

Analytical Geometry and Linear Algebra I, Lab 6

Line in plane



Questions from the class

No questions for today



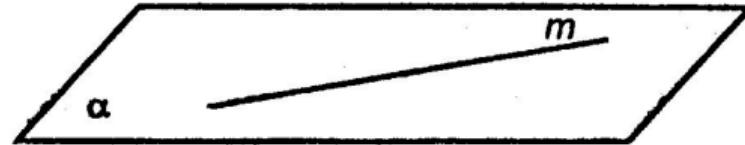
Objectives

- To see the structure of all formulas, which were covered
- To understand how to transform one from to another and vice-versa
- How to apply the knowledge

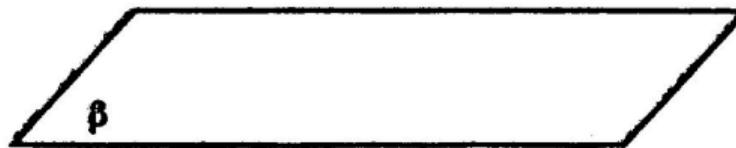


Type of elements

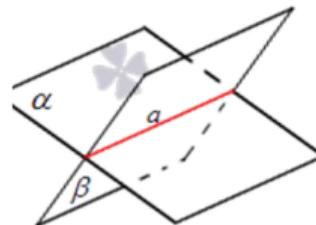
Line in plane



Plane



Line in space





Computer Aided Design

Form types

| Type | Form | Example | Description |
|------------|---|--|----------------|
| Explicit | $y = f(x)$ | $y = mx + b$ | Line |
| Implicit | $f(x, y) = 0$ | $(x - a)^2 + (y - b)^2 = r^2$ | Circle |
| Parametric | $x = \frac{g(t)}{w(t)}$; $y = \frac{h(t)}{w(t)}$ | $x = a_0 + a_1 t; y = b_0 + b_1 t$ $x = a + r \cos t; y = b + r \sin t$ | Line Circle |



Line in plane

Formulas

1. **General equation** $Ax + By + C = 0$, where
 $C = -Ax_0 - By_0$
2. **Slope-intercept** $y = kx + b$, where
 $k = \tan(X\text{-axis} \hat{y}(x))$ - slope
3. **Passing through two points** $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$; slope
is $k = \frac{y_2 - y_1}{x_2 - x_1}$
where $x_{1,2}, y_{1,2}$ are some particular coordinates of points on the line
4. **Canonical** $\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y}$,
where x_0, y_0 is a point on the line and a_x, a_y - direction vector coefficients on a basis

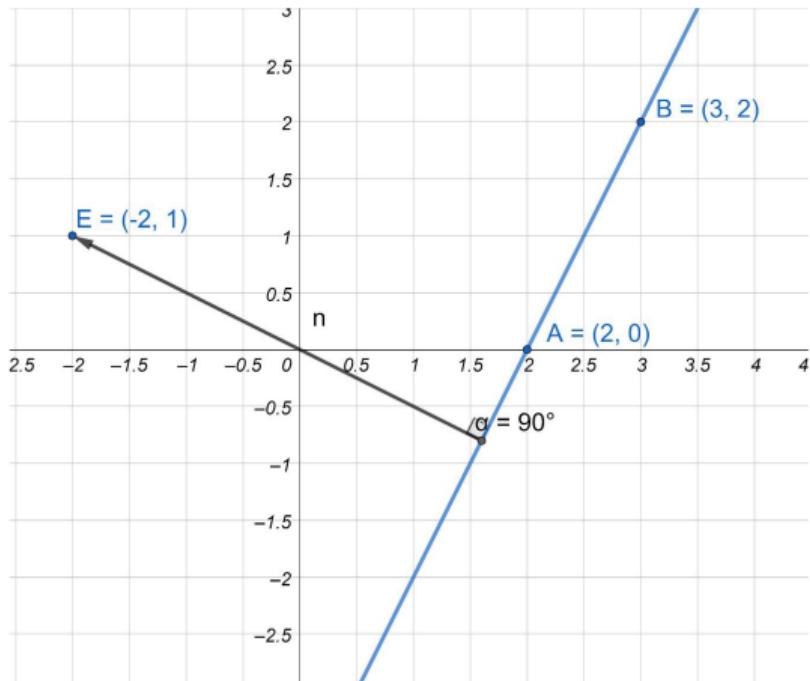
5. **Parametric** $\vec{r} = \vec{r}_0 + \tau \vec{a} = \begin{cases} x = x_0 + \tau a_x \\ y = y_0 + \tau a_y \end{cases}$, where τ is parameter,
which can be received from canonical form
 $(\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \tau)$
6. **Using normal line** $(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$, where $\vec{n} = \begin{bmatrix} A \\ B \end{bmatrix}$,
 $\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\vec{r}_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$



