Mechanisms & Machines

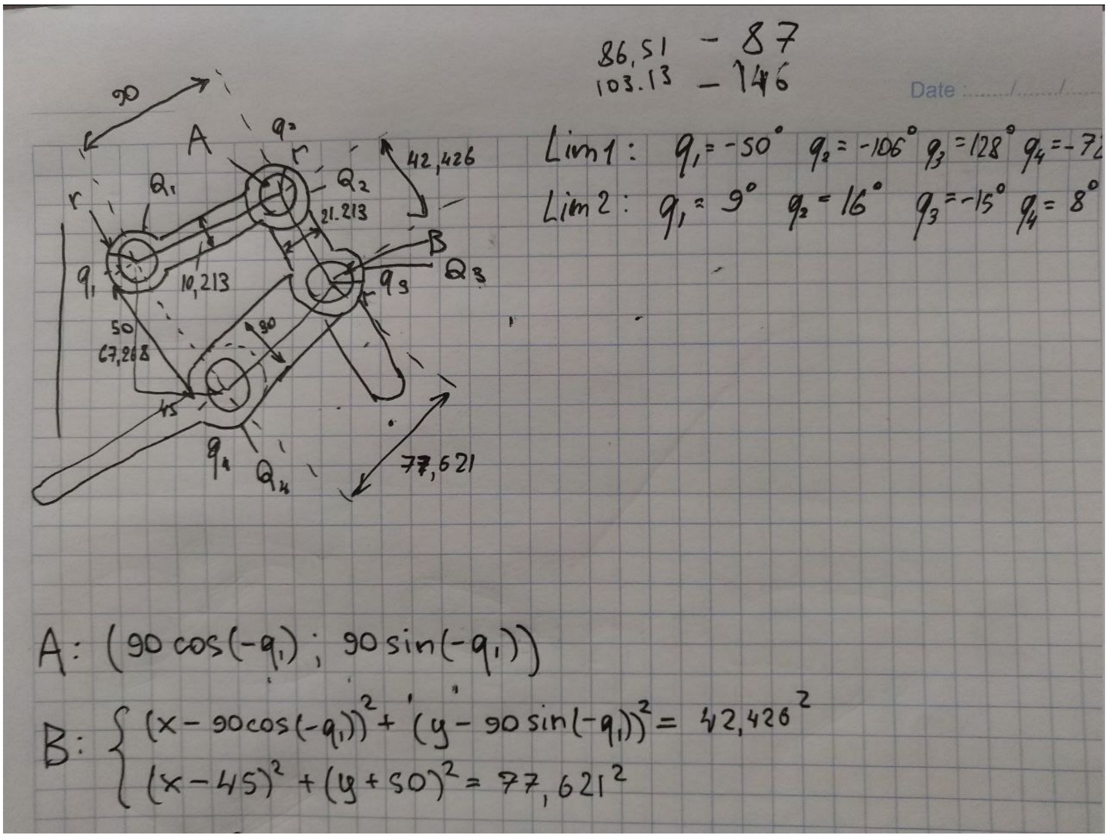
Homework 2_2

by @Jud1cator

Tasks:
1) Find angle limits:
    - Analytically
    - Using Fusion
2) Find torque for the 1st joint
    - Analytically
    - Using ANSYS

