Lectures (parts I-III)

Problems of description and compensation of kinetic friction in robotic and mechatronic drives that use feedback control

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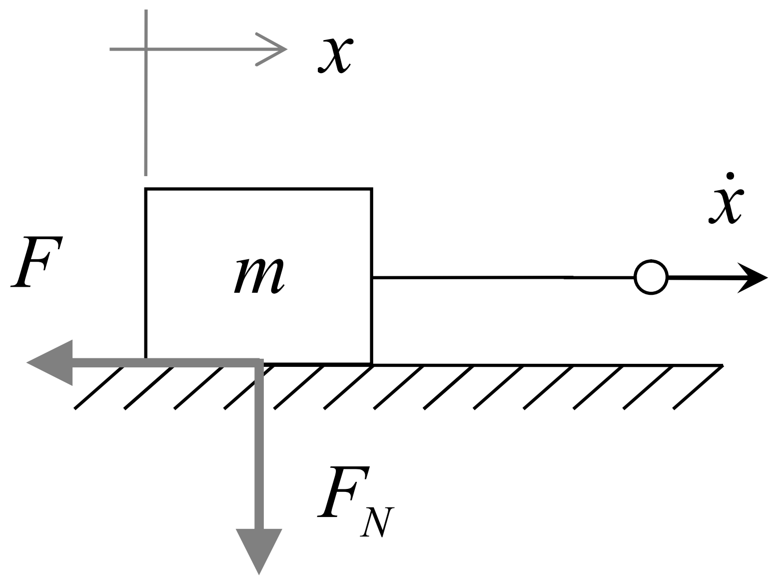
Lectures outline

- Part I – Phenomena of kinetic friction in drives
- Part II – Dynamics with friction and feedback control
- Part III – Compensation of friction in motion control

1-DOF motion on a surface

- Basic setup with generalized coordinates and forces

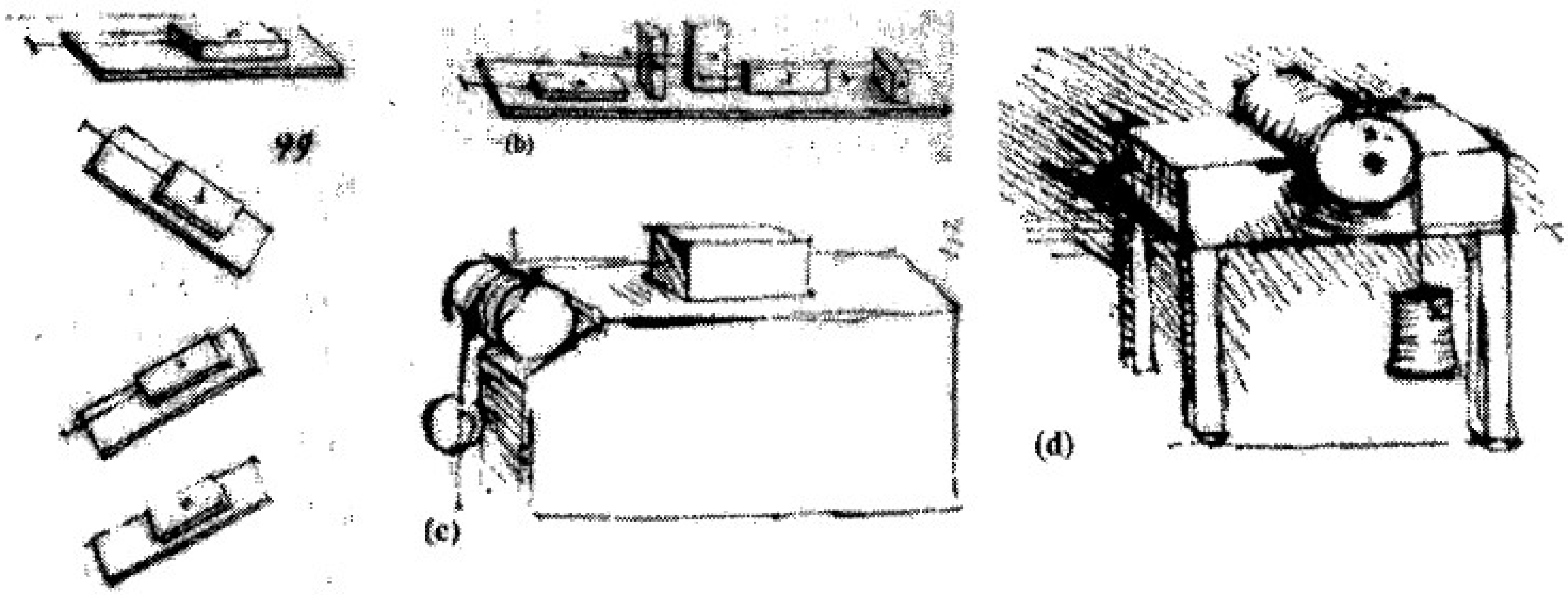


- Normal constraint, expressed by the normal force (F_N), gives rise to the kinetic friction force opposed to the relative motion

$$\mathrm{sign}(F) = -\mathrm{sign}(\dot{x}) \qquad\qquad |F| \sim \left(|\dot{x}|, F_N\right)$$

Friction experiments by Leonardo da Vinci

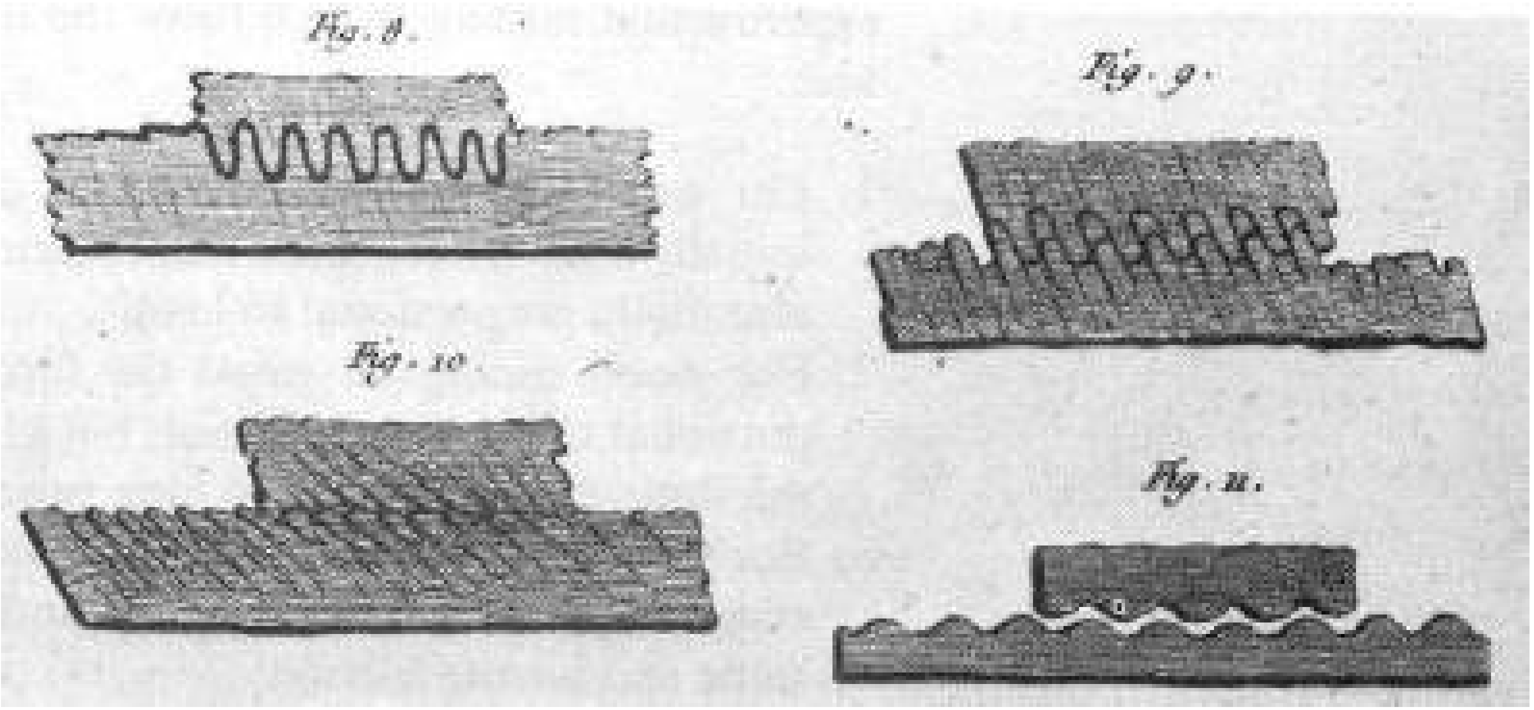
Leonardo da Vinci (1452-1519) observed the angle of inclined surfaces at the onset of the relative motion of a body, and the weight required to move it



Source: E. Meyer, Nanoscience: Friction And Rheology on the Nanometer Scale, World Scientific, 1998

Coulombs' representation of the rough surfaces in contact

C.A. Coulomb: Theorie des machines simples, en ayant egard au frottement de leurs parties, et a la roideur des cordages. Memoire de Mathematique et de physics de l'academie Royal, 1875



Source: F. Al-Bender et al.: Lift-up Hysteresis Butterflies in Friction, Tribology Letters, 2012

Historical remarks on kinetic friction (cont.)

Coulomb's friction law is usually stated as: *The resistance force exerted on a body by a plane surface along which the body is sliding is proportional to the force pressing the body to the surface; the resistance force depends neither on the contact region nor on the sliding velocity. This force is directed oppositely to the velocity vector and lies in the contact plane. The coefficient of proportionality is called the dry friction coefficient.*

$$F = \begin{cases} \mu F_N \dfrac{\dot{x}}{|\dot{x}|}, & \text{if } \dot{x} \neq 0, \\ [-F_s, F_s], \; F_s \geq \mu F_N, & \text{if } \dot{x} = 0. \end{cases}$$

Sometimes used is an (additional) correction term *C*

$$|F| = \mu F_N + C$$

Historical remarks on kinetic friction (cont.)

Coulomb and Euler[6]

V. P. Zhuravlev, "On the history of the dry friction law," *Mechanics of solids*, vol. 48, no. 4, pp. 364–369, 2013.

Experience shows that the friction force is always equal to part of the pressure acting to press the body to the plane along which it is sliding so that the friction depends neither on the area of contact nor on the velocity value; the friction influence in various machines can easily be considered as a constant force opposite to the direction of motion and lying in the plane of contact of interacting bodies.

SUR LA DIMINUTION
DE LA RESISTENCE DU FROTTEMENT,
PAR M. EULER.

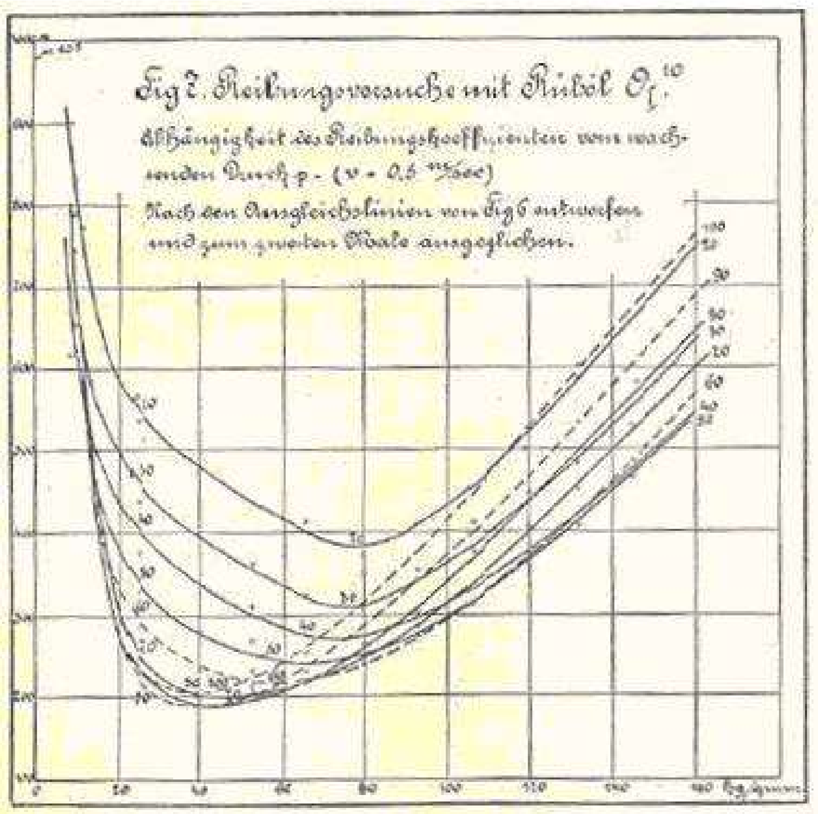
I.

L'Experíence nous ayant fait voir que la force du frottement eſt toujours égale à une certaine partie de la preſſion, dont un corps eſt preſſé contre la ſurface, ſur laquelle il ſe meut, de ſorte que le frottement ne dé-

L. Euler, "Sur la Diminution de la Résistance du Frottement," Histoire de l'Académie Royale des Sciences et Belles-Lettres de Berlin, 4, 133–148 (1748).

Historical remarks on kinetic friction (cont.)

Stribeck characteristic friction curve (Richard Stribeck, 1861-1950)



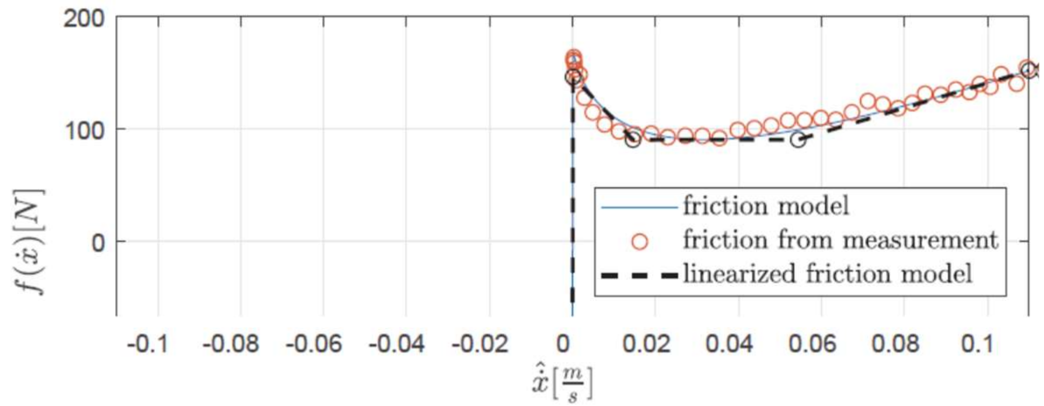
Source: R. Stribeck. Die Wesentlichen Eigenschaften der Gleit- und Rollenlager. VDI-Zeitschrift, Vol. 46, 1902

Dependency on the relative velocity in case on hydrodynamic friction behavior

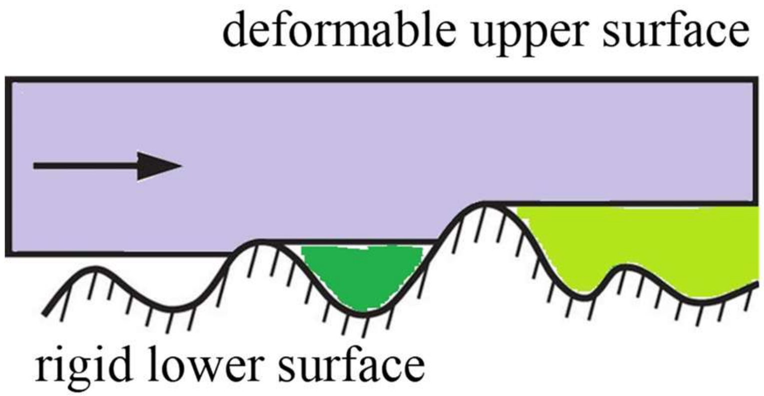
An often-used analytic expression:

$$F \sim F_s \, \mathrm{e}^{-\left(\frac{|\dot{x}|}{S}\right)^{\alpha}} + F_c + \sigma |\dot{x}|$$

Example from practice[7]:



- Surface pair with lubricant penetration & typical surface topology



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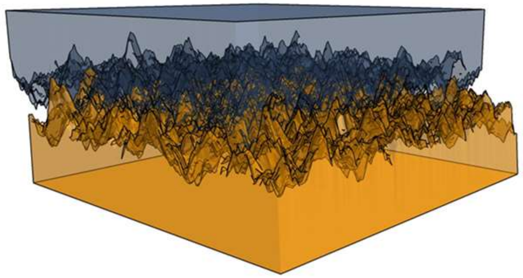
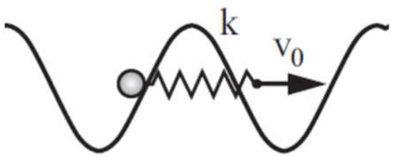
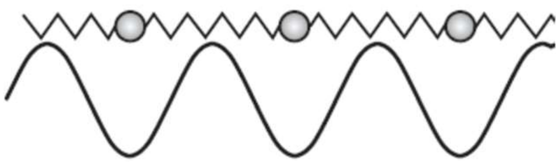


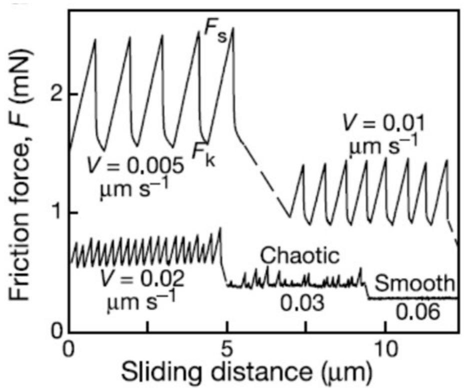
- Complex tribological/physical approaches (we do not consider)



Prandtl model of the point mass dragged in a periodic potential



Kontorova-Frenkel model of an elastic continuum



'Nano-scale' friction[9]

Simplified kinetic friction for modeling & control

- We make following basic assumptions for kinetic friction:

i. Coulomb friction force (at steady-state) depending on the motion direction only

$$F_c = \gamma \operatorname{sign}(\dot{x})$$

ii. Viscous friction force (at steady-state) depending on the motion rate only

$$F_v = \sigma \dot{x}$$

iii. Coulomb and viscous friction coefficients are weakly known (uncertain)

$$0 < \sigma, \gamma \neq \text{const} \qquad \sigma(t) < \sigma_{\max}, \quad \gamma(t) < \gamma_{\max}$$

iv. Total kinetic friction is a superposition of Coulomb and viscous friction forces

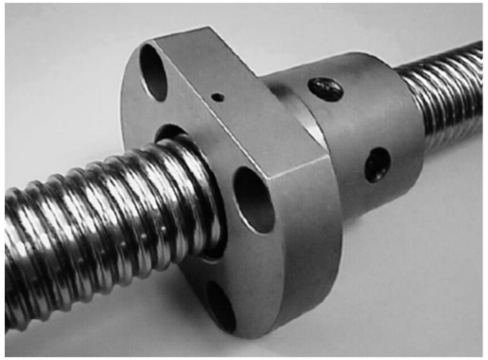
$$F(t) = F_c + F_v$$

v. Frictional transients are essential at motion onset/stop or large accelerations

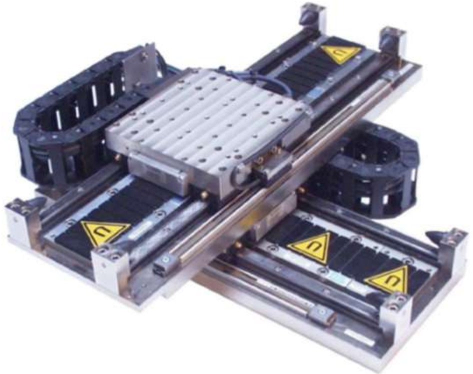
$$\frac{d}{dt} F(t) \neq 0, \quad \text{if} \quad \frac{d}{dt} \operatorname{sign}(\dot{x}) \neq 0 \ \vee \ |\ddot{x}| \gg 0$$

Kinetic friction in robotic mechanisms

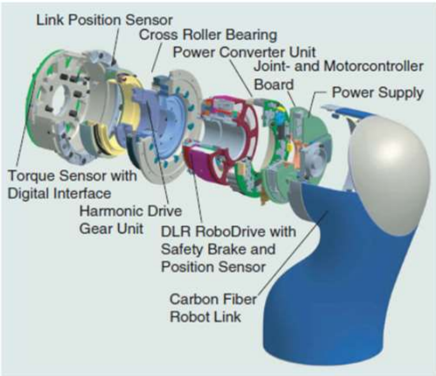
- Moving mechanical "pairs" with contact surfaces



