



# Mechanics and Machines, Lecture 4

Synthesis of planar mechanisms

# Difference between Analysis and Synthesis



## Analysis

Analysis allows determining whether a given system will comply with certain requirements or not.

During «Theoretical Mechanics» we only analyzed systems. We knew all dimensions and tried to find positions, velocities, accelerations.

## Synthesis

Synthesis is the design of a mechanism so that it complies with previously specified requirements.

We know, that it should work on uneven terrain, and we are trying to design the robot with such possibilities.



# Types of Synthesis

## Structural

This synthesis deals with the **topological** and **structural study** of mechanisms.

It only considers the interconnectivity pattern of the links so that the results are unaffected by the changes in the geometric properties of the mechanisms.

## Dimensional

It focuses on the problem of **obtaining the dimensions of a predefined mechanism** that has to comply with certain given requirements.

It will be necessary to define the dimension of the links and the position of the supports, among others.

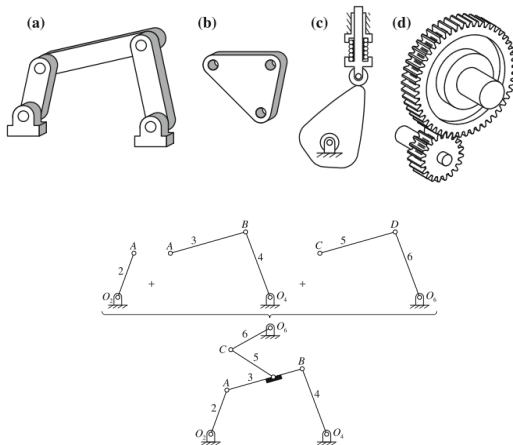
# Structural Synthesis

*2 questions, which should be answered*

**1) Synthesis of type or Reuleaux synthesis:** What type of mechanism is more suitable? What type of elements will it be made of? Can it be formed by linkages, gears, flexible elements or cams?

Different configurations are developed according to the pre-established requirements. The criteria to value the different characteristics of the mechanism are set.

**2) Synthesis of number or Grübler synthesis:** In the case of a linkage, it determines the number of links and their configuration.



# Structural Synthesis: Case Study

*Structural synthesis problem*



## Question

What the optimal number of legs should be in such robot mover?

# Structural Synthesis: Case Study

*Structural synthesis problem*



## Question

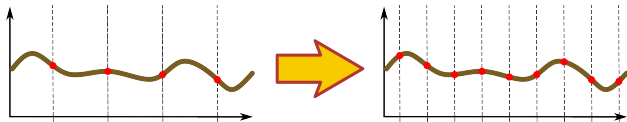
What the optimal number of legs should be in such robot mover?

## Answer

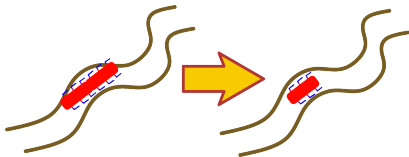
Robot should have **8-14 legs** in total!

# Structural Synthesis: Case Study

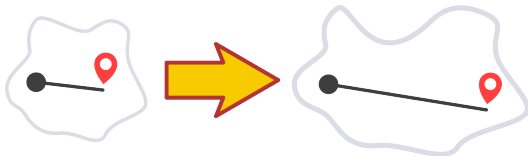
*Criteria*



More legs  $\rightarrow$  higher data discretisation



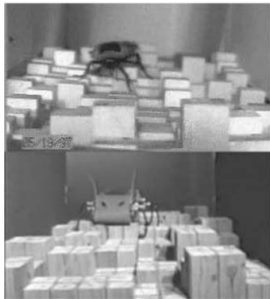
More legs  $\rightarrow$  longer robot  $\rightarrow$  cannot pass through crooked terrains



Amount of legs nonlinearly correlates of maximal terrain passability

# Structural Synthesis: Case Study

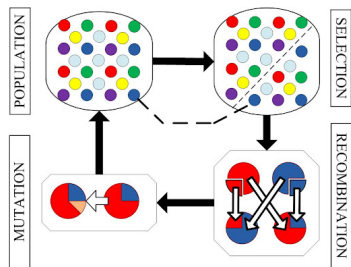
*Technological stack*



Generating terrain approach  
(Robot traverse an **artificial terrain** based on **generating parameters**)