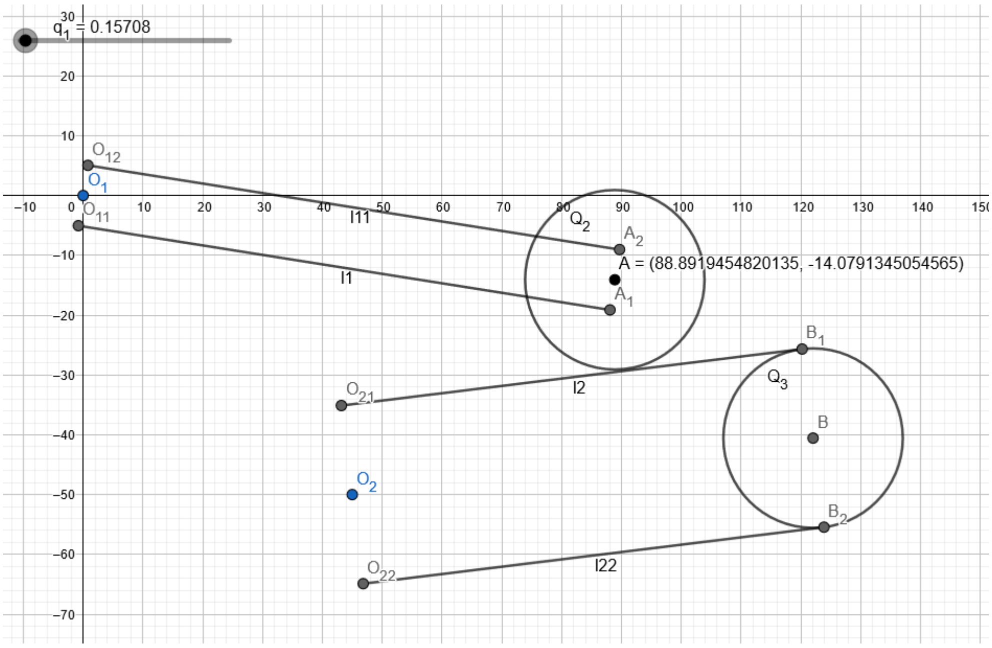
Angle limits

To find angle limits, we may represent a given mechanism as a 4-link bar and simulate it as a double-rocker (we can check the 4-link bar inequality to identify its type). Additionally, we have geometric constraints which put even more limits to motion of the mechanism (link width, joint radius).
We have q1 (angle of the first joint) as an input variable. Knowing it, we can get the equation of point A (see picture below). Then using numerical solution for nonlinear system of intersection of two circles we can get the position of point B.

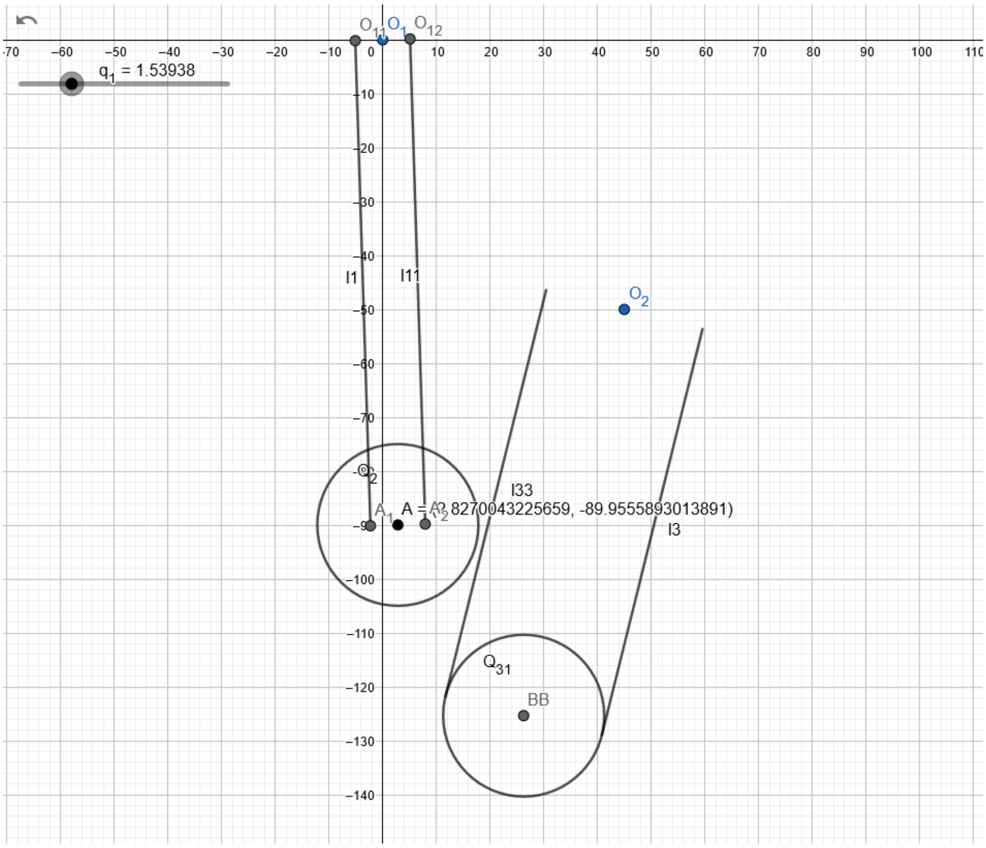


Thus, for any given angle q1 we can get the position of any point of the mechanism - knowing points A and B and link geometry. We will find the intersection points of links and joints (which will define joint limits) using GeoGebra (https://www.geogebra.org/calculator/g6qrehar)

