Analytical Geometry and Linear Algebra II, Lab 8

Fourier Series

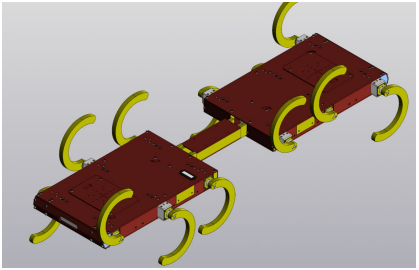
Fast Fourier Transform (FFT)

Discrete Fourier Transform (DFT)

Innopolis University

How I spent last weekend

(a) Minecraft RAGE club meeting



(b) StriRus robot

(c) Museum of Modern Art

Course VS My Goal

Course Goal

Is to give you a knowledge how to calculate FFT and DFT by hand. It's an application for using Complex Numbers and Matrices.

My Goal

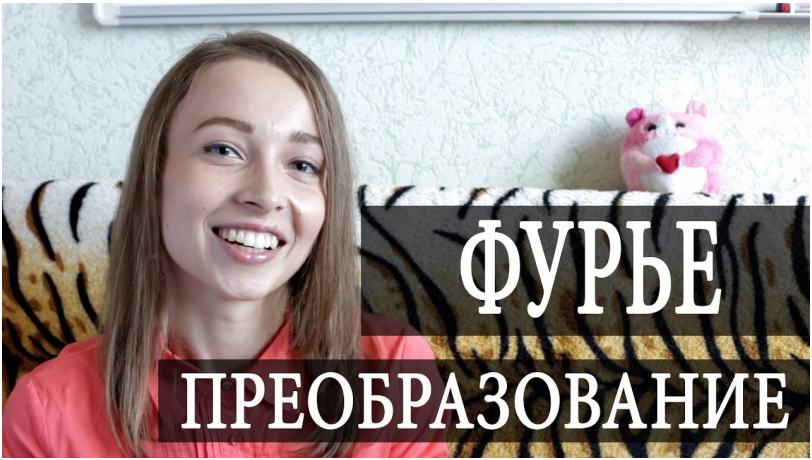
Is to give you the application and the concept why do we need it. It won't be on the exam.

Outline

1. Fourier Series, intuition
2. Fourier Series, math
3. Discrete Fourier Transform
4. Fast Fourier Transform

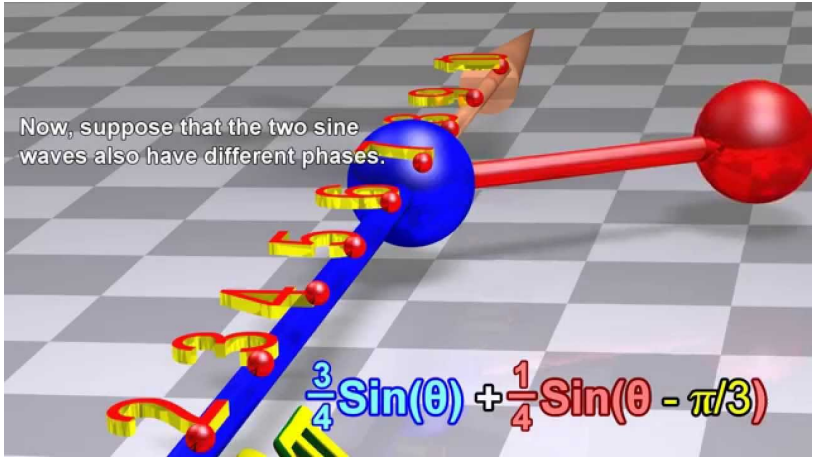
Why do we need Fourier Transform

Video (rus)



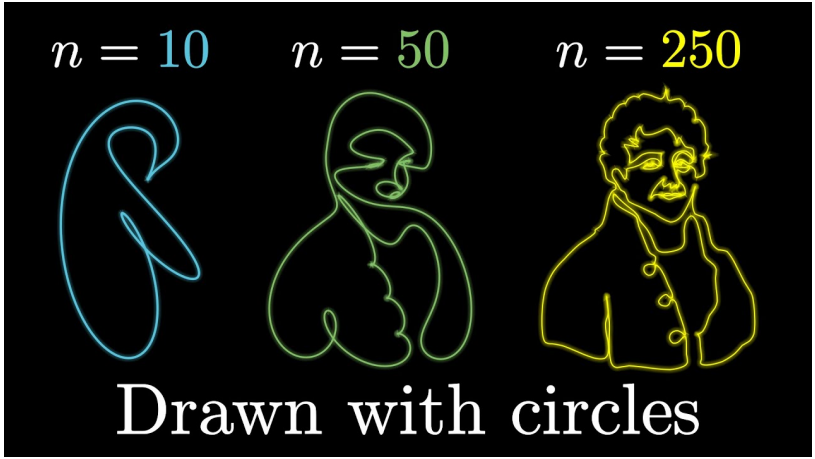
How to sum up sines (Spectrum)

