Analytical Geometry and Linear Algebra I, Lab 1

Introduction to AGLA

Vector

Basis and Subspace

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About Me (1)

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About Me (2)

Education

- *Bachelor* — BMSTU (МГТУ им Н.Э. Баумана) «CAD developer»
- *Master & PHD* — Innopolis University (IU) «Robotics»

Job

- *Senior Lecturer* — IU: Linear Algebra, Theoretical Mechanics, Mechanics and Machines
- *Course Author* — Skillbox: Mathematics for Robotics
- *Trainer* — IU: RAGE (ДИЧь) club – Hiking / Kayaking, Historical fencing, Folkgames, Archery

Link to the channel

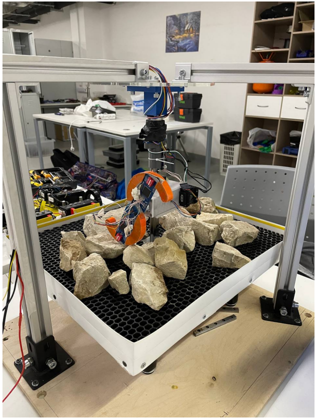
Where did I use linear algebra?

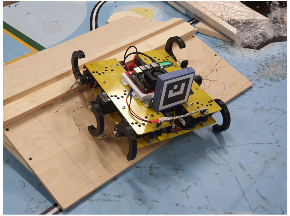
Problem 1

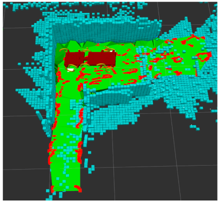
To determine geometrical and physical properties of passed terrain.

Linear Algebra topics:

- All core topics from LA (vector, matrix, etc)
- Change of basis
- Projections, regression







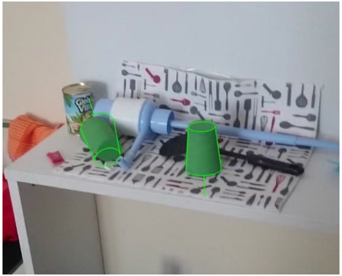
Where did I use linear algebra?

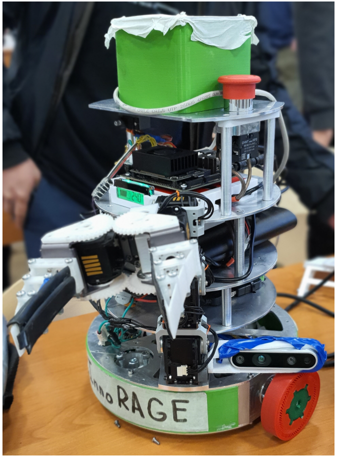
Problem 2

To determine the position and orientation of a mug using camera.

Linear Algebra topics:

- All core topics from LA (vector, matrix, etc)
- Change of basis, affine transformations
- Projections





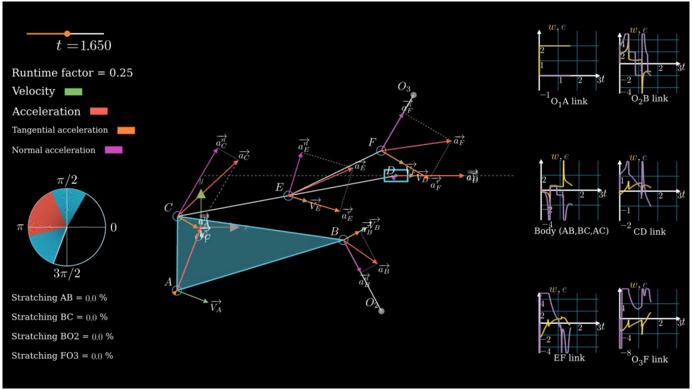
Where did I use linear algebra?

Problem 3

To simulate the mechanism.

Linear Algebra topics:

- Vectors, Matrices
- Change of basis



Short Summary

1. I used to use the course knowledge in my research career
2. It's not a useless course and I'll try to show it by presenting practical cases (not only from robotics))
3. I will give you an intuition of some basics

Agreements among the group (for now)

Pretest

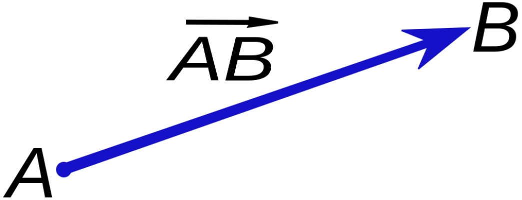
- Vector
- Span
- Linear combination
- Linear independence
- Vector length
- Subspace

Vector

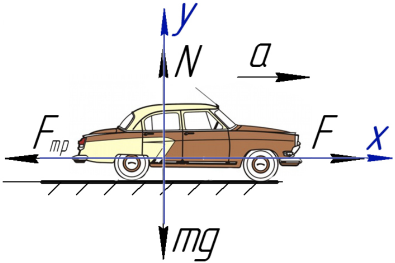
Definition from school

In the simplest case, it is a mathematical object characterized by magnitude and direction.