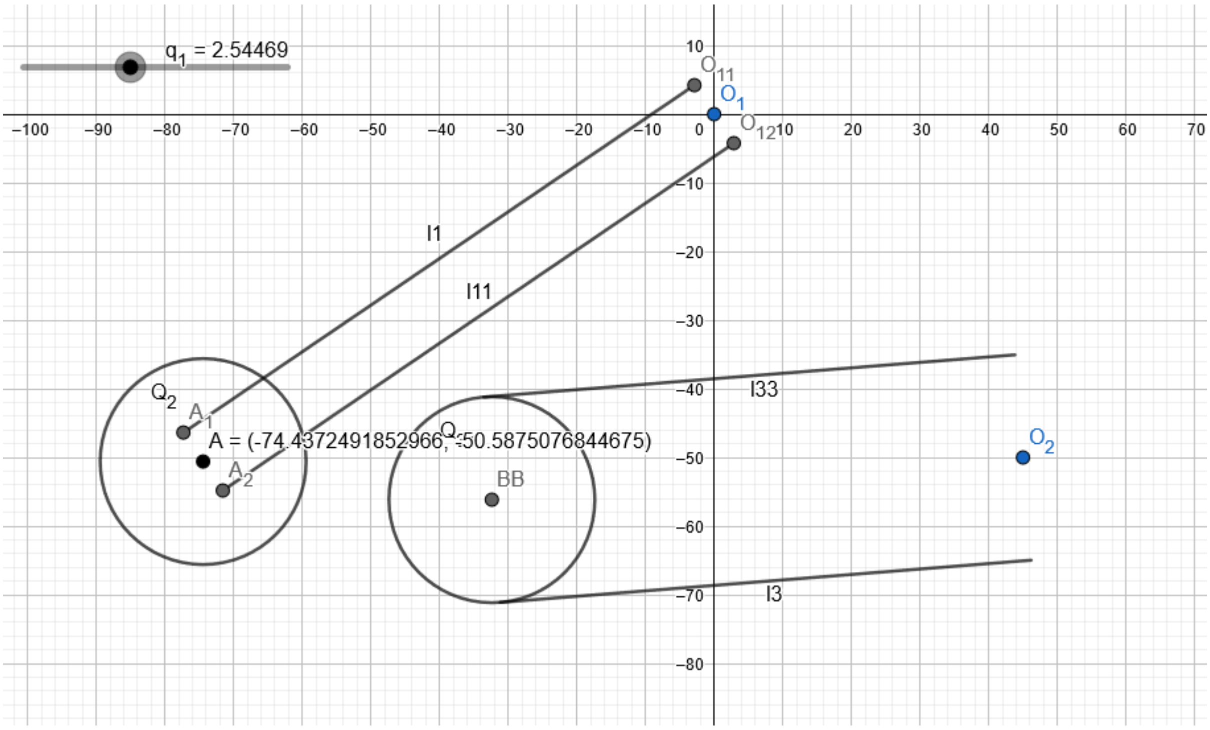
Limit 1: q1 = 0.157 rad = 8.64 deg (limit by geometry constraints)