Line in plane

Task 0

Write down all forms of the line





Line in plane

Task 0, Answer

$$\begin{cases} 0 = k \cdot 2 + b \\ 2 = k \cdot 3 + b \end{cases} \Rightarrow \begin{aligned} b &= -2k \Rightarrow b = -4 \\ &\text{(or using ③)} \end{aligned}$$

$$② y = 2x - 4$$

$$① -2x + 1y + 4 = 0$$

\nearrow
 4

\nearrow
 B

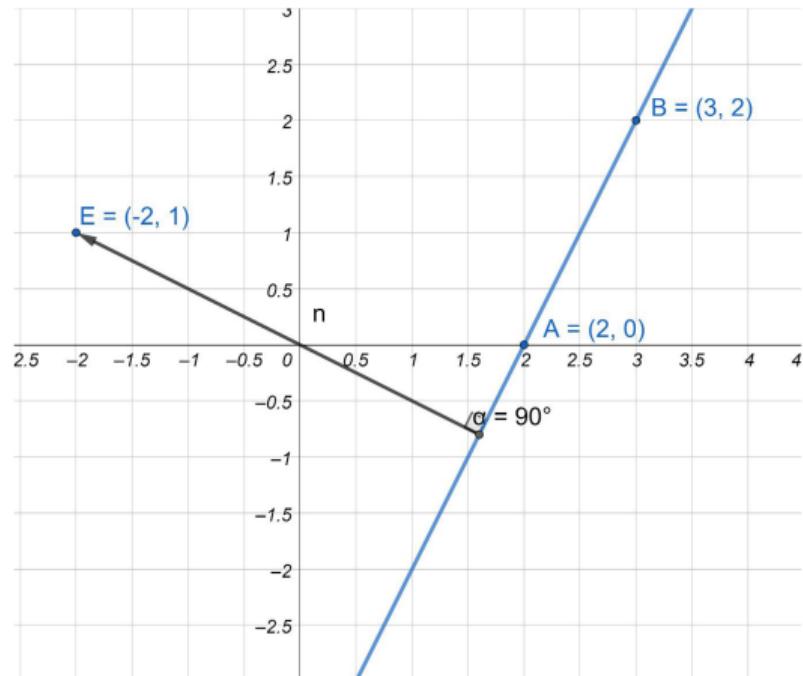
\nearrow
 C

$$a = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$④ \frac{y}{2} = \frac{x-2}{1}; \quad ④ \rightarrow ⑤ \frac{y}{2} = \frac{x-2}{1} = \tau$$

$$⑤ \begin{cases} x = 2 + \tau \\ y = 2\tau \end{cases}$$

$$⑥ \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = -4; \quad \begin{bmatrix} x-2 \\ y \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 0$$





Task 1

Find the slope of the line joining the points $(2, 3)$ and $(4, -5)$.



Task 1

Answer

The slope of the line joining the two given points (x_1, y_1) and (x_2, y_2)

is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Therefore, the slope of the line joining the two given points is

$$m = \frac{-5 - 3}{4 - 2} = \frac{-8}{2} = -4.$$



Task 2

Find the slope of the line $2x - 3y + 7 = 0$.



Task 2

Answer

The equation of the line is $2x - 3y + 7 = 0$ (i.e.) $3y = 2x + 7$.

$$\text{(i.e.) } y = \frac{2}{3}x + \frac{7}{3}.$$

Therefore, slope of the line = $\frac{2}{3}$.



Task 3

Find the equation of the straight line, the portion of which between the axes is bisected at the point $(2, -5)$.



Task 3

Answer

Let the line meet the x and y axes at A and B , respectively. Then the coordinates of A and B are $(a, 0)$ and $(0, b)$. The midpoint of AB is

$\left(\frac{a}{2}, \frac{b}{2}\right)$. However, the midpoint is given as $(2, -5)$.