Linear ball screws

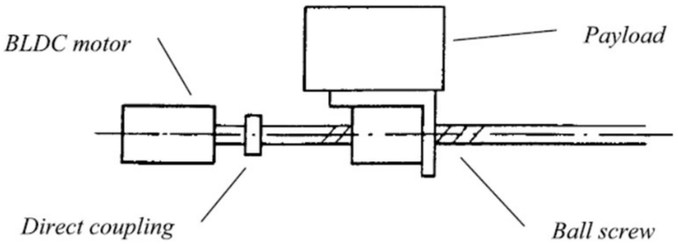


Linear sliding guides

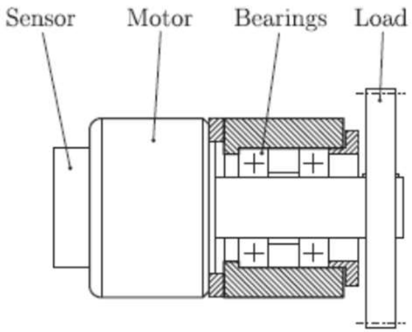


Rotary robot joints[10]

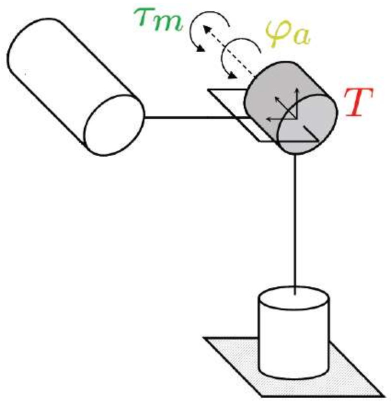
- Origin of the frictional forces / torques in drives



Contact forces between balls-set and screw guide



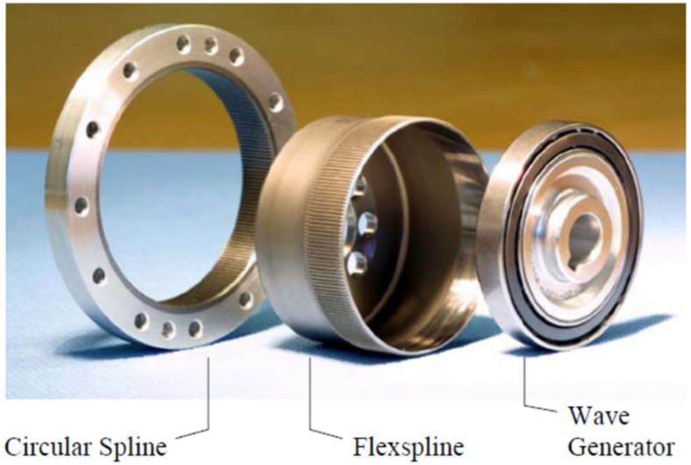
Contact forces between bearings and rotary shafts



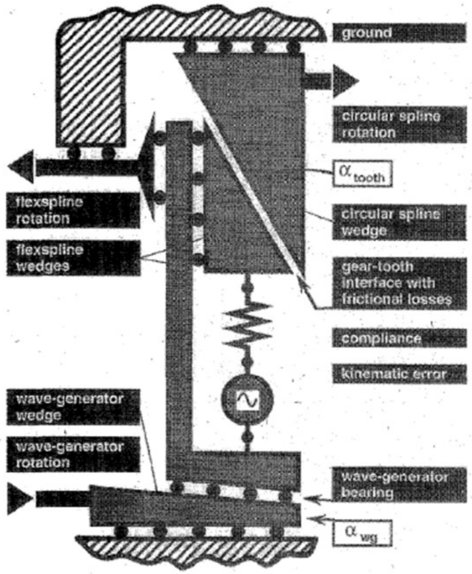
Σ of various contact forces

Kinetic friction in robotic mechanisms (cont.)

- Exemplified structure of the geared robotic joint



Harmonic drive (also as wave-motion) gears



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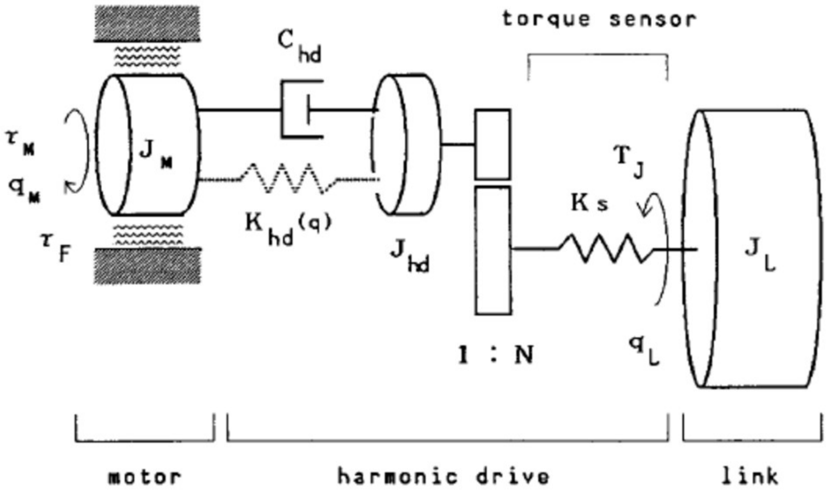
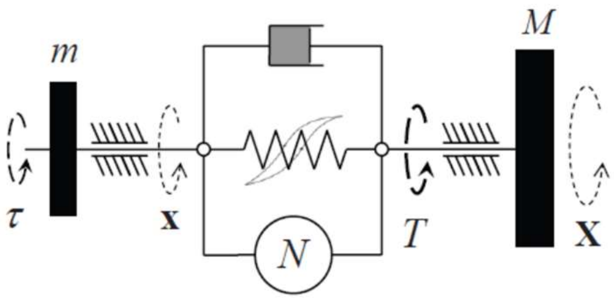


[12]

Generic structure of robotic joint



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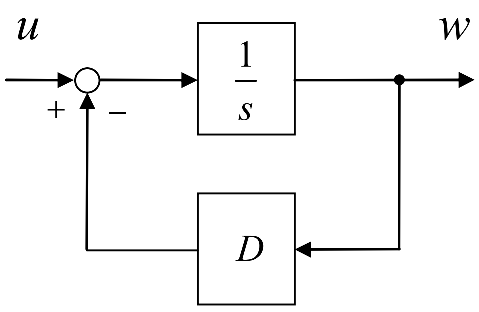
Some "signatures" of friction

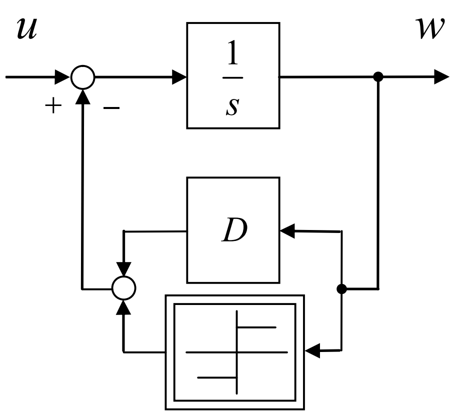
- Linear approach

$$\text{(i)} \qquad \dot{w}(t) + D\,w(t) = u(t)$$

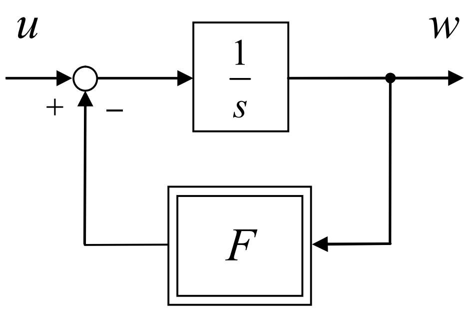


- Nonlinear approaches

$$\text{(ii)} \qquad \dot{w}(t) + D\,w(t) + \operatorname{sign}\big(w(t)\big) = u(t)$$

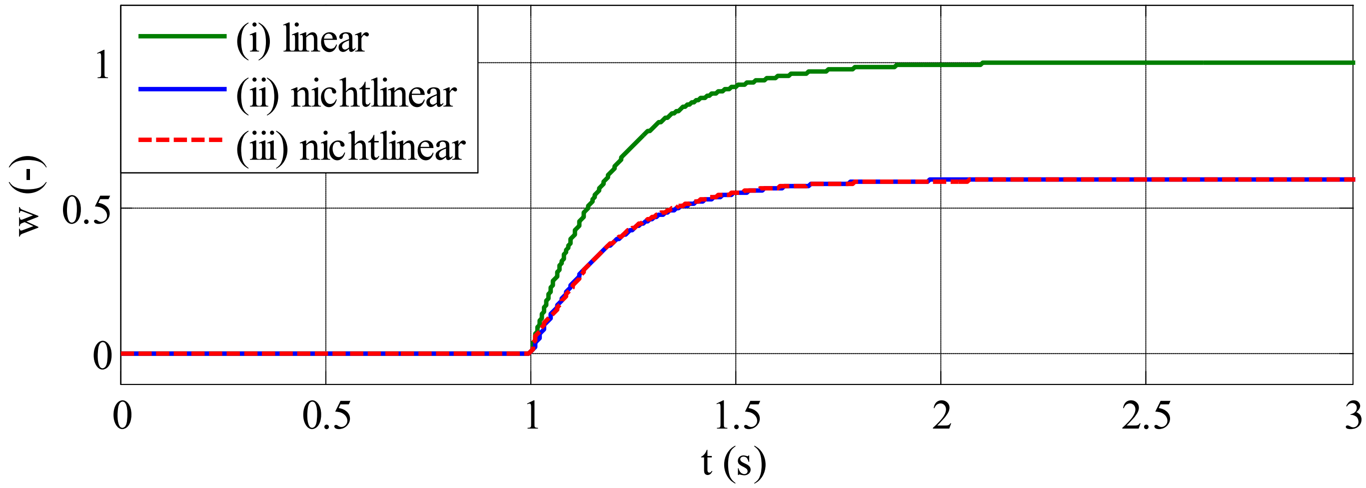


$$\text{(iii)} \qquad \dot{w}(t) + F\big(w(t), t\big) = u(t)$$

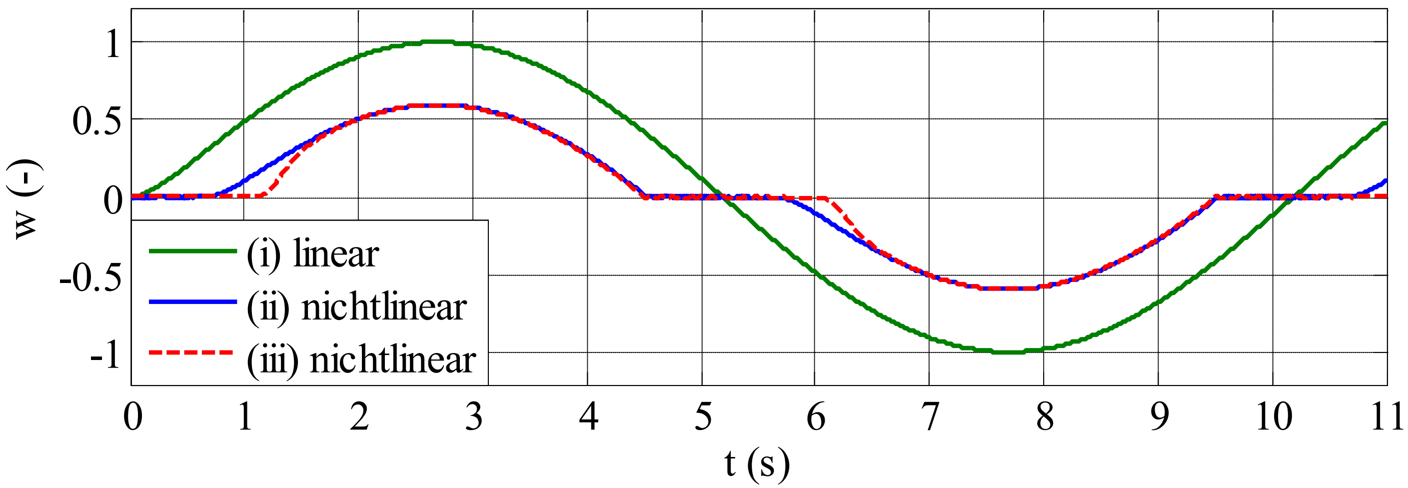


Some “signatures” of friction (cont.)

- Step response – no signature of nonlinear friction

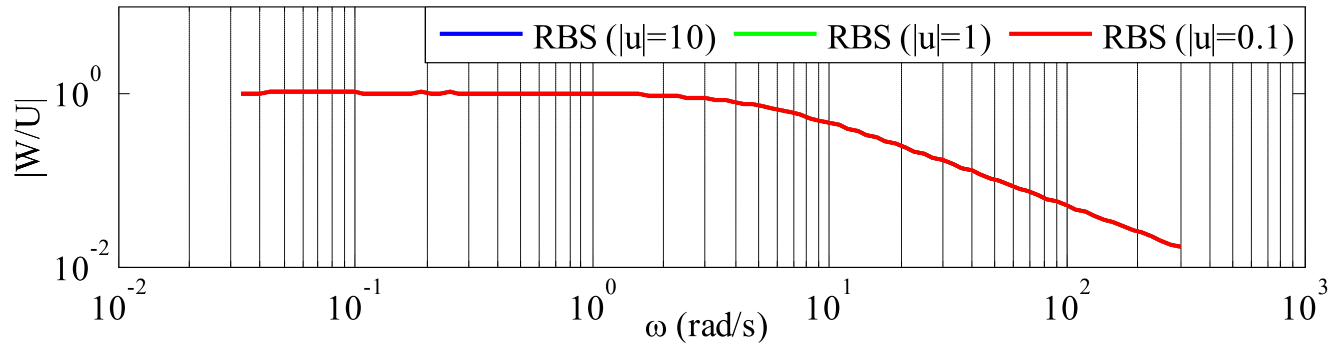


- Single harmonic response – “light” signature of nonlinear friction

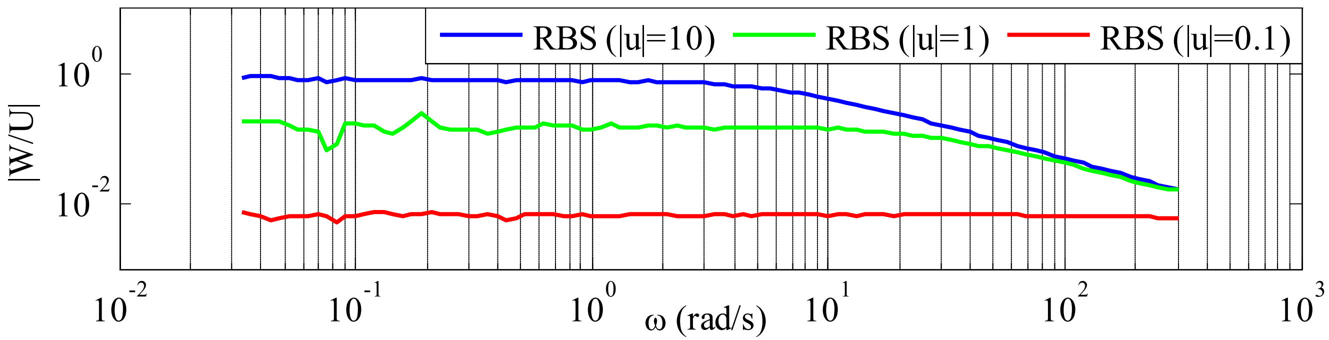


- Frequency response function (numerical simulation)

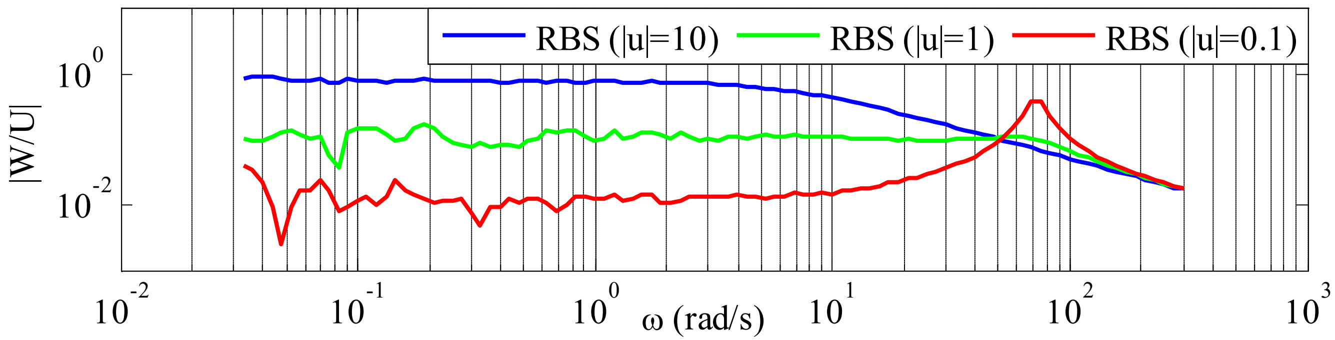
(i) Linear case



(ii) Nonlinear case



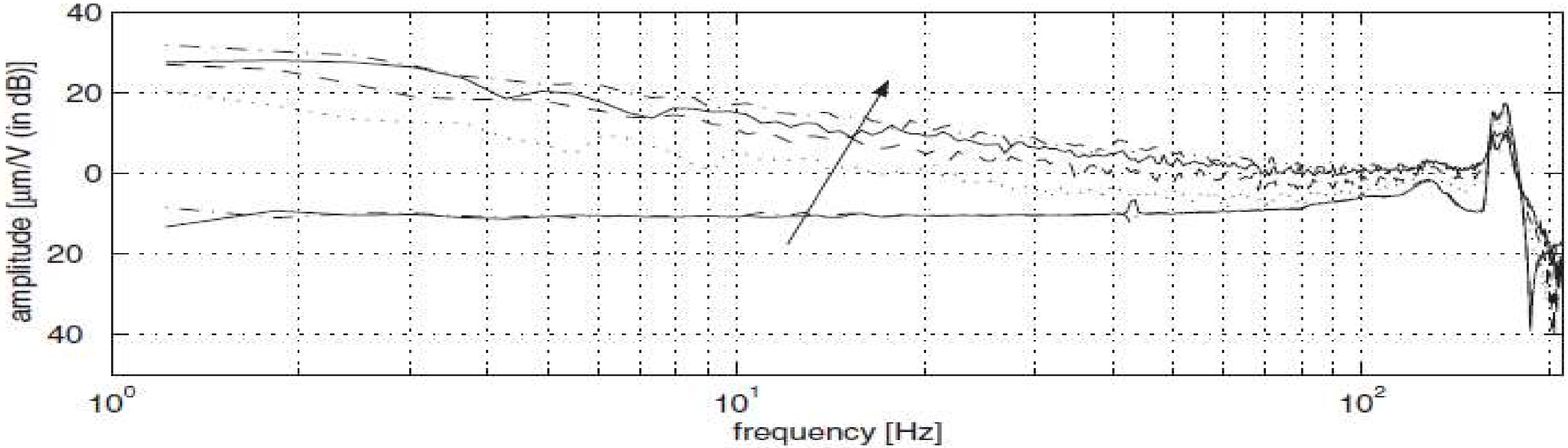
(iii) Nonlinear case



Some “signatures” of friction (cont.)