It is a directed segment with a certain length.



How a vector is seen by



Physics student

$$\overline{\mathbf{v}}$$

Mathematician

$$\begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

CS student

Vector operations

- Summation
- Scalar multiplication
- Norm / magnitude / vector length
- Transpose
- Dot (inner) product
- Cross (outer) product

Task 1

We have $\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}, \vec{c} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$

You should find $2\vec{a} + 3\vec{b} - \vec{c}$ и $16\vec{a} + 5\vec{b} - 9\vec{c}$

Task 2

Check if the result of each of the following operations is a vector or not. Explain your answer.

1. $\mathbf{a} + \mathbf{b}$, if $\mathbf{a}$ and $\mathbf{b}$ are vectors
2. $\mathbf{a} - \mathbf{a}$, if $\mathbf{a}$ is a vector
3. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
4. $\begin{bmatrix} 2x + 15 - 4y \\ y - x \end{bmatrix}$, if x and y are integer numbers
5. $\begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix} - \begin{bmatrix} x + y \\ 2y + 122 - 3x \end{bmatrix}$, if x and y are real numbers

Linear combination

Vector $\vec{w} \in V$ is a linear combination of $\vec{v}_1, \dots, \vec{v}_n \in V$ vectors,
if $\exists c_k \in \mathbb{R}; (k = 1..n)$
such as,

$$\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$$

Linear indepence

Two vectors $\vec{a}$ и $\vec{b}$ are *linear independent*
if for $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1\vec{a} + \alpha_2\vec{b} = \mathbf{0}$ if and only if $\alpha_1 = \alpha_2 = 0$.

Span

Let $S = \{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\} \subset V$.

$$span(S) \equiv \left\{ \mathbf{w} \in V : \mathbf{w} = \sum_{k=1}^{n} c_k \mathbf{v_k}, \quad \forall c_k \in \mathbb{R} \right\}$$

In words, $W = span(S)$ is the set of all (possible) linear combinations of the vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$.

Note that W is a subspace of V.

Basis

A **set** of vectors is a *basis* of $\mathbb{R}^2$ if it spans $\mathbb{R}^2$ and this set is **linear independent**.

Standard basis in $\mathbb{R}^2$

$\{\vec{i}, \vec{j}\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ — is basis of $\mathbb{R}^2$. It's a classical (canonical) basis in $\mathbb{R}^2$.

Standard basis in $\mathbb{R}^3$

$\{\vec{i}, \vec{j}, \vec{k}\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ — basis in $\mathbb{R}^3$.

Task 3

Check for each case if the following set of vectors is a basis or not. Explain your answer.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$
5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$
6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Task 3

Answer

A **set** of vectors is a *basis* of $\mathbb{R}^2$ if it spans $\mathbb{R}^2$ and this set is **linear independent**.

1. $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ NOT
2. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
3. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ NOT
4. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$
5. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$ NOT
6. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ NOT

Task 4

Points $\mathbf{A}(3,-2)$ and $\mathbf{B}(1,4)$ are given. The **M** point is on the line **AB** in the way that $|\mathbf{AM}| = 3|\mathbf{AB}|$. Find coordinates of the **M** point, if:

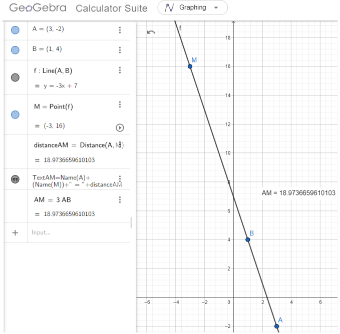
1. The points **M** and **B** are from the same side from A.
2. The points **M** and **B** are from the different sides from A.

Task 4

One of possible solutions

1. Find the distance between points **A** and **B**
2. Find the equation for the line **AB**
3. Find 2 point on the line with distance 3 |**AB**| from **A**

Another of possible solutions



Link to the Geogebra

Task 5 and 6

5. Find the coordinates of the gravity center of a triangular plate **ABC** with vertices in points $\mathbf{A}(3, 1)$, $\mathbf{B}(6, 3)$, $\mathbf{C}(0, 2)$.

