



Analytical Geometry and Linear Algebra I, Lab 1

Introduction to AGLA
Vector
Basis and Subspace



About Me (1)



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About Me (2)

Education

- *Bachelor* — BMSTU (МГТУ им Н.Э. Баумана) «CAD developer»
- *Master & PHD* — Innopolis University (IU) «Robotics»

Job

- *Senior Lecturer* — IU: Linear Algebra, Theoretical Mechanics, Mechanics and Machines
- *Course Author* — Skillbox: Mathematics for Robotics
- *Trainer* — IU: RAGE (ДИЧЬ) club – Hiking / Kayaking, Historical fencing, Folkgames, Archery



[Link to the channel](#)



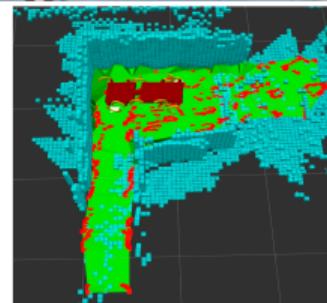
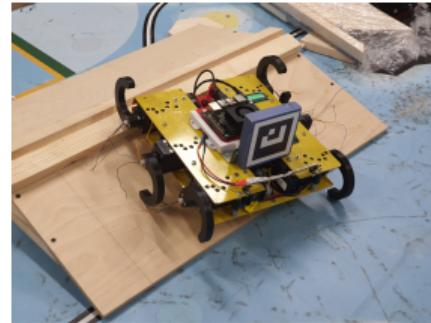
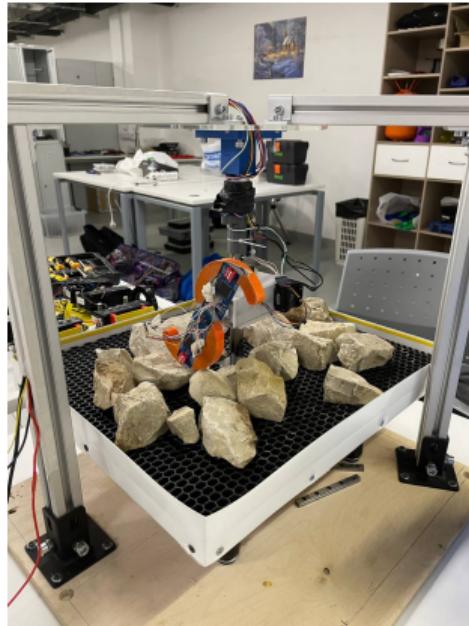
Where did I use linear algebra?

Problem 1

To determine geometrical and physical properties of passed terrain.

Linear Algebra topics:

- All core topics from LA
(vector, matrix, etc)
- Change of basis
- Projections, regression





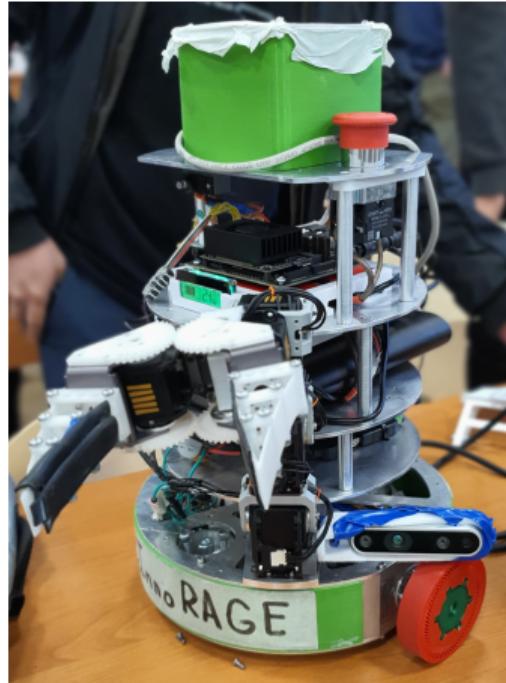
Where did I use linear algebra?

Problem 2

To determine the position and orientation of a mug using camera.

