Mid Term Exam. Variant 1 **Innopolis University**
Full Name: ________________________ **AG&LA II**
Group #: __________________________ **Spring 2023**

Duration: 120 Minutes **23.03.2023**

You need to perform this exam alone and without the use of any equipment, apart from this booklet and pens. You can use only 1 sheet of A4 paper with formulas on both sides. Please do not consult any person or use any equipment, except for a simple non-programmable calculator. Otherwise, you will be disqualified from this exam at the first attempt.

Good Luck!

Grade Table (for teacher use only)

Question	1	2	3	4	5	6	7	Total
Points	3	6	4	6	6	6	4	35
Score								

1. (3 points) True or False statement.

For matrix $A(m\text{x}n)$ with rank$(A) < m$ & rank$(A) < n$ is	**True**	**False**
$\text{Dim}[C(A)] = \text{Dim}[C(A^T)]$		
$C(A) = C(A^T)$		
Dim $C(A)$ + Dim $N(A^T) = m$.		

2. Considering the matrix, A and the vector b,

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \\ 2 & 1 \\ -1 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 2 \end{bmatrix}$$

(a) (4 points) Find the projection of b onto the column space of A.
(b) (1 points) Split b into $p + e$, with p in the column space and e orthogonal to that space.
(c) (1 point) Which of the four fundamental spaces of A contains e.

3. (4 points) Let $S_1 = \{x, y, z : x - 2y - 4z = 8\}$ and $S_2 = \{x, y, z : x - y + z = 3\}$.

$\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ – lies at the line of intersection of the planes S_1 and S_2. Find $\vec{v}$.

4. Considering the following measurements:

t	-2	-1	1	2
b	-2	0	4	6

(a) (4 points) Find the best straight-line fit (Least squares) to the measurements,
(b) (2 point) Find the projection matrix of vector $b = [-2, 0, 4, 6]^T$ onto the column space of matrix A:

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

5. (6 points) Find the dimensions of the four fundamental subspaces associated with A, depending on the parameters α and β.

$$A = \begin{bmatrix} 7 & \alpha & 5 & 10 \\ 1 & 2 & 1 & 2 \\ 1 & 3 & \beta & 4 \\ 5 & 8 & 3 & 6 \end{bmatrix}$$

6. Find an orthonormal basis for the subspace spanned by the vectors: a_1, a_2 and a_3 (4 points).

Then express $A = [a_1, a_2, a_3]$ in the form of QR (2 points).

$$a_1 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}, a_3 = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}.$$

7. (4 points) Prove that the matrix $A^T A$ has the same nullspace as $A(m \times n)$