Full name: Group:

Task:	1	2	3	4	5	Extra	Total
Score:							of 30 pts extra

- 1. (6 points) Find the equation of a plane which is perpendicular to the plane x + 4y 3z + 7 = 0 and crosses the straight line $\frac{x-2}{5} = \frac{y-3}{1} = \frac{z+1}{2}$.
- 2. (6 points) There are three points $A(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}), B(2\mathbf{i}), C(-2\mathbf{i} + 3\mathbf{j} 5\mathbf{k})$. Find the equation of the triangle's altitude passing through the point B.
- 3. (6 points) Solve the system

(a)
$$\mathbf{A}\mathbf{w} = \mathbf{b}$$
,

where
$$\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mathbf{A} = \begin{bmatrix} 1 & -3 & 1 \\ -1 & 2 & -5 \\ 5 & -13 & 13 \end{bmatrix}$$
, $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix}$.

- (b) Find rank of the matrix A.
- (c) Explain relative positions of the planes that correspond to equations.
- 4. (6 points) In the triangle ABC points K, L, M are placed on the sides AB, BC and AC respectively such that AK : KB = 2 : 1, BL : LC = 4 : 1, CM : MA = 3 : 1. Find the transition matrix from the basis \overrightarrow{AC} , \overrightarrow{AB} to the basis \overrightarrow{ML} , \overrightarrow{KL} . Express new basis vectors in terms of the old ones using matrix-vector notation.
- 5. (6 points) The coordinates of the vertices of a tetrahedron are given: A(3,2,1), B(1,4,-2), C(3,7,6), D(5,8,4). Find the coordinates of the intersection of the altitude dropped down from the vertex B to the plane ACD and its length. The answer may contain roots.
- 6. (Extra, 5 points) Draw figure that represents the set of points of the plane (x, y) which satisfies the inequality $x^3 + xy^2 \ge x$.

AGLA 2022. MIDTERM. II VARIANT

Full name: Group:

Task:	1	2	3	4	5	Extra	Total
Score:							of 30 pts extra

- 1. (6 points) Find the equation of a plane which is perpendicular to the plane x + 5y 3z + 12 = 0 and crosses the straight line $\frac{x-4}{6} = \frac{y-9}{1} = \frac{z-5}{3}$
- 2. (6 points) There are three points $A(2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}), B(3\mathbf{i}), C(-2\mathbf{i} + \mathbf{j} 5\mathbf{k})$. Find the equation of the triangle altitude passing through the point B.
- 3. (6 points) Solve the system

(a)
$$\mathbf{A}\mathbf{w} = \mathbf{b}$$
,

where
$$\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
, $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 9 & 15 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

- (b) Find rank of the matrix A.
- (c) Explain relative positions of the planes that correspond to equations.
- 4. (6 points) In the triangle ABC points K, L, M are placed on the sides AB, BC and AC respectively such that AK : KB = 4 : 1, BL : LC = 3 : 2, CM : MA = 1 : 3. Find the transition matrix from the basis \overrightarrow{CA} , \overrightarrow{AB} to the basis \overrightarrow{LK} , \overrightarrow{ML} . Express new basis vectors in terms of the old ones using matrix-vector notation.
- 5. (6 points) The coordinates of the vertices of a tetrahedron are given: A(3,2,1), B(1,4,-2), C(3,7,6), D(5,8,4). Find the coordinates of the intersection of the altitude dropped down from the vertex C to the plane ABD and its length. The answer may contain roots.
- 6. (Extra, 5 points) Draw figure that represents the set of points of the plane (x, y) which satisfies the inequality $y^3 + yx^2 \ge y$.