Analytical Geometry and Linear Algebra I, Visiting Lecture

Splines: What is it, B-Spline, NURBS, point modification

Surfaces: Linear, B-Spline

Innopolis University

Analytical Geometry and Linear Algebra I, Visiting Lecture

Splines: What is it, B-Spline, NURBS, point modification
Surfaces: Linear, B-Spline

1.

План

Disclaimer

Goal

The goal of this lecture is to get aquatinted with splines, surfaces, their applications. Obtain some basic intuition.

Constraints

1. Only really necessary proofs (others can be found in reference material).
2. I show today topics only from practical perspective (how to use it as a user and a programmer, not as a creator of new algorithms).
3. I am using Matlab code snippets.
4. I have to make a small recap of some topics due to the reason of your misunderstanding of some concepts.

Lecture Objectives

- To get the main benefit of parametric form.
- To have an intuition where and how splines can be used.
- To understand a relationship between splines and conic sections.
- How to make a surfaces using curves.


THE ADIDASAURUS REX WAS BORN

Computer Aided Design

Form types

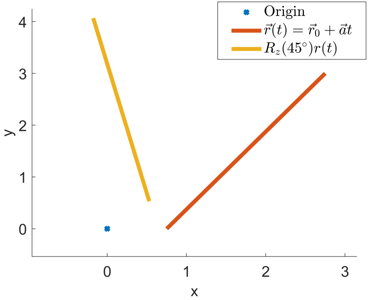
Type	Form	Example	Description
Explicit	$y = f(x)$	$y = mx + b$	Line
Implicit	$f(x, y) = 0$	$(x - a)^2 + (y - b)^2 = r^2$	Circle
Parametric	$x = \dfrac{g(t)}{w(t)}; y = \dfrac{h(t)}{w(t)}$	$x = a_0 + a_1 t; y = b_0 + b_1 t$	Line
		$x = a + r\cos t; y = b + r\sin t$	Circle

Benefits of parametric form

Definition

A parametric description of a curve is called such if the coordinates of the curve point are **continuous** and **unambiguous** functions of the parameter t.

- The result is a point cloud, which can be easily discretized.
- It can be easily controlled.
- We can work with our parametric curves as with coordinates (change basis, apply affine transformation).



Segment line in parametric form

AGLA (6th lab) — $\vec{r}(t) = \vec{p}_0 + a(t)$ or

$$\begin{cases} x = p_{0x} + a_x t \\ y = p_{0y} + a_y t \\ z = p_{0z} + a_z t \end{cases}$$

Not easy to make a segment line (we have only one clear point and a direction)

$\vec{r}(t) = \vec{p}_0(1 - t) + \vec{p}_1 t$ or

$$\begin{cases} x = p_{0x}(1 - t) + p_{1x} t \\ y = p_{0y}(1 - t) + p_{1y} t \\ z = p_{0z}(1 - t) + p_{1z} t \end{cases}$$

It is really convenient, if you know 2 points and want to make a segment line. We will meet this form a lof of times today

Polyline (Polygonal chain)

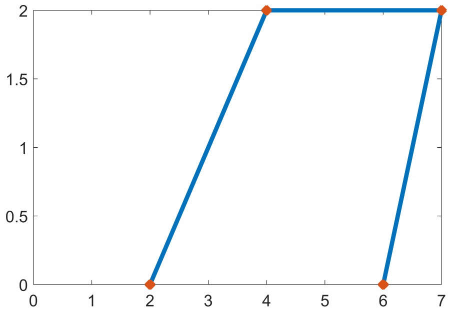
$\vec{r}(t) = \vec{p}_i(1 - w) + \vec{p}_{i+1}w$, where w is a local parameter $w = \dfrac{t - t_i}{t_{t+1} - t_i}$, $t_i \le t \le t_{i+1}$

Polyline passes through given control points $\vec{p}_0,\ \vec{p}_1, \dots, \vec{p}_n$. $t_i \le t_{i+1}$

