# Linear algebra and analytical geometry II, Lab 1

# Task 1

*Warm up*

Rewrite the following system in the matrix form:

$$\begin{cases} 3x + 4y - 2z = 1 \\ 3y - 2z + x = -2 \\ 5x - 7z - 2y = 3 \end{cases}$$

# Task 2

Describe geometricaly (line, plane, or whole space) all linear combinations of:

1. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$
2. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$
3. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

# Task 3

Draw $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v + w$, $v - w$ in a single $xy$ plane

# Task 4

Explain, why the system

$$\begin{cases} u + v + w = 2 \\ u + 2v + 3w = 1 \\ v + 2w = 0 \end{cases}$$

is singular?

What value should replace that last zero on the right side, to allow the equations to have solutions, and what is one of the solutions?

# What does it mean, singular?

*Lec 1, page 11*

**Example 2.** Singular (incurable)

$$\begin{cases} u+v+w= \\ 2u+2v+5w= \\ 4u+4v+8w= \end{cases} \Rightarrow \begin{cases} u+v+w= \\ 3w= \\ 4w= \end{cases}$$

There is no exchange of equations that can avoid zero in the second pivot position. The equations themselves may be solvable or unsolvable. If the last two equations are $3w=6$ and $4w=7$, there is no solution. If those two equations happen to be consistent as in $3w=6$ and $4w=8$ then this singular case has an infinity of solutions. We know that $w=2$, but the first equation cannot decide both $u$ and $v$.

# Task 4

Explain, why the system

$$\begin{cases} u + v + w = 2 \\ u + 2v + 3w = 1 \\ v + 2w = 0 \end{cases}$$

is singular?

What value should replace that last zero on the right side, to allow the equations to have solutions, and what is one of the solutions?

# Task 5

1. Choose a coefficient $b$ that makes this system singluar.
2. Then choose a right-hand side $g$ that makes it solvable.
3. Find two solutions in that singular case.

$$\begin{cases} 2x + by = 16 \\ 4x + 8y = g \end{cases}$$

# Task 6

Give 3x3 examples (not just the zero matrix):

1. a diagonal matrix: $a_{ij} = 0$, if $i \neq j$;
2. a symmetric matrix: $a_{ij} = a_{ji}$ for all $i$ and $j$;
3. an upper trianglular matrix: $a_{ij} = 0$, if $i > j$ ;
4. a skew-symmetric matrix: $a_{ij} = -a_{ji}$ for all $i$ and $j$.

# Task 7

Make a rref of

$$\begin{cases} 2u + 3v + 0w = 0 \\ 4u + 5v + w = 3 \\ 2u - 1v - 3w = 5 \end{cases}$$

# Reference material

- Lectures 1 – 3