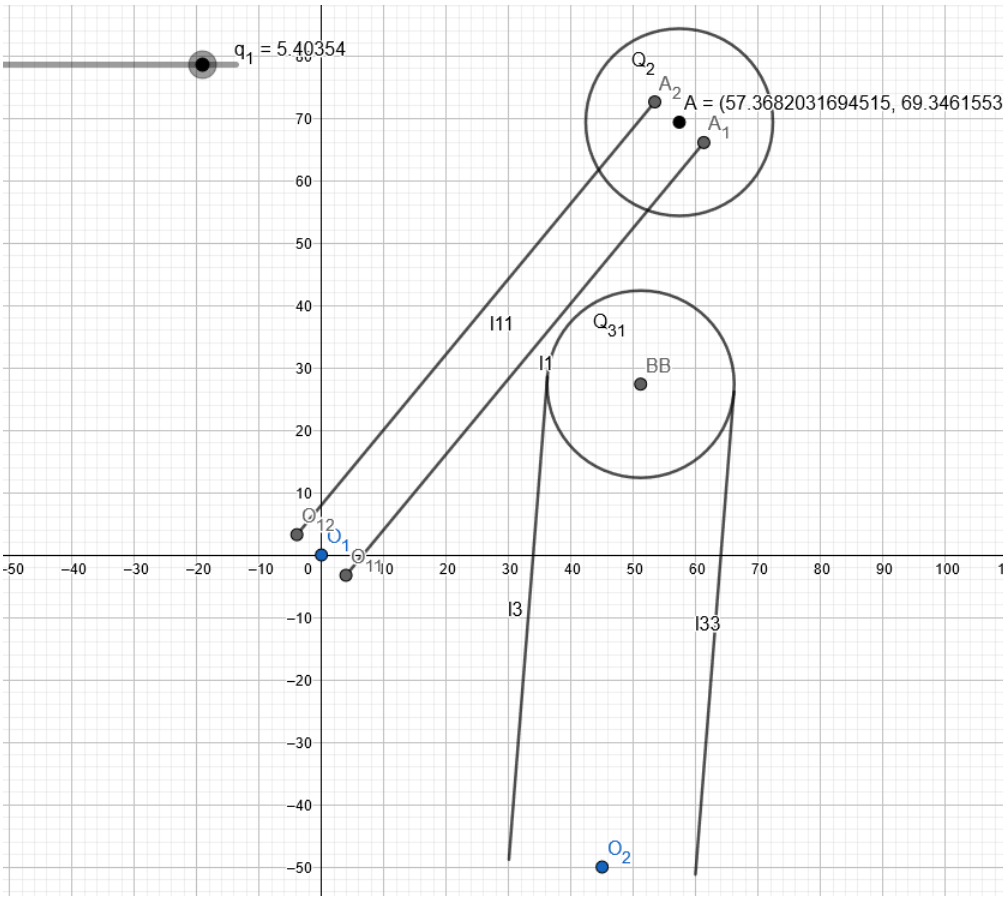
Limit 2: q1 = 1.539 rad = 88.18 deg (limit by geometry constraints)



Limit 3: q1 = 2.545 rad = 145.82 deg (limit by double-rocker constraints)