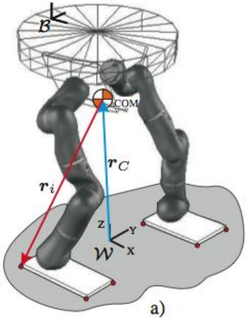
6. In the plane of the triangle **ABC** find the point **O** such that $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \mathbf{0}$. Are there such points outside of the triangle?

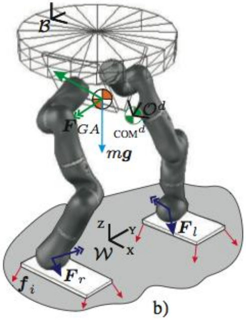
Note: $\mathbf{0}$ is a zero-vector.

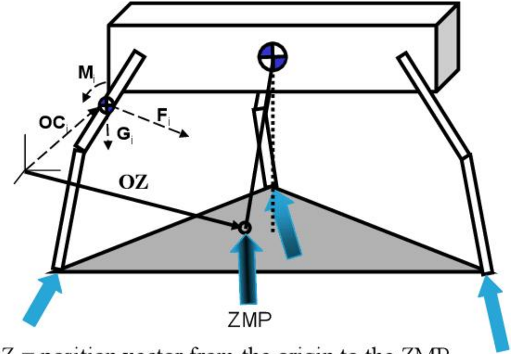
Center of gravity VS Center of mass

For classical mechanics - it’s the same. More info here

Where a center of mass can be used?

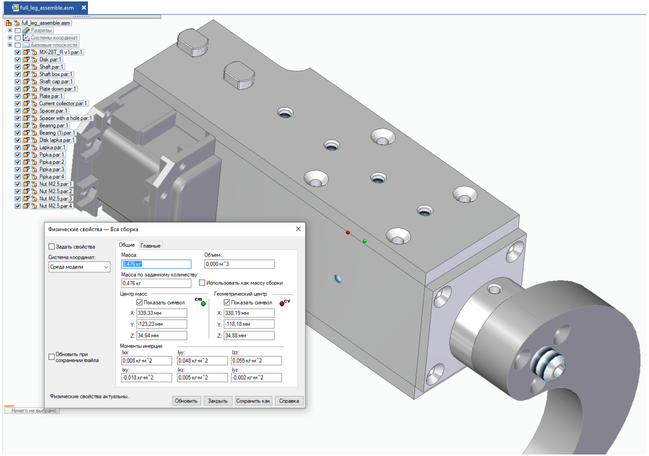






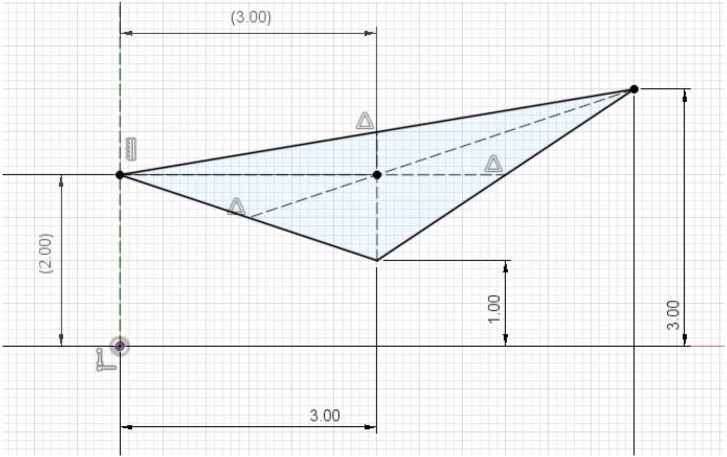
OZ = position vector from the origin to the ZMP

How to find a CoM in real life



Task 5

Solution made in CAD



Subspace

Definition

W is a <u>subspace</u> of V if

a)	$W \subset V$	(subset)
b)	$\mathbf{u}, \mathbf{v} \in W \Rightarrow \mathbf{u} + \mathbf{v} \in W$	(closure under addition)
c)	$\mathbf{u} \in W, \lambda \in \mathbb{R} \Rightarrow \lambda \mathbf{u} \in W$	(closure under scalar multiplication)

Task 7

Check for each case if the following set of vectors is a subspace or not. Explain your answer.

1. Part of the plane $x > 0$
2. Entire plane
3. Part of the plane $y < 0$
4. Part of the plane $x > 0, \; y > 0$
5. Inner circle with the radius $r = 5$

Reference material

OnlineMschool

- Vectors | Chapter 1, Essence of linear algebra - YouTube
- Linear combinations, span, and basis vectors | Chapter 2, Essence of linear algebra - YouTube
- Vectors Definition. Main information
- Векторы для чайников. Действия с векторами. Координаты вектора.
- Линейная зависимость и независимость векторов. Базис векторов

Deserve "A" grade!

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Room 105 (Underground robotics lab)