Linear Algebra topics:

- All core topics from LA (vector, matrix, etc)
- Change of basis, affine transformations
- Projections





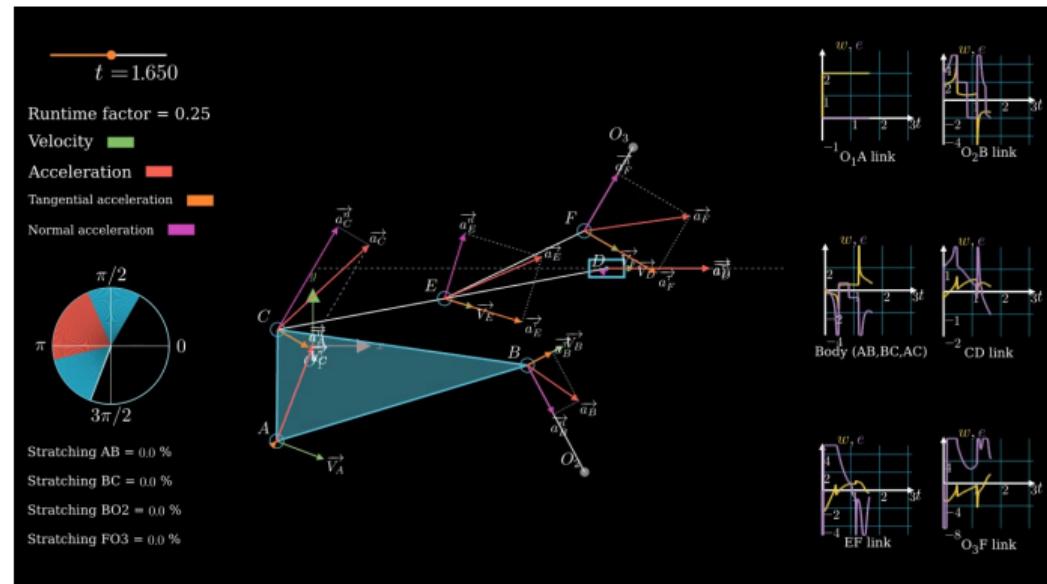
Where did I use linear algebra?

Problem 3

To simulate the mechanism.

Linear Algebra topics:

- Vectors, Matrices
- Change of basis





Short Summary

1. I used to use the course knowledge in my research career
2. It's not a useless course and I'll try to show it by presenting practical cases (not only from robotics))
3. I will give you an intuition of some basics



Agreements among the group (for now)



Pretest

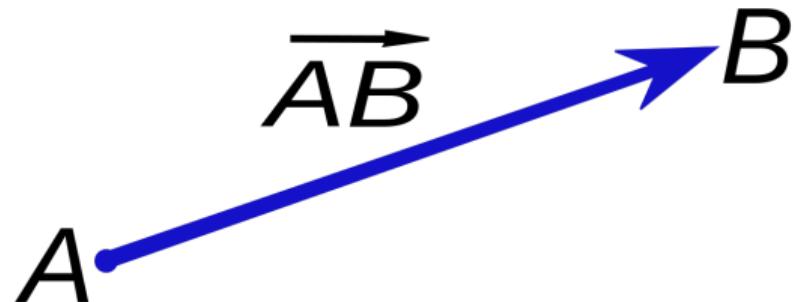
- Vector
- Span
- Linear combination
- Linear independence
- Vector length
- Subspace

Vector

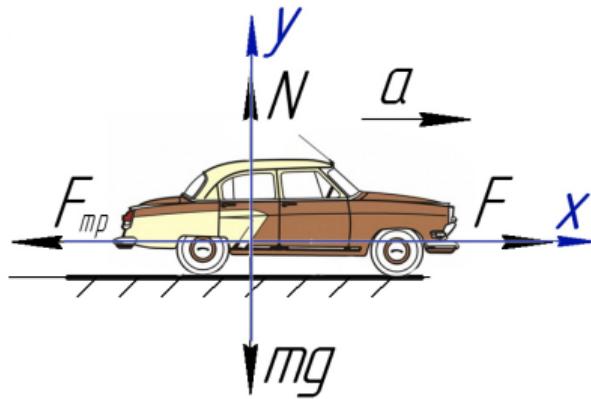
Definition from school

In the simplest case, it is a mathematical object characterized by magnitude and direction.

It is a directed segment with a certain length.