- Evidence from experiments

[14]



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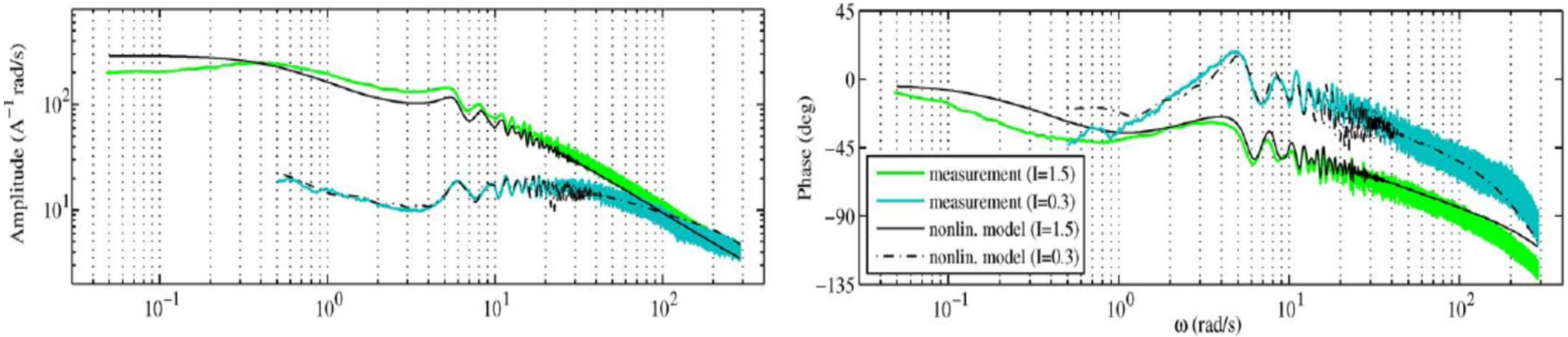
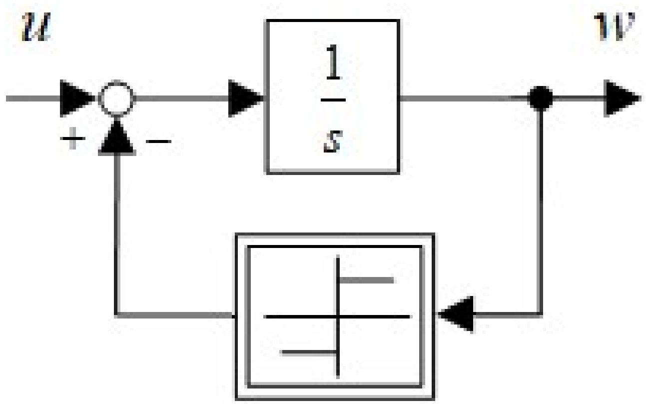
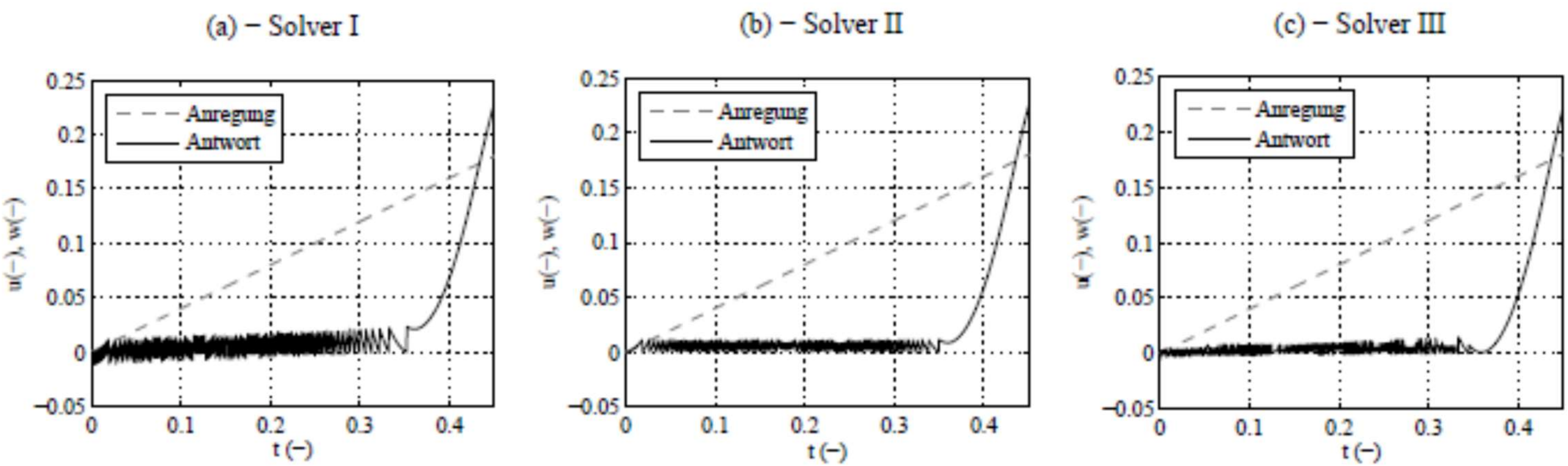


How an actuated motion begins?

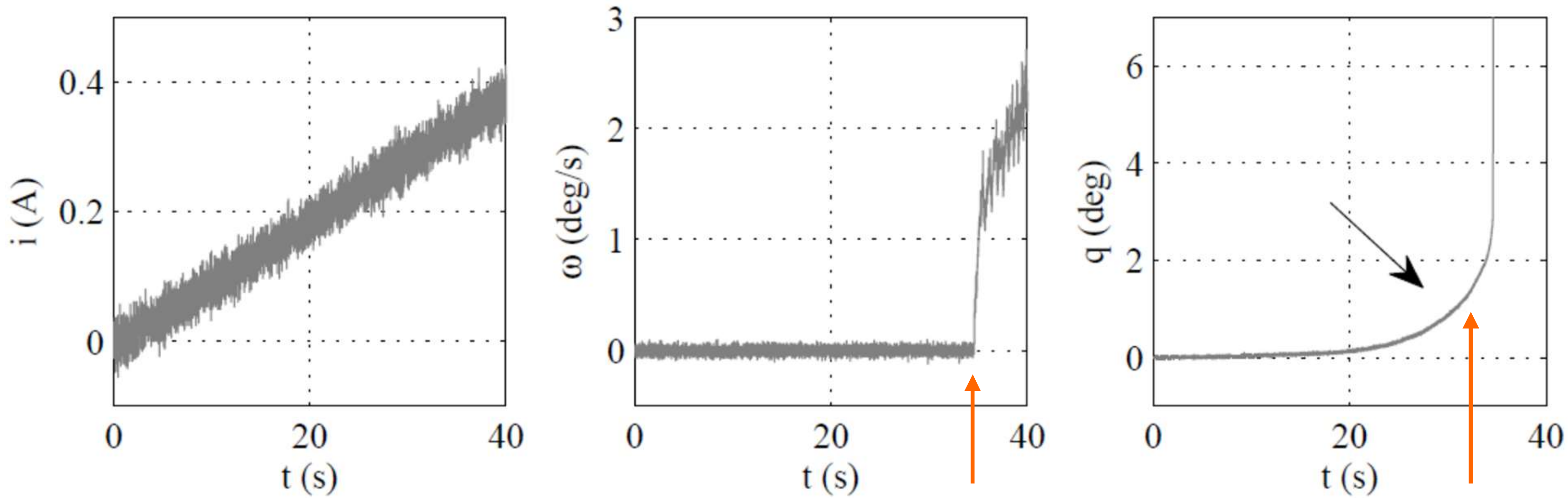
- Simple numerical test



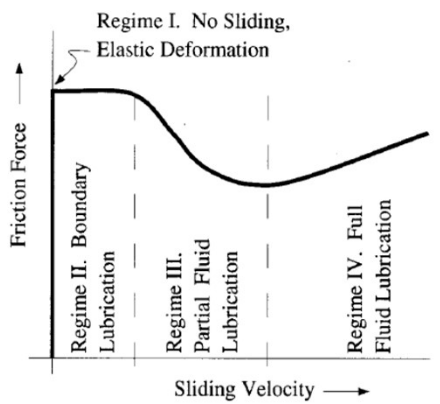


When a relative motion “breaks away”?

- Some motivating experiments

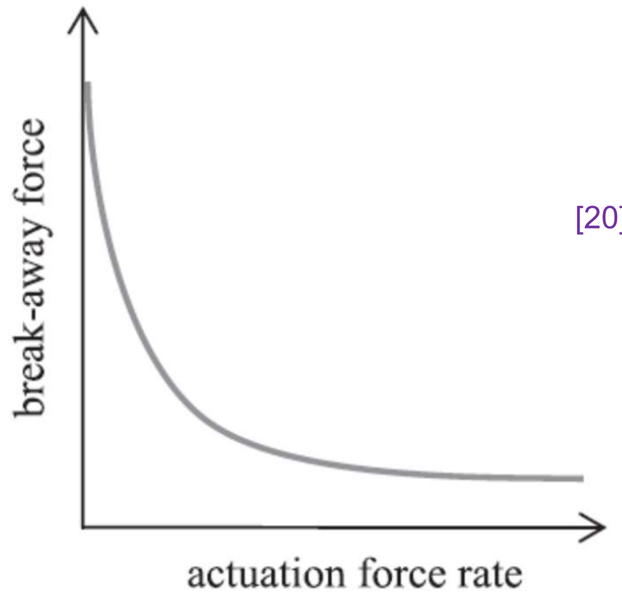


- So-called <u>break-away</u> force, depending on the applied force rate:



[19]

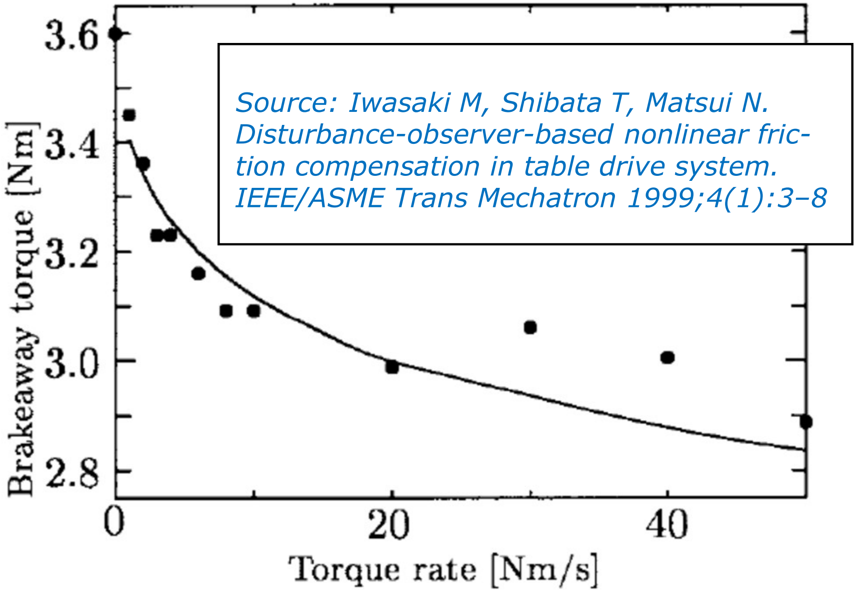
provided that the Stribeck effect applies



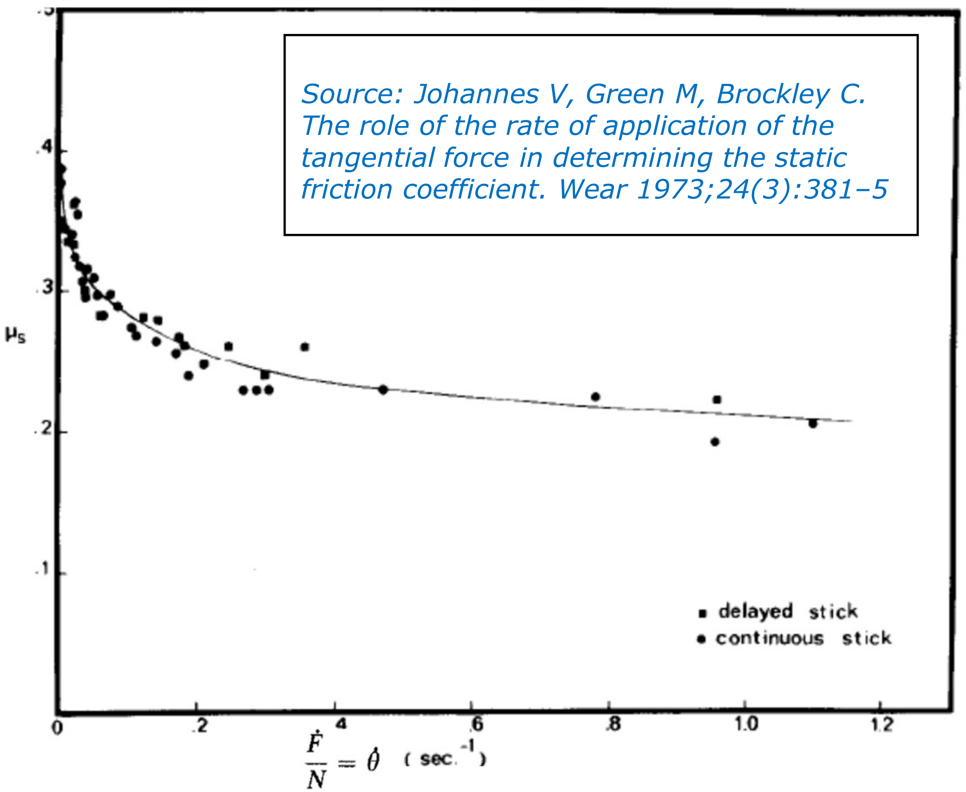
[20]

When a relative motion "breaks away"? (cont.)

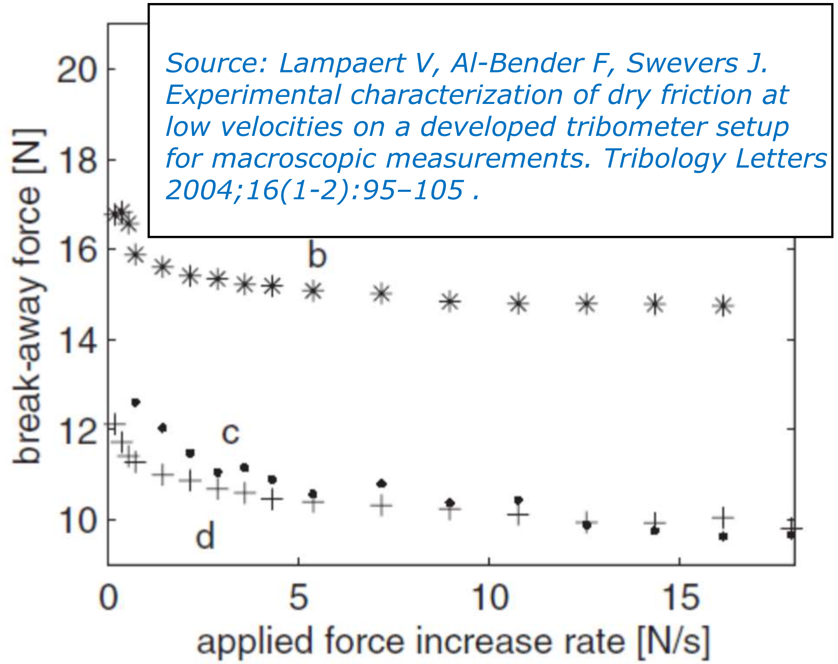
Break-away forces, as maximal actuation forces at which a sticking system begins to slide and thus passes over to a steady (macro) motion, are well known from the engineering practice but still not fully understood in their cause-effect relationship



Source: Iwasaki M, Shibata T, Matsui N. Disturbance-observer-based nonlinear friction compensation in table drive system. IEEE/ASME Trans Mechatron 1999;4(1):3–8

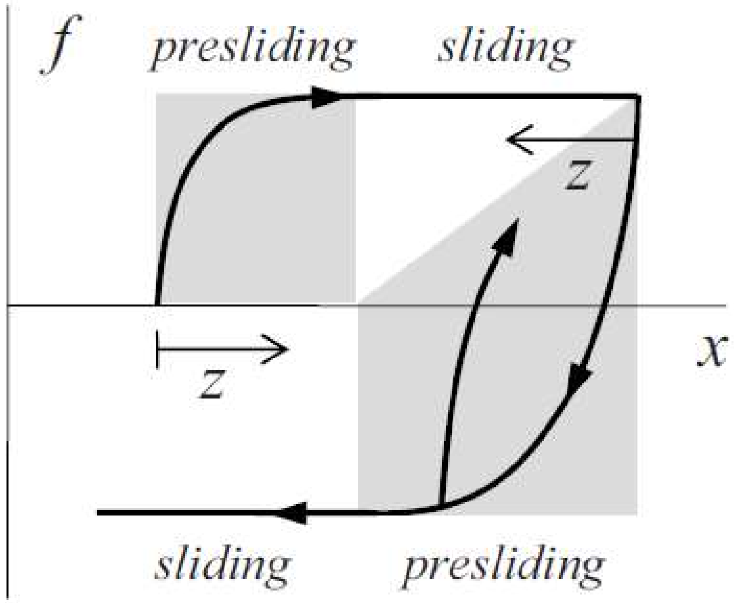


Source: Johannes V, Green M, Brockley C. The role of the rate of application of the tangential force in determining the static friction coefficient. Wear 1973;24(3):381–5

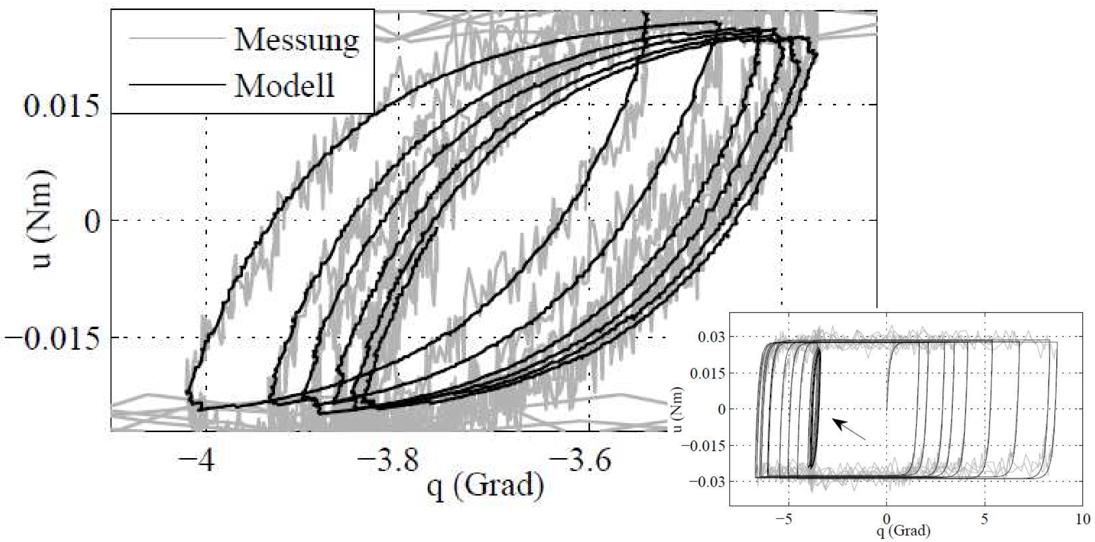


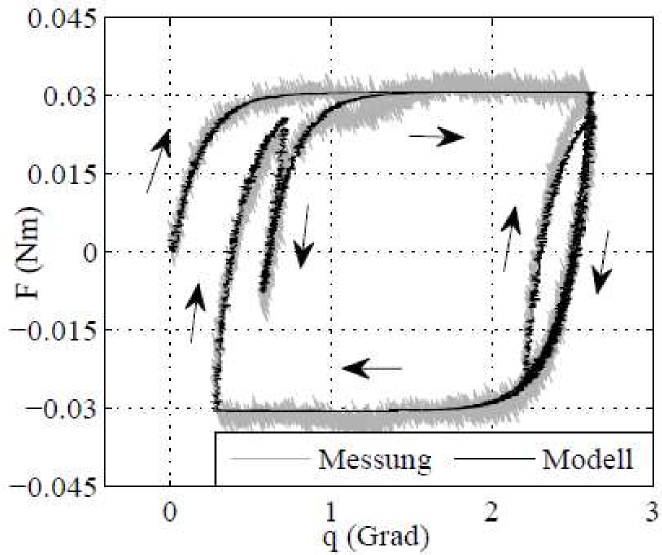
Source: Lampaert V, Al-Bender F, Swevers J. Experimental characterization of dry friction at low velocities on a developed tribometer setup for macroscopic measurements. Tribology Letters 2004;16(1-2):95–105 .

Presliding mode of kinetic friction



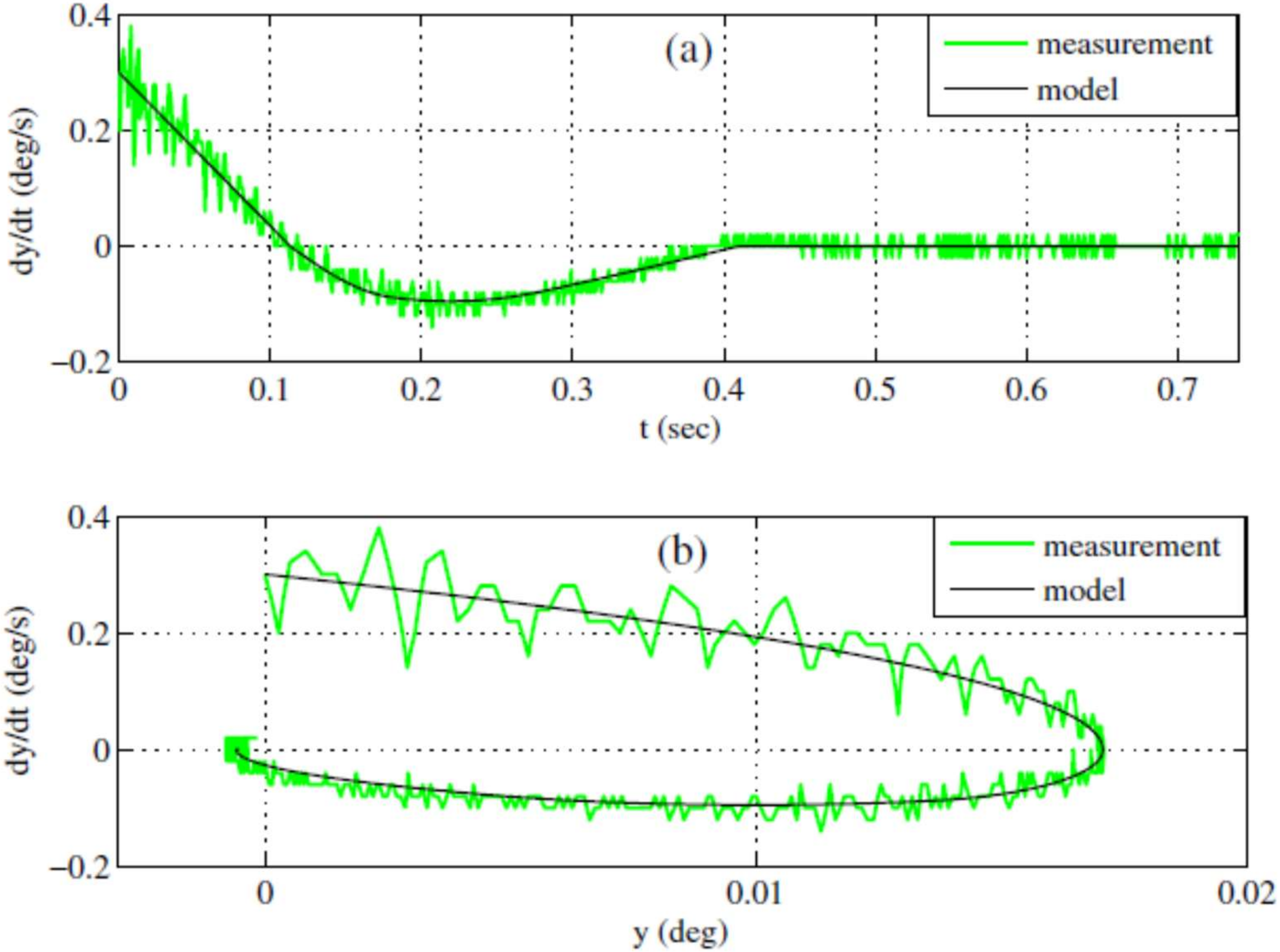
- In pre-sliding regime, the tangential friction force depends rather on the relative displacement than the relative velocity
- Each motion reversal induces new hysteresis branch in the force-displacement coordinates
- Relative distance *(z)* is characteristic for the pre-sliding friction, i.e. after reversals





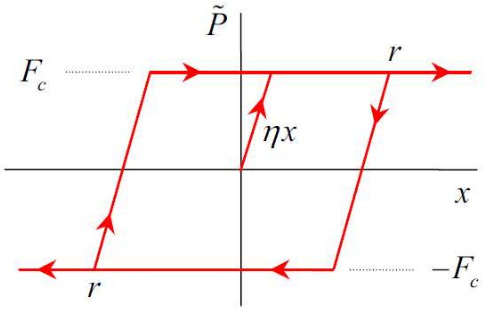
But how then the motion stops?

