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Original source

The lecture is based on the "
Fundamentals of Machine Theory and Mechanisms" [Fundamentals of Machine Theory and Mechanisms | SpringerLink](). We are interested in Chapter 10. Link is here. [Synthesis of planar mechanisms.pdf — Яндекс.Диск]()

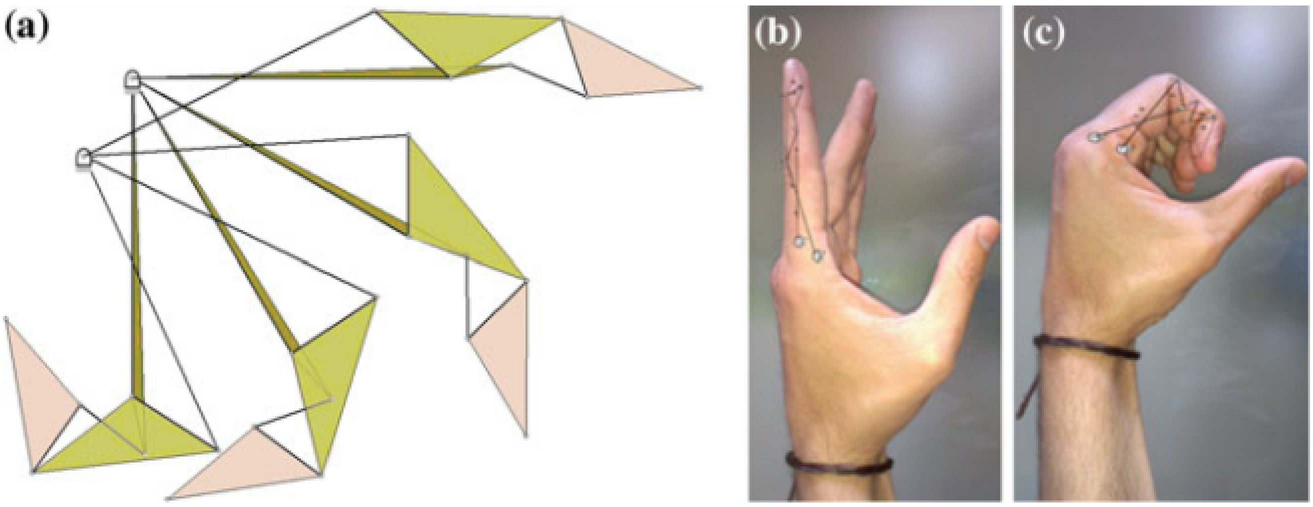
Difference between Analysis and Synthesis

Analysis allows determining whether a given system will comply with certain requirements or not.

Synthesis is the design of a mechanism so that it complies with previously specified requirements.

