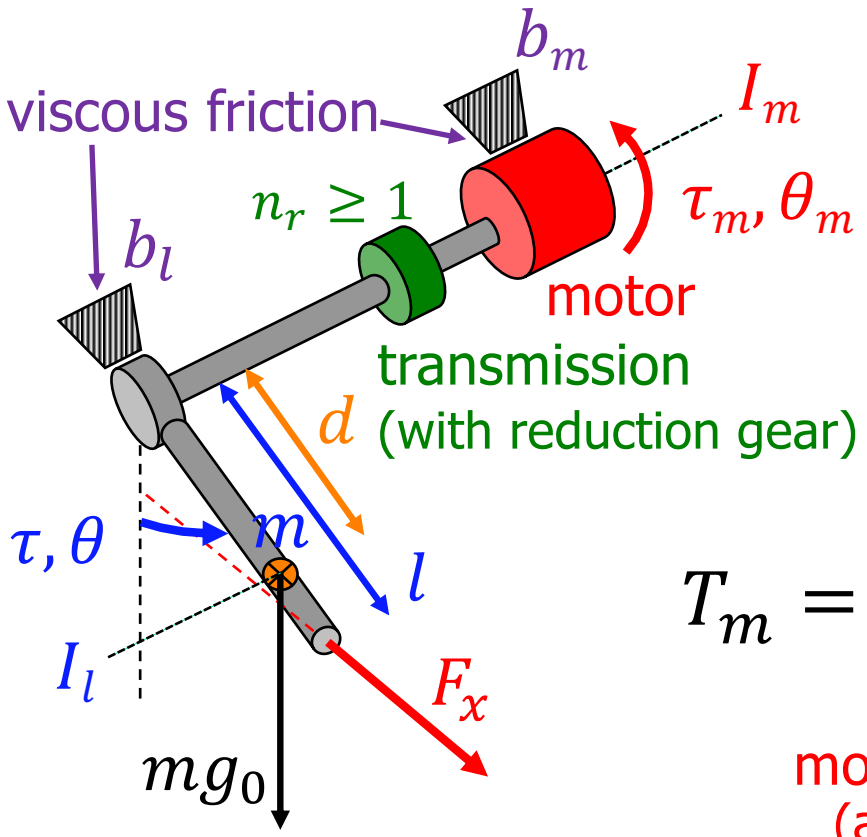
Dynamics of an actuated pendulum

a first example

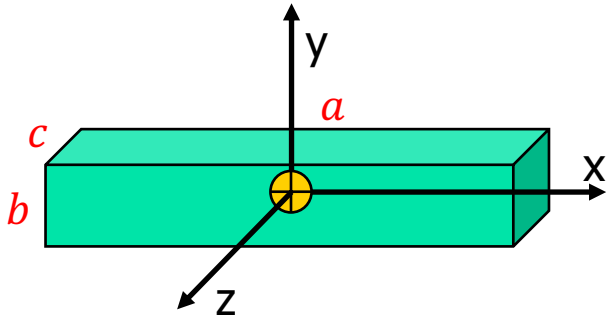


$$T_m = \frac{1}{2} I_m \dot{\theta}_m^2$$

motor inertia (around its spinning axis)

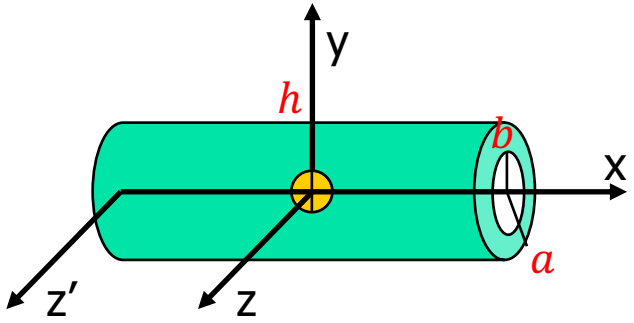
Examples of body inertia matrices

homogeneous bodies of mass m, with axes of symmetry



parallelepiped with sides
a (length/height), b and c (base)

$$I_c = \begin{pmatrix} I_{xx} & & \\ & I_{yy} & \\ & & I_{zz} \end{pmatrix} = \begin{pmatrix} \frac{1}{12}m(b^2+c^2) & & \\ & \frac{1}{12}m(a^2+c^2) & \\ & & \frac{1}{12}m(a^2+b^2) \end{pmatrix}$$



empty cylinder with length h,
and external/internal radius a and b

$$I_c = \begin{pmatrix} \frac{1}{2}m(a^2+b^2) & & \\ & \frac{1}{12}m(3(a^2+b^2)+h^2) & \\ & & I_{zz} \end{pmatrix} \qquad I_{zz} = I_{yy}$$

Kinetic energy of a rigid body (cont)

$$= \frac{1}{2} m \, v_c^T v_c$$

translational kinetic energy (point mass at CoM)

+

rotational kinetic energy (of the whole body)

$$= \frac{1}{2} \omega^T \, I_c \, \omega$$

body inertia matrix (around the CoM)

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