Linear Algebra. Final - Variant#1

Name	1	2	3	4	5	6	7	Total

Problem 1: Write down three equations for the line, which is go through points (-1, 6), (1, 6), (2, 18), find least squares solution and draw the closest line. (**5** points)

Problem 2: Is A a projection matrix? If so, find basis of the subspace of R^3 onto which A is projecting, and the basis of its orthogonal complement. (**6** points):

$$A = \frac{1}{2}\begin{bmatrix}1 & 0 & 1\\ 1 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 2\\ 1 & -1 & 0\end{bmatrix}$$

Problem 3: What values of α produce instability $v_{n+1} = \alpha(v_n - w_n), w_{n+1} = \alpha(w_n - v_n)$. (**6** points).

Problem 4: Matrix A has singular values 100, 64, and 2. Using SVD, we can find the best rank 2 approximation of A: the matrix $\tilde{A}$. What is upper bound of $|A\boldsymbol{x} - \tilde{A}\boldsymbol{x}|$ for all possible unit vectors $\boldsymbol{x}$? (**6** points).

Problem 5: Find A^k for the matrix $A = \begin{bmatrix}6 & 9\\ 4 & 1\end{bmatrix}$. (**5** points)

Problem 6: Transformation T first performs translation by vector (1,1), then rotation by π radians counterclockwise, then translation by vector (1,1) again, then another rotation by π radians, but this time clockwise, and finally performs projection on axis OX, and takes the length of the resulting vector. Find matrix corresponding to T, or show why it can not be done. (**6** points)

Problem 7: Solve the differential equation, (**6** points):

$$4y'' - 3y = 0, y(0) = 1, y'(0) = 1.$$

What happens to y(t) as $t \to \infty$?

Linear Algebra. Final - Variant#2

Name	1	2	3	4	5	6	7	Total

Problem 1: Write down three equations for the line, which is go through points (1, 7), (-1, 7), (-2, 21), find least squares solution and draw the closest line. (**5** points)

Problem 2: Is A a projection matrix? If so, find basis of the subspace of R^3 onto which A is projecting, and the basis of its orthogonal complement. (**6** points):

$$A = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Problem 3: What values of α produce stability $v_{n+1} = \alpha(v_n - w_n), w_{n+1} = \alpha(w_n - v_n)$. (**6** points).

Problem 4: Matrix A has singular values 10, 9, and 1. Using SVD, we can find the best rank 2 approximation of A: the matrix $\tilde{A}$. What is upper bound of $|A\boldsymbol{x} - \tilde{A}\boldsymbol{x}|$ for all possible unit vectors $\boldsymbol{x}$? (**6** points).

Problem 5: Find A^k for the matrix $A = \begin{bmatrix} 6 & 9 \\ 1 & 6 \end{bmatrix}$. (**5** points)

Problem 6: Transformation T first performs rotation by π radians clockwise, then translation by vector (-1,1), then another rotation by π radians (counterclockwise), another translation by (-1,1), and finally performs projection on axis OY, and takes the length of the resulting vector. Find matrix corresponding to T, or show why it can not be done. (**6** points)

Problem 7: Solve the differential equation, (**6** points):

$$5y'' - 2y = 0, y(0) = 1, y'(0) = 1.$$

What happens to y(t) as $t \to \infty$?