

Example of MidTerm tasks

1. Considering the matrix, A and the vector b ,

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

- (a) Find the projection of b onto the column space of A .
(b) Split b into $p + e$, with p in the column space and e orthogonal to that space.
2. Let $S_1 = \{x, y, z : x - 5y + 8z = 11\}$ and $S_2 = \{x, y, z : x - y = 3\}$.

$\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ – lies at the line of intersection of the planes S_1 and S_2 . Find \vec{v} .

3. Considering the following measurements:

$$\begin{array}{c|ccc} t & -1 & 1 & 2 \\ \hline b & -1 & 3 & 5 \end{array}$$

Find the best straight-line fit (Least squares) to the measurements,

4. Find the dimensions of the four fundamental subspaces associated with A , depending on the parameter α .

$$A = \begin{bmatrix} 7 & 8 & 5 \\ 4 & \alpha & 3 \\ 6 & 7 & 4 \end{bmatrix}$$

5. Find a vector \vec{x} orthogonal to the $C(A^T)$ space, and a vector \vec{y} orthogonal to the $C(A)$, and a vector \vec{z} orthogonal to the $N(A)$:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$$

6. Find an orthonormal basis for the subspace spanned by the vectors: a_1, a_2 and a_3 . Then express $A = [a_1, a_2, a_3]$ in the form of QR .

$$a_1 = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} -1 \\ 1 \\ -4 \end{bmatrix}, a_3 = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}.$$