Robot simulator

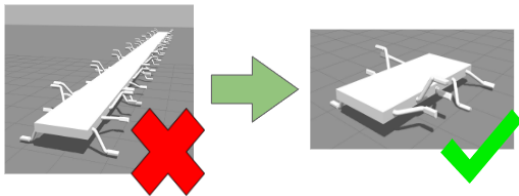


Genetic algorithm  
(OpenAI-ES)

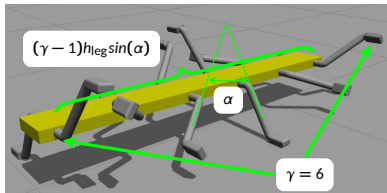


# Structural Synthesis: Case Study

*Proposed solution*



**Idea:** Minimize number of legs without losing off-road passability

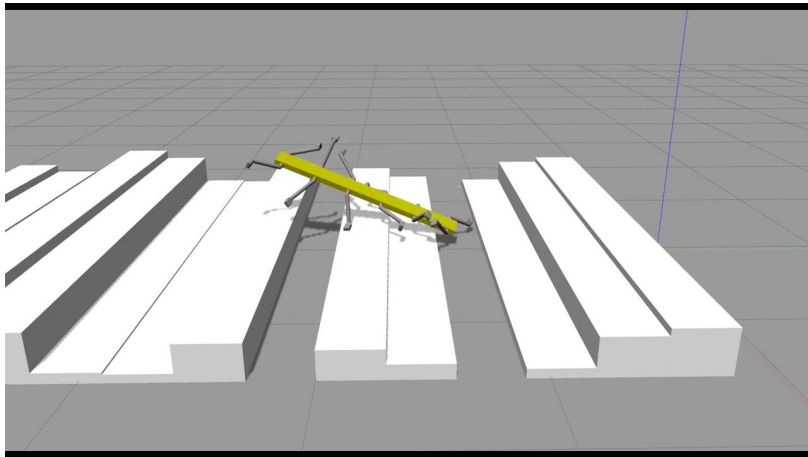


$$F \rightarrow \max = \beta \left( \omega_1 \cdot \overbrace{\delta}^{\text{Distance}} + \omega_2 \cdot \frac{\overbrace{1}^{\text{Simplified body length}}}{(\gamma - 1)h_{\text{leg}}\sin(\alpha)} \right) + (1 - \beta)\delta^{\omega_1} \left( \frac{1}{(\gamma - 1)h_{\text{leg}}\sin(\alpha)} \right)^{\omega_2}$$

$\beta$  is adaptive parameter,  
 $\omega_{1,2} \in [0..1]$  are the weight coefficients.

# Structural Synthesis: Case Study

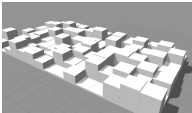
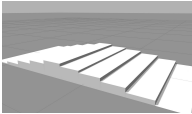
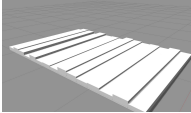
*Video: The story of one generated robot*



# Structural Synthesis: Case Study

Particular results:  $\omega_1 = 0.6$ ,  $\omega_2 = 0.4$



	Terrain types	No. Legs	Angle b/w neighbor legs	No. individuals
Phase 1		12	73	200
Phase 2		12	72	55
		10	68	
		12	77	

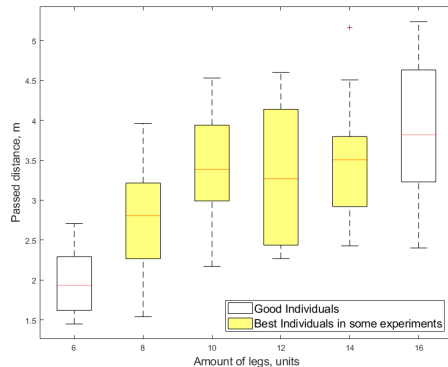
# Structural Synthesis: Case Study

## *Global results*

Based on fitness function the number of legs range starts from 8 till 14 for different  $\omega$  values.

It can be explained by static stability criteria.

In such case 4 legs will touch the ground.

