Analytical Geometry and Linear Algebra I, Lab 6

Line in plane

Questions from the class

No questions for today

Objectives

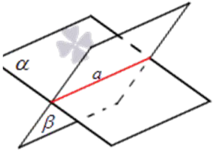
- To see the structure of all formulas, which were covered
- To understand how to transform one from to another and vise-versa
- How to apply the knowledge

Type of elements

Line in plane

Plane

Line in space



Computer Aided Design

Form types

Type	Form	Example	Description
Explicit	$y = f(x)$	$y = mx + b$	Line
Implicit	$f(x, y) = 0$	$(x - a)^2 + (y - b)^2 = r^2$	Circle
Parametric	$x = \dfrac{g(t)}{w(t)}; y = \dfrac{h(t)}{w(t)}$	$x = a_0 + a_1 t; y = b_0 + b_1 t$	Line
		$x = a + r\cos t; y = b + r\sin t$	Circle

Line in plane

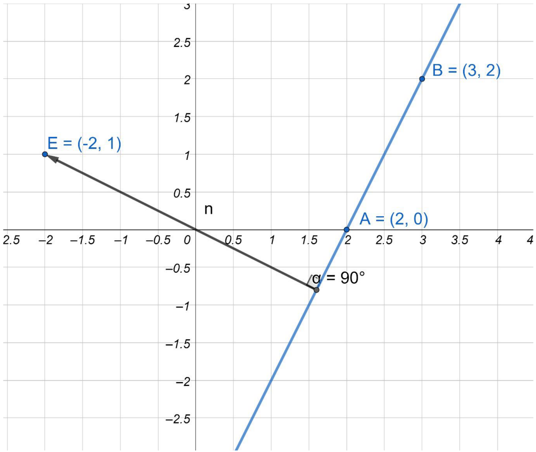
Formulas

1. ***General equation*** $Ax + By + C = 0$, where $C = -Ax_0 - By_0$
2. ***Slope-intercept*** $y = kx + b$, where $k = tan(X\text{-axis}\hat{\ }y(x))$ – slope
3. ***Passing through two points*** $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$; slope is $k = \frac{y_2 - x_1}{x_2 - x_1}$
   where $x_{1,2}, y_{1,2}$ are some particular coordinates of points on the line
4. ***Canonical*** $\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y}$,
   where x_0, y_0 is a point on the line and a_x, a_y – direction vector coefficients on a basis
5. ***Parametric*** $\vec{r} = \vec{r_0} + \tau\vec{a} = \begin{cases} x = x_0 + \tau a_x \\ y = y_0 + \tau a_y \end{cases}$, where τ is parameter,
   which can be received from canonical form ($\frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \tau$)
6. ***Using normal line*** $(\vec{r} - \vec{r_0}) \cdot \vec{n} = 0$, where $\vec{n} = \begin{bmatrix} A \\ B \end{bmatrix}$,
   $\vec{r} = \begin{bmatrix} x \\ y \end{bmatrix}, \vec{r_0} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

Line in plane

Task 0

Write down all forms of the line



Line in plane

Task 0, Answer

$$\begin{cases} 0 = k\cdot 2 + b \\ 2 = k\cdot 3 + b \end{cases} \Rightarrow \begin{matrix} b = -2k \\ k = 2 \end{matrix} \Rightarrow b = -4$$

(or using ③)

② $y = 2x - 4$

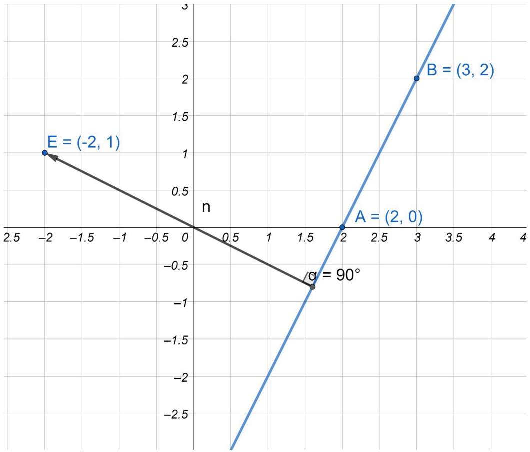
① $\underset{A}{-2}x + \underset{B}{1}y + \underset{C}{4} = 0$

$a = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

④ $\dfrac{y}{2} = \dfrac{x-2}{1}$; ④ → ⑤ $\dfrac{y}{2} = \dfrac{x-2}{1} = \tau$

⑤ $\begin{cases} x = 2 + \tau \\ y = 2\tau \end{cases}$

⑥ $\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = -4$; $\begin{bmatrix} x-2 \\ y \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \end{bmatrix} = 0$



Task 1

Find the slope of the line joining the points $(2, 3)$ and $(4, -5)$.