Types of synthesis

Two branches: structural synthesis and dimensional synthesis



Dimensional synthesis

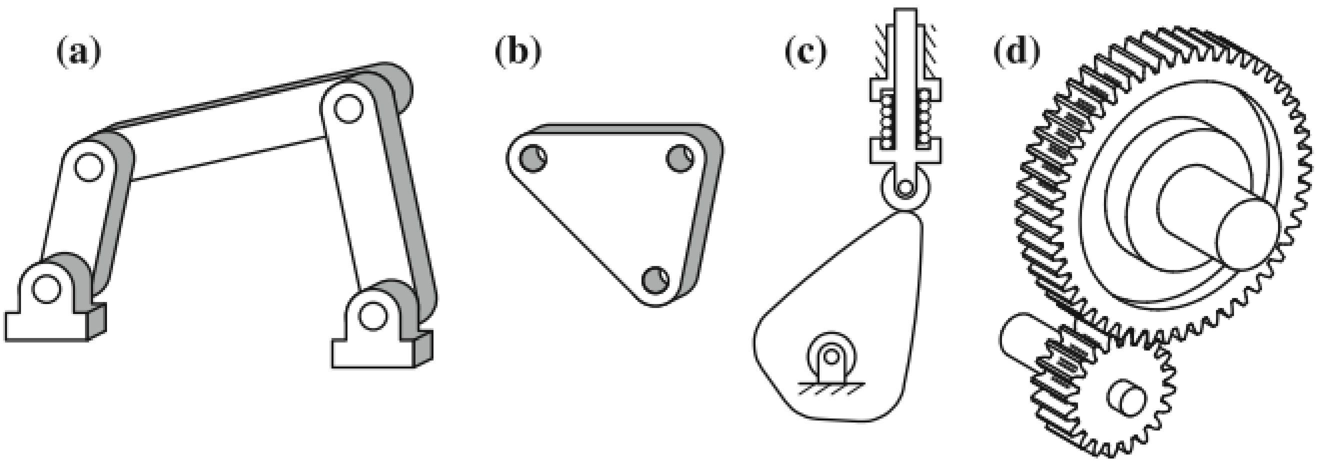
Structural synthesis

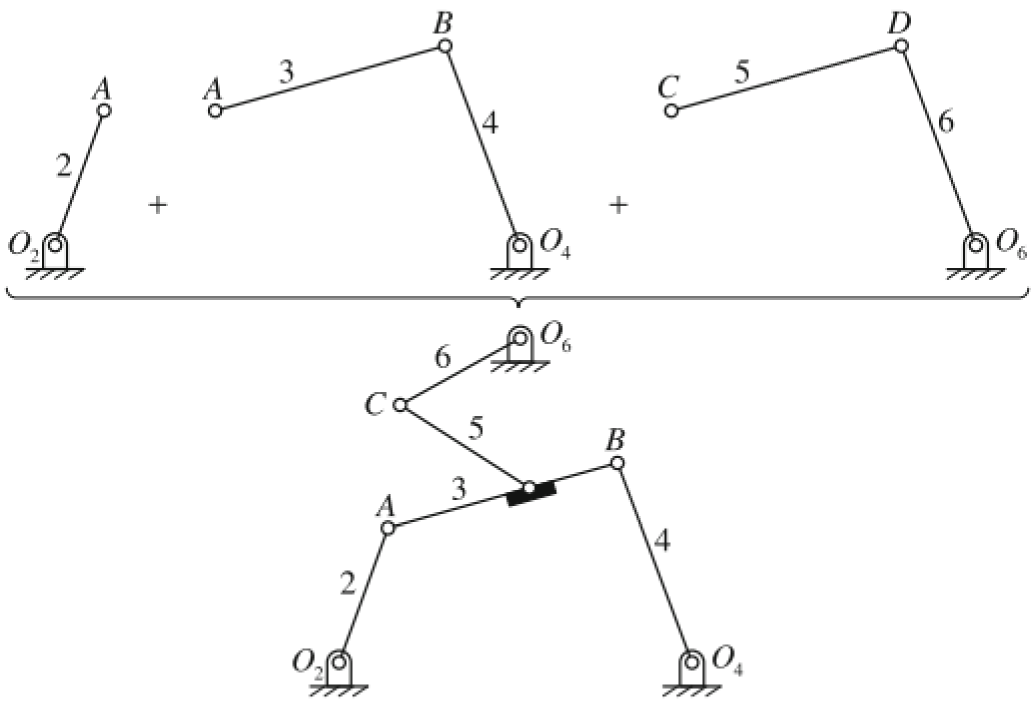
Defenition

This synthesis deals with the topological and structural study of mechanisms. It only considers the interconnectivity pattern of the links so that the results are unaffected by the changes in the geometric properties of the mechanisms.

Structural synthesis includes the following
Synthesis of type or Reuleaux synthesis:
Questions:
What type of mechanism is more suitable? What type of elements will it be made of?
Can it be formed by linkages, gears, flexible elements or cams?
Different configurations are developed according to the pre-established requirements.
The criteria to value the different characteristics of the mechanism are set

Synthesis of number or Grübler synthesis
In the case of a linkage, it determines the number of links and their configuration.





Structural Synthesis

case study from my Master Thesis

Разработка робота
Структурный синтез

Вопрос

Какое оптимальное количество ног должен иметь такой движитель?

Ответ

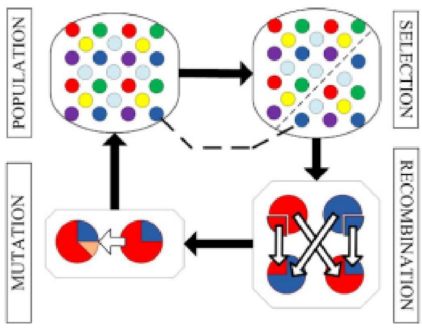
Решив задачу структурного синтеза,
результатом которого является движитель с **8—14 ногами**

Разработка робота
Используемые технологии

*Генерация поверхности (Параметризованная **искусственная территория**)*
Проблема формализации сложности поверхности

GAZEBO

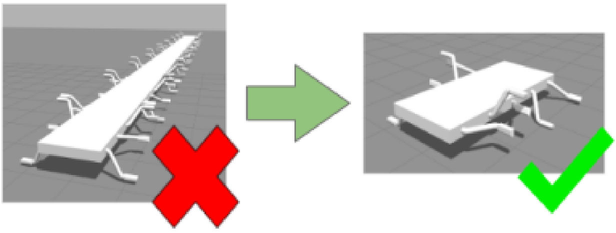
Робосимулятор (Неявная математическая модель)
Громоздкость явной модели

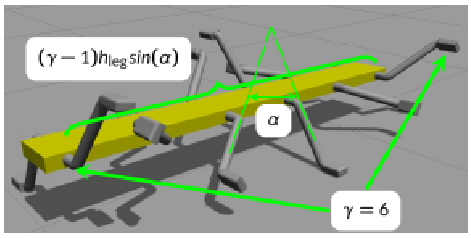


Генетический алгоритм
Отличен для дискретной глобальной мультикритериальной задачи оптимизации

Разработка робота

Предлагаемое решение





$$F \to max = \beta\left(\omega_1 \cdot \overbrace{\delta}^{\text{Дистанция}} + \omega_2 \cdot \overbrace{\frac{1}{(\gamma-1)h_{leg}sin(\alpha)}}^{\text{Упр. длина корпуса}}\right) + (1-\beta)\delta^{\omega_1}\left(\frac{1}{(\gamma-1)h_{leg}sin(\alpha)}\right)^{\omega_2}$$

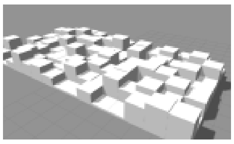
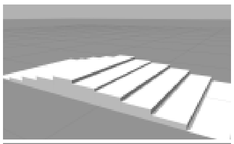
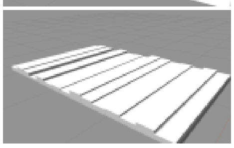
Идея: *Минимизировать кол-во ног без потери проходимости*