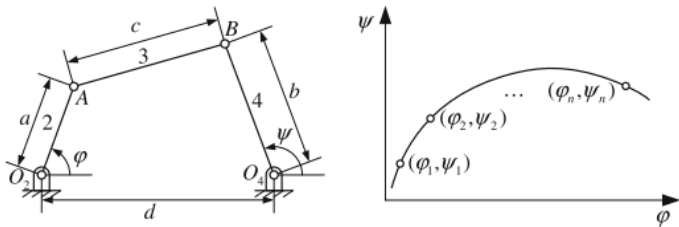


Correlation between amount of legs and passed distance by best robot individuals from several experiments

# Dimensional Synthesis

## *Functional Generation*

Pre-established conditions refer to the relation between the input and output motions. These are defined by variables  $\phi$  and  $\psi$ , that identify their positions.

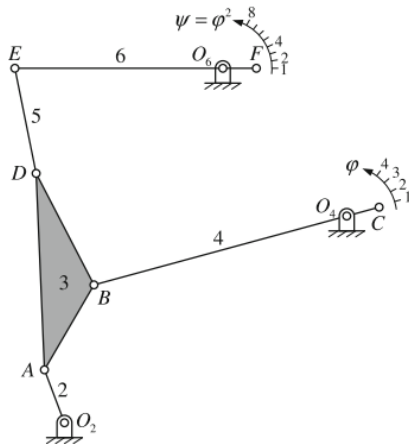


# Functional Generation

## Case Study

Function generation can be used to design mechanisms that carry out mathematical operations: addition, differentiation, integration or a combination of them.

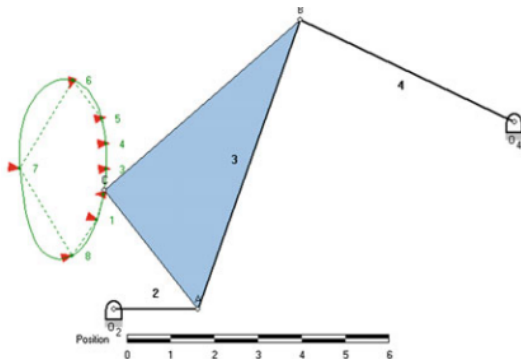
The first computers were mechanical devices based on this type of mechanisms.



# Dimensional Synthesis

## *Trajectory Generation*

It studies and provides methods in order to obtain mechanisms in which one of the points describes a given trajectory



# Trajectory Generation

*Case Study: Video*







# Types of solving methods

## Graphical

These methods are very didactic and help us to understand the problem in an easy way. However, they offer a limited range of possibilities.

## Analytical

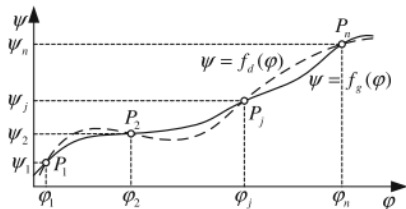
They solve the problem by means of mathematical equations based on the requirements.

## Optimization-technique-based

They can find the optimal solution to the problem by means of the minimization of an objective function and the establishment of a series of restrictions. Different optimization techniques can be used.

# Function Generation Synthesis

*How to represent data*

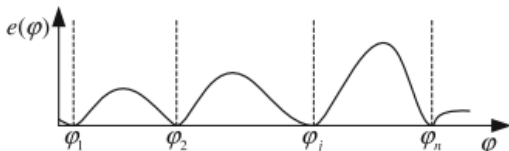


1) Continuous function (desired and generated)

2) Precision points

*It's representation for optimization based algorithms.* Error function (difference between generated and desired)

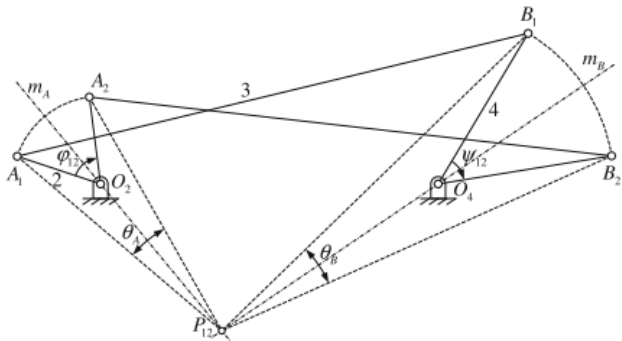
$\varphi$	$\varphi_1$	$\varphi_2$	...	$\varphi_n$
$\psi$	$\psi_1$	$\psi_2$	...	$\psi_n$