Example

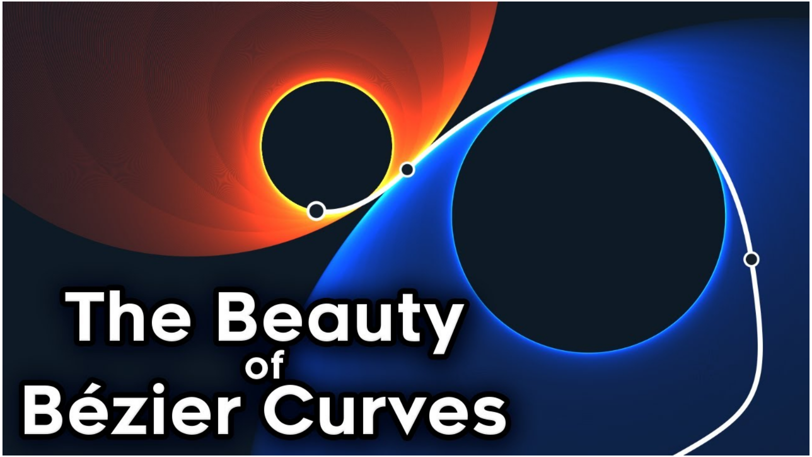
$$\vec{p}_0 = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ \vec{p}_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix},\ \vec{p}_2 = \begin{bmatrix} 7 \\ 2 \end{bmatrix},\ \vec{p}_3 = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

If each knot will be the same and equal to 1, then $\vec{r}(t) = \vec{p}_i(1 - t) + \vec{p}_{i+1}t$, $i = 0 \dots n - 1$



Intro to splines

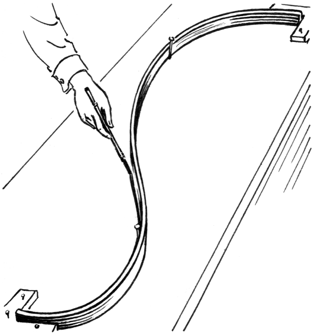
Video



Splines

Informal Definition

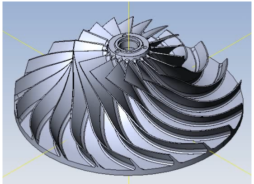
Splines (*piecewise polynomial* functions) are awesome tool to construct *smooth* and flexible shapes in computer graphics.



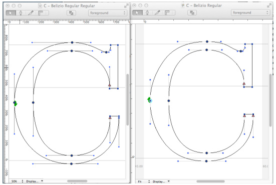
Starting 15th century, ship hull designers used splines for making a smooth shape

French curve (Лекало)

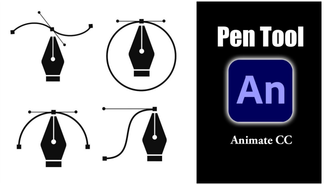
Splines: Applications



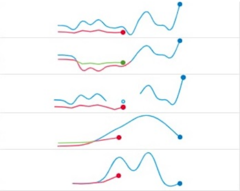
User: Car shape design, aircrafting



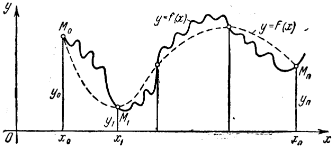
User: Make fonts



User: Pen tool in PhotoShop

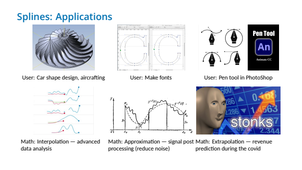


Math: Interpolation — advanced data analysis



Math: Approximation — signal post processing (reduce noise)

Math: Extrapolation — revenue prediction during the covid



Использовал сплайны при прогнозировании выручки во время ковида: был бейзлайн прогноз, на который я умножал на кривую (сигмоиду), которую рассчитывал в соответствии с последними точками факта. Часть параметров кривой была зафиксирована, а часть обновлялась каждый день. Так мы получили более-менее точный прогноз во время кризиса.

Ну вот тут интерполяция, да. У тебя есть набор точек, но для красивого графика ты рассчитываешь промежуточные точки.

Ну приходят тебе дискретные сигналы с датчика, ты восстанавливаешь функцию, вот чтобы эта функция была похожа на реальность надо юзать сплайн

Reference material

- Geometrical Modeling, Golovanov N.N. (book, rus)
- The Beauty of Bézier Curves (video, eng)
- Computer Graphics course, lectures notes 12 and 13 (Imperial College London)
- 12 Spline Curves (video, eng)
- Data Fitting: Polynomial Fitting and Splines, Part 4 (video, eng)
- Qubic spline (habr, rus)

Deserve “A” grade!

– Oleg Bulichev

o.bulichev@innopolis.ru

@Lupasic

Room 105 (Underground robotics lab)