

# Essentials of Analytical Geometry and Linear Algebra I, Class #4

Innopolis University, September 2022

## 1 Inverse Matrix

- Find inverse matrices for the following matrices:

(a)  $\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix};$

(b)  $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix};$

- Solve matrix equations:

(a)  $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix};$

(b)  $X \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix};$

## 2 Matrix Rank

- Calculate the ranks of the following matrices:

(a)  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}.$

## 3 Changing Basis and Coordinates

- If vectors **a** and **b** form a basis (you should check it), it is needed to find coordinates **c** and **d** in the basis.

$$\mathbf{a} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}.$$

- There are two bases in  $R^3$ :

$$e_1 = i, e_2 = j, e_3 = k \text{ and } e'_1 = i + j + k, e'_2 = i + j, e'_3 = i$$

Find coordinates of  $x = 2i - 3j + k$  in the basis  $e'_1, e'_2, e'_3$ .

- There are 4 vectors  $f_1, f_2, f_3, x$  and the basis

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \text{ Find the coordinates of } x \text{ in the basis}$$

$$(f_1, f_2, f_3), \text{ if } f_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, f_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, f_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$