# Function Generation Synthesis

## Graphical Method (1)

As an example, we will generate a four-bar mechanism in which a rotated angle of the input link between positions 1 and 2,  $\phi_{12}$ , corresponds to a rotated angle of output link  $\psi_{12}$



# Function Generation Synthesis

## *Graphical Method (2): Algorithm*

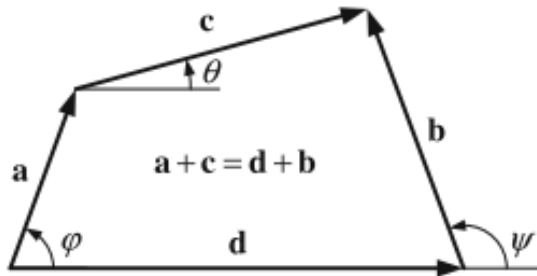
1. We take an arbitrary point and name it pole  $P_{12}$ . We draw line  $m_A$  and any point  $O_2$  in it.
2. Next we choose a value for the length of the crank and, taking point  $O_2$  as the origin, we draw points  $A_1$  and  $A_2$  in symmetric positions with respect to line  $m_A$ . The angle formed by the points and the origin, has to be equal to specified angle  $\varphi_{12}$ .
3. Points  $A_1, A_2$  and  $P_{12}$  are connected so that they form angle  $\theta_A = \widehat{A_1 P_{12} A_2}$ .
4. Taking  $P_{12}$  as the origin, we draw a new arbitrary line,  $m_B$ , and take point  $O_4$  in it.
5. Taking point  $P_{12}$  as the origin, we draw two lines that comply with two conditions, the angle they form is equal to  $\theta_B = \widehat{B_1 P_{12} B_2} = \theta_A$  and  $m_B$  is their bisector.
6. Taking point  $O_4$  as the origin we draw two lines with the condition that the angle they form is equal to  $\psi_{12}$  and that  $m_B$  is their bisector.
7. The intersection points of these four lines define points  $B_1$  and  $B_2$ .



# Function Generation Synthesis

*Analytical Method (1): Freudenstein's method*

Find all lengths of four link bar mechanism if we know 3 related positions of input and output links.



# Function Generation Synthesis



*Analytical Method (2): full solution in pdf page 11*

$$\vec{a} + \vec{c} = \vec{d} + \vec{b} \quad (1)$$

$$\begin{cases} a \cos(\phi) + c \cos(\theta) = d + b \cos(\psi) \\ a \sin(\phi) + c \sin(\theta) = b \sin(\psi) \end{cases} \quad (2)$$

$$\text{In order to find the relation } \psi = f(\phi) \quad (3)$$

$$c^2 = d^2 + b^2 + a^2 + 2bd \cos(\psi) - 2ab \cos(\psi - \phi) - 2da \cos(\phi) \quad (4)$$

$$R_1 \cos(\psi) - R_2 \cos(\phi) + R_3 = \cos(\psi - \phi), \text{ where:} \quad (5)$$

$$\begin{cases} R_1 = \frac{d}{a}; R_2 = \frac{d}{b} \\ R_3 = \frac{d^2 + b^2 + a^2 - c^2}{2ab} \end{cases} \quad (6)$$

# Function Generation Synthesis



## *Analytical Method (3)*

Equation is known as Freudenstein's equation and it is an effective tool to carry out function generation synthesis. We can obtain the length of links  $a$ ,  $b$ ,  $c$  and  $d$  in a four-bar mechanism, provided that we know three related positions of the input and output links. These positions are defined by pairs  $(\phi_1, \psi_1)$ ,  $(\phi_2, \psi_2)$  and  $(\phi_3, \psi_3)$  which are known as precision points.

$$\left. \begin{aligned} R_1 \cos \psi_1 - R_2 \cos \phi_1 + R_3 &= \cos(\psi_1 - \phi_1) \\ R_1 \cos \psi_2 - R_2 \cos \phi_2 + R_3 &= \cos(\psi_2 - \phi_2) \\ R_1 \cos \psi_3 - R_2 \cos \phi_3 + R_3 &= \cos(\psi_3 - \phi_3) \end{aligned} \right\}$$

This system is linear and independent. It can easily be solved to obtain unknowns  $R_1$ ,  $R_2$  and  $R_3$ . Using the calculated values in the mathematical definitions of these parameters, we can find the length of links  $a$ ,  $b$ ,  $c$  and  $d$  by assigning an arbitrary value to one of them, for example,  $d = 1$ . In this case, the size of the mechanism obtained will depend on the value given to  $d$ , but it can be escalated to any size.