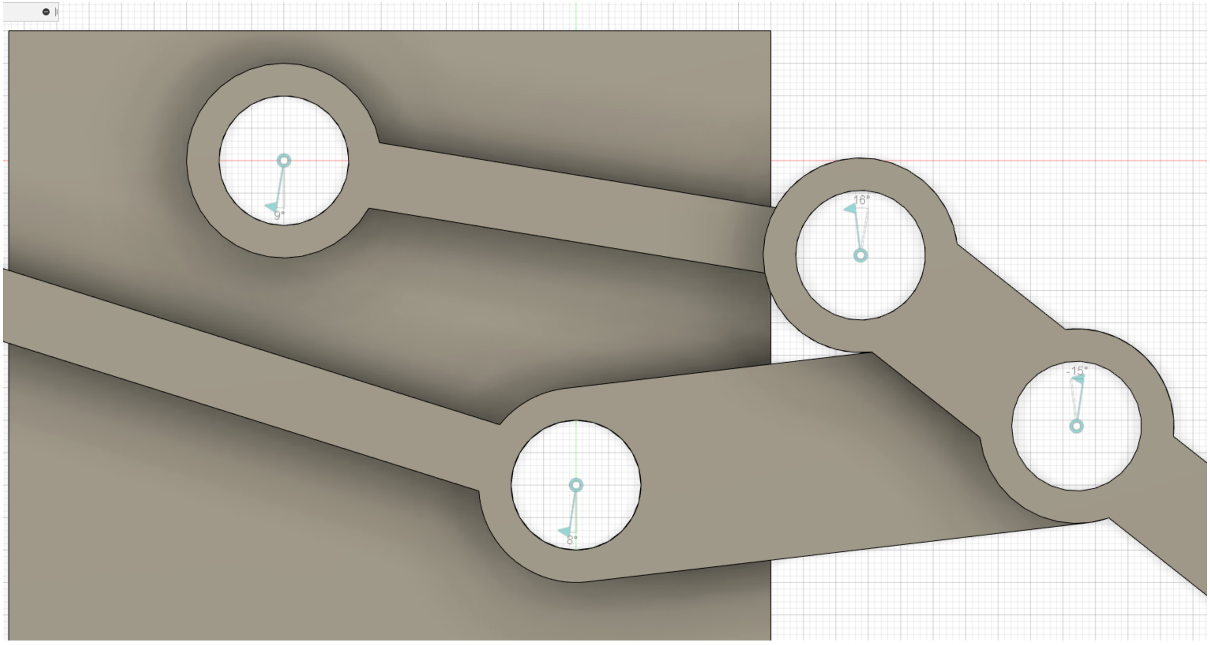


Limit 4: q1 = 5.404 rad = 309.63 deg (limit by geometry constraints)

