Full Name: ______________________
Mid Term Exam
Variant 1

Innopolis University
AG&LA II
Spring 2021

Duration: 120 Minutes

16.03.2021

You need to perform this exam alone and without the use of any equipment or book, apart from this booklet and pens. Please do not consult any person or use any equipment, otherwise you will be disqualified from this exam at the very first attempt.

Good Luck!

Grade Table (for teacher use only)

Question	1	2	3	4	5	6	Total
Points	3	7	5	6	4	5	30
Score							

1. (3 points) True or False statement.

Statement	**True**	**False**
The vectors *b* that are not in the column space *C(A)* form a subspace.		
The column space of 2*A* equals the column space of *A*.		
dim(row space) + dim(nullspace) = number of columns.		

2. Considering the matrix, *A* and the vector *b*,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$$

   (a) (3 points) Find the projection of *b* onto the column space of *A*.
   (b) (3 points) Split *b* into $p + q$, with *p* in the column space and *q* perpendicular to that space.
   (c) (1 point) Which of the four subspaces contains q.

3. Considering the vectors, *a*, *b*, *c*.

$$a = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \qquad c = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$

   (a) (3 points) Apply the Gram Schmidt process to find the orthonormal basis for the subspace spanned by the vectors a, b, c.
   (b) (2 points) Express $A = [a \quad b \quad c]$ in the form of QR ($A = QR$).

4. Considering the following measurements:

t	-2	-1	1	2
b	-4	-2	-2	0

   (a) (4 points) Find the best straight-line fit (Least squares) to the measurements,
   (b) (1 point) and draw its graph,
   (c) (1 point) and write E^2

5. (4 points) Find the dimensions of the four fundamental subspaces associated with *A*, depending on the parameters α and β.

$$A = \begin{bmatrix} 0 & \alpha & 2 \\ 2 & -1 & 1 \\ 4 & -2 & \beta \end{bmatrix}$$

6. Construct a matrix with the required property or say why that is impossible.
   (a) (3 points) Column space contains $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ and nullspace contains $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.
   (b) (2 points) With (1, 2, 3) in the row space and column space.

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Mid Term Exam

Variant 2

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>
> Good Luck!

Grade Table (for teacher use only)

Question	1	2	3	4	5	6	Total
Points	3	7	5	6	4	5	30
Score							

1. (3 points) True or False statement.

Statement	**True**	**False**
The null space of *A* is the solution set of the equation = 0.		
The null space of an $m \times n$ matrix is in R^m		
Col *A* is the set of all vectors that can be written as *Ax* for some *x*.		

2. Considering the matrix, *A* and the vector *b*,

$$A = \begin{bmatrix} -1 & 1 \\ -3 & 0 \\ -1 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

(a) (3 points) Find the projection of *b* onto the column space of *A*.

(b) (3 points) Split *b* into $p + q$, with *p* in the column space and *q* perpendicular to that space.

(c) (1 point) Which of the four subspaces contains *q*.

3. Considering the vectors, *a*, *b*, *c*.

$$a = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}, \qquad c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) (3 points) Apply the Gram Schmidt process to find the orthonormal basis for the subspace spanned by the vectors *a*, *b*, *c*.

(b) (2 points) Express $A = [a \;\; b \;\; c]$ in the form of QR ($A = QR$).

4. Considering the following measurements:

t	-1	0	1	2
b	-1	2	4	5

(a) (4 points) Find the best straight-line fit (Least squares) to the measurements,

(b) (1 point) and draw its graph,

(c) (1 point) and write E^2

5. (4 points) Find the dimensions of the four fundamental subspaces associated with *A*, depending on the parameters α and β. (without row exchange)

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 1 & \alpha & -1 \\ -1 & 1 & \beta \end{bmatrix}$$

6. Construct a matrix with the required property or say why that is impossible.

(a) (3 points) Column space contains $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$ and nullspace contains $\begin{bmatrix} 14 \\ 5 \\ 1 \end{bmatrix}$.

(b) (2 points) With (3, 4, 5) in the row space and column space.

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Mid Term Exam

Variant 3

Innopolis University

AG&LA II

Spring 2021

Duration: 120 Minutes

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Good Luck!

Grade Table (for teacher use only)

Question	1	2	3	4	5	6	Total
Points	3	7	5	6	4	5	30
Score							

1. (3 points) True or False statement.

Statement	**True**	**False**
If matrix A is invertible, then its rows are independent		
The Column space of an $m \times n$ matrix is in R^m		
dim(nullspace) = number of pivot columns		

2. Considering the matrix, A and the vector b,

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}, \qquad b = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

   (a) (3 points) Find the projection of b onto the column space of A.

   (b) (3 points) Split b into $p + q$, with p in the column space and q perpendicular to that space.

   (c) (1 point) Which of the four subspaces contains q.

3. Considering the vectors, a, b, c.

$$a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \qquad c = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

   (a) (3 points) Apply the Gram Schmidt process to find the orthonormal basis for the subspace spanned by the vectors a, b, c.

   (b) (2 points) Express $A = [a \;\; b \;\; c]$ in the form of QR ($A = QR$).

4. Considering the following measurements:

t	1	2	3	4
b	0	3	2	5

   (a) (4 points) Find the best straight-line fit (Least squares) to the measurements,

   (b) (1 point) and draw its graph,

   (c) (1 point) and write E^2

5. (4 points) Find the dimensions of the four fundamental subspaces associated with A, depending on the parameters α and β. (without row exchange)

$$A = \begin{bmatrix} 2 & 5 & 1 \\ -1 & \alpha & 0 \\ -2 & -5 & \beta \end{bmatrix}$$

6. Construct a matrix with the required property or say why that is impossible.

   (a) (3 points) Column space contains $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -3 \end{bmatrix}$ and nullspace contains $\begin{bmatrix} -6 \\ -2 \\ 1 \end{bmatrix}$.

   (b) (2 points) With (2, 1, 3) in the row space and column space.