How a vector is seen by



$$\bar{\mathbf{v}}$$

$$\begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

Physics student

Mathematician

CS student



Vector operations

- Summation
- Scalar multiplication
- Norm / magnitude / vector length
- Transpose
- Dot (inner) product
- Cross (outer) product



Task 1

We have $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

You should find $2\vec{a} + 3\vec{b} - \vec{c}$ и $16\vec{a} + 5\vec{b} - 9\vec{c}$



Task 2

Check if the result of each of the following operations is a vector or not. Explain your answer.

1. $\mathbf{a} + \mathbf{b}$, if \mathbf{a} and \mathbf{b} are vectors

2. $\mathbf{a} - \mathbf{a}$, if \mathbf{a} is a vector

3. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

4. $\begin{bmatrix} 2x + 15 - 4y \\ y - x \end{bmatrix}$, if x and y are integer numbers

5. $\begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix} - \begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix}$, if x and y are real numbers



Linear combination

Vector $\vec{w} \in V$ is a linear combination of $\vec{v}_1, \dots, \vec{v}_n \in V$ vectors,
if $\exists c_k \in \mathbb{R}; (k = 1..n)$
such as,

$$\vec{w} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \cdots + c_n \vec{v}_n$$



Linear independence

Two vectors \vec{a} и \vec{b} are *linear independent*
if for $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1\vec{a} + \alpha_2\vec{b} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = 0$.



Span

Let $S = \{v_1, v_2, \dots, v_n\} \subset V$.

$$span(S) \equiv \left\{ w \in V : w = \sum_{k=1}^n c_k v_k, \quad \forall c_k \in \mathbb{R} \right\}$$

In words, $W = span(S)$ is the set of all (possible) linear combinations of the vectors v_1, v_2, \dots, v_n .

Note that W is a subspace of V .



Basis

A **set** of vectors is a *basis* of \mathbb{R}^2 if it spans \mathbb{R}^2 and this set is **linear independent**.

Standard basis in \mathbb{R}^2

$\{\vec{i}, \vec{j}\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ — is basis of \mathbb{R}^2 . It's a classical (canonical) basis in \mathbb{R}^2 .

Standard basis in \mathbb{R}^3

$\{\vec{i}, \vec{j}, \vec{k}\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ — basis in \mathbb{R}^3 .



Task 3

Check for each case if the following set of vectors is a basis or not. Explain your answer.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



Task 3

Answer

A **set** of vectors is a *basis* of \mathbb{R}^2 if it spans \mathbb{R}^2 and this set is **linear independent**.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ NOT

2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ NOT

4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ NOT

6. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ NOT



Task 4

Points **A**(3, -2) and **B**(1, 4) are given. The **M** point is on the line **AB** in the way that $|AM| = 3|AB|$. Find coordinates of the **M** point, if:

1. The points **M** and **B** are from the same side from A.
2. The points **M** and **B** are from the different sides from A.

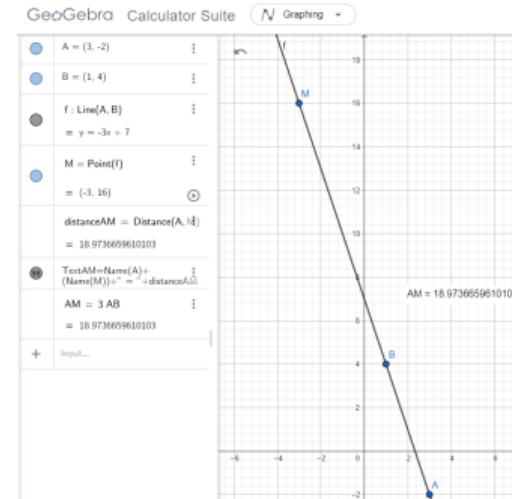


Task 4

One of possible solutions

1. Find the distance between points **A** and **B**
2. Find the equation for the line **AB**
3. Find 2 point on the line with distance $3 |AB|$ from **A**

Another of possible solutions



[Link to the Geogebra](#)



Task 5 and 6

5. Find the coordinates of the gravity center of a triangular plate **ABC** with vertices in points **A(3, 1)**, **B(6, 3)**, **C(0, 2)**.
6. In the plane of the triangle **ABC** find the point **O** such that $\vec{OA} + \vec{OB} + \vec{OC} = \mathbf{0}$. Are there such points outside of the triangle?

Note: **0** is a zero-vector.

