Analytical Geometry and Linear Algebra I, Lab 4

Innopolis University, September 2023

1 Changing Basis and Coordinates

1. Two bases are given in the plane: $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}'_1$, $\mathbf{e}'_2$. The vectors of the first basis have coordinates $(-1; 3)$ and $(2; -7)$ in the second basis.

   (a) Compose transition matrices from the old basis to the new and vice versa. **Ans:**
   $e = \begin{bmatrix} -1 & 3 \\ 2 & -7 \end{bmatrix} e'$

   $e' = \begin{bmatrix} -7 & -3 \\ -2 & -1 \end{bmatrix} e$

   (b) Find the coordinates of a vector in the old basis given that it has coordinates α'_1, α'_2 in the new basis.

   (c) Find the coordinates of a vector in the new basis given that it has coordinates α_1, α_2 in the old basis.

2. Let us consider two coordinate systems in the plane: O, $\mathbf{e}_1$, $\mathbf{e}_2$ and O', $\mathbf{e}'_1$, $\mathbf{e}'_2$. Point O' has coordinates $(7; -2)$ in the old coordinate system, and vectors $\mathbf{e}'_1$, $\mathbf{e}'_2$ can be obtained from vectors $\mathbf{e}_1$, $\mathbf{e}_2$ by rotating them $60°$ (a) clockwise; (b) counterclockwise. Find the old coordinates of a point x, y given its new coordinates x', y'.
   Ans: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} 7 \\ -2 \end{bmatrix}$, (a) $\theta = 60°$, (b) $\theta = -60°$

3. If vectors **a** and **b** form a basis (you should check it), it is needed to find coordinates **c** and **d** in the basis.

   $\mathbf{a} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{d} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$.
   Ans: $\mathbf{c} = \frac{1}{16}\mathbf{a} + \frac{11}{16}\mathbf{b}$
   $\mathbf{d} = -2\mathbf{b}$

4. There are 4 vectors $f_1,\ f_2,\ f_3,\ x$ and the basis
   $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Find the coordinates of x in the basis $(f_1,\ f_2,\ f_3)$, if $f_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $f_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $f_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

   Ans: $\mathbf{x} = 1\mathbf{f}_1 + 1\mathbf{f}_2 - 1\mathbf{f}_3$

2 Inverse Matrix

1. Find inverse matrices for the following matrices:

   (a) $\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix}$; **Ans:** $\begin{bmatrix} \frac{9}{2} & -\frac{5}{2} \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$; **Ans:** $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}$

2. Solve matrix equations:

   (a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$; **Ans:** $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

   (b) $X \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$; **Ans:** $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$