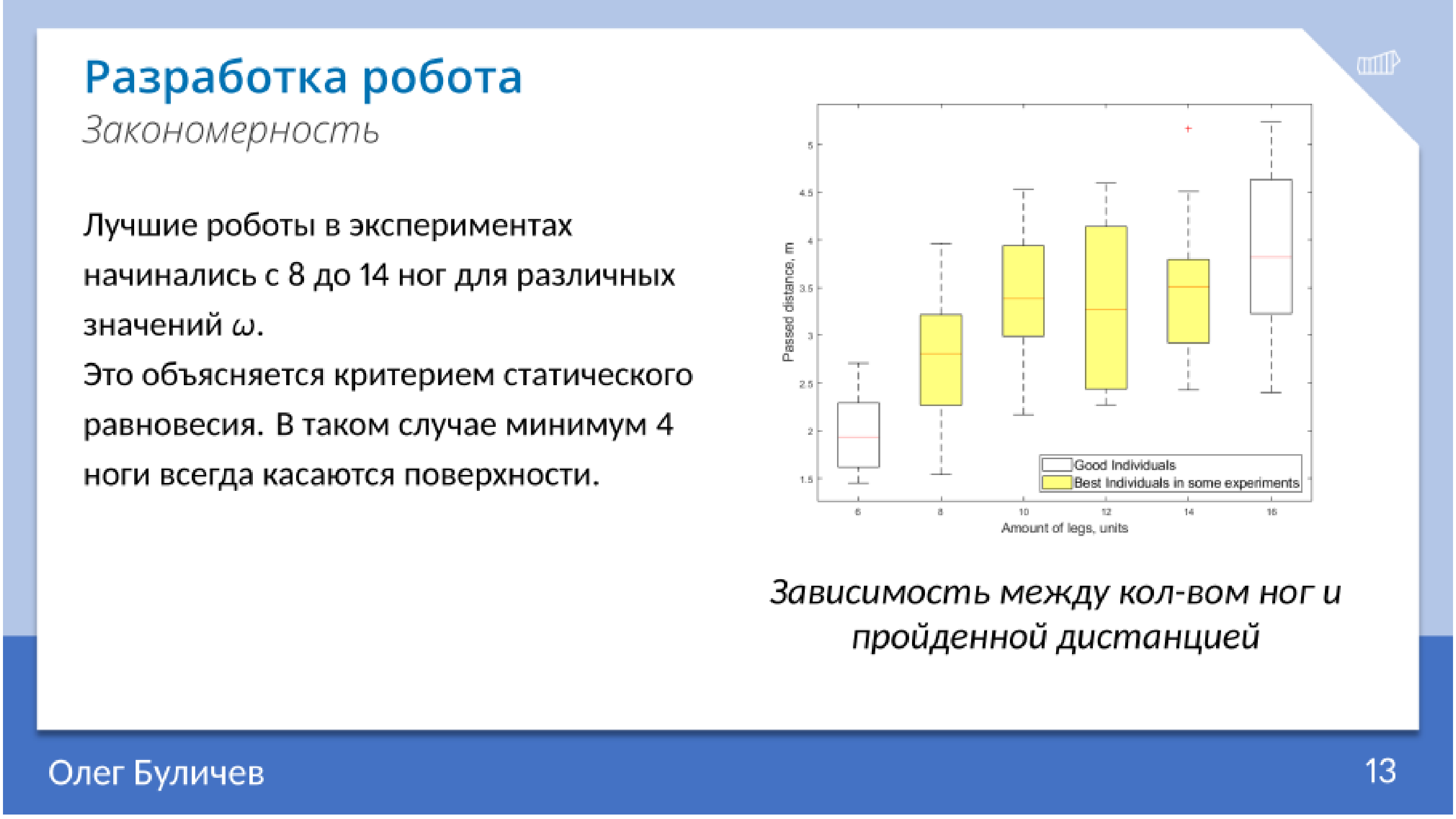
β – адаптивный параметр,

$\omega_{1,2} \in [0..1]$ – весовые коэффициенты.

Разработка робота

Конкретные результаты: $\omega_1 = 0.6$, $\omega_2 = 0.4$

	Тип территории	Кол-во ног	Угол между соседними ногами	Кол-во индивидов
Этап 1		12	73	200
Этап 2		12	72	55
		10	68	
		12	77	



Dimensional Synthesis

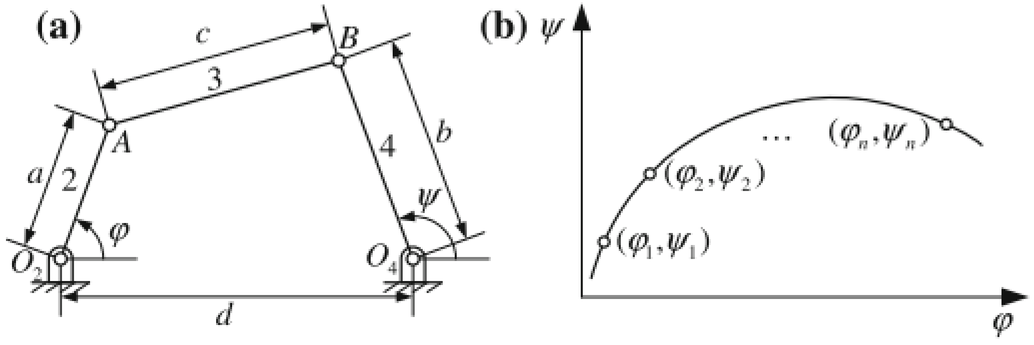
It focuses on the problem of obtaining the dimensions of a predefined mechanism that has to comply with certain given requirements. It will be necessary to define the dimension of the links and the position of the supports, among others.

2 types: function generation, trajectory generation

Function generation

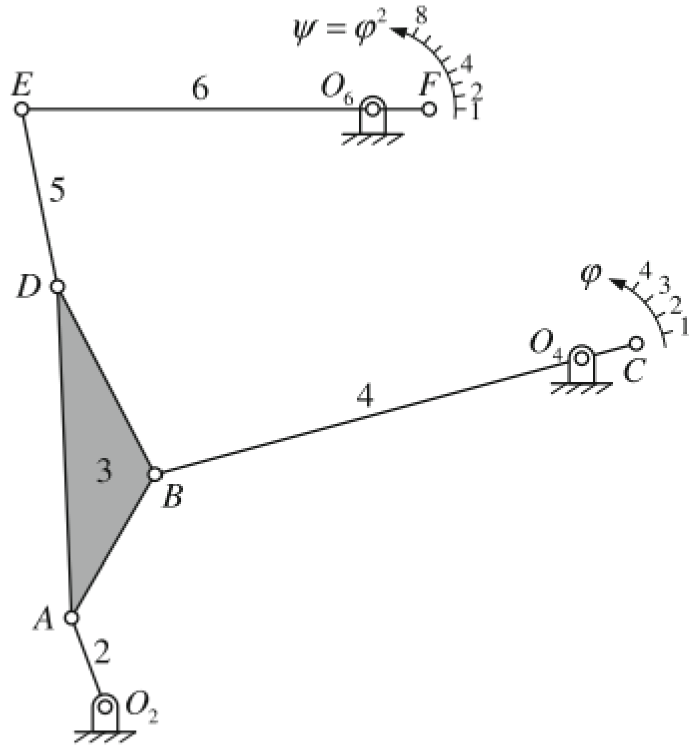
Problem explanation

Pre-established conditions refer to the relation between the input and output motions. These are defined by variables φ nd ψ that identify their positions (Fig. 10.7a). These parameters, normally used in synthesis of mechanisms, are equivalent to angles θ_2 and θ_4 used so far in this book. The relationship between φ and ψ can be defined by means of (Table 10.1) in which n pairs of these values are specified. These pairs can be set manually or according to a mathematical function. In this case, a series of precision points is used to generate the mechanism with exact correspondence between points or with a maximum error measured by means



