Analytical Geometry and Linear Algebra I, Lab 4

Inverse Matrix

Change of basis

Innopolis University

1. Смену базиса давать через вывод формулы: вектор - фреймлесс, а координаты вектора - нет. Поэтому можно вот выразить ручку по разному. Все на основе линейных комбинаций.

   Есть 2 твои формулы - бро $E' = EA$ и $Ex = E'x'$. Разновидность бро $Ex = Eb + E'x'$, где $E = [e_1\ e_2\ e_3]$, $E = [e'_1\ e'_2\ e'_3]$

2. Забить на слайды и объяснять на маркерах, ручках и доске про смену базиса

Questions from the class

No questions for today

Inverse Matrix

Why do we need it?

Say we want to find matrix X, and we know matrix A and B:

$$XA = B$$

It would be nice to divide both sides by A (to get X=B/A), but remember **we can't divide.**

But what if we multiply both sides by A^{-1} ?

$$XAA^{-1} = BA^{-1}$$

And we know that $AA^{-1} = I$, so:

$$XI = BA^{-1}$$

We can remove I (for the same reason we can remove "1" from 1x = ab for numbers):

$$X = BA^{-1}$$

And we have our answer (assuming we can calculate A^{-1})

Inverse Matrix

Properties

1. $det(A^{-1}) = \frac{1}{det(A)}$
2. $(AB)^{-1} = A^{-1}B^{-1}$
3. $(A^{-1})^T = (A^T)^{-1}$
4. $(kA)^{-1} = \frac{A^{-1}}{k}$
5. $(A^{-1})^{-1} = A$

Inverse Matrix

How to find

There are 2 ways:

1. Classical approach
2. Gauss-Jordan

Inverse Matrix: Classical Approach

Theory

$\underset{2\times 2}{A}^{-1} = \dfrac{C^T}{det(A)}$, where C is a matrix of *cofactors*.

$\underset{2\times 2}{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$, where $C_{ij} = (-1)^{i+j} M_{ij}$ — (we met it on previous lab (lab 3))

Inverse Matrix: Classical Approach

Case Study

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Let's find A^{-1}.

1. Find a determinant (shouldn't be equal to 0, otherwise → stop calculations).
   $det(A) = 1 \cdot 4 - 2 \cdot 3 = -2$
2. Find Cofactor matrix
   $C_{11} = (-1)^{1+1} M_{11} = (-1)^2 |4| = 4$
   $C_{12} = (-1)^{1+2} M_{12} = (-1)^3 |3| = -3$
   $C_{21} = (-1)^{2+1} M_{21} = (-1)^3 |2| = -2$
   $C_{22} = (-1)^{2+2} M_{22} = (-1)^4 |1| = 1$

$$C = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

3. Transpose cofactor matrix
   $$C^T = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$
4. Substitute it to the main formula
   $$A^{-1} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

Inverse Matrix: Gauss-Jordan

Core Idea & Case study (2×2)

The

Inverse Matrix: Gauss-Jordan
Core Idea & Case study (2×2)

The

Не забудь, нужно подумать как оформить эту штуку честь по чести. А то вроде ее и понимают, но времени не много. Как это оформить нормально.

Reference material

- Inverse Matrix (OnlineMschool)
- Gauss-Jordan (Wiki)
- Matrix Rank (OnlineMschool)

Deserve “A” grade!

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@Lupasic

Room 105 (Underground robotics lab)