Video



How to draw pictures using Furier Transform

Video



Some Math words

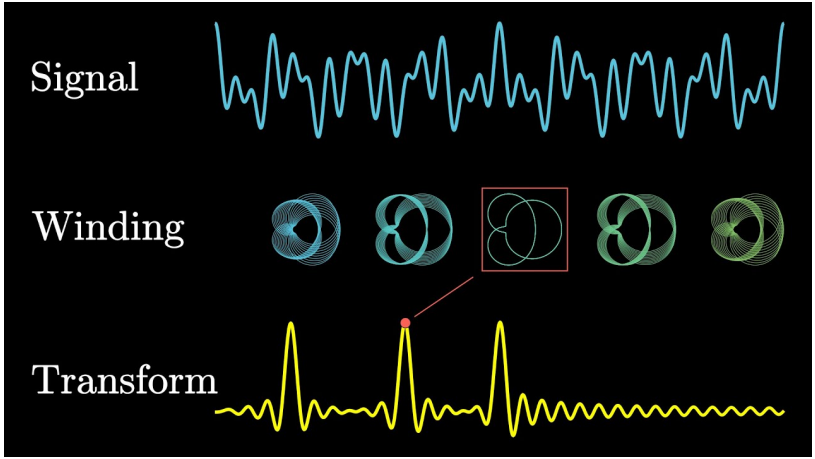
On a whiteboard

FFT in Sound (rus)

Video

Fourier Transform in sound (watch at home)

Video

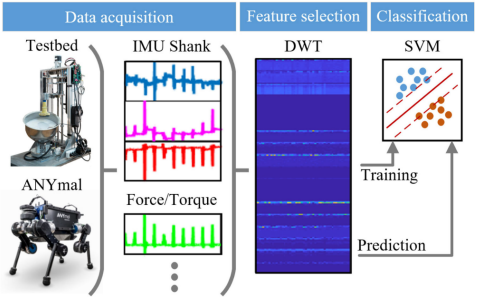


Fourier Transform Application

Terrain Classification: Feature Extraction step

Problem

How to put such data (2) in ML algorithm (for instance SVM)?



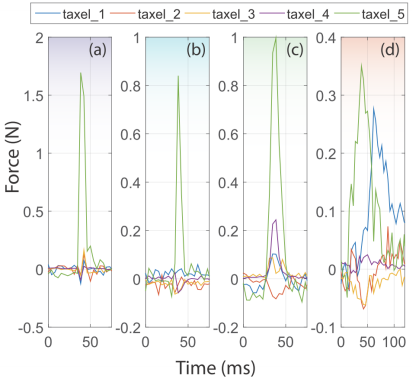


Figure 2: Individual taxel forces recorded on different surfaces at 10 Hz stride frequency

Math words again

On a whiteboard

Task 1

What are F^2 and F^4 for the 4 by 4 Fourier matrix F?

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Answer

$$F^2 = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix}, \quad F^4 = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix} = 4^2 I.$$

Task 2

$$Fc = y \qquad \begin{array}{rcrcrcrcl} c_0 & + & c_1 & + & c_2 & + & c_3 & = & 2 \\ c_0 & + & ic_1 & + & i^2c_2 & + & i^3c_3 & = & 4 \\ c_0 & + & i^2c_1 & + & i^4c_2 & + & i^6c_3 & = & 6 \\ c_0 & + & i^3c_1 & + & i^6c_2 & + & i^9c_3 & = & 8. \end{array}$$

Solve the 4 by 4 system if the right-hand sides are $y_0 = 2$, $y_1 = 0$, $y_2 = 2$, $y_3 = 0$. In other words, solve $F_4 c = y$.

Task 2

$$Fc = y \qquad \begin{aligned} c_0 + c_1 + c_2 + c_3 &= 2 \\ c_0 + ic_1 + i^2c_2 + i^3c_3 &= 4 \\ c_0 + i^2c_1 + i^4c_2 + i^6c_3 &= 6 \\ c_0 + i^3c_1 + i^6c_2 + i^9c_3 &= 8. \end{aligned}$$

Solve the 4 by 4 system if the right-hand sides are $y_0 = 2, y_1 = 0, y_2 = 2, y_3 = 0$. In other words, solve $F_4 c = y$.

Answer

$$c = (1, 0, 1, 0).$$

Task 3

Find all solutions to the equation $e^{ix} = -1$, and all solutions to $e^{i\theta} = i$.

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Find all solutions to the equation $e^{ix} = -1$, and all solutions to $e^{i\theta} = i$.

Answer

$e^{ix} = -1$ for $x = (2k+1)\pi$, $e^{i\theta} = i$ for $\theta = 2k\pi + \pi/2$, k is integer.

Task 4

All entries in the factorization of F_6 involve powers of $w =$ sixth root of 1:

$$F_6 = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_3 & \\ & F_3 \end{bmatrix} \begin{bmatrix} \\ P \\ \\ \end{bmatrix}.$$

Write these factors with $1, w, w^2$ in D and $1, w^2, w^4$ in F_3. Multiply!

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Answer

$$D = \begin{bmatrix} 1 & & \\ & e^{2\pi i/6} & \\ & & e^{4\pi i/6} \end{bmatrix} \text{ and } F_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{2\pi i/3} & e^{4\pi i/3} \\ 1 & e^{4\pi i/3} & e^{2\pi i/3} \end{bmatrix}.$$

Reference material

- Fourier Series
- Lecture 26, 2nd part
- ”*Linear Algebra and Applications*”, *pdf pages 221–234*
  Fast Fourier Transform
- ”*Introduction to Linear Algebra*”, *pdf pages 456–462*
  Fast Fourier Transform
- ”*Introduction to Linear Algebra*”, *pdf pages 501–506*
  Fourier Series: Linear Algebra for Functions

Deserve “A” grade!

– Oleg Bulichev

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Room 105 (Underground robotics lab)