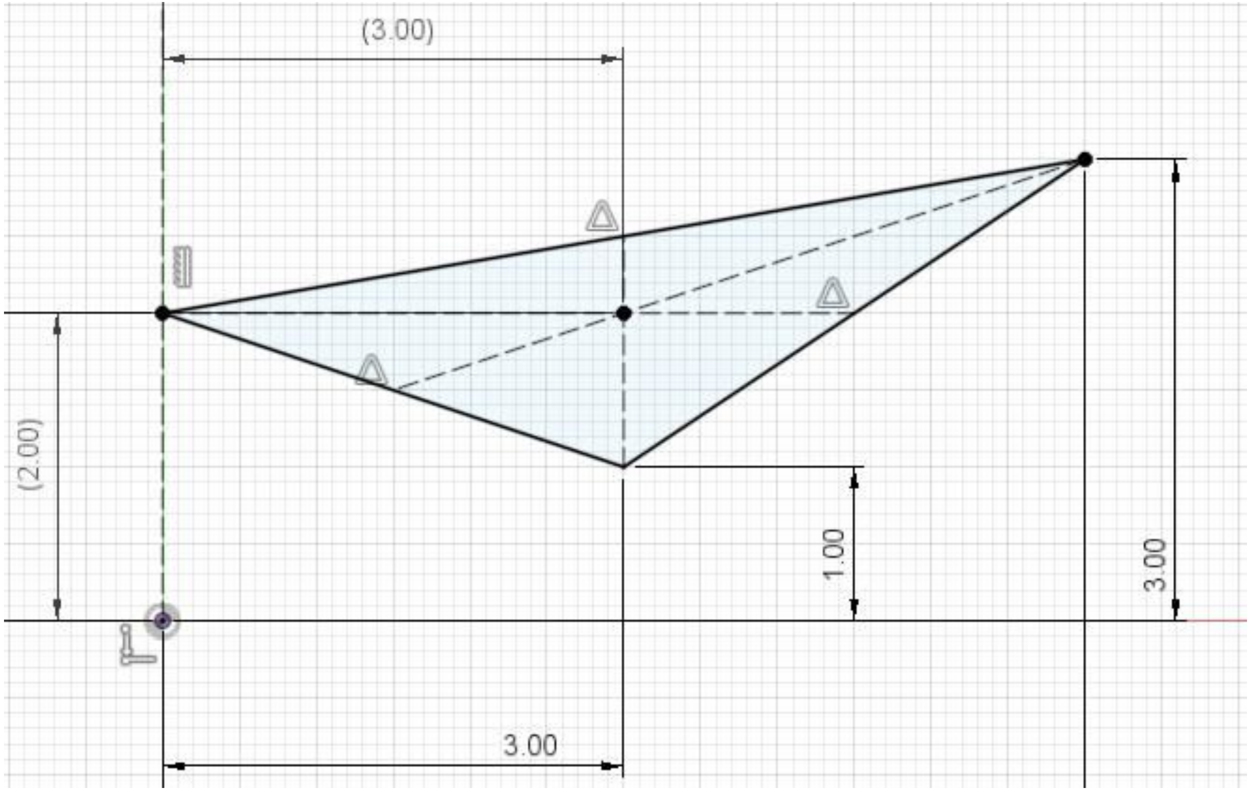




How to find the center of mass in real life



Task 5 + 6 sol



Task 7

Check for each case if the following set of vectors is a coplanar or not. Explain your answer.

1. $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\1\\5\end{bmatrix}, \begin{bmatrix}-1\\3\\2\end{bmatrix}$
2. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\4\end{bmatrix}, \begin{bmatrix}1\\3\\3\end{bmatrix}$
3. $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\end{bmatrix}, \begin{bmatrix}3\\3\\3\end{bmatrix}$

Task 8

Check for each case if the following set of vectors is a subspace or not. Explain your answer.

1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0,\ y > 0$
5. Inner circle with the radius $r = 5$

Subspace

Definition

W is a subspace of V if

a) $W \subset V$ (subset)

b) $\mathbf{u}, \mathbf{v} \in W \Rightarrow \mathbf{u} + \mathbf{v} \in W$ (closure under addition)

c) $\mathbf{u} \in W, \lambda \in \mathbb{R} \Rightarrow \lambda\mathbf{u} \in W$ (closure under scalar multiplication)

Extra material (3bluebrown)

[Vectors]

[Basis, linear independence]

Best extra extra materials

1) RUS - Матпрофи
2) ENG - OnlineMSchool - not only explanation, but also online calculators
3) ENG Videos - 3Blue1Brown
4) ENG Lectures - Gilbert Strang - AGLA2 based on the course

Deserve “A” grade!

– Oleg Bulichev

o.bulichev@innopolis.ru
@Lupasic
Room 105 (Underground robotics lab)