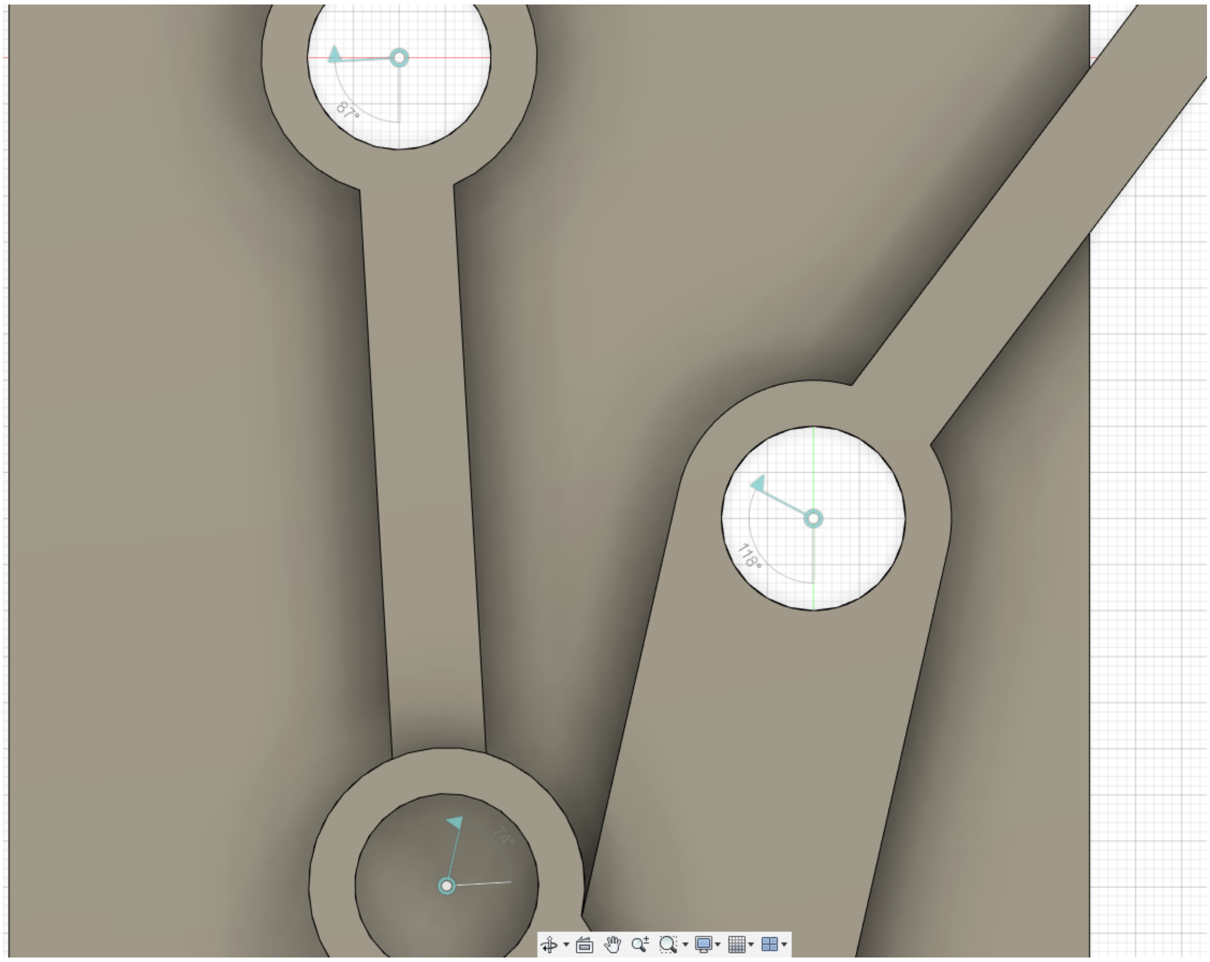


Angle limits from Fusion

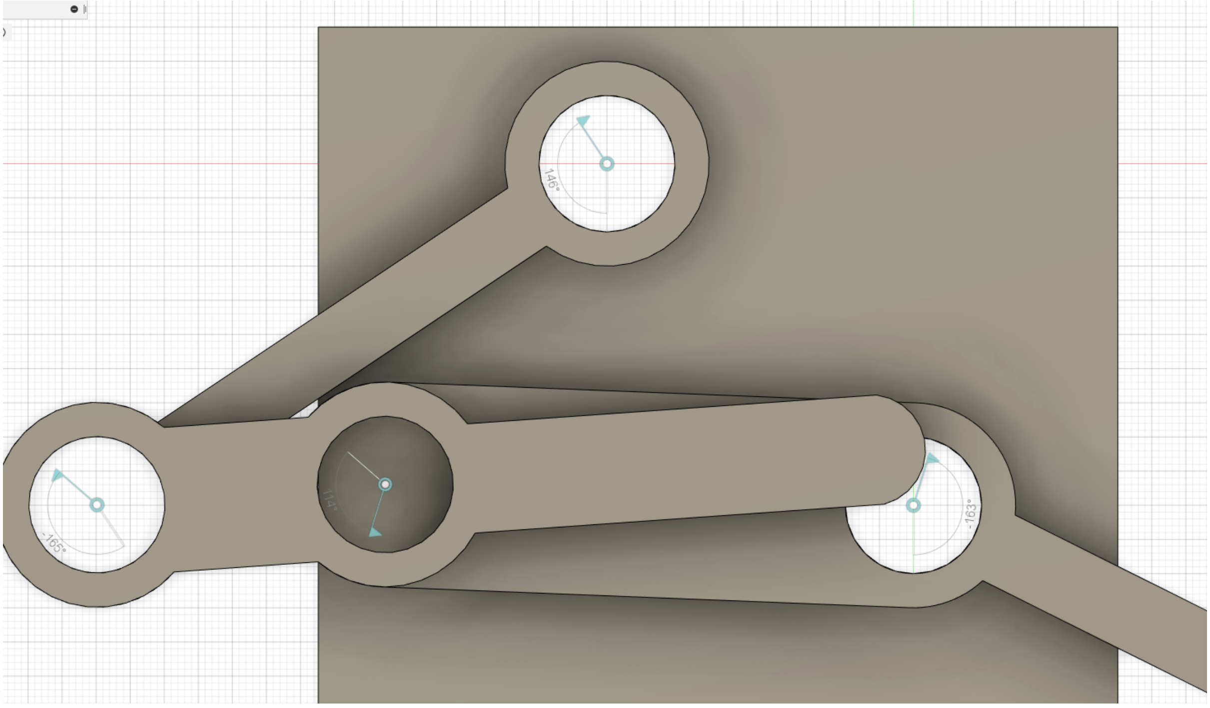
Limit 1: 9 deg



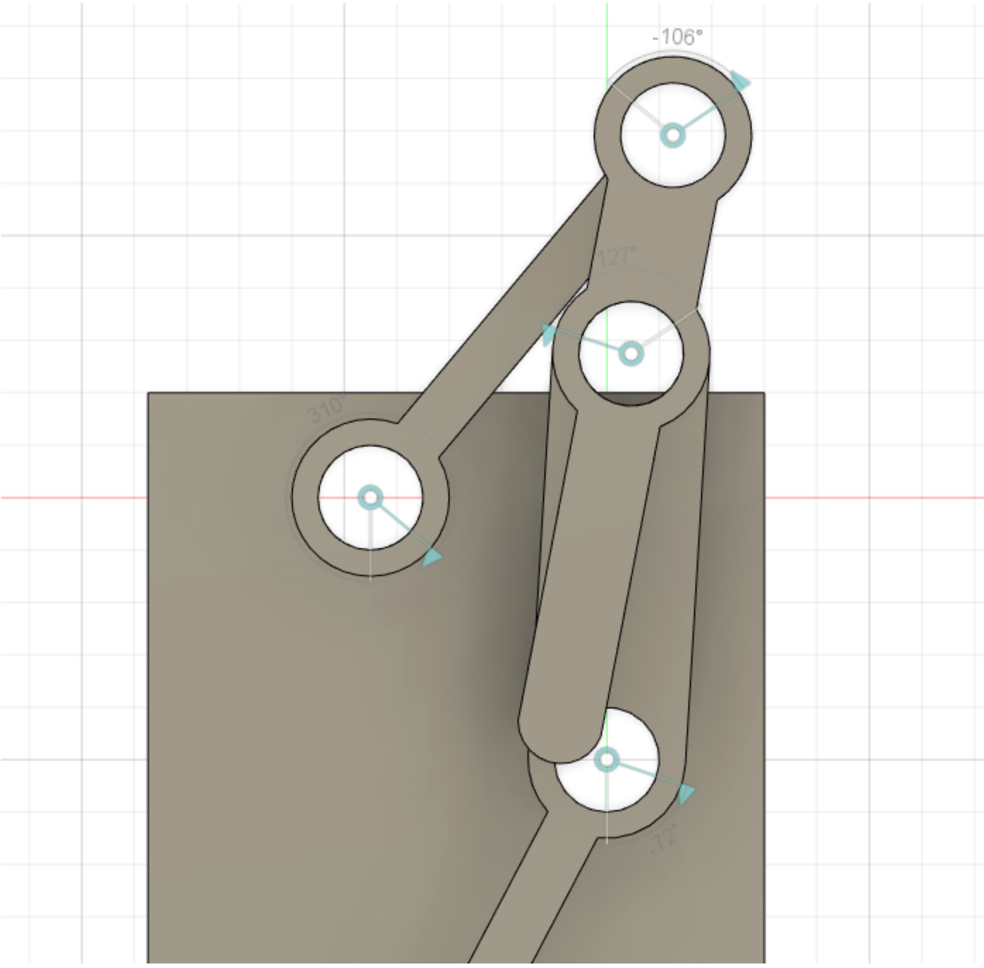
Limit 2: 87 deg



Limit 3: 146 deg

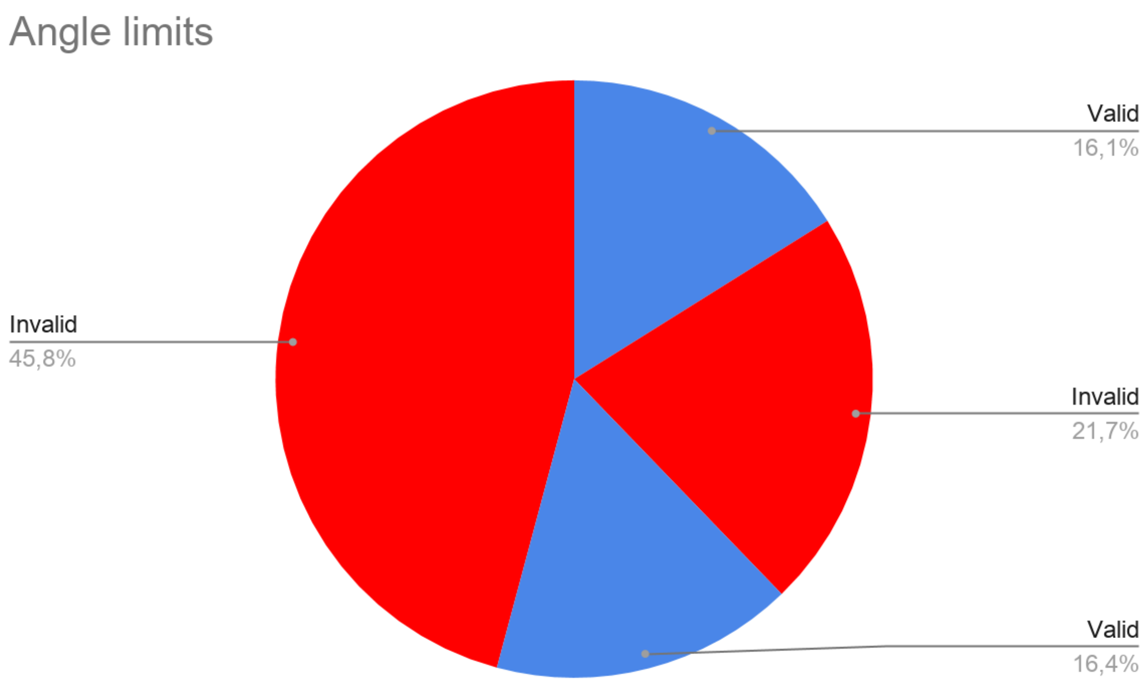


Limit 4: 310 deg



As we can see, angels we have calculated using GeoGebra correspond to ones from Fusion with sufficient precision.

Unfortunately, Google does not provide adequate pie chart editor, so it won't represent the 1st joint's rotational space (first valid sector starts at 310 deg)



Joint torque

The second task is to export given model to ANSYS, set bronze alloy for all materials, add gravity (directed in negative Y direction, downward looking as on screenshots from Fusion)