- Some motivating experimental observations[21]



But how then the motion stops? (cont.)

- When using, for example, stop-type operator $m\ddot{x} + \tilde{P}[x] = 0$

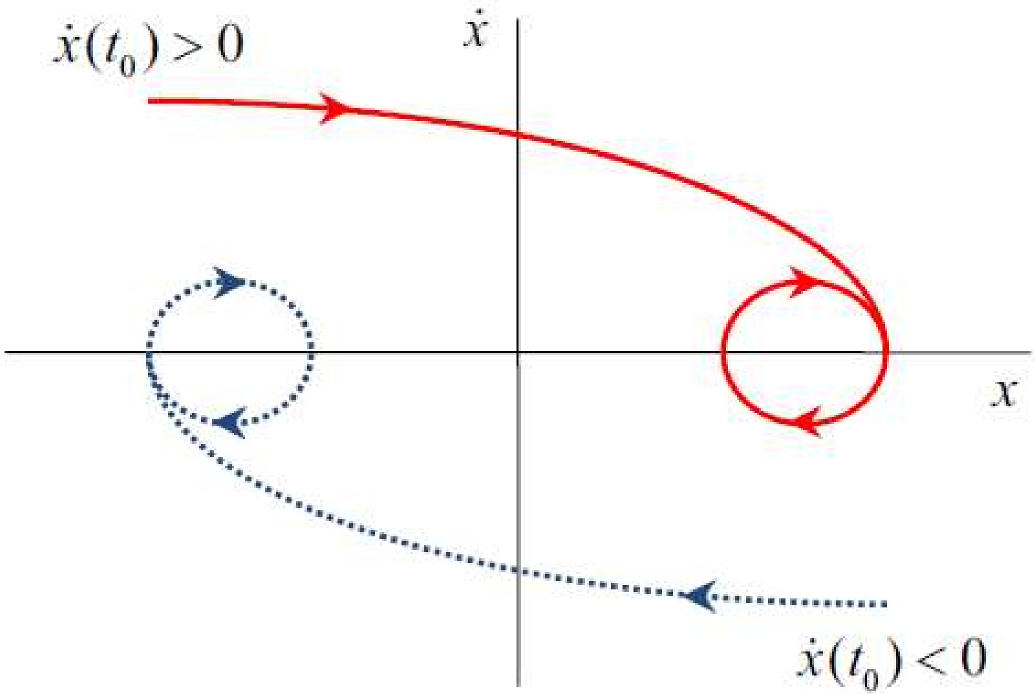


(i) Sliding until next motion reversal:

$$\boxed{m\ddot{x} = -F_c \operatorname{sign}(\dot{x})}$$

$$\frac{d\dot{x}}{dx} = -\frac{\operatorname{sign}(\dot{x})F_c}{m\dot{x}}$$

$$x = -\frac{\dot{x}^2 m}{2\operatorname{sign}(\dot{x})F_c} + \Lambda x(t_r)$$



(ii) Limit cycles after reversal:

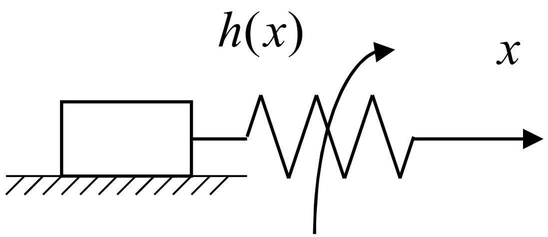
$$\boxed{m\ddot{\underline{x}} = -\eta \underline{x}}$$

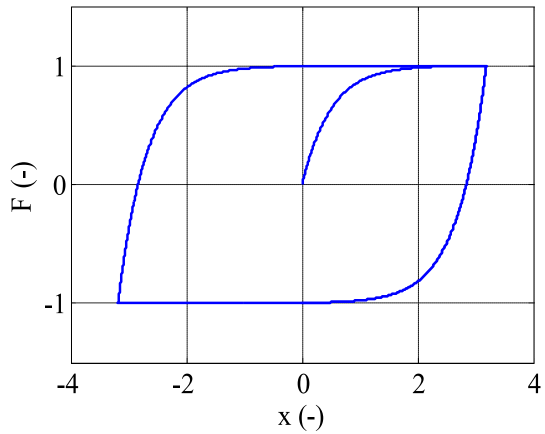
$$\underline{x} = x - x(t_r) + \operatorname{sign}(\ddot{x}(t_r))\,\frac{F_c}{\eta}$$

But how then the motion stops? (cont.)

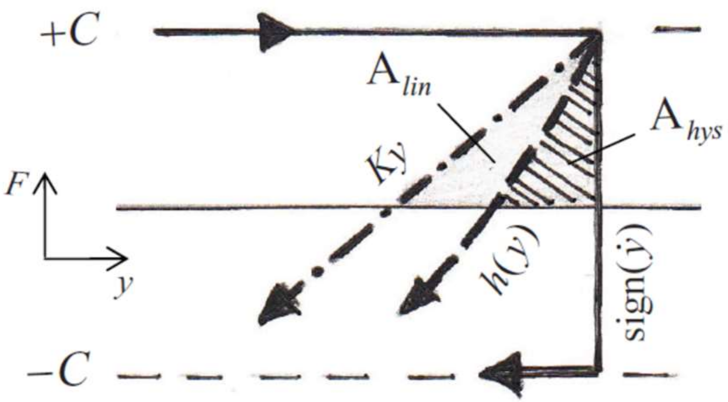
- Using other pre-sliding friction models:

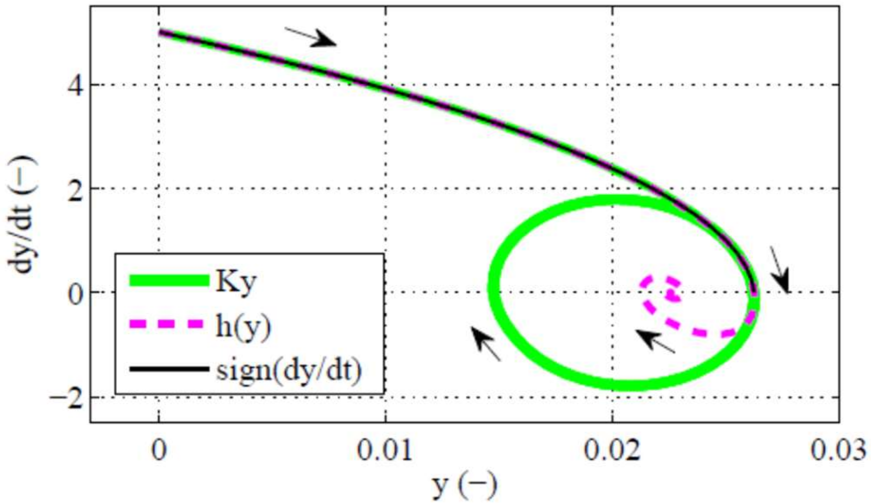
For example, a regressive spring after each motion reversal [22]





- Differing approaches for transients at motion reversals:
  - Zero damping rate (boundary case – limit cycle)
  - Varying, hysteresis-dependent damping rate
  - Const. damping rate (boundary case – Coulomb)





Lectures outline

- Part I – Phenomena of kinetic friction in drives
- Part II – Dynamics with friction and feedback control
- Part III – Compensation of friction in motion control

Lagrange dynamics without dissipation

- Lagrangian as difference between kinetic and potential energy

$$L(q,\dot{q},t) = T(q,\dot{q},t) - V(q,t)$$

- Lagrange's equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = U$$

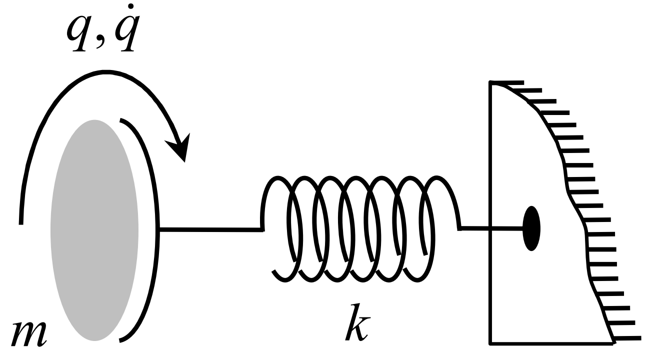
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = U + \frac{\partial L}{\partial q} \quad \Leftrightarrow \quad \frac{d}{dt}\left(\text{momentum}\right) = \sum(\text{forces})$$

- Autonomous system, i.e. $U=0$, (here for the sake of simplicity)

$\rightarrow$ Conservative forces only $\rightarrow$ energy lossless system $\quad \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$

Lagrange dynamics without dissipation (cont.)

- <u>Illustrative example</u>: harmonic oscillator (disk on rotary spring)



Kinetic and potential energy $$T = \frac{1}{2} m \dot{q}^2, \quad V = -\int_0^q (-kq)dq = \frac{1}{2} kq^2$$

result in the Lagrangian and, afterwards, in the Lagrange's equation as

$$L = T - V = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} kq^2 \qquad \Rightarrow \qquad m\ddot{q} + kq = 0$$

$\rightarrow$ well-known second-order (Newtonian) dynamics of a mass-spring system

Lagrange dynamics without dissipation (cont.)

- Illustrative example (cont.)

Why it is **energy lossless**?

$$E = T + V = \frac{1}{2} m \dot{q}^2 + \frac{1}{2} k q^2$$

$$\dot{E} = m \dot{q} \ddot{q} + k q \dot{q} = \dot{q} \underbrace{(m \ddot{q} + k q)}_{\equiv 0 (\text{always!})} = 0$$

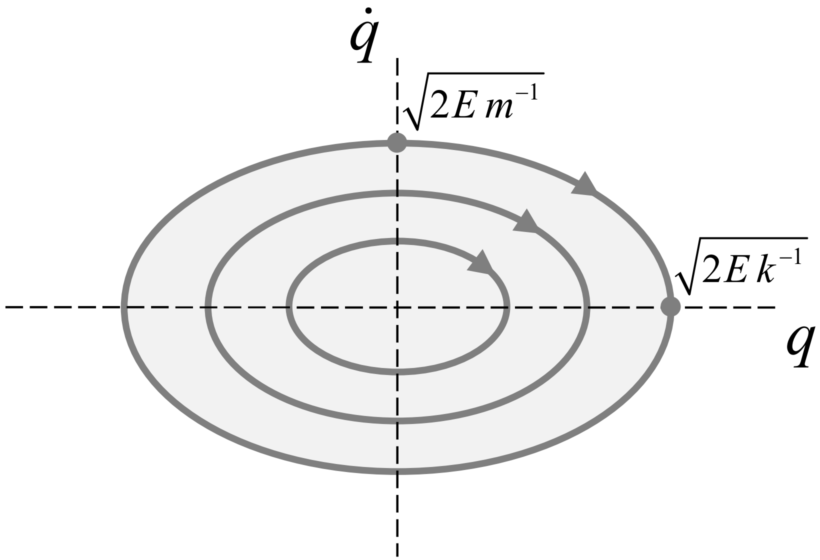
i.e. independent of the velocity

Let's have a look on state trajectories

from total energy equation's viewpoint:

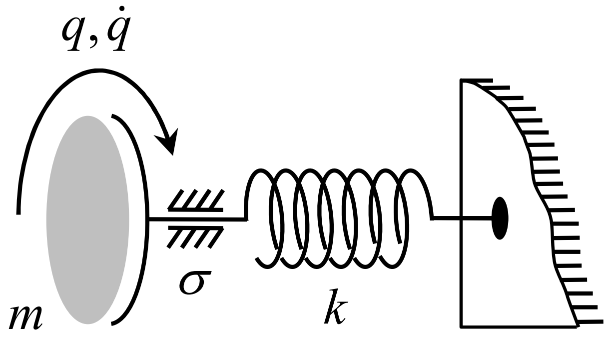
$$\frac{\dot{q}^2}{2E/m} + \frac{q^2}{2E/k} = 1$$

$\rightarrow$ ellipse with axes length depending on *E*

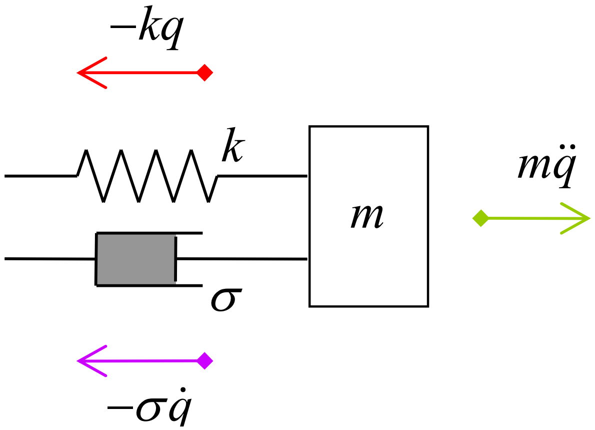


Lagrange dynamics with dissipation (cont.)

- What is to do when the motion is damped?



Newton-Euler approach of (free-body) dynamics



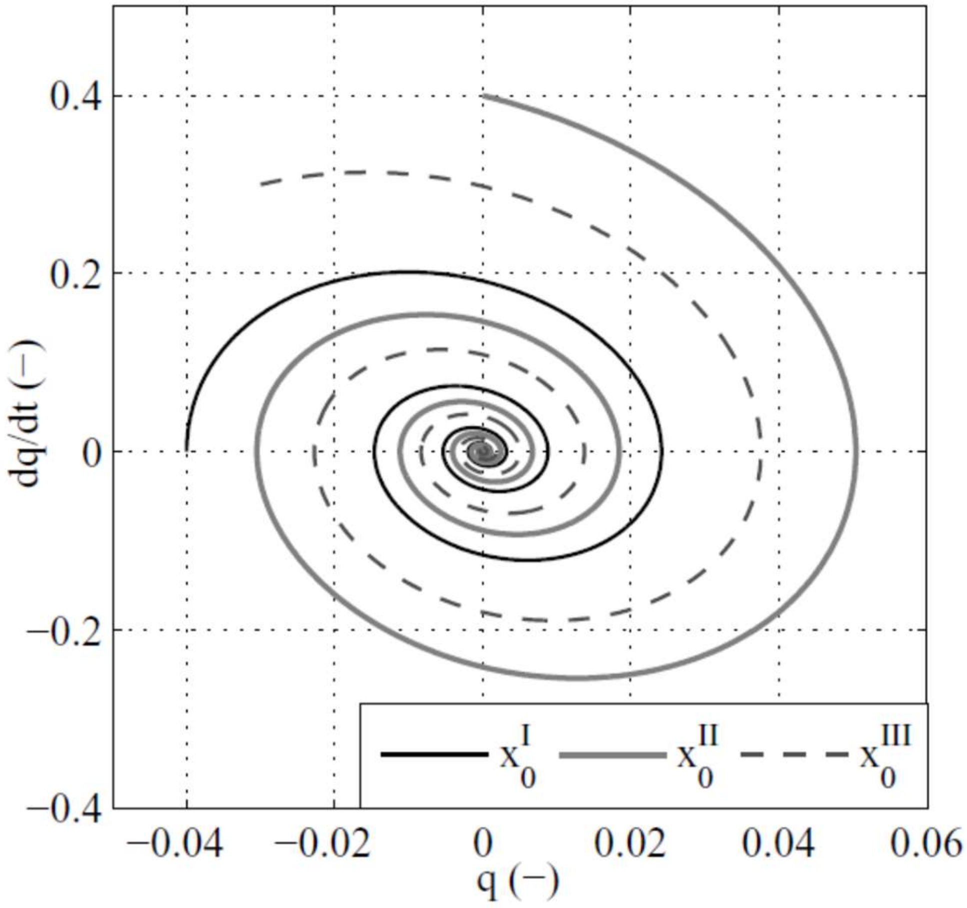
$$\sum \text{forces} = 0 \;\Rightarrow\; m\ddot{q} + \sigma\dot{q} + kq = 0$$

Energy consideration yields: $\dot{E} = \dot{q}\left(m\ddot{q} + kq\right) = -\sigma\dot{q}^2 < 0 \quad \forall \dot{q} \neq 0$

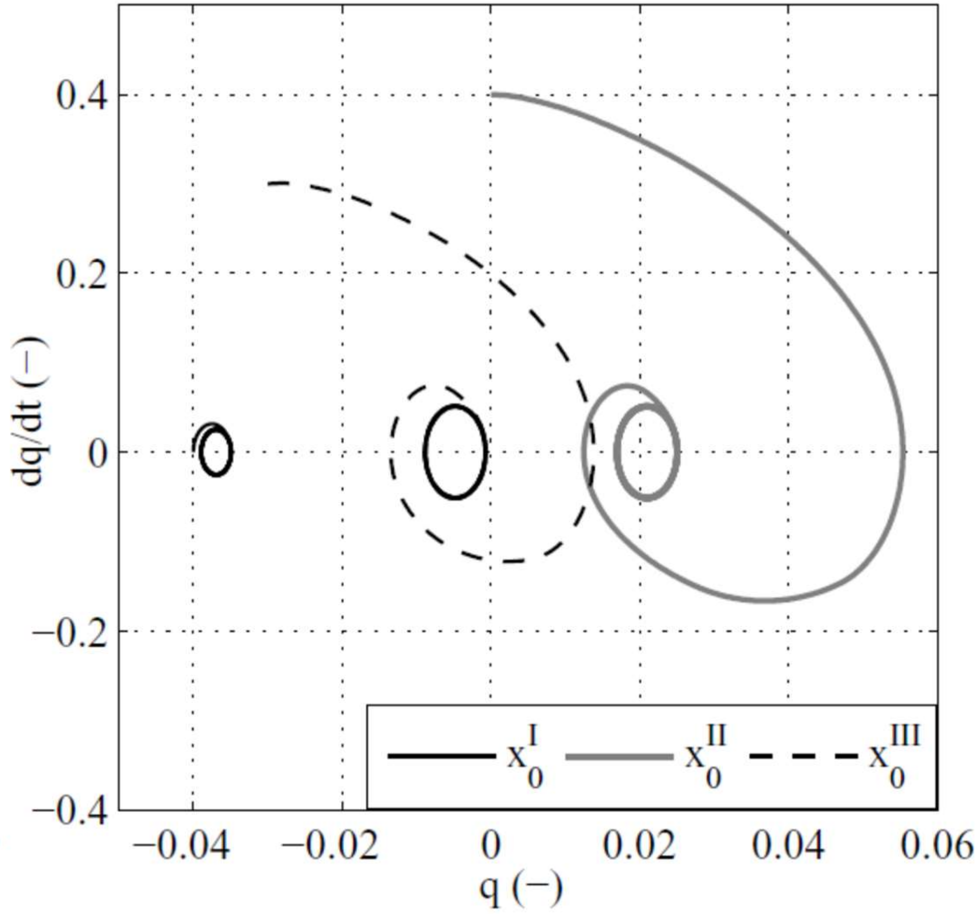
Lagrange dynamics with dissipation (cont.)

- State trajectories of the damped oscillator

System with linear damping



System with nonlinear damping (also memory affected)



Lagrange dynamics with dissipation (cont.)

- How to include damping into Lagrangian dynamics?