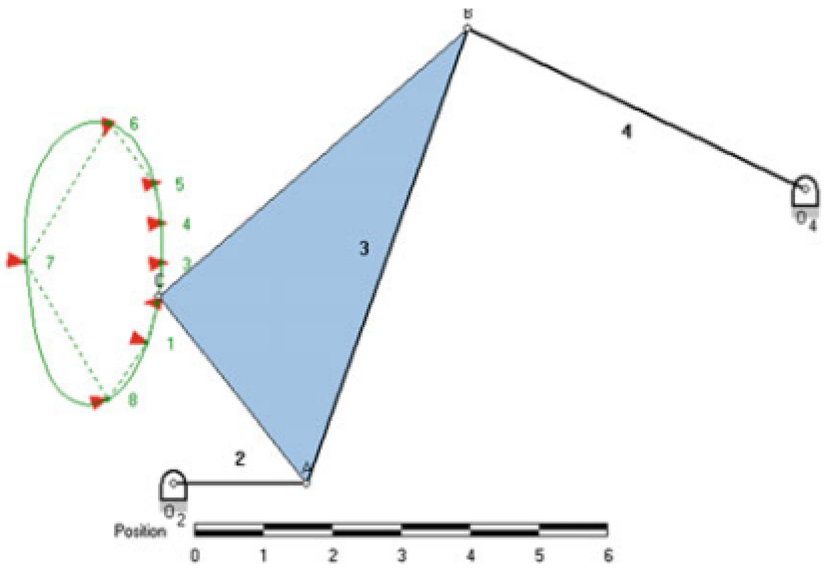
Application

Function generation can be used to design mechanisms that carry out mathematical operations: addition, differentiation, integration or a combination of them.
The first computers were mechanical devices based on this type of mechanisms.



Trajectory generation

Problem explanation



It studies and provides methods in order to obtain mechanisms in which one of the points describes a given trajectory

Types of solving methods

Graphical. These methods are very didactic and help us to understand the problem in an easy way. However, they offer a limited range of possibilities.
Analytical methods. They solve the problem by means of mathematical equations based on the requirements.

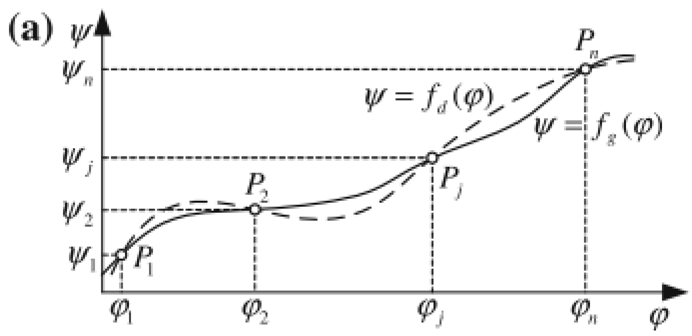
Optimization-technique-based methods. They can fi nd the optimal solution to the problem by means of the minimization of an objective function and the establishment of a series of restrictions. Different optimization techniques can be used.

Function Generation Synthesis

Formal definition

It can be defined as the part of synthesis that studies how the position of the input and output links in a mechanism relate to each other.

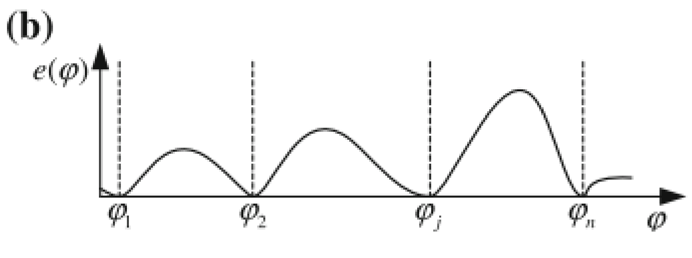
2 types of representation:
Continious function (desired and generated)