We will use Euler Lagrange method to find the torque of the first joint. Thus we will need to calculate kinetic and potential energy for our system. To calculate potential energy, we need positions of centers of masses of all links of our mechanism, and to calculate kinetic energy we need their velocities. The "easiest" way to find those functions is to obtain equations for positions of CMs from kinematics and differentiate them w.r.t. time to obtain velocities. The main difficulty here is point B and CM of the 2nd link. Previously we used numerical solution provided by GeoGebra, but now we need analytical expression: (https://math.stackexchange.com/questions/256100/how-can-i-find-the-points-at-which-two-circles-intersect)

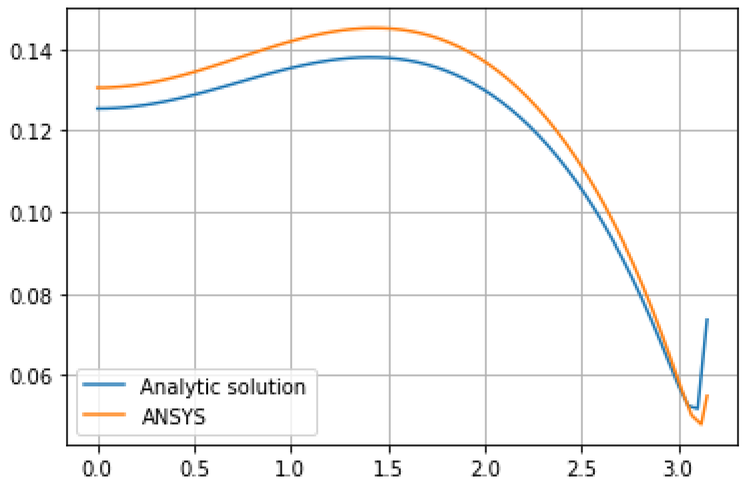
$$R = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

$$(x, y) = \frac{1}{2}(x_1 + x_2, y_1 + y_2) + \frac{r_1^2 - r_2^2}{2R^2}(x_2 - x_1, y_2 - y_1)$$
$$\pm \frac{1}{2}\sqrt{2\frac{r_1^2 + r_2^2}{R^2} - \frac{(r_1^2 - r_2^2)^2}{R^4} - 1}(y_2 - y_1, x_1 - x_2),$$

After obtaining analytical expressions for positions of all COMs, we can construct the Lagrangian and Euler-Lagrange equation to get the desired torque. For the details, you can see the code:
https://colab.research.google.com/drive/1LFHx4ZJjB0ALOuJds0oRKKUy_uRaqt29?usp=sharing

Below you can see our simulation results and results from ANSYS:

Our results vs ANSYS:



ANSYS results: