Analytical Geometry and Linear Algebra II, Lab 12

Similar matrices

Singular value decomposition - SVD

Left and right inverses. Pseudoinverse

Innopolis University

Similar Matrices

The primary goal of SVD

To “X-RAY” matrix

(To understand the structure of matrix)

Singular Value Decomposition (SVD)

3 ways of explanation

1. Linear transformation – Kutz video
2. Algebraic – MIT video (Strang), Aaron Greiner video
3. As a tool for DS – Stanford video, Brunton video

According to Kholodov words, SVD was created for *finding an inverse for any matrices*. It is needed in linear transformation related operations. Other properties were found afterwards.

Singular Value Decomposition (SVD)

How to calculate it (2 possible ways)

First approach

1. Find eigenpairs for A^TA. Result is Σ and V. ($A^TA = V\Sigma^2V^T$)
2. Find U, using Σ and V ($AV\Sigma^+ = U$)

Second approach

1. Find eigenpairs for A^TA and AA^T. ($A^TA = V\Sigma^2V^T$) ($AA^T = U\Sigma^2U^T$)
2. Put signs correctly

Singular Value Decomposition (SVD)

Obtain SVD for $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$*, using second approach (task was taken)*

1. Eigenpairs of AA^T.

$$AA^T = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}.\ \lambda_1 = 25, \lambda_2 = 9.\ x_{\lambda_1} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, x_{\lambda_2} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.\ U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

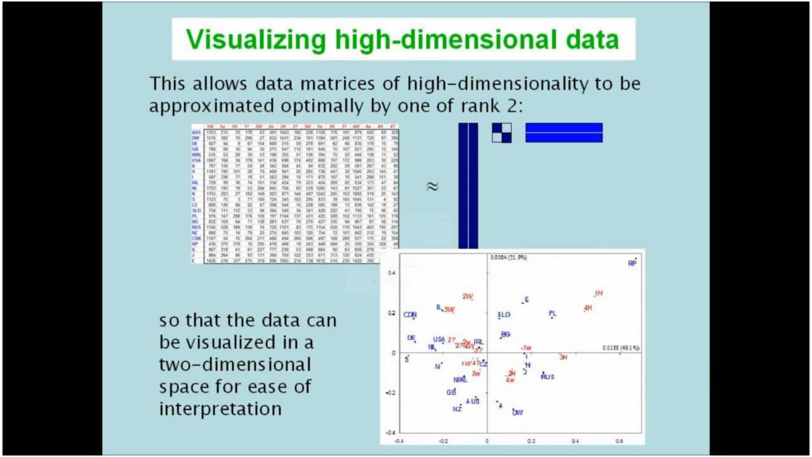
2. Eigenpairs of A^TA.

$$A^TA = \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}.\ \lambda_1 = 25, \lambda_2 = 9, \lambda_3 = 0.\ V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{18}} & \frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{18}} & -\frac{2}{3} \\ 0 & \frac{4}{\sqrt{18}} & -\frac{1}{3} \end{bmatrix}$$

3. Result. $A\Sigma V^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{18}} & -\frac{1}{\sqrt{18}} & \frac{4}{\sqrt{18}} \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

It had to be *U*

Video



Singular Value Decomposition (SVD)

Properties

TODO

Singular Value Decomposition (SVD)

Where it can be used

- Image compression (slides)
- Pseudo-inverse (slides)
- Dimensionality reduction (code + comments in pdf)
- Least square (code + comments in pdf)
- Principal Component Analysis (PCA) (video)
- Eigenfaces algorithms (video)

SVD Applications

Image compression

Task: We want to compress our image for reducing the size/

Solution: We can represent our picture as a matrix.

Next step is using SVD for reducing matrix rank.

Pseudo-Inverse

Reference material

- Lecture 28: Similar Matrices and Jordan Form.
- Lecture 29: Singular Value Decomposition
- Lecture 33: Left and Right Inverses; Pseudoinverse
- 6. Singular Value Decomposition (SVD)
- *"Introduction to Linear Algebra", pdf pages 375–411*
  7 Singular Value Decomposition (SVD)
- *"Linear Algebra and Applications", pdf pages 335–345*
  5.6 Similarity Transformations
- *"Linear Algebra and Applications", pdf pages 377–386*
  6.3 Singular Value Decomposition

Deserve “A” grade!

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Room 105 (Underground robotics lab)