Introducing Rayleigh dissipation function (cf. e.g. [16])

$$\mathbf{\Phi} = \frac{1}{n+1} \sum_{i=1}^{N} c_i \dot{q}_i^{\,n+1}$$ for a system of N particles (13)

$$\Phi = \frac{1}{n+1} c \dot{q}^{\,n+1}$$ for a single point-mass (lumped parameter), *n=0* is Coulomb friction, *n=1* is viscous friction (14)

added into nonconservative kinetic potential results in the modified Lagrange's equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \frac{\partial \Phi}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \qquad (15)$$

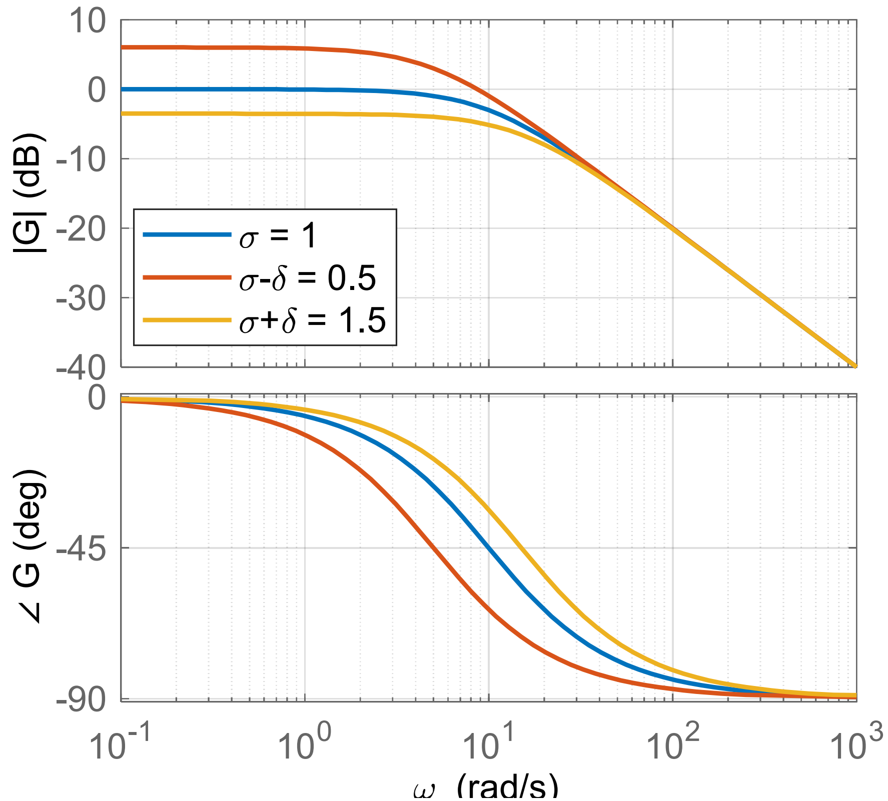
- For previous example of a damped harmonic oscillator:

$$m\ddot{q} + \underbrace{c\,\dot{q}}_{\text{viscous friction}} + kq = 0 \qquad m\ddot{q} + \underbrace{c\,\mathrm{sign}(\dot{q})}_{\text{Coulomb friction}} + kq = 0$$

- Uncertain viscous friction

$$m\ddot{q} + \underbrace{(\sigma \pm \delta)}_{>0}\dot{q} = u$$

$$u \longrightarrow \boxed{\frac{1}{ms+(\sigma\pm\delta)}} \longrightarrow \dot{q}$$



leads to uncertain system damping and, as implication, uncertain steady-state gain

$$G(s) = \frac{1}{ms+(\sigma\pm\delta)} = \frac{(\sigma\pm\delta)^{-1}}{m(\sigma\pm\delta)^{-1}s+1}$$

Effects of frictional damping/disturbance (cont.)

- Uncertain viscous friction (cont.)

Assume a standard PI velocity control (with K_P and K_i gains)

$$m\ddot{q} + (K_p + \sigma \pm \delta)\dot{q} + K_i \int \dot{q}\,dt = 0$$

The characteristic polynomial is

$$ms^2 + (K_p + \sigma \pm \delta)s + K_i = 0$$

The roots (correspondingly the poles of the closed-loop system) are

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{with} \quad a = m,\, b = \left(K_p + \sigma \pm \delta\right),\, c = K_i$$

For physical mass and damping and K_P, K_i > 0 the control is always stable since

$$-\frac{b}{2a} = -\frac{K_p + \sigma \pm \delta}{2m} < 0 \quad \text{and} \quad \mathrm{Re}\left\{-b + \sqrt{b^2 - 4ac}\right\} < 0$$

but (!) the transient control response may suffer under $\pm\delta$

Effects of frictional damping/disturbance (cont.)

- Uncertain viscous friction (cont.)

For a critically damped (i.e. double real pole) control parameterization

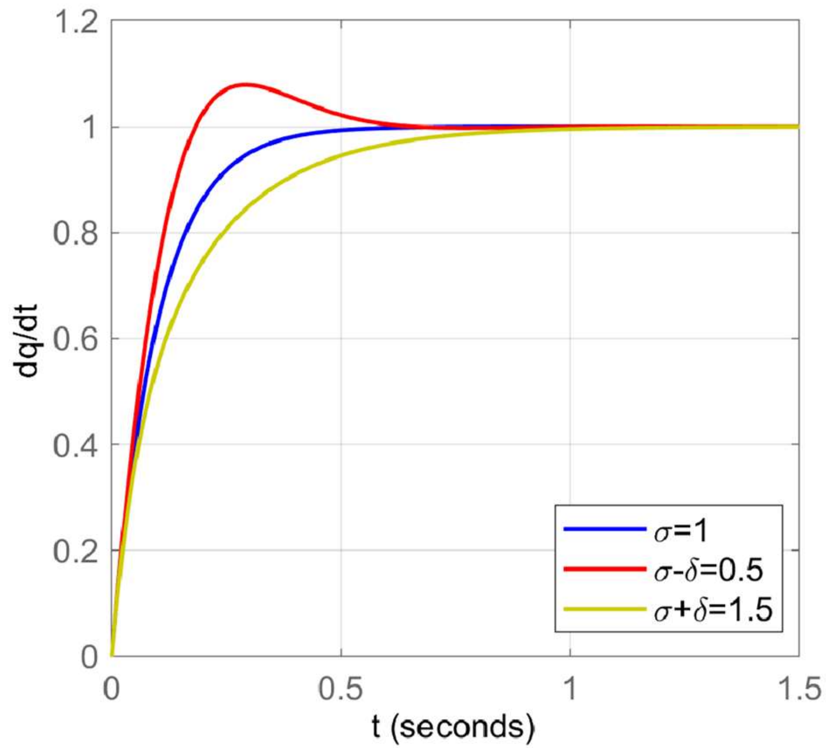
$$b^2 = 4ac \quad \Rightarrow \quad K_p + \sigma = \sqrt{4mK_i}$$

At the same time, K_p determines the real poles (or real part of conjugate complex poles)

$$\mathrm{Re}\left\{\lambda_{1,2}\right\} = -\frac{K_p + \sigma \pm \delta}{2m}$$

$if \;\; \sigma - \delta \Rightarrow$ the control loop becomes underdamped, that leads to transient oscillations

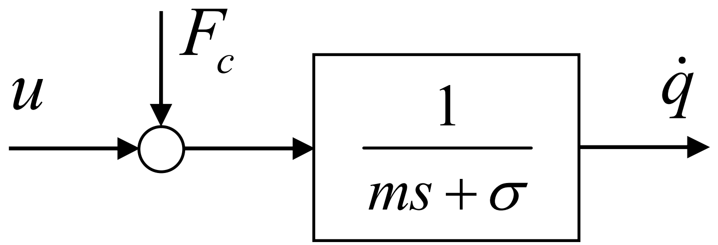
$if \;\; \sigma + \delta \Rightarrow$ the control loop becomes overdamped, that slows down transient response



Effects of frictional damping/disturbance (cont.)

- Uncertain Coulomb friction

$$m\ddot{q} + \sigma\dot{q} = u - F_c$$

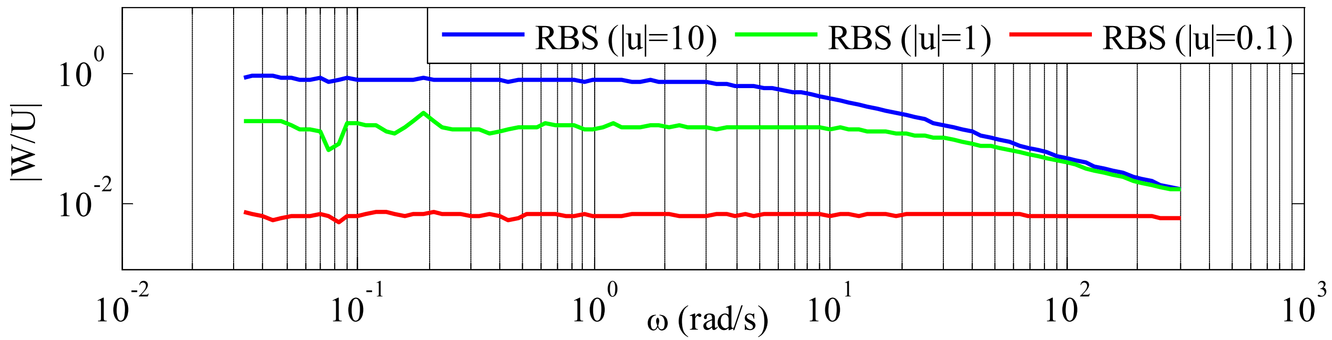


For unidirectional motion (i.e. sign*(dq/dt)*=const) → constant matched (input) disturbance

$$if \quad |u| > |F_c| \quad \Rightarrow$$ reduced driving force / reduced control action

$$if \quad |u| < |F_c| \quad \Rightarrow$$ system stiction / discontinuous solution (in Filippov sense)

For periodic motion (i.e. harmonic excitation) → reduced gain over entire bandwidth



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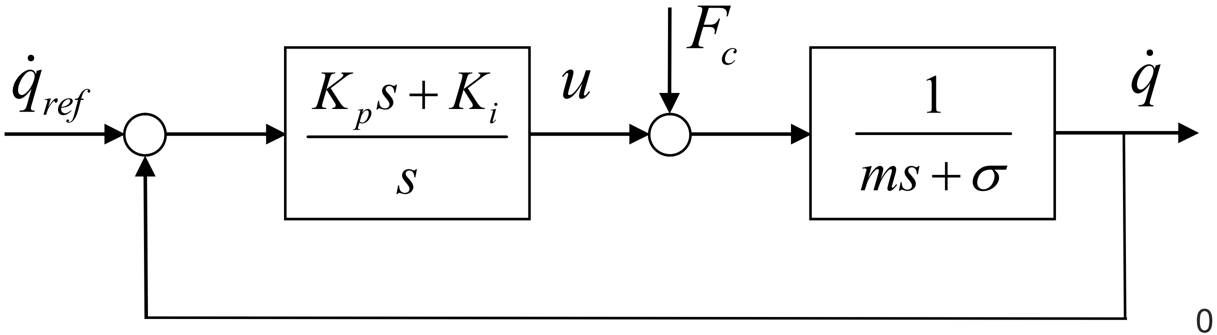
Effects of frictional damping/disturbance (cont.)

- ## Uncertain Coulomb friction (cont.)

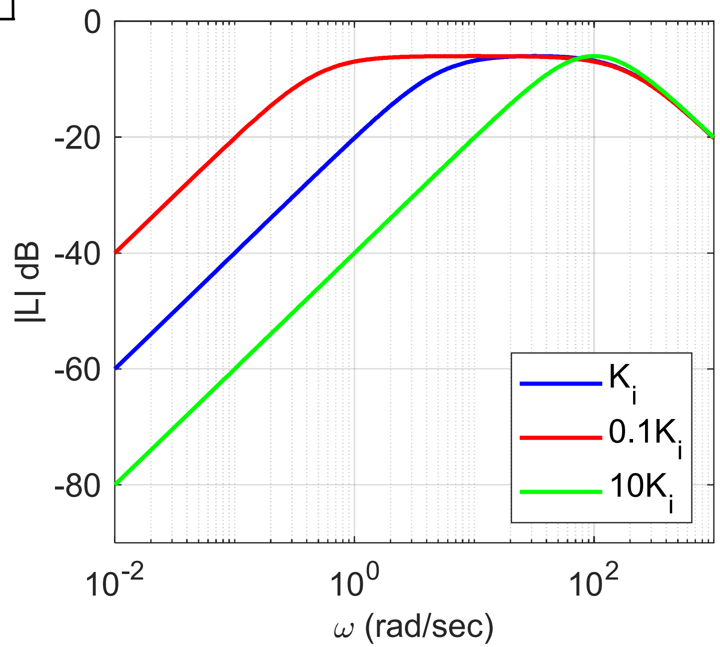
Assume a standard PI velocity control (with K_P and K_i gains)



Disturbance sensitivity function

$$L(s) = \frac{s \cdot q}{F_c} = \frac{G(s)}{1 + PI(s) \cdot G(s)}$$

$$= \frac{s}{ms^2 + (K_p + \sigma)s + K_i}$$



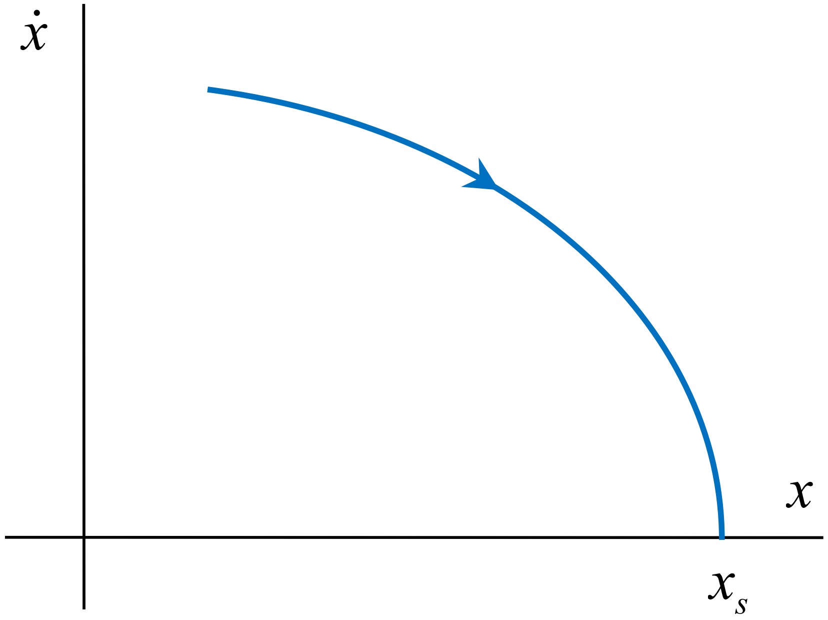
Coulomb friction is fully compensated (by integral part) at the steady-state motion

Effects of frictional damping/disturbance (cont.)

- Feedback controlled positioning with Coulomb friction

First, consider a free motion with non-zero initial conditions

$$m\ddot{x} + F_c \operatorname{sign}(\dot{x}) = 0, \quad \dot{x}_0 = \dot{x}(t_0) \neq 0$$



State trajectories in the phase plane

$$x = -\frac{0.5m\dot{x}^2}{F_c \operatorname{sign}(\dot{x})} + x_s$$

$\ddot{x} = \text{const} < 0 \;\Rightarrow$ constant damping rate

$\ddot{x}\big|_{\dot{x}=0} = \pm F_c \;\Rightarrow$ once reaching zero velocity, the system stays for always there

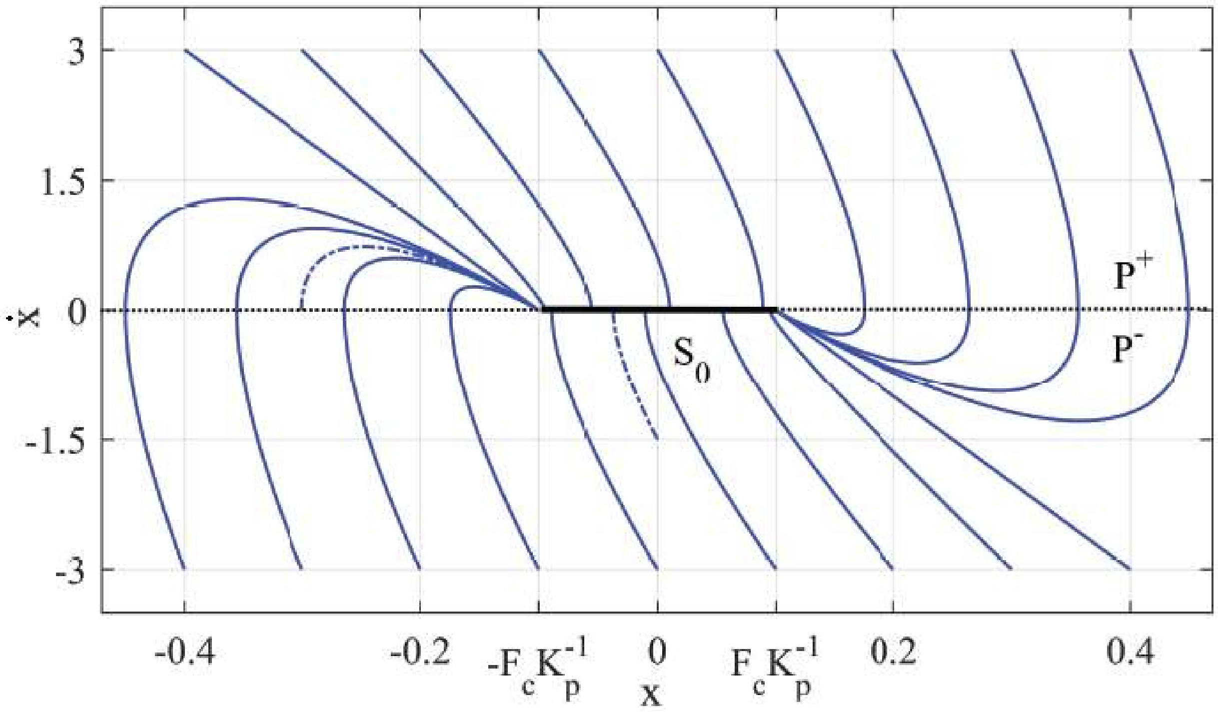
Infinite set of equilibria by invariance principle[17], depending on initial conditions

$$E = \left\{ (x = x_s, \dot{x} = 0) \mid x_s \in \left[-\infty, \infty \right] \right\}$$

- Feedback controlled positioning with Coulomb friction (cont.)

Assume a standard PD position control (with K_P and K_D gains)

$$m\ddot{x} + F_c \operatorname{sign}(\dot{x}) + K_D \dot{x} + K_P x = 0, \quad \dot{x}_0, x_0 \neq 0$$



[18]

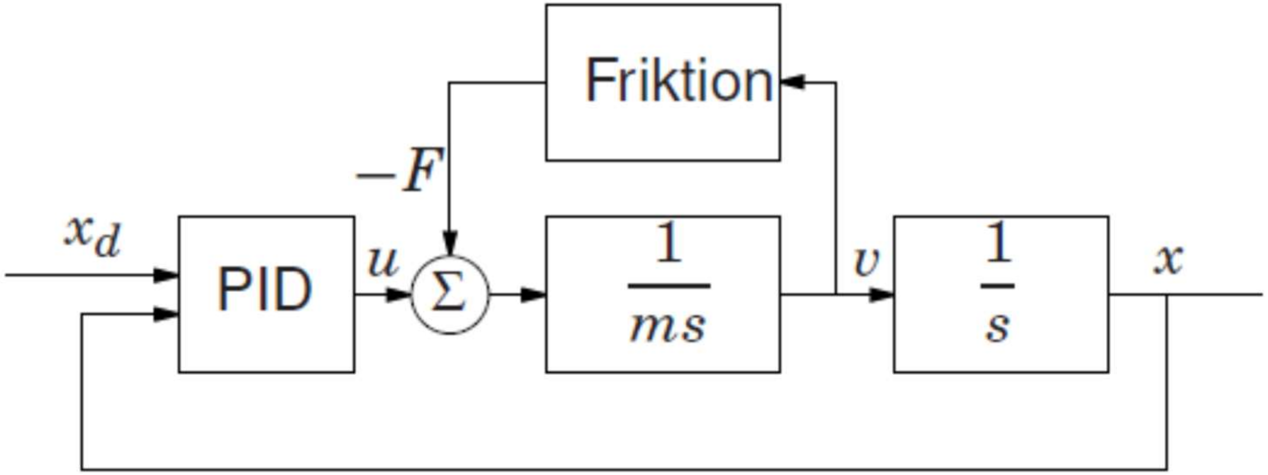
Largest invariant set of equilibria:

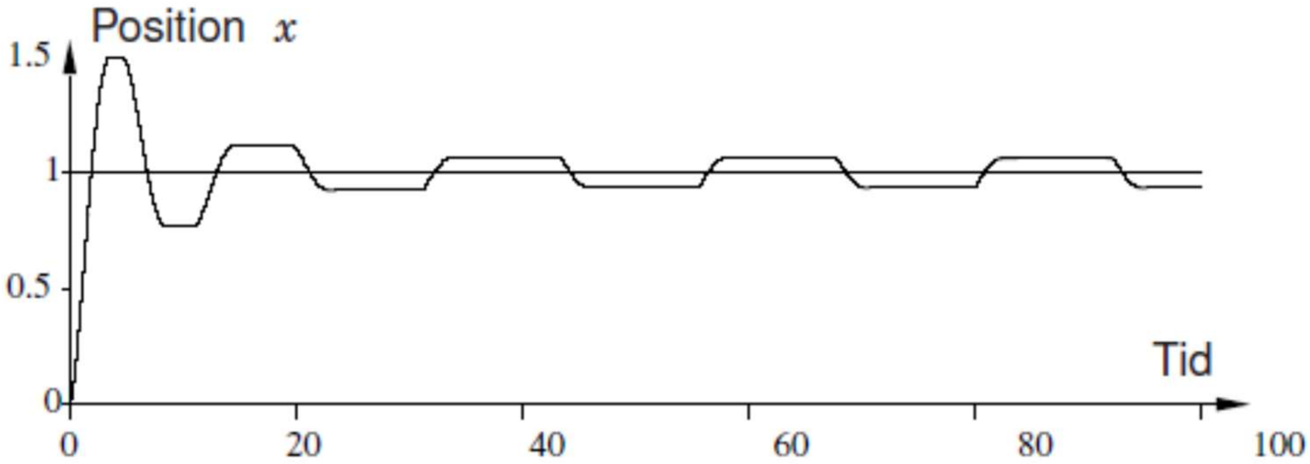
$$S_0 = \left\{ (x = x_s, \dot{x} = 0) \mid x_s \in \left[-F_c K_p^{-1}, F_c K_p^{-1} \right] \right\}$$

J. Alvarez, I. Orlov, and L. Acho, "An invariance principle for discontinuous dynamic systems with application to a coulomb friction oscillator," *Journal of Dynamic Systems, Measurement, and Control*, vol. 122, no. 4, pp. 687–690, 2000.

