Pseudoinverse

- $J^{\#}$ always exists, and is the unique matrix satisfying

$$J\,J^{\#}\,J = J \qquad J^{\#}\,J\,J^{\#} = J^{\#}$$
$$(J\,J^{\#})^T = J\,J^{\#} \qquad (J^{\#}\,J)^T = J^{\#}\,J$$

- if J is full (row) rank, $J^{\#} = J^T (J\,J^T)^{-1}$; else, it is computed numerically using the SVD (Singular Value Decomposition) of J (`pinv` of Matlab)

Computation of pseudoinverses

- show that the pseudoinverse of J is equal to

$$J = U\Sigma V^T \quad \Rightarrow \quad J^{\#} = V\Sigma^{\#}U^T \qquad \Sigma^{\#} = \left(\begin{array}{cccc} \frac{1}{\sigma_1} & & & \\ & \ddots & & \\ & & \frac{1}{\sigma_\rho} & \\ & & & 0_{(M-\rho)\times(M-\rho)} \\ \hline & & 0_{(N-M)\times M} & \end{array}\right)$$

for any rank ρ of J