Therefore, $\frac{a}{2} = 2$ and $\frac{b}{2} = -5$.

$$\therefore a = 4 \text{ and } b = -10.$$

Hence, the equation of the straight line is $\frac{x}{4} - \frac{y}{10} = 1$.

$$\text{(i.e.) } 5x - 2y = 20.$$

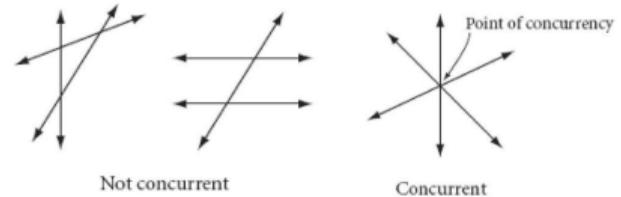


Task 4

Find the equation of the straight line concurrent^a with the lines $2x + 3y = 3$ and $x + 2y = 2$ and also concurrent with the lines $3x - y = 1$ and $x + 5y = 11$.

^a Lines are said to be concurrent if they are intersect at a single point

- **Concurrent – Lines or segments that have three or more points in common**





Task 4

Answer

The point of intersection of the lines $2x + 3y = 3$ and $x + 2y = 2$ is obtained by solving the following two equations:

$$2x + 3y = 3 \quad (2.35)$$

$$x + 2y = 2 \quad (2.36)$$

$$(2.35) \qquad \qquad 2x + 3y = 3$$

$$(2.36) \times 2 \qquad \qquad 2x + 4y = 4$$

On subtracting, we get $y = 1$ and hence $x = 0$.

Therefore, the point of intersection is $(0, 1)$.

$$3x - y = 1 \quad (2.37)$$

$$x + 5y = 11 \quad (2.38)$$

$$(2.37) \times 5 \qquad \qquad 15x - 5y = 5$$

$$(2.38) \times 1 \qquad \qquad x + 5y = 11$$

On adding, we get $16x = 16$ which gives $x = 1$ and hence $y = 2$. The point of intersection of the second pair of lines is $(1, 2)$. The equation of the line joining the two points $(0, 1)$ and $(1, 2)$ is

$$\frac{y-1}{x-0} = \frac{1-2}{0-1} = 1 \Rightarrow y - 1 = x \quad (\text{i.e.}) \quad x - y + 1 = 0.$$



**WHY
SLAVS
WEAR ADIDAS**



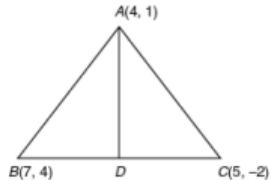
Task 5

$A(4, 1)$, $B(7, 4)$, and $C(5, -2)$ are the vertices of a triangle. Find the line equation which goes from A and perpendicular to BC .



Task 5

Answer



The slope of the line BC is

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 + 2}{7 - 5} \\ = \frac{6}{2} = 3.$$

Therefore, the slope of the perpendicular AD to BC is $-\frac{1}{3}$. Hence, the

equation of the perpendicular from $A(4, 1)$ on BC is $y - y_1 = m(x - x_1)$

$$\text{(i.e.)} \quad y - 1 = \frac{-1}{3}(x - 4) \Rightarrow 3y - 3 = -x + 4 \\ \therefore x + 3y = 7.$$



Task 6

Find the equation of the straight line making an angle 135° with the positive direction of x-axis and cutting off an intercept 5 on the y-axis.



Task 6

Find the equation of the straight line making an angle 135° with the positive direction of x-axis and cutting off an intercept 5 on the y-axis.

Answer

$$\begin{aligned}m &= \tan \theta = \tan 135^\circ \\&= \tan(180 - 45) \\&= -\tan 45^\circ \\&= -1.\end{aligned}$$

y intercept $= c = 5$. Therefore, the equation of the straight line is

$$y = mx + c \quad (\text{i.e.}) \quad y = -x + 5 \quad \text{or} \quad x + y = 5.$$



Reference material

- Parametric equation (Wiki)
- Benefits of parametric form
- Line in plane (OnlineMschool)

Deserve “A” grade!

– Oleg Bulichev

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↗ @Lupasic

🚪 Room 105 (Underground robotics lab)