Analytical Geometry and Linear Algebra II, Lab 3

Quiz

Four Fundametal Subspaces

Quiz

1) Obtain P, L, U matrices from A, using $PA = LU$ factorization.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \qquad (1)$$

$$A = \begin{bmatrix} 1 & 4 & 0 & 0 \\ 3 & 12 & 1 & 5 \\ 2 & 8 & 1 & 5 \\ 0 & 2 & 2 & 3 \end{bmatrix} \qquad (2)$$

2) For

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix} x = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

for 5th point

$Ax = [0,\ 6,\ -6]$

1. Reduce $[A\ b]$ to $[U\ c]$, to reach a triangular system.
2. Find the condition on b_1, b_2, b_3 to have a soultion.
3. Describe the column space of A. Find the basis of the column space.
4. Describe the nullspace of A. Declare free variables.
5. Find a particular solution and the complete solution $x_p + x_n$

Task 1

Reduce these matrices to their ordinary echelon forms U:

(a) $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$.

Which are the free variables and which are the pivot variables?

Task 1

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$$\text{(a)}\quad A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \qquad \text{(b)}\quad B = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}.$$

Which are the free variables and which are the pivot variables?

Answer

(a) $U = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ Free variables x_2, x_4, x_5
Pivot variables x_1, x_3

(b) $U = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ Free x_3
Pivot x_1, x_2

Task 2

Construct 3 by 3 matrices A to satisfy these requirements (if possible):

(a) A has no zero entries but $U = I$.

(b) A has no zero entries but $R = I$.

(c) A has no zero entries but $R = U$.

(d) $A = U = 2R$.

Task 2

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(b) A has no zero entries but $R = I$.

(c) A has no zero entries but $R = U$.

(d) $A = U = 2R$.

Answer

(a) Impossible row 1 (b) $A =$ invertible (c) $A =$ all ones (d) $A = 2I$, $R = I$.

Task 3

If the special solutions to $Rx = \mathbf{0}$ are in the columns of these N, go backward to find the nonzero rows of the reduced matrices R:

$$N = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} \ \end{bmatrix} \text{ (empty 3 by 1).}$$

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Answer

Any zero rows come after these rows: $R = [\,1 \quad -2 \quad -3\,]$, $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $R = I$.

Reference material

- Lecture 9 and 10
- *"Linear Algebra and Applications", pdf pages 139–149*
  The application of four fundamental subspaces in CS
- Matrix Transpose and the Four Fundamental Subspaces
  Video is about how A transpose appeared
- Matrix online calculator in russian

Deserve “A” grade!

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Room 105 (Underground robotics lab)