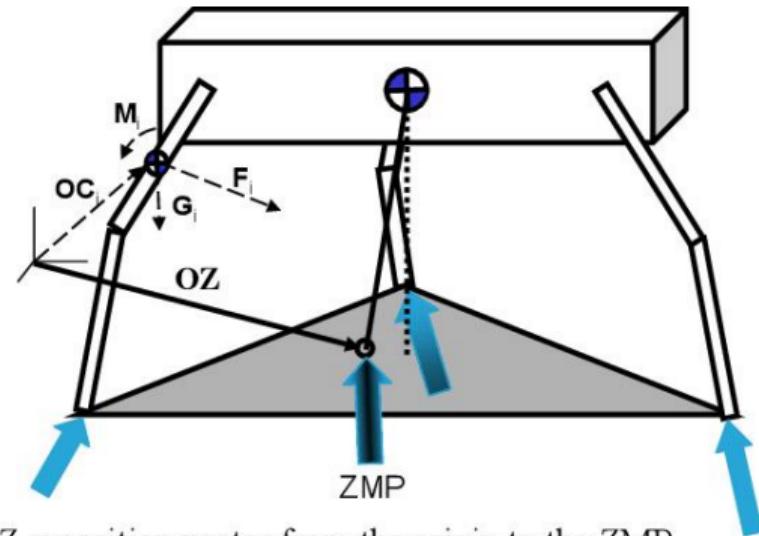
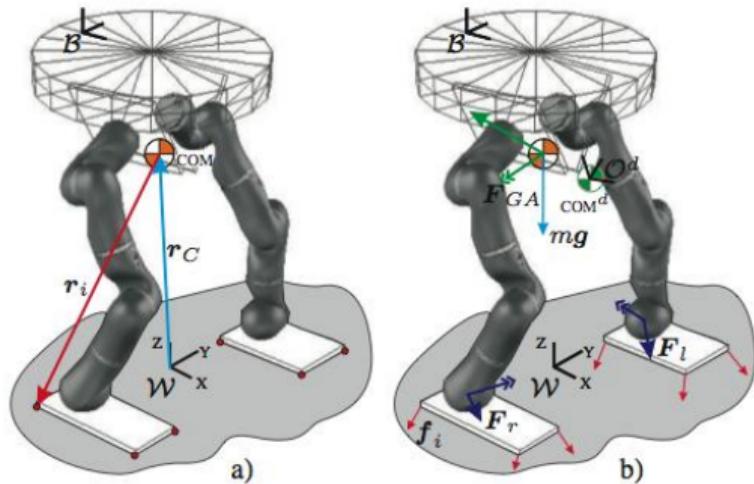


Center of gravity VS Center of mass

For classical mechanics - it's the same. More info [here](#)



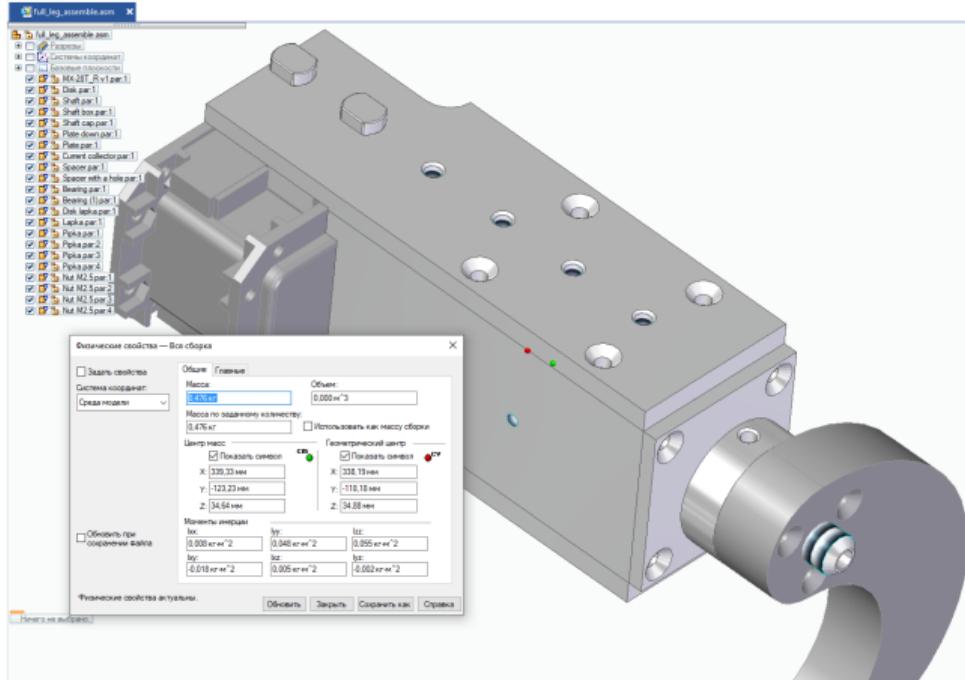
Where a center of mass can be used?