- So-called hunting limit cycles (see e.g. in [18], [19])

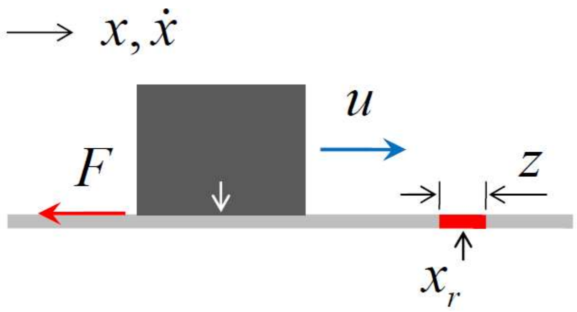
[23]





Friction compensation problems (cont.)

- Integral action gives rise to “creeping” in vicinity to setpoint (x_r)



$$\ddot{x}(t) + F(\dot{x}, t) = u(t)$$

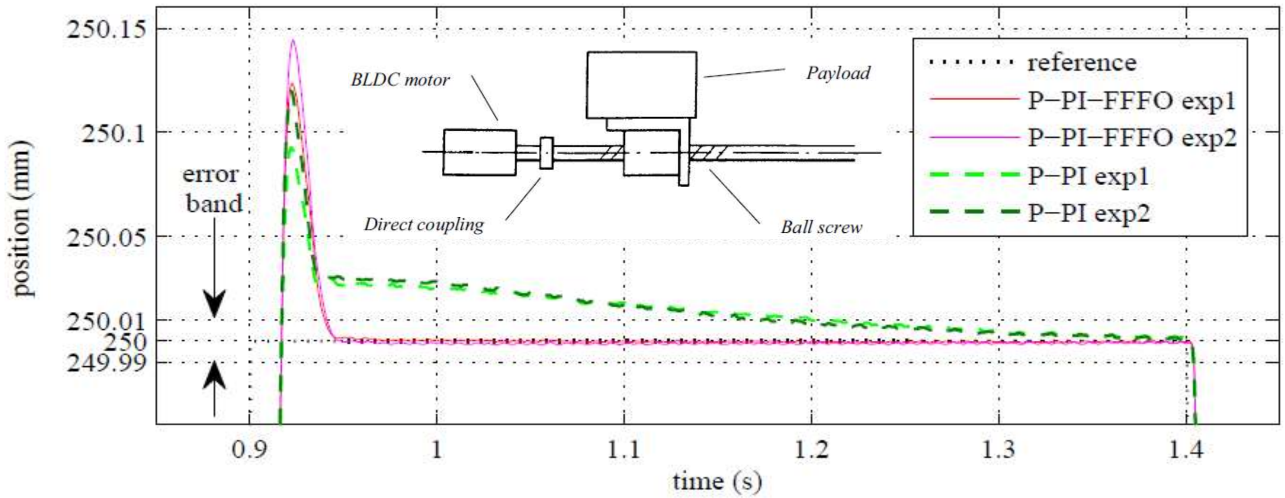
Apparently trivial positioning problem with apparently non-trivial frictional issues

Closed-loop PID-control system with feedback gains K_p, K_i, K_d

$$\dddot{x} + K_d \ddot{x} + K_p \dot{x} + K_i x + \dot{F}(\dot{x}, t) = K_d \ddot{x}_r + K_p \dot{x}_r + K_i x_r.$$

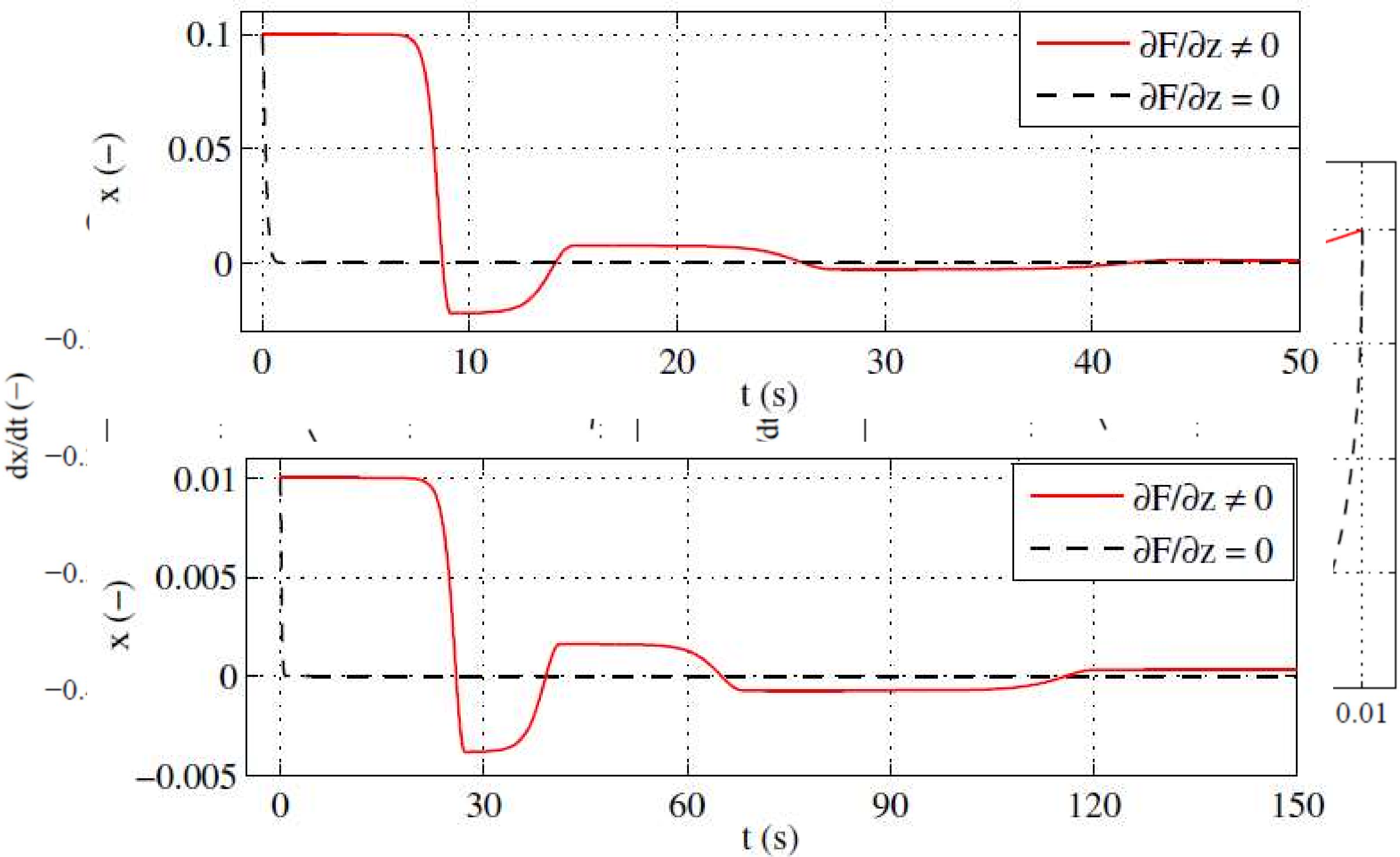
$$\dot{F}(\dot{x}, z) = \underbrace{\frac{\partial F}{\partial \dot{x}}}_{\approx 0} \ddot{x} + \frac{\partial F}{\partial z} \dot{x}$$

From control experiments (μ-positioning) [24]



Friction compensation problems (cont.)

cf. [18]

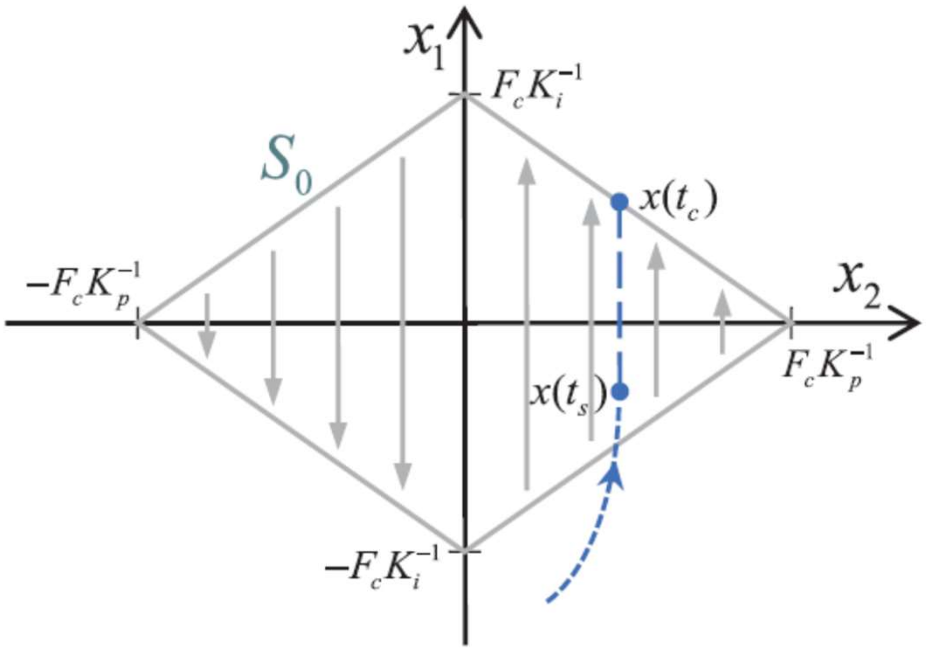


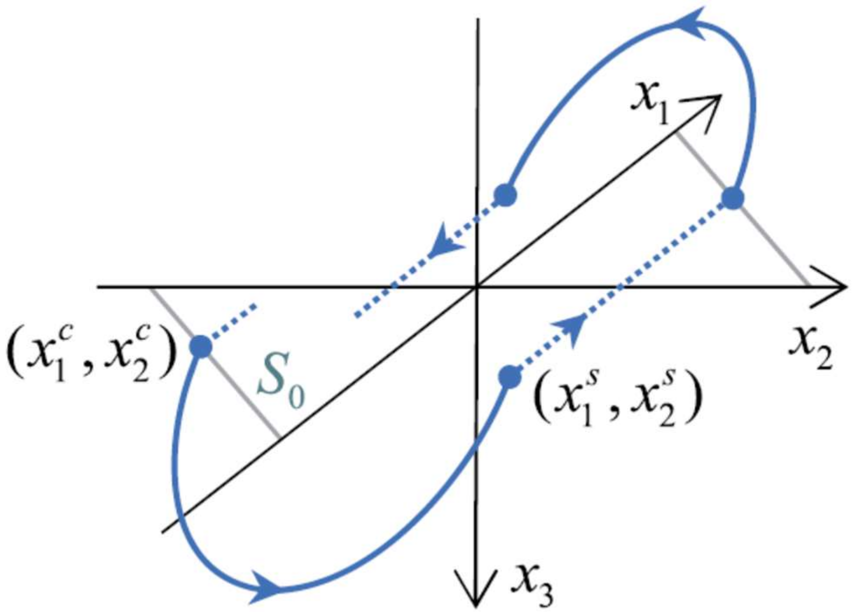
What happens in vicinity to reference setpoint?

Analyzing PID feedback-controlled dynamics with friction

$$\ddot{\phi}(t) + K_d\dot{\phi}(t) + K_p\phi(t) + K_i \int \phi(t)dt + F(t) = 0 \qquad F = F_c \,\mathrm{sign}\left(\dot{\phi}\right)$$

for introduced state variables: $\left[x_1, x_2, x_3\right]^T \equiv \left[\int \phi, \phi, \dot{\phi}\right]^T$





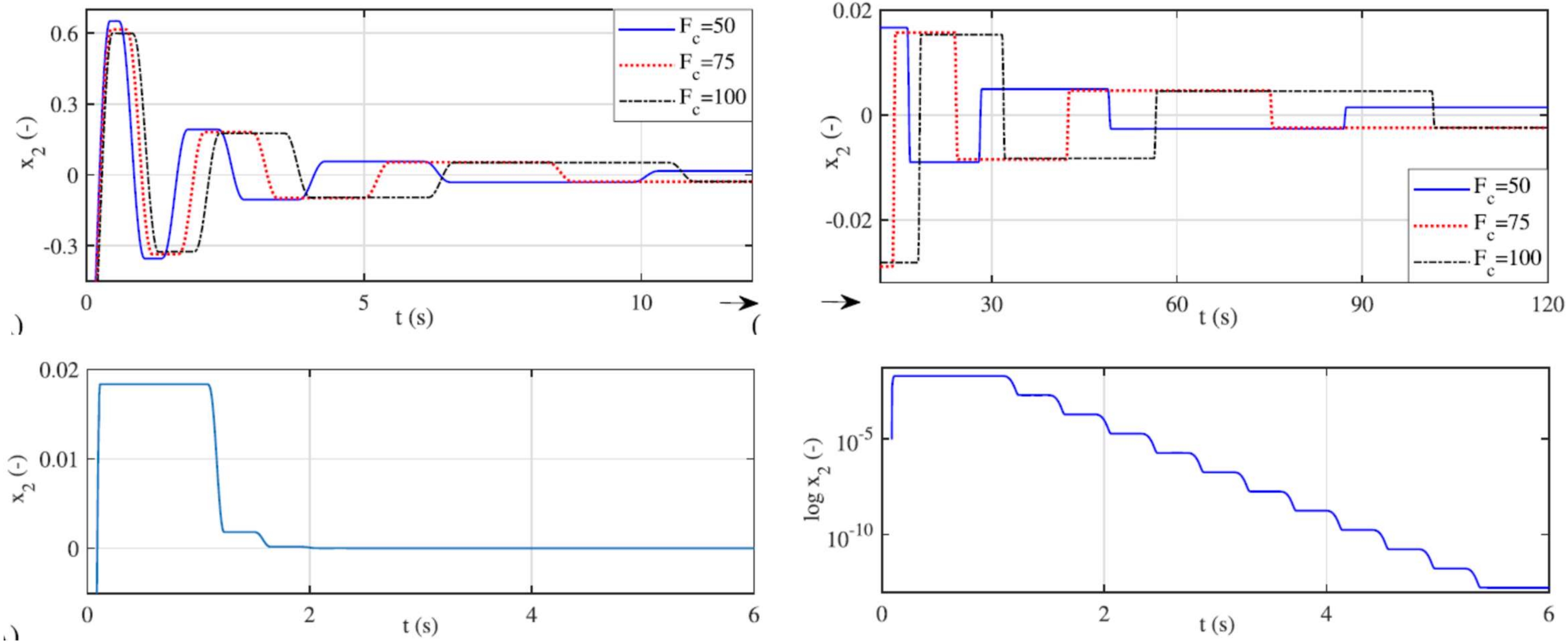
Theorem 1. *Given is the control system (3)–(7) with the Coulomb friction. The system is sticking at $x_3 = 0$ iff*

$$|K_i x_1| + |K_p x_2| \leq F_c. \tag{12}$$

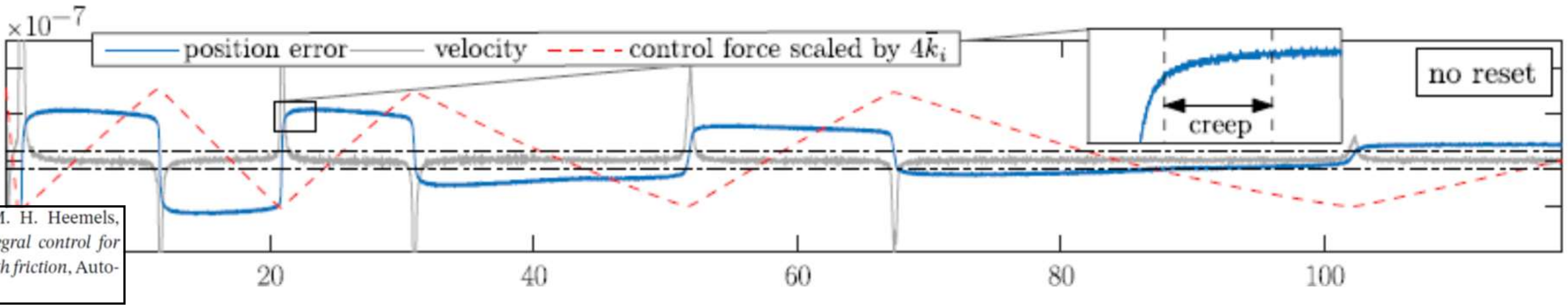
M. Ruderman, "Stick-slip and convergence of feedback-controlled systems with Coulomb friction," *Asian Journal of Control*, vol. Inprint. [Online]. Available: https://onlinelibrary.wiley.com/doi/full/10.1002/asjc.2718

What happens in vicinity to reference setpoint?

Possible convergence scenarios (for various parameters and initial conditions) [18]



Experimental evidence e.g.



R. Beerens, A. Bisoffi, L. Zaccarian, W. P. M. H. Heemels, H. Nijmeijer, and N. van de Wouw, *Reset integral control for improved settling of PID-based motion systems with friction*, Automatica **107** (2019), 483–492.