# Function Generation Synthesis

*Analytical Method (4): Comparison with graphical*

```
1  theta_1 = deg2rad(173.9);
2  theta_2 = deg2rad(83.9);
3  theta_3 = deg2rad(141.1958);
4  psi_1 = deg2rad(7.6);
5  psi_2 = deg2rad(72.5);
6  psi_3 = deg2rad(60.1145);
7
8  a = 1;
9
10 A = [cos(psi_1) -cos(theta_1) 1 ;
11      cos(psi_2) -cos(theta_2) 1;
12      cos(psi_3) -cos(theta_3) 1];
13 B = [cos(psi_1 - theta_1); cos(psi_2 - theta_2); cos(psi_3 - theta_3)];
14
15 X = linsolve(A,B)
16
17 syms b c d;
18 eqns = [X(1) == d/a; X(2) == d/b; X(3) == (d^2 + b^2 + a^2 - c^2)/(2*a*b)];
19 res = solve(eqns);
20 abs(double(res.b))
21 abs(double(res.c))
22 abs(double(res.d))
```

```
ans =
      4.4520
      4.4520
```

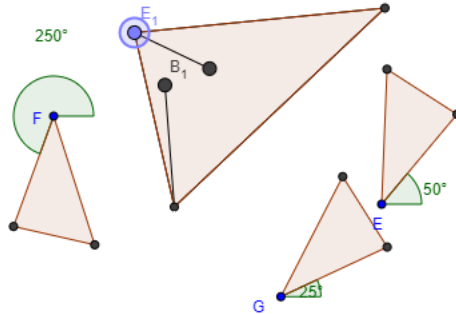
```
ans =
      3.3606
      3.3606
```

```
ans =
      2.0814
      2.0814
```

# Trajectory Generation Synthesis

## *Formal definition*

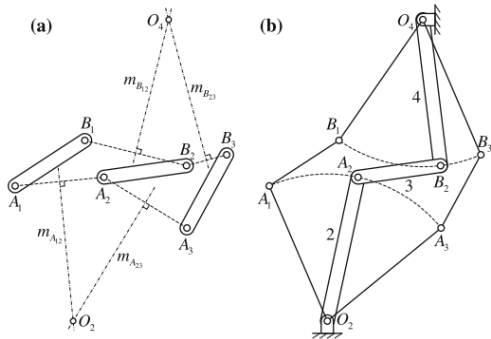
Relationship between the trajectory described by a point in a link and the motion of another link, usually the input one.



# Trajectory Generation Synthesis

## *Graphical Method (1)*

This method allows finding a four-bar mechanism in which the coupler link passes through the three specified positions. The steps to follow are the next: ones:



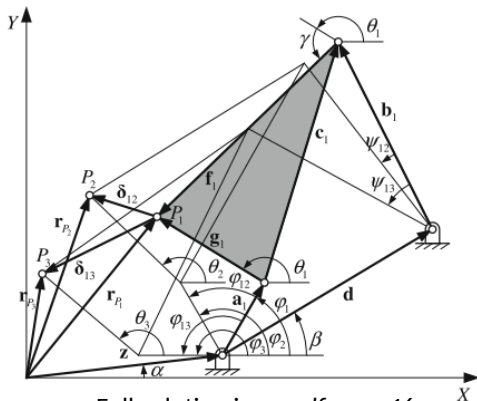
# Trajectory Generation Synthesis

## *Graphical Method (2): Algorithm*

1. The three positions of link  $AB$  are considered known and they are identified as  $A_1B_1, A_2B_2$  and  $A_3B_3$ .
2. We draw a segment between points  $A_1$  and  $A_2$  and then its perpendicular bisector  $m_{A_{12}}$ . The same way we draw a segment between points  $A_2$  and  $A_3$  and then its perpendicular bisector  $m_{A_{23}}$ .
3. The intersection point of both bisectors is point  $O_2$ .
4. We operate the same way drawing segments  $B_1B_2$  and  $B_2B_3$  with their perpendicular bisectors  $m_{B_{12}}$  and  $m_{B_{23}}$ .
5. Their intersection point defines the position of point  $O_4$ .
6. Hence we obtain the mechanism we were looking for,  $\{O_2, A, B, O_4\}$

