I. Prove that for any square matrix $A(n \times n)$ with eigenvalues $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ the multiplication: $(A - \lambda_1 I)(A - \lambda_2 I) \cdots (A - \lambda_n I)$ produces the zero matrix:

$$(A-\lambda_1 I)(A-\lambda_2 I)\cdots(A-\lambda_n I) = S(\Lambda-\lambda_1 I)S^{-1}S(\Lambda-\lambda_2 I)S^{-1}\cdots S(\Lambda-\lambda_n I)S^{-1} =$$

$$= S(\Lambda-\lambda_1 I)(\Lambda-\lambda_2 I)\cdots(\Lambda-\lambda_n I)S^{-1} =$$

$$= S\begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & \lambda_2-\lambda_1 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \lambda_n-\lambda_1 \end{bmatrix}\begin{bmatrix} \lambda_1-\lambda_2 & \cdots & \cdots & 0 \\ \vdots & 0 & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & \lambda_n-\lambda_2 \end{bmatrix}\cdots\begin{bmatrix} \lambda_1-\lambda_n & \cdots & \cdots & 0 \\ \vdots & \lambda_2-\lambda_n & 0 & \vdots \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}S^{-1} =$$

$$= S\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}S^{-1} = [0]$$

II. Prove that any real square matrix can be factored into $A = QS$, where Q is orthogonal and S is symmetric positive semidefinite.

Remember it's not enough to present $A = (UV^T)(V\Sigma V^T) = QS$ but it is also necessary to show that $Q = UV^T$ is orthogonal: $QQ^T = UV^T V U^T = I$ and
$S = V\Sigma V^T$ is positive semidefinite:
$\forall x \neq 0$: $x^T V \Sigma V^T x = \left(x^T V\sqrt{\Sigma}\right)\left(\sqrt{\Sigma}V^T x\right) = y^T y \geq 0$ where $y = \sqrt{\Sigma}V^T x$.