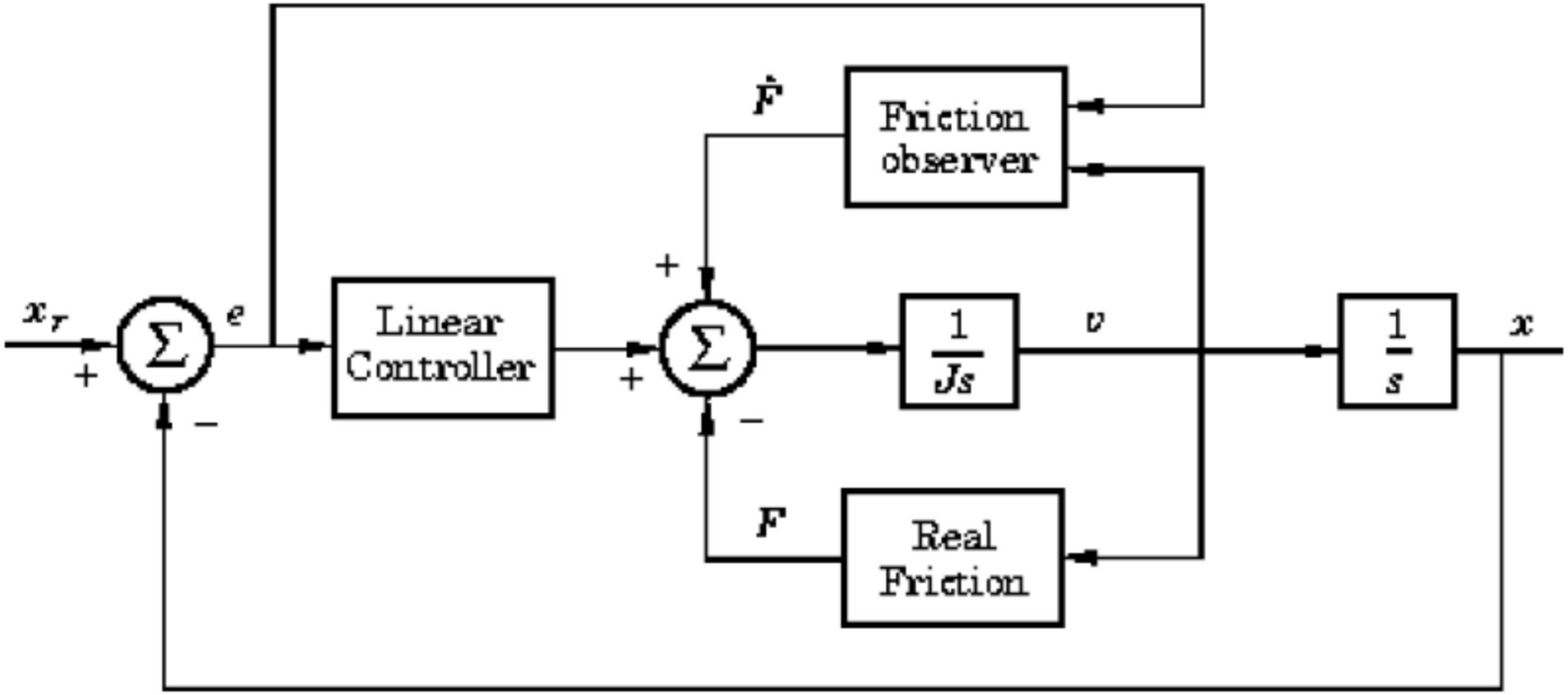
Lectures outline

- Part I – Phenomena of kinetic friction in drives
- Part II – Dynamics with friction and feedback control
- Part III – Compensation of friction in motion control

Possible observer-based solution

- Observer-based friction compensation

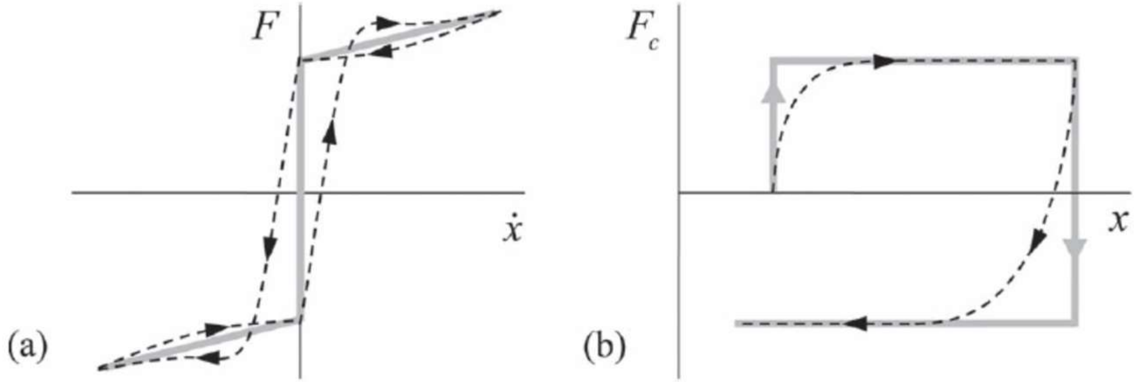
[23]

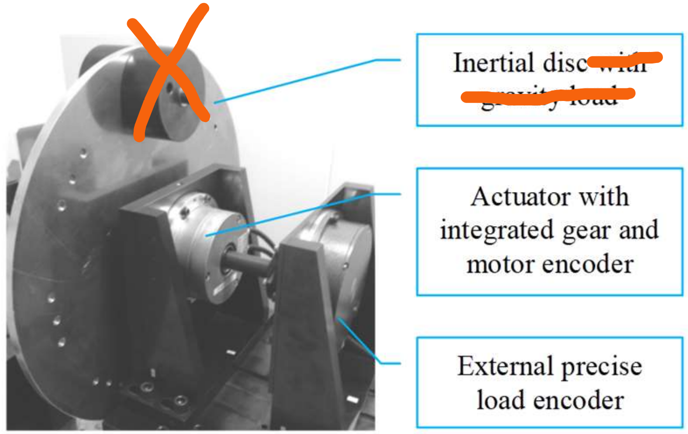


- Large number of the related approaches published since <u>nineties</u>

Possible observer-based solution (cont.)

- Observer-based friction compensation [25]





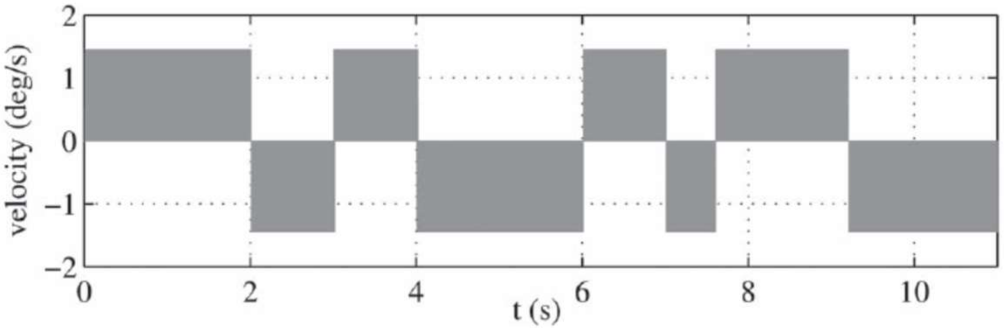
From steady-state friction law (grey solid line)

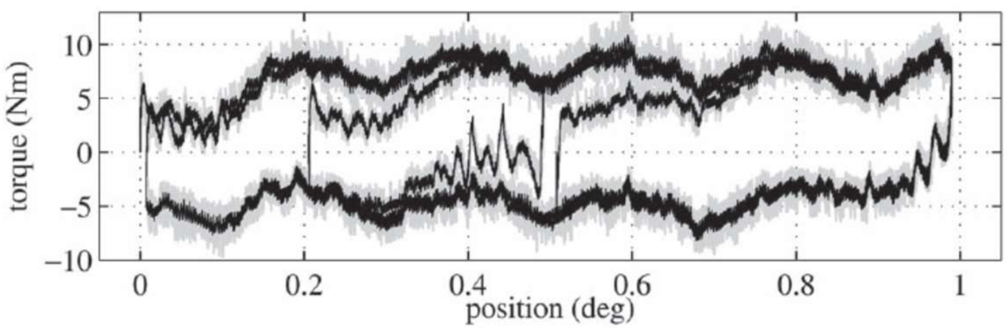
$$F = F_v + F_c$$
$$F = D\dot{x} + C\,\mathrm{sign}(\dot{x})$$

To dynamic-extended friction law (black dash line)

$$b\dot{F}_v + F_v = D\dot{x}$$
$$\dot{F}_c = \frac{\partial F_c}{\partial z}\dot{x}$$

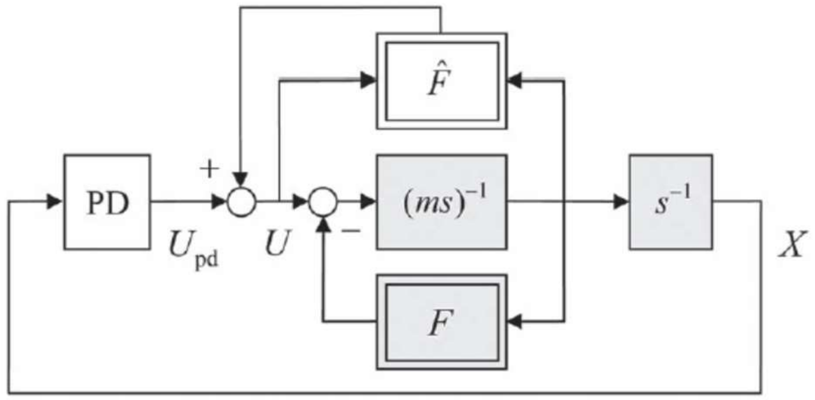
Then, one can construct an observer for online estimating $F_c(t)$ and $F_v(t)$

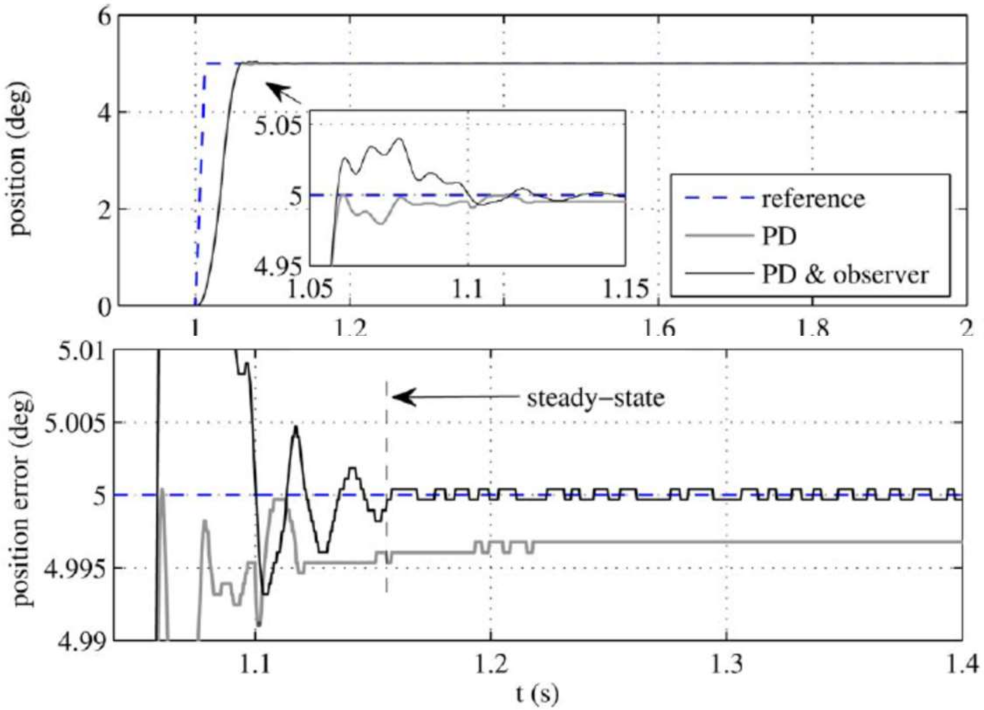


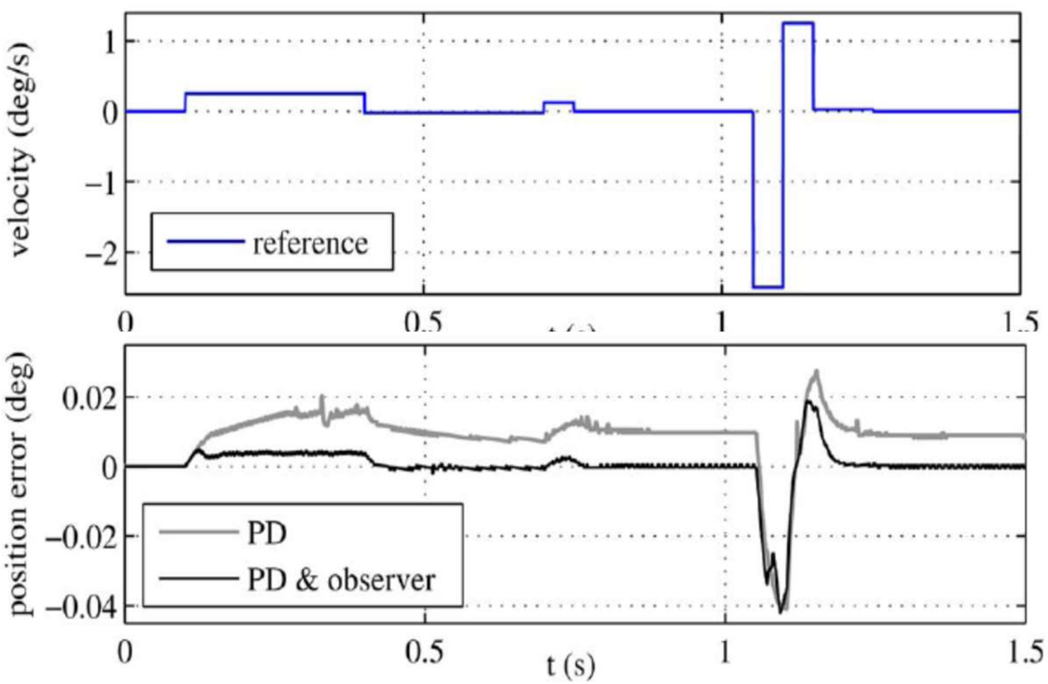


Possible observer-based solution (cont.)

- Observer-based friction compensation [25]

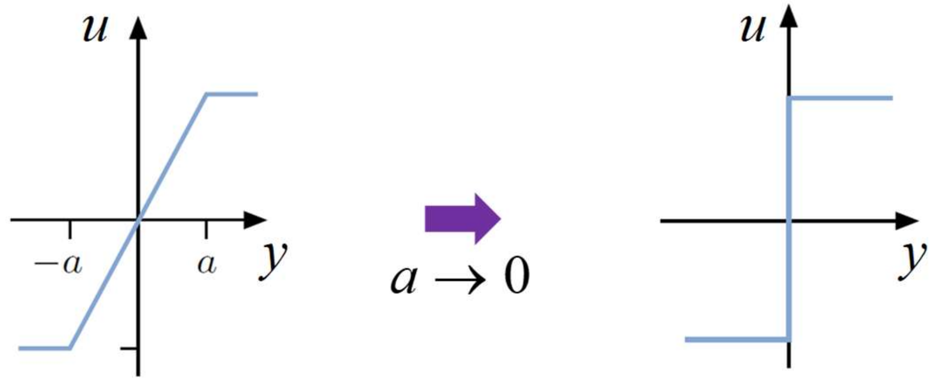




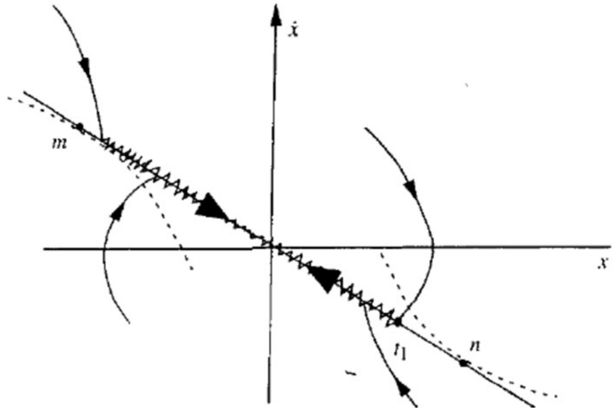


Possible robust solution: sliding-mode control

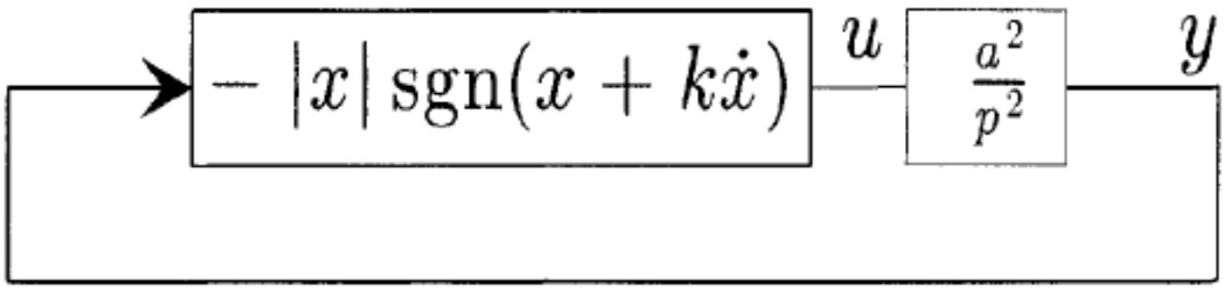
- High-gain feedback (like increasing proportional gain), but the actuator saturations can be a problem for real drive systems
- Use ideal relay for switching (i.e. discrete) control actions

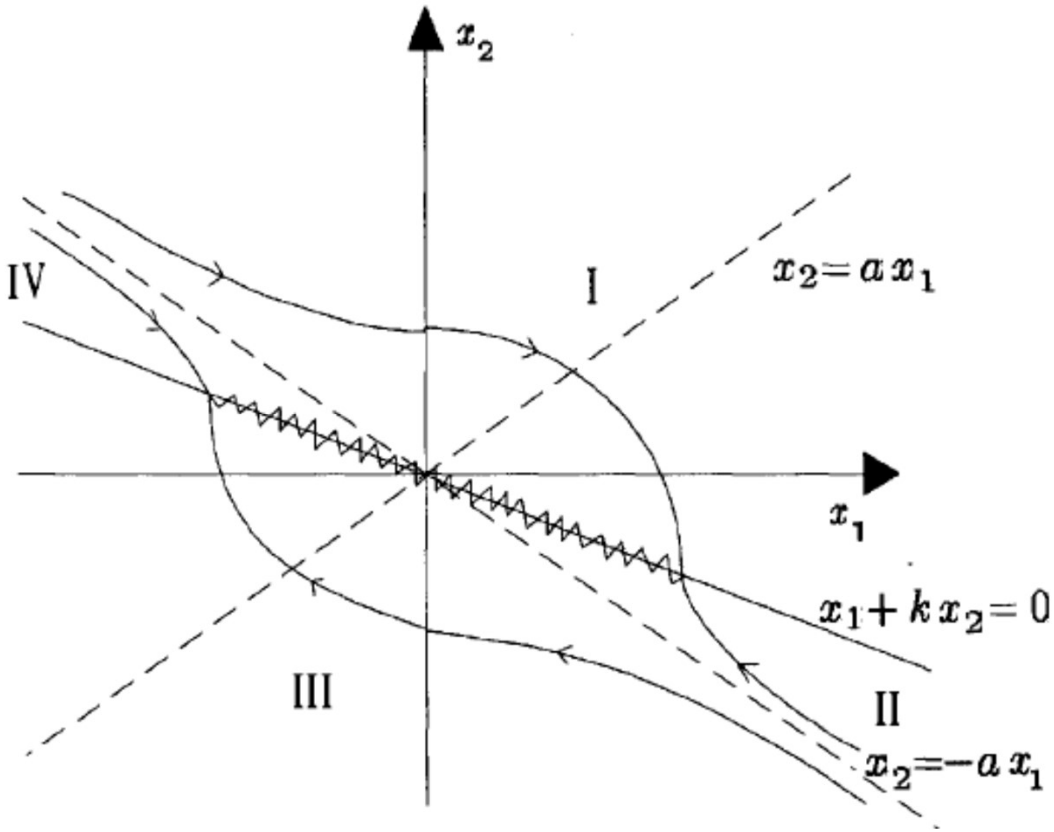


- Basic idea: if you sufficiently “beat” (via control) an uncertain system from both directions, you can force a desired trajectory



Possible robust solution: SMC (cont.)

