Mid Term Exam. Variant 2 | **Innopolis University**
Full Name: ___________________________ | **AG&LA II**
Group #: _____________________________ | **Spring 2023**

Duration: 120 Minutes | **23.03.2023**

You need to perform this exam alone and without the use of any equipment, apart from this booklet and pens. You can use only 1 sheet of A4 paper with formulas on both sides. Please do not consult any person or use any equipment, except for a simple non-programmable calculator. Otherwise, you will be disqualified from this exam at the first attempt.

Good Luck!

Grade Table (for teacher use only)

Question	1	2	3	4	5	6	7	Total
Points	3	6	4	6	6	6	4	35
Score								

1. (3 points) True or False statement.

For matrix $A(m\text{x}n)$ with $\text{rank}(A) < m$ & $\text{rank}(A) < n$ is	**True**	**False**
$C(A^T) \neq C(A)$		
$\text{Dim}[C(A^T)] \neq \text{Dim}[C(A)]$		
$\text{Dim } C(A^T) + \text{Dim } N(A) = n.$		

2. Considering the matrix, A and the vector b,

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

   (a) (4 points) Find the projection of b onto the column space of A.
   (b) (1 points) Split b into $p + e$, with p in the column space and e orthogonal to that space.
   (c) (1 point) Which of the four fundamental spaces of A contains e.

3. (4 points) Let $S_1 = \{x, y, z : x + 2y - 4z = 8\}$ and $S_2 = \{x, y, z : x + y + z = 3\}$.

   $\vec{v} = \begin{bmatrix} \text{x} \\ \text{y} \\ \text{z} \end{bmatrix}$ – lies at the line of intersection of the planes S_1 and S_2. Find $\vec{v}$.

4. Considering the following measurements:

t	-1	0	1	2
b	1	-1	1	-2

   (a) (4 points) Find the best straight-line fit (Least squares) to the measurements,
   (b) (2 point) Find the projection matrix of vector $b = [1, -1, 1, -2]^T$ onto the column space of matrix A:

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}.$$

5. (6 points) Find the dimensions of the four fundamental subspaces associated with A, depending on the parameters α and β.

$$A = \begin{bmatrix} \alpha & 6 & 8 & 12 \\ 2 & 1 & 1 & 2 \\ 1 & \beta & 1 & 0 \\ 9 & 4 & 5 & 8 \end{bmatrix}$$

6. Find an orthonormal basis for the subspace spanned by the vectors: a_1, a_2 and a_3 (4 points).

   Then express $A = [a_1, a_2, a_3]$ in the form of QR (2 points).

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, a_2 = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}, a_3 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}.$$

7. (4 points) Prove that if the matrix $A(n \times n)$ is symmetric the dot product of the vectors $x \in \mathbb{R}^n$ and Ay is the same as the vectors Ax and $y \in \mathbb{R}^n$