Task 1

Answer

The slope of the line joining the two given points (x_1, y_1) and (x_2, y_2)

is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Therefore, the slope of the line joining the two given points is

$$m = \frac{-5-3}{4-2} = \frac{-8}{2} = -4.$$

Task 2

Find the slope of the line $2x - 3y + 7 = 0$.

Task 2

Answer

The equation of the line is $2x - 3y + 7 = 0$ (i.e.) $3y = 2x + 7$.

$$\text{(i.e.)} \quad y = \frac{2}{3}x + \frac{7}{3}.$$

Therefore, slope of the line = $\frac{2\pi}{3}$.

Task 3

Find the equation of the straight line, the portion of which between the axes is bisected at the point $(2, -5)$.

Task 3

Answer

Let the line meet the x and y axes at A and B, respectively. Then the coordinates of A and B are (a, 0) and (0, b). The midpoint of AB is

$\left(\frac{a}{2}, \frac{b}{2}\right)$. However, the midpoint is given as (2, −5).

Therefore, $\frac{a}{2} = 2$ and $\frac{b}{2} = -5$.

$\therefore$ a = 4 and b = −10.

Hence, the equation of the straight line is $\frac{x}{4} - \frac{y}{10} = 1$.

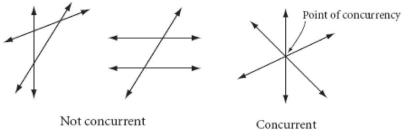
(i.e.) $5x - 2y = 20$.

Task 4

Find the equation of the straight line concurrent[a] with the lines $2x + 3y = 3$ and $x + 2y = 2$ and also concurrent with the lines $3x - y = 1$ and $x + 5y = 11$.

[a] Lines are said to be cuncurrent if they are intersect at a single point

- **Concurrent** – **Lines or segments that have three or more points in common**



Task 4

Answer

The point of intersection of the lines $2x + 3y = 3$ and $x + 2y = 2$ is obtained by solving the following two equations:

$$2x + 3y = 3 \qquad (2.35)$$
$$x + 2y = 2 \qquad (2.36)$$

$$(2.35) \qquad 2x + 3y = 3$$
$$(2.36) \times 2 \qquad 2x + 4y = 4$$

On subtracting, we get $y = 1$ and hence $x = 0$.
Therefore, the point of intersection is (0, 1).

$$3x - y = 1 \qquad (2.37)$$
$$x + 5y = 11 \qquad (2.38)$$

$$(2.37) \times 5 \qquad 15x - 5y = 5$$
$$(2.38) \times 1 \qquad x + 5y = 11$$

On adding, we get $16x = 16$ which gives $x = 1$ and hence $y = 2$. The point of intersection of the second pair of lines is (1, 2). The equation of the line joining the two points (0, 1) and (1, 2) is

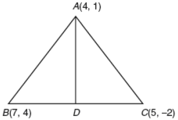
$$\frac{y-1}{x-0} = \frac{1-2}{0-1} = 1 \Rightarrow y - 1 = x \quad \text{(i.e.)} \quad x - y + 1 = 0.$$

Task 5

$A(4,1)$, $B(7,4)$, and $C(5,-2)$ are the vertices of a triangle. Find the line equation which is goes from A and perpendicular to BC.

Task 5

Answer



The slope of the line *BC* is

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4+2}{7-5}$$
$$= \frac{6}{2} = 3.$$

Therefore, the slope of the perpendicular *AD* to *BC* is $-\frac{1}{3}$. Hence, the equation of the perpendicular from *A*(4, 1) on *BC* is $y - y_1 = m(x - x_1)$

$$\text{(i.e.)} \quad y - 1 = \frac{-1}{3}(x-4) \Rightarrow 3y - 3 = -x + 4$$
$$\therefore x + 3y = 7.$$

Reference material

- Parametric equation (Wiki)
- Benefites of parametric form
- Line in plane (OnlineMschool)

Deserve “A” grade!

– Oleg Bulichev

o.bulichev@innopolis.ru

@Lupasic

Room 105 (Underground robotics lab)