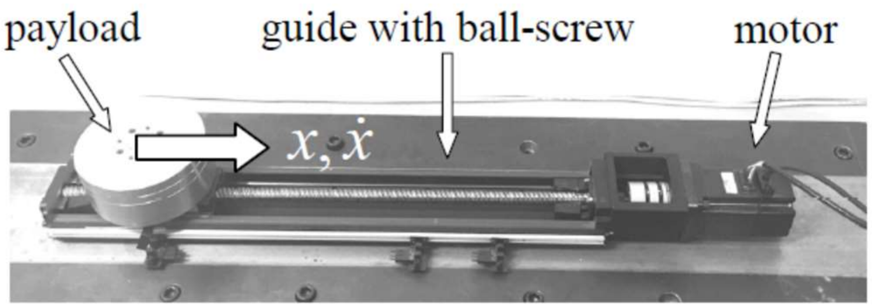


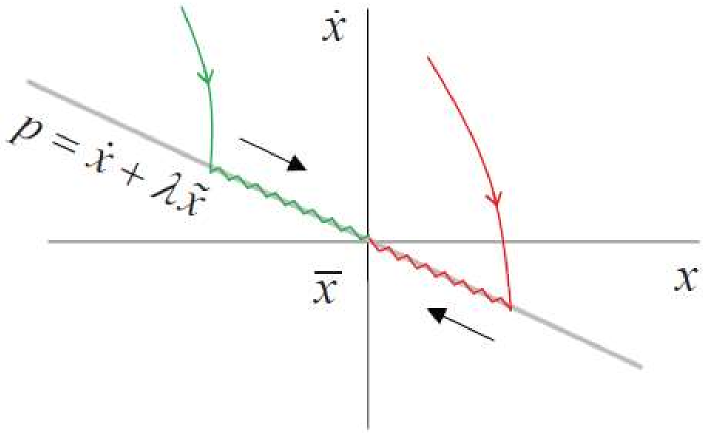


[26]

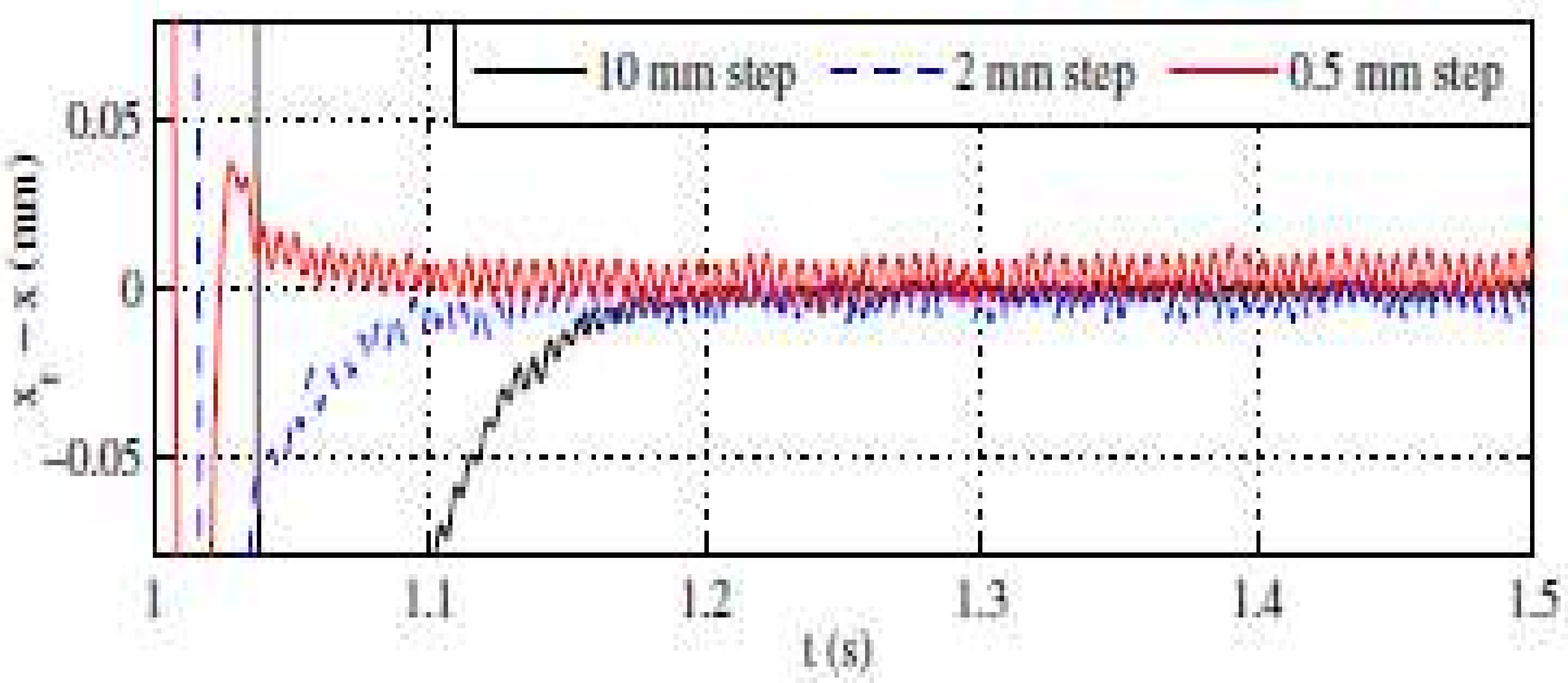
Possible robust solution: SMC (cont.)

- Experimental example





Sliding-mode control (measured control error)



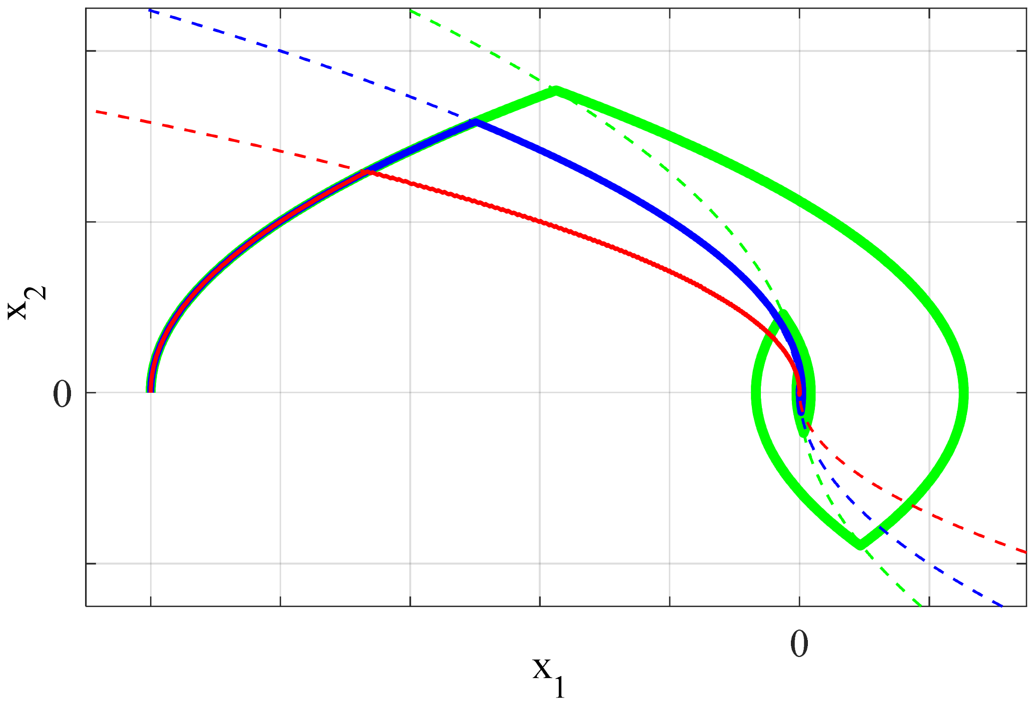
Possible robust solution: SMC (cont.)

- Motion systems (2nd order) with terminal sliding mode control [27]

❑ Perturbed motion system

$$m\ddot{x}(t) = u(t) + \xi(t)$$

with upper bounded perturbation ξ and control saturations $u \in [-U, +U]$



❑ So-called terminal sliding mode control

$$u = -k \,\mathrm{sign}(s)$$

with a surface of the type

$$s = \dot{x} + \beta x^{q/p}$$

parameterized by $k, \beta, p, q > 0$

❑ Objective: optimal (one-parameter) terminal sliding mode control, for q/p=0.5, k=U, and *single free parameter* α of surface; with α=0.5 for boundary layer of *twisting mode*

more specifically, for the surface of the type**

$$s = \dot{x} + \beta \sqrt{|\,x\,|}\,\mathrm{sign}(x)$$

** A. Levant, "Sliding order and sliding accuracy in sliding mode control," *International journal of control*, vol. 58, no. 6, pp. 1247–1263, 1993.

Possible robust solution: SMC (cont.)

- Time-optimal terminal SMC

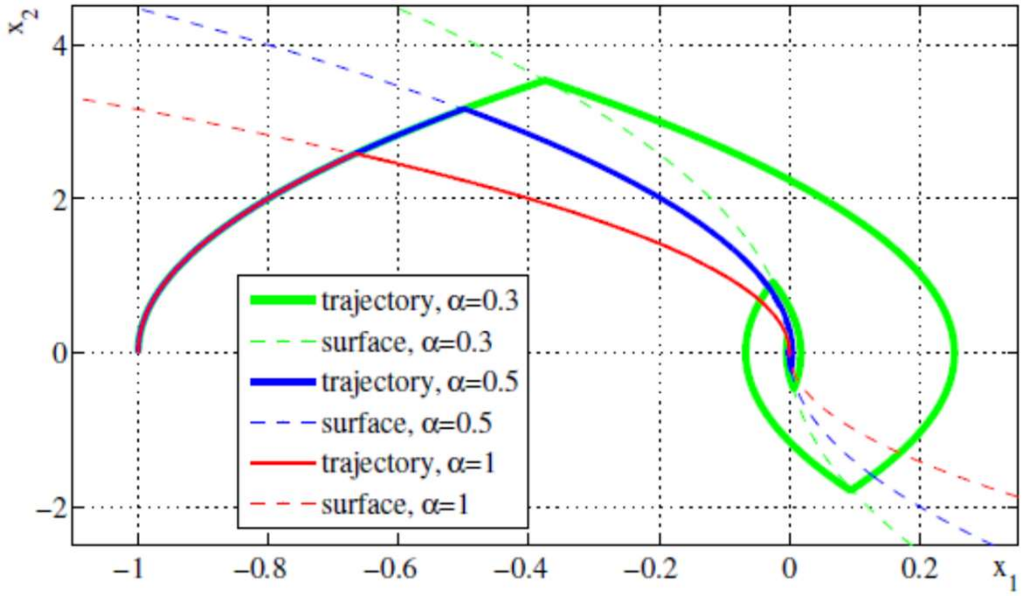
$$u(t) = -U \operatorname{sign}(s)$$

$$s = x_1 + \alpha m U^{-1} x_2^2 \operatorname{sign}(x_2)$$

Sliding mode exists for

$\alpha > 0.5$ (for proof of existence see [27])

$0 < \alpha \leq 0.5$ (twisting mode, since the control is still switching at *s*-crossing)



- Reachability condition for global stability

$s\dot{s} < -\eta|s| \;\Rightarrow\; \operatorname{sign}(s)\dot{s} < -\eta$, substituting $\dot{s} = x_2 - 2\alpha|x_2|\operatorname{sign}(s) \;\Rightarrow\; x_2 \operatorname{sign}(s) < -\eta + 2\alpha|x_2|$

It is always possible to find a small positive constant η such that the last condition is fulfilled

- Single boundness condition for perturbations $|\xi| < U$

$$\ddot{x}(t) = m^{-1}\big(u(t) + \xi(t)\big)$$

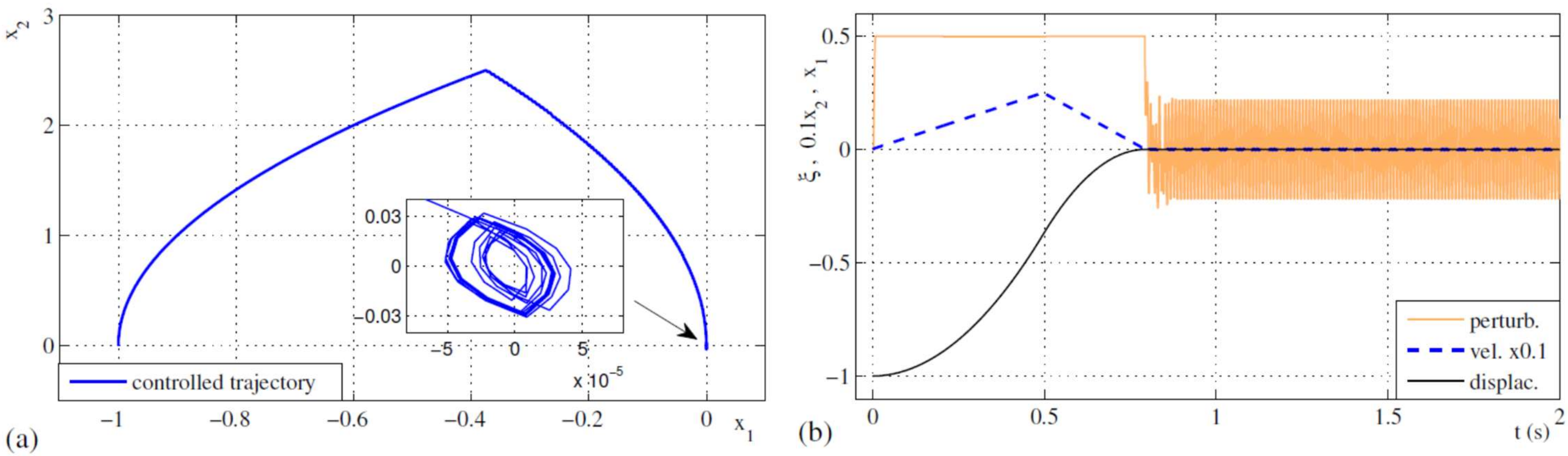
Possible robust solution: SMC (cont.)

- Simulation framework:

The assumed numerical parameters, here and for the rest of the paper, are $m = 0.1$, $U = 1$, $\alpha = 0.6$. All the controlled motion trajectories start at the initial state $\big(x_1(t_0), x_2(t_0)\big) = (-1, 0)$, while the control set point is placed in the origin, meaning $\big(x_1(t_f), x_2(t_f)\big)_{ref} = \mathbf{0}$. The implemented and executed simulations are with 1kHz sampling rate and with the (most simple) first-order Euler numerical solver.

[27]

- Numerical control example with nonlinear Coulomb friction

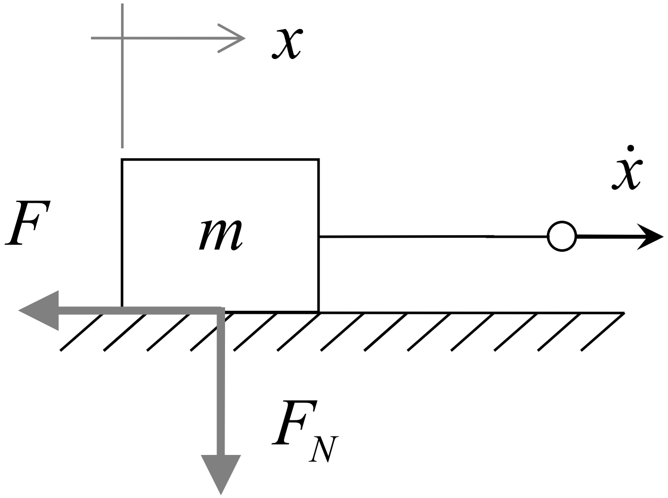


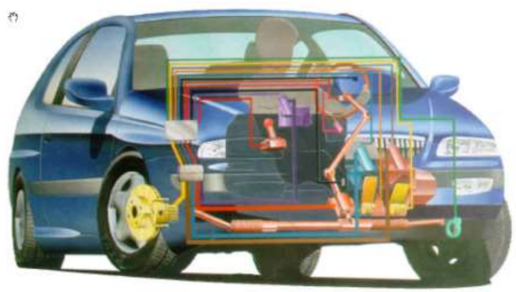
Summing up: friction related issues

- Kinetic friction in (almost) all dynamic systems

+ Natural system damping

− Nonlinear disturbance





- At all measurable / controllable levels (macro-, micro-, nano-)

Summing up: friction related issues (cont.)

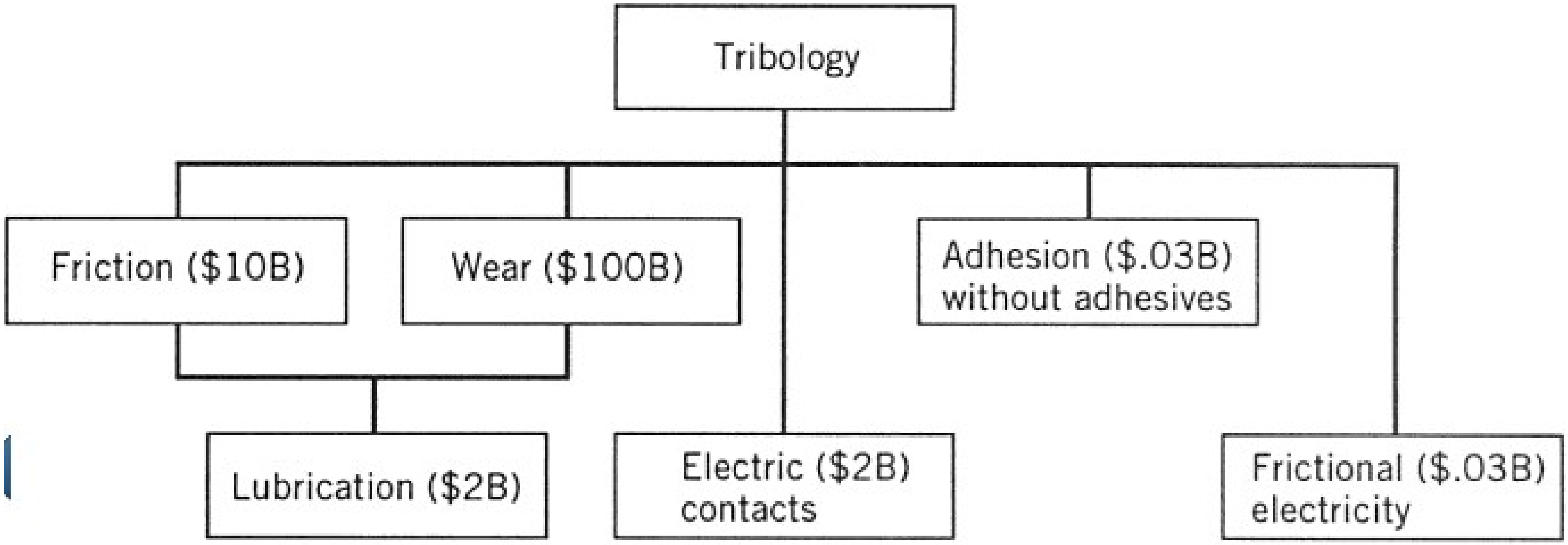


Fig. 1.2 The various mechanical surface interaction phenomena included in the field of tribology. A rough estimate is given of the economic importance of each (in billions of 1976 U.S. dollars).

Source: E. Rabinowicz. Friction and wear of materials. John Wiley and Sons, New York, USA, 2nd ed., 1995

Summing up: friction related issues (cont.)

- Actuality of friction control issues (in robotics, mechatronics, precision engineering) does not appear to dimmish with time

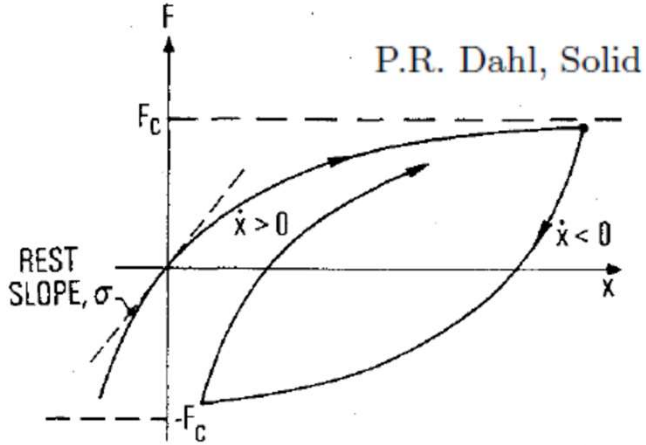
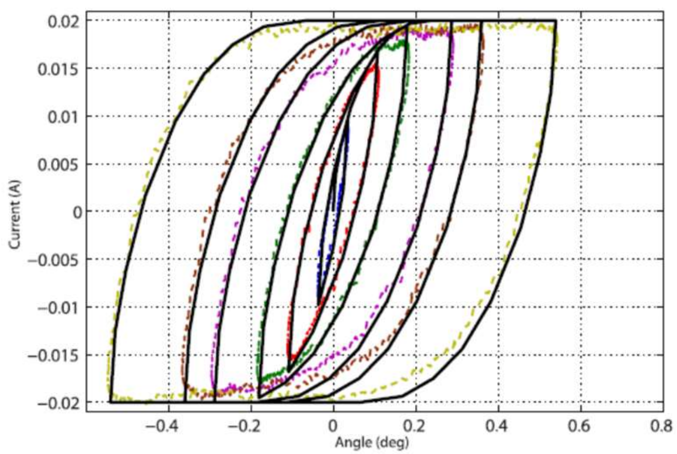
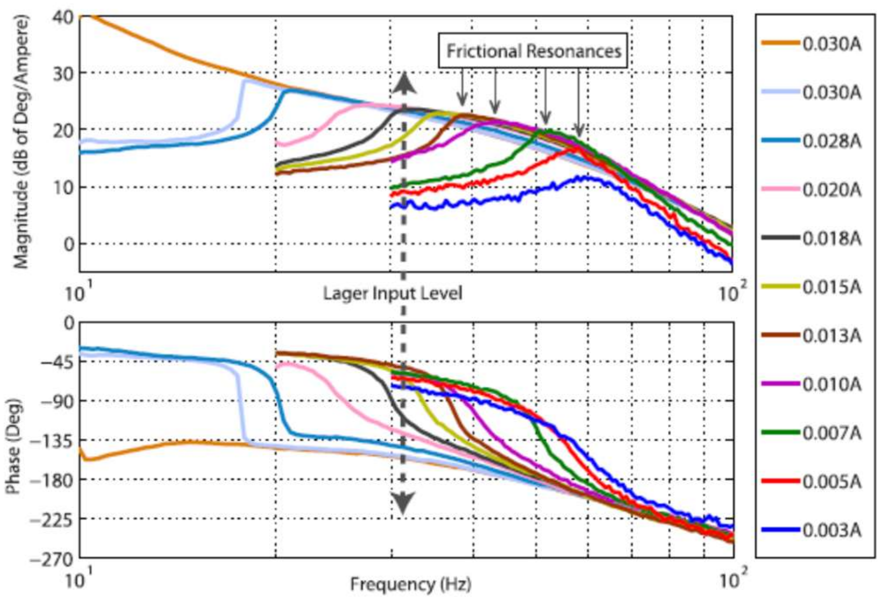


Fig. 1 Typical solid friction force function.

P.R. Dahl, Solid friction damping of mechanical vibrations. *AIAA J.* 14 (1976) 1675–1682



J. Y. Yoon and D. L. Trumper, *Friction microdynamics in the time and frequency domains: Tutorial on frictional hysteresis and resonance in precision motion systems*, Precis. Eng. 55 (2019),



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Thank you for attention