Analytical Geometry and Linear Algebra I. Midterm Exam. October 27, 2023

VARIANT 1

Full name:	Group:

Task:	Theory	1a	1b	2a	2b	3	4	5	Total
Score:									_____ of 30 pts.

In each sheet, you **should** write your last name, first name, variant number, and group number in the **upper right** corner. Unsigned sheets or sheets without the information above will NOT BE graded. This assignment sheet must also be submitted along with your solution. glhf!

Theory. Maximum 5 points

- *Definitions, simple proofs (from lectures).*

  1. (1 point) Vectors $v_1, v_2, ..., v_n$ are linearly independent if...
  2. (2 points) Give a condition of coplanarity of three vectors.
  3. (2 points) Give definition of a trace, $Tr(A)$, of matrix and prove linearity of trace.

Practice. Maximum 25 points

1. *Vector operations / Matrices*

   (a) (2 points) Find the determinant of the 3x3 matrix:
   $$\begin{bmatrix} 2 & 5 & -3 \\ 1 & 4 & -2 \\ -7 & 3 & 0 \end{bmatrix}$$
   (b) (4 points) Find a vector that is orthogonal to both $v_1 = (1, 0, 1)$ and $v_2 = (1, 3, 0)$ and which dot product with vector $v_3 = (1, 1, 0)$ equals to 8.

2. *Lines / Planes*

   (a) (2 points) Find the angle between the planes $2x - y + z = 6$, $x + y + 2z = 3$.
   (b) (4 points) What is the general equation of the plane which contains the following two parallel lines: $\frac{x+1}{6} = \frac{y-2}{7} = z$ and $\frac{x-3}{6} = \frac{y+4}{7} = z - 1$

3. (4 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x + y + z = 1$ and $2x - y - 5z = 1$.

4. (4 points) Two vertices of a triangle are $(4, -3)$ and $(-2, 5)$. If the orthocenter (intersection of altitudes) of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

5. (5 points) In a regular tetrahedron ABCD, find the coordinates of the point M in the basis $\{D, \mathbf{DA}, \mathbf{DB}, \mathbf{DC}\}$, if the point M has coordinates $(0, 1/3, 1/3)$ in the basis $\{A, \mathbf{AD}, \mathbf{AB}, \mathbf{AC}\}$.

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VARIANT 2

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Score:									_____ of 30 pts.

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Theory. Maximum 5 points

1. *Definitions, simple proofs.*

   (a) (1 point) Let $V = \{v_1, v_2, ..., v_n\}$ be a set of vectors. Give a definition of a span $S(V)$

   (b) (2 points) What is the geometrical interpretation of the magnitude of $\mathbf{a} \times \mathbf{b}$?

   (c) (2 points) Given that BC and CB are valid, prove that $Tr(BC) = Tr(CB)$

Practice. Maximum 25 points

1. *Vector operations / Matrices*

   (a) (2 points) Find the determinant of matrix
   $$\begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & -1 \\ 4 & -3 & 5 \end{bmatrix}$$

   (b) (4 points) Find a vector that is orthogonal to both $v_1 = (1, -1, 1)$ and $v_2 = (6, -3, 0)$ and which dot product with a vector $v_3 = (3, 2, 3)$ equals to 10.

2. *Lines / Planes*

   (a) (2 points) Find the angle between the planes $x + y - 4z = 8$, $4x + y - z = 8$.

   (b) (4 points) What is the general equation of the plane which contains the following two parallel lines: $\frac{x-1}{5} = \frac{y+2}{3} = z$ and $\frac{x+3}{5} = \frac{y-4}{3} = z - 1$

3. (4 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x + y + z = 1$ and $2x - y - 5z = 1$.

4. (4 points) Two vertices of a triangle are $(6, -1)$ and $(-2, 5)$. If the orthocenter (intersection of altitudes) of the triangle is at $(1, 2)$, find the coordinates of the third vertex.

5. (5 points) In a regular tetrahedron ABCD, find the coordinates of the point M in the basis $\{A, \mathbf{AD}, \mathbf{AB}, \mathbf{AC}\}$, if the point M has coordinates $(1/3, 1/3, 1/3)$ in the basis $\{D, \mathbf{DA}, \mathbf{DB}, \mathbf{DC}\}$.