# Trajectory Generation Synthesis

*Analytical Method (1): Based on complex numbers*



Full solution is on pdf page 16

# Optimal Synthesis

## Formal definition (1)

In general, the solution of an optimization problem determines the value of the variables  $(x_1, x_2, \dots, x_n)$  that minimize objective function  $f(x)$  subject to a set of constraints. This can be written as:

$$\min f(x_1, x_2, \dots, x_n)$$

Subject to:

$$h_j(x_1, x_2, \dots, x_n) \leq 0 \quad j = 1, 2, \dots, m$$

$$g_k(x_1, x_2, \dots, x_n) = 0 \quad k = 1, 2, \dots, p$$

Function  $f(x)$  is called objective function and functions  $h_j(x)$  and  $g_k(x)$  are called constraints of the problem. We can have both inequality and equality constraints. In the context of engineering design, the above mentioned concepts are defined as:

# Optimal Synthesis

## *Formal definition (2)*

Function  $f(x)$  is called objective function and functions  $h_j(x)$  and  $g_k(x)$  are called constraints of the problem. We can have both inequality and equality constraints. In the context of engineering design, the above mentioned concepts are defined as:

- Objective function: A function that expresses a fundamental aspect of the problem. An extreme value (minimum or maximum) is sought along the process of optimization. This function is often called merit function. Multifunctional functions, in which several features are optimized, can also be formulated. In this case, each one of them is weighted depending on their importance.
- Independent design variables: Such variables represent the geometry of the model. They are usually the dimensions of the mechanism such as the length or width of the links.

# Optimal Synthesis

## *Formal definition (3)*

- **Dependent variables:** These are parameters that have to be included in the formulation of the objective function or the constraints but that depend on the design variables.
- **Constraints:** They are mathematical functions that define the relationships between the design variables that have to be met by every set of values that define a possible design. These relationships can be of three types.
  - **Inequality restrictions:** They are usually limitations to the behavior of the mechanism or security restrictions to prevent failure under certain conditions.
  - **Variable limits:** They are a specific case of the previous ones.
  - **Equality restrictions:** They are conditions that have to be met strictly in order for the design to be acceptable.



# Optimal Synthesis

## Function Generation: objective function

We will use Freudenstein's equation and the method developed in of this book to obtain the relationship between the output angle,  $\psi$ , and input one,  $\varphi$

$$\psi = 2 \arctan \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \psi(a, b, c, d, \varphi)$$

$$\left. \begin{aligned} A &= \cos \varphi - R_1 - R_2 \cos \varphi + R_3 \\ B &= -2 \sin \varphi \\ C &= R_1 - (R_2 + 1) \cos \varphi + R_3 \end{aligned} \right\}$$

where  $R_1, R_2$  and  $R_3$  are known functions of  $a, b, c$ , and  $d$ .

$$\min \sum_{i=1}^N (\psi_i(X, \varphi_i^d) - \psi_i^d)^2$$

Subject to:

$$x_i \in [li_i, ls_i] \quad \forall \quad x_i \in X = [a, b, c, d]$$

# Optimal Synthesis

*Trajectory Generation: objective function*



Starting *pdf* page 23

# Optimal Synthesis

*Evolutionary Algorithms*



Starting *pdf page 28*

# Reference material



1. Synthesis of planar mechanisms (chapter from book)
2. *"Fundamentals of Machine Theory and Mechanisms"* book
3. Collection of applets on the Synthesis of Mechanisms (Geogebra)
4. Mechanics of Machinery (MOM) Module 6 Synthesis of Mechanisms (YouTube)

# Deserve "A" grade!

– Oleg Bulichev

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🏢 Room 105 (Underground robotics lab)