OZ = position vector from the origin to the ZMP



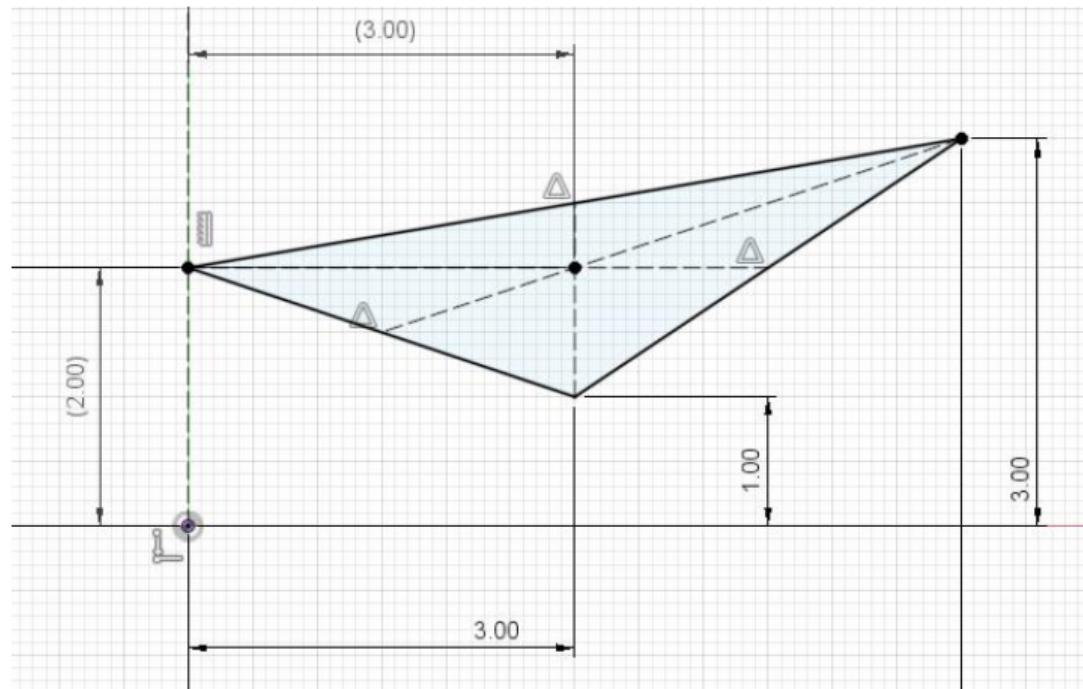
How to find a CoM in real life





Task 5

Solution made in CAD





Subspace

Definition

W is a subspace of V if

- a) $W \subset V$ (subset)
- b) $\mathbf{u}, \mathbf{v} \in W \Rightarrow \mathbf{u} + \mathbf{v} \in W$ (closure under addition)
- c) $\mathbf{u} \in W, \lambda \in \mathbb{R} \Rightarrow \lambda \mathbf{u} \in W$ (closure under scalar multiplication)



Task 7

Check for each case if the following set of vectors is a subspace or not. Explain your answer.

1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0, y > 0$
5. Inner circle with the radius $r = 5$



Reference material

OnlineMschool

- Vectors | Chapter 1, Essence of linear algebra - YouTube
- Linear combinations, span, and basis vectors | Chapter 2, Essence of linear algebra - YouTube
- Vectors Definition. Main information
- Векторы для чайников. Действия с векторами. Координаты вектора.
- Линейная зависимость и независимость векторов. Базис векторов

Deserve “A” grade!

– Oleg Bulichev

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↗ @Lupasic

🚪 Room 105 (Underground robotics lab)