Precision points

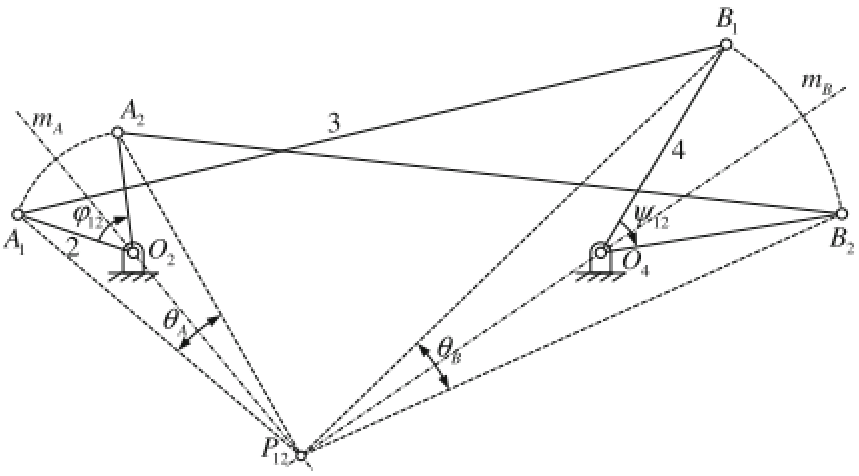
φ	φ_1	φ_2	...	φ_n
ψ	ψ_1	ψ_2	...	ψ_n

error function



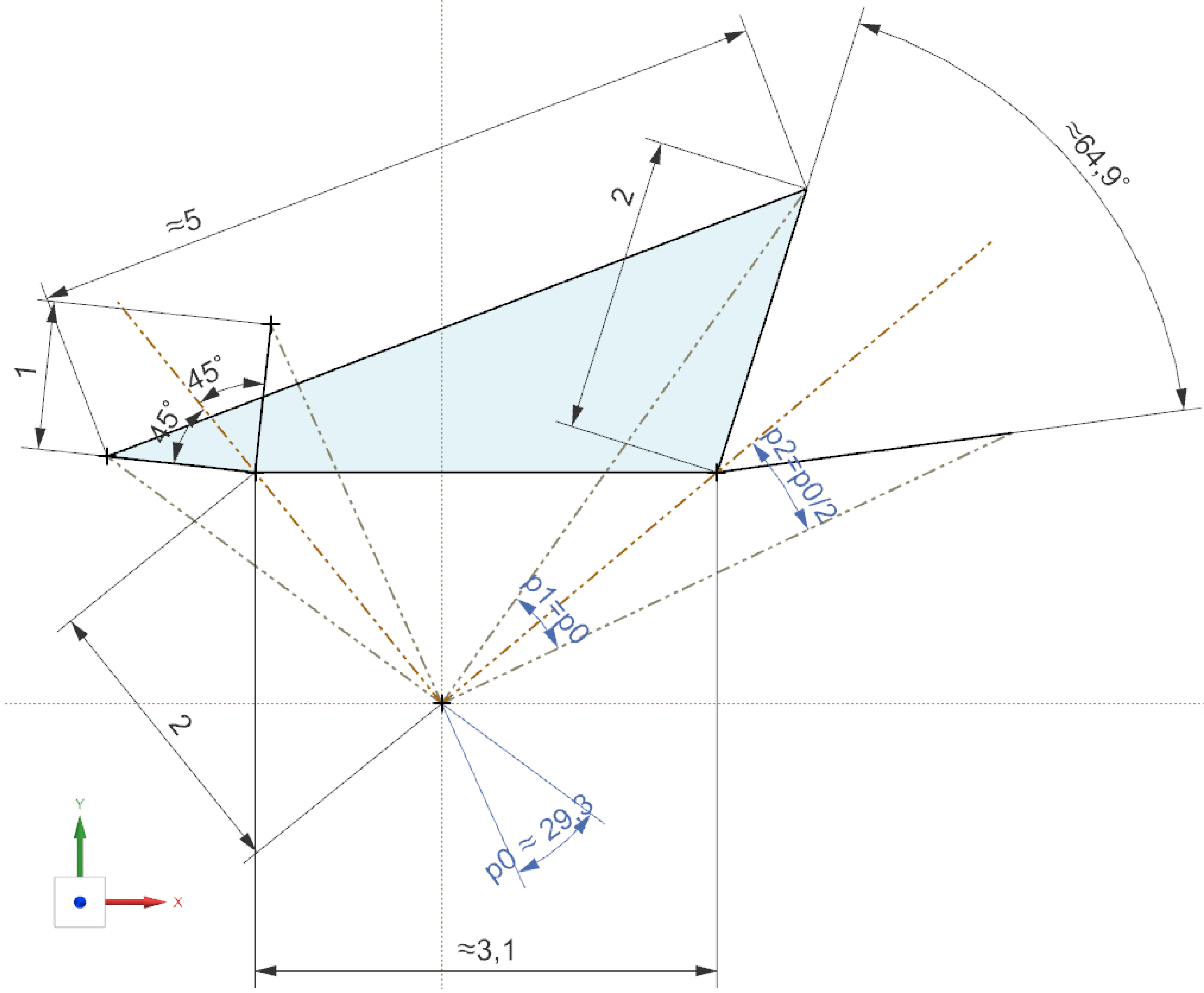
Graphical method

As an example, we will generate a four-bar mechanism in which a rotated angle of the input link between positions 1 and 2, u 12
, corresponds to a rotated angle of output link w 12



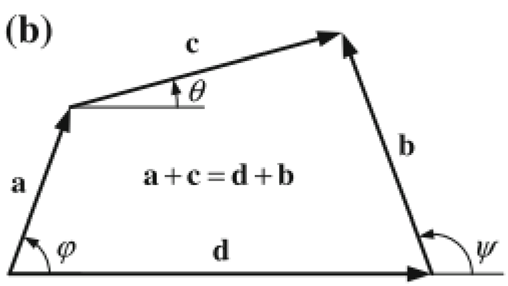
1. We take an arbitrary point and name it pole P_{12}. We draw line m_A and any point O_2 in it.
2. Next we choose a value for the length of the crank and, taking point O_2 as the origin, we draw points A_1 and A_2 in symmetric positions with respect to line m_A. The angle formed by the points and the origin, has to be equal to specified angle φ_{12}.
3. Points A_1, A_2 and P_{12} are connected so that they form angle $\theta_A = A_1\widehat{P_{12}}A_2$.
4. Taking P_{12} as the origin, we draw a new arbitrary line, m_B, and take point O_4 in it.
5. Taking point P_{12} as the origin, we draw two lines that comply with two conditions, the angle they form is equal to $\theta_B = B_1\widehat{P_{12}}B_2 = \theta_A$ and m_B is their bisector.
6. Taking point O_4 as the origin we draw two lines with the condition that the angle they form is equal to ψ_{12} and that m_B is their bisector.
7. The intersection points of these four lines define points B_1 and B_2.

8. The solution obtained is mechanism $\{O_{2,}, A_1, B_1, O_4\}$ as well as its second position, given by $\{O_{2,}, A_2, B_2, O_4\}$.



Analytical method (Freudenstein ’ s Method)

Выкладки (pdf pages 11)



a+b = d + b

$$\left.\begin{aligned} a\cos\phi + c\cos\theta &= d + b\cos\psi \\ a\sin\phi + c\sin\theta &= b\sin\psi \end{aligned}\right\}$$

In order to find the relation $\psi = f(\varphi)$.

$$c^2 = d^2 + b^2 + a^2 + 2bd\cos\psi - 2ab\cos(\psi - \phi) - 2da\cos\phi$$

$$\left.\begin{aligned} R_1 &= \frac{d}{a} \\ R_2 &= \frac{d}{b} \\ R_3 &= \frac{d^2 + b^2 + a^2 - c^2}{2ab} \end{aligned}\right\}$$

$$R_1 \cos\psi - R_2 \cos\phi + R_3 = \cos(\psi - \phi)$$

Equation (10.8) is known as Freudenstein's equation and it is an effective tool to carry out function generation synthesis. We can obtain the length of links a, b, c and d in a four-bar mechanism, provided that we know three related positions of the input and output links. These positions are defined by pairs $(\varphi_1, \psi_1), (\varphi_2, \psi_2)$ and (φ_3, ψ_3) which are known as precision points.

$$\left.\begin{aligned} R_1 \cos\psi_1 - R_2 \cos\phi_1 + R_3 &= \cos(\psi_1 - \phi_1) \\ R_1 \cos\psi_2 - R_2 \cos\phi_2 + R_3 &= \cos(\psi_2 - \phi_2) \\ R_1 \cos\psi_3 - R_2 \cos\phi_3 + R_3 &= \cos(\psi_3 - \phi_3) \end{aligned}\right\}$$

This system is linear and independent. It can easily be solved to obtain unknowns R_1, R_2 and R_3. Using the calculated values in the mathematical definitions of these parameters, we can find the length of links a, b, c and d by assigning an arbitrary value to one of them, for example, $d = 1$. In this case, the size of the mechanism obtained will depend on the value given to d, but it can be escalated to any size.

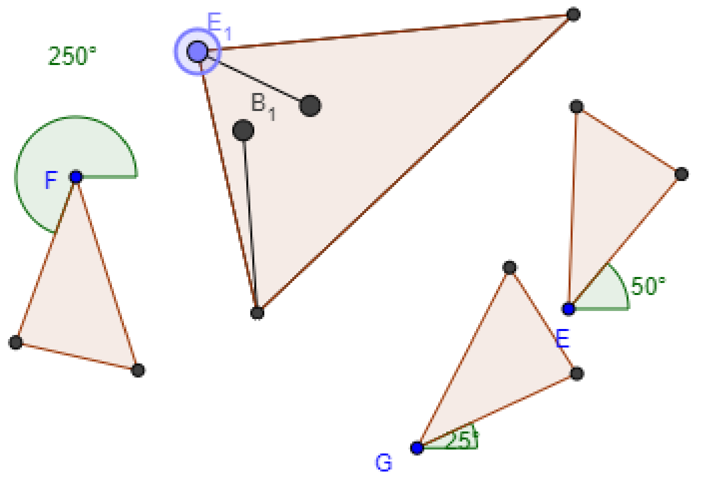
```
ans =

    4.4520
    4.4520


ans =

    3.3606
    3.3606


ans =

    2.0814
    2.0814
```

```
theta_1 = deg2rad(173.9);
theta_2 = deg2rad(83.9);
theta_3 = deg2rad(141.1958);
psi_1 = deg2rad(7.6);
psi_2 = deg2rad(72.5);
psi_3 = deg2rad(60.1145);

a = 1;

A = [cos(psi_1) -cos(theta_1) 1 ;
    cos(psi_2) -cos(theta_2) 1;
    cos(psi_3) -cos(theta_3) 1];
B = [cos(psi_1 - theta_1); cos(psi_2 - theta_2); cos(psi_3 - theta_3)];

X = linsolve(A,B)

syms b c d;
eqns = [X(1) == d/a; X(2) == d/b; X(3) == (d^2 + b^2 + a^2 - c^2)/(2*a*b)];
res = solve(eqns);
abs(double(res.b))
abs(double(res.c))
abs(double(res.d))
```

Trajectory Generation Synthesis

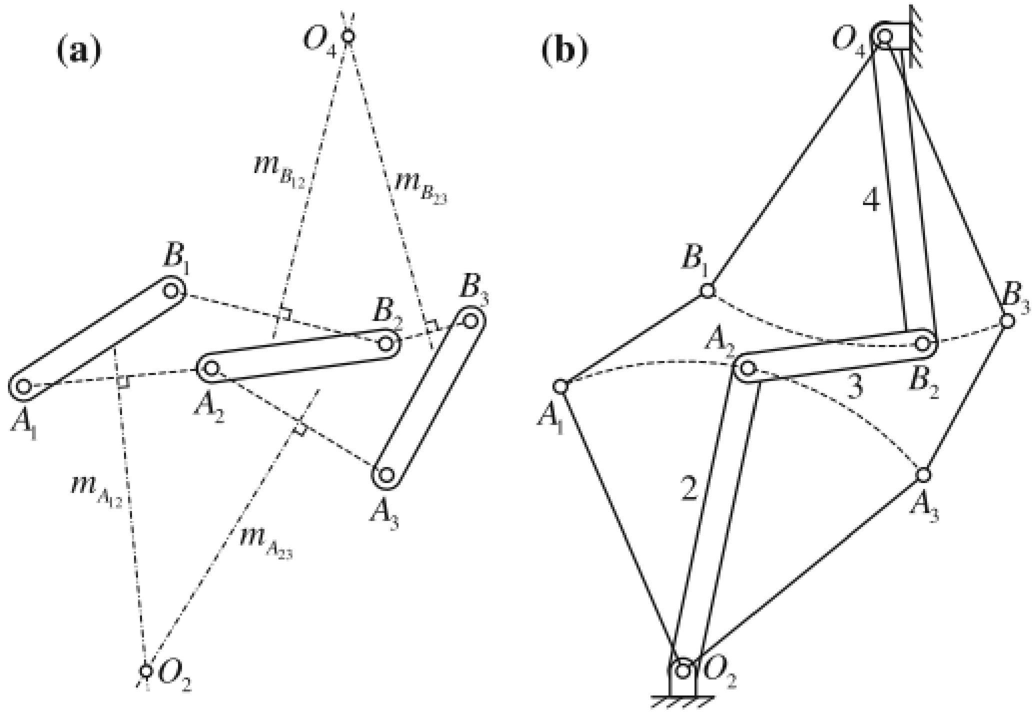
Formal definition

Relationship between the trajectory described by a point in a link and the motion of another link, usually the input one.



Graphical methods

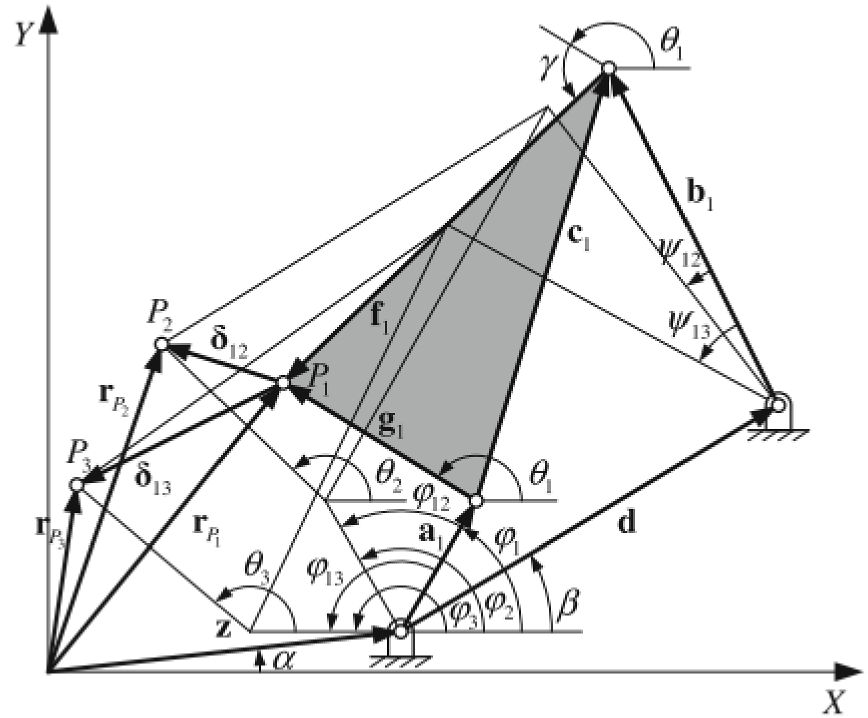
his method allows finding a four-bar mechanism in which the coupler link passes through the three specified positions. The steps to follow are the next ones:



1. The three positions of link AB are considered known and they are identified as A_1B_1, A_2B_2 and A_3B_3.
2. We draw a segment between points A_1 and A_2 and then its perpendicular bisector $m_{A_{12}}$. The same way we draw a segment between points A_2 and A_3 and then its perpendicular bisector $m_{A_{23}}$.
3. The intersection point of both bisectors is point O_2 (Fig. 10.16a).

4. We operate the same way drawing segments B_1B_2 and B_2B_3 with their perpendicular bisectors $m_{B_{12}}$ and $m_{B_{23}}$.
5. Their intersection point defines the position of point O_4.
6. Hence we obtain the mechanism we were looking for, $\{O_2, A, B, O_4\}$ (Fig. 10.16b).

Analytical method (based on complex numbers)



$$\mathbf{r}_{P_1} = \mathbf{z} + \mathbf{a}_1 + \mathbf{g}_1$$

$$\mathbf{r}_{P_1} = \mathbf{z} + \mathbf{d} + \mathbf{b}_1 + \mathbf{f}_1$$

$$\left.\begin{aligned} \mathbf{r}_{P_1} &= z_1 \mathrm{e}^{i\alpha} + a\mathrm{e}^{i\varphi_1} + g\mathrm{e}^{i\theta_1} \\ \mathbf{r}_{P_2} &= z_1 \mathrm{e}^{i\alpha} + a\mathrm{e}^{i\varphi_2} + g\mathrm{e}^{i\theta_2} \end{aligned}\right\}$$

$$\boldsymbol{\delta}_{12} = \mathbf{r}_{P_2} - \mathbf{r}_{P_1} = a\left(\mathrm{e}^{i\varphi_2} - \mathrm{e}^{i\varphi_1}\right) + g\left(\mathrm{e}^{i\theta_2} - \mathrm{e}^{i\theta_1}\right)$$

$$\boldsymbol{\delta}_{13} = \mathbf{r}_{P_3} - \mathbf{r}_{P_1} = a\left(\mathrm{e}^{i\varphi_3} - \mathrm{e}^{i\varphi_1}\right) + g\left(\mathrm{e}^{i\theta_3} - \mathrm{e}^{i\theta_1}\right)$$

$$\mathbf{a}_1 = \frac{\begin{vmatrix} \boldsymbol{\delta}_{12} & \mathrm{e}^{i\theta_{12}} - 1 \\ \boldsymbol{\delta}_{13} & \mathrm{e}^{i\theta_{13}} - 1 \end{vmatrix}}{\begin{vmatrix} \mathrm{e}^{i\varphi_{12}} - 1 & \mathrm{e}^{i\theta_{12}} - 1 \\ \mathrm{e}^{i\varphi_{13}} - 1 & \mathrm{e}^{i\theta_{13}} - 1 \end{vmatrix}}$$

$$\mathbf{g}_1 = \frac{\begin{vmatrix} \mathrm{e}^{i\varphi_{12}} - 1 & \boldsymbol{\delta}_{12} \\ \mathrm{e}^{i\varphi_{13}} - 1 & \boldsymbol{\delta}_{13} \end{vmatrix}}{\begin{vmatrix} \mathrm{e}^{i\varphi_{12}} - 1 & \mathrm{e}^{i\theta_{12}} - 1 \\ \mathrm{e}^{i\varphi_{13}} - 1 & \mathrm{e}^{i\theta_{13}} - 1 \end{vmatrix}}$$

In order to obtain $\mathbf{b}_1$ and $\mathbf{f}_1$, we write Eq. (10.14) for positions 1 and 2 (Eq. 10.26):

$$\left.\begin{aligned} \mathbf{r}_{P_1} &= z e^{i\alpha} + d e^{i\beta} + b e^{i\psi_1} + f e^{i(\theta_1+\gamma)} \\ \mathbf{r}_{P_2} &= z e^{i\alpha} + d e^{i\beta} + b e^{i\psi_2} + f e^{i(\theta_2+\gamma)} \end{aligned}\right\} \tag{10.26}$$

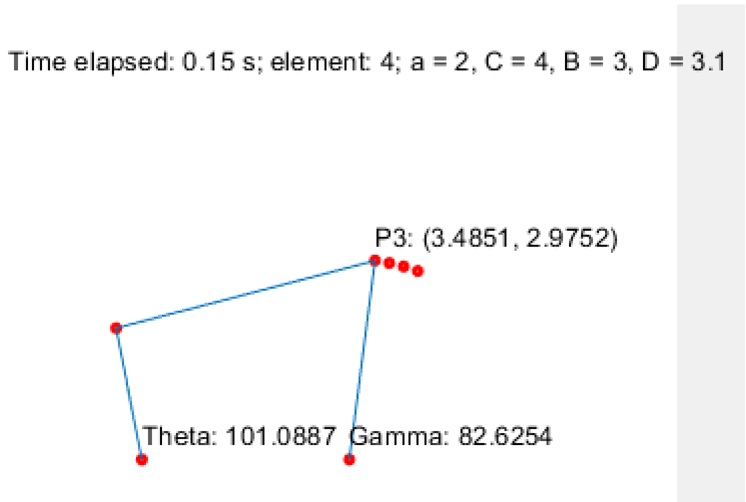
$$\boldsymbol{\delta}_{12} = \mathbf{r}_{P_2} - \mathbf{r}_{P_1} = b(e^{i\psi_2} - e^{i\psi_1}) + f(e^{i(\theta_2+\gamma)} - e^{i(\theta_1+\gamma)})$$

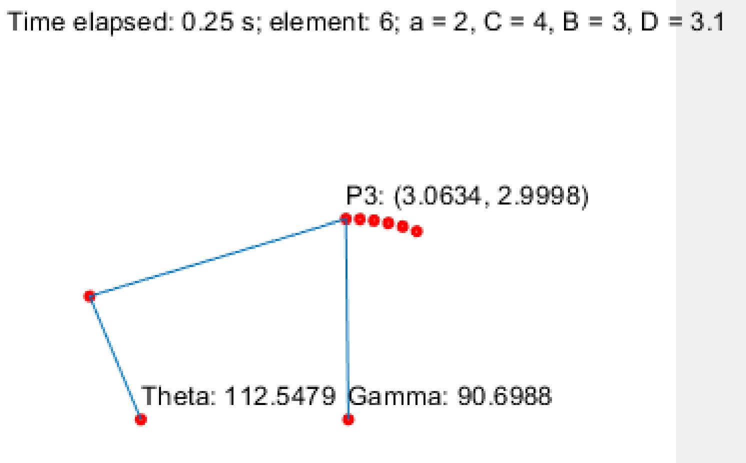
$$\left.\begin{aligned} \boldsymbol{\delta}_{12} &= \mathbf{b}_1(e^{i\psi_{12}} - 1) + \mathbf{f}_1(e^{i\theta_{12}} - 1) \\ \boldsymbol{\delta}_{13} &= \mathbf{b}_1(e^{i\psi_{13}} - 1) + \mathbf{f}_1(e^{i\theta_{13}} - 1) \end{aligned}\right\}$$

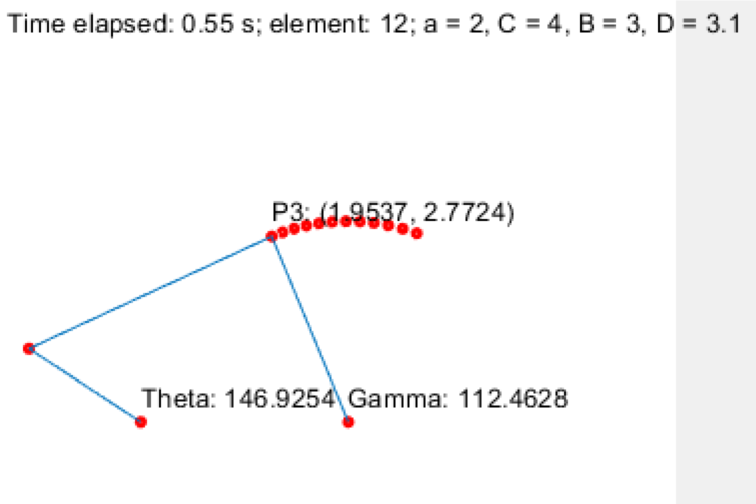
$$\mathbf{b}_1 = \frac{\begin{vmatrix} \boldsymbol{\delta}_{12} & e^{i\theta_{12}} - 1 \\ \boldsymbol{\delta}_{13} & e^{i\theta_{13}} - 1 \end{vmatrix}}{\begin{vmatrix} e^{i\psi_{12}} - 1 & e^{i\theta_{12}} - 1 \\ e^{i\psi_{13}} - 1 & e^{i\theta_{13}} - 1 \end{vmatrix}}$$

$$\mathbf{g}_1 = \frac{\begin{vmatrix} e^{i\psi_{12}} - 1 & \boldsymbol{\delta}_{12} \\ e^{i\psi_{13}} - 1 & \boldsymbol{\delta}_{13} \end{vmatrix}}{\begin{vmatrix} e^{i\psi_{12}} - 1 & e^{i\theta_{12}} - 1 \\ e^{i\psi_{13}} - 1 & e^{i\theta_{13}} - 1 \end{vmatrix}}$$

$$\left.\begin{aligned} &\mathbf{z} = \mathbf{r}_{P_1} - \mathbf{a}_1 - \mathbf{g}_1 \\ &\mathbf{d} = \mathbf{r}_{P_1} - \mathbf{z} - \mathbf{b}_1 - \mathbf{f}_1 \\ &\mathbf{c}_1 = \mathbf{d} + \mathbf{b}_1 - \mathbf{a}_1 \end{aligned}\right\}$$







Optimal Synthesis of Mechanisms

Formal definition

In general, the solution of an optimization problem determines the value of the variables $(x_1, x_2, ..., x_n)$ that minimize objective function $f(x)$ subject to a set of constraints (Eq. 10.37). This can be written as:

$$\begin{aligned}
&\min f(x_1, x_2, ..., x_n) \\
&\text{Subject to:} \\
&\quad h_j(x_1, x_2, ..., x_n) \le 0 \quad j = 1, 2, ..., m \\
&\quad g_k(x_1, x_2, ..., x_n) = 0 \quad k = 1, 2, ..., p
\end{aligned} \tag{10.37}$$

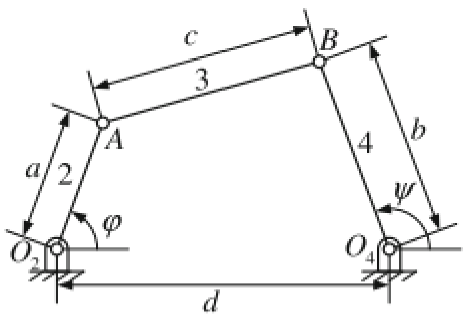
Function $f(x)$ is called objective function and functions $h_j(x)$ and $g_k(x)$ are called constraints of the problem. We can have both inequality and equality constraints. In the context of engineering design, the above mentioned concepts are defined as:

Function $f(x)$ is called objective function and functions $h_j(x)$ and $g_k(x)$ are called constraints of the problem. We can have both inequality and equality constraints. In the context of engineering design, the above mentioned concepts are defined as:

- Objective function: A function that expresses a fundamental aspect of the problem. An extreme value (minimum or maximum) is sought along the process of optimization. This function is often called merit function. Multifunctional functions, in which several features are optimized, can also be formulated. In this case, each one of them is weighted depending on their importance.
- Independent design variables: Such variables represent the geometry of the model. They are usually the dimensions of the mechanism such as the length or width of the links.
- Dependent variables: These are parameters that have to be included in the formulation of the objective function or the constraints but that depend on the design variables.
- Constraints: They are mathematical functions that define the relationships between the design variables that have to be met by every set of values that define a possible design. These relationships can be of three types.

  - Inequality restrictions: They are usually limitations to the behavior of the mechanism or security restrictions to prevent failure under certain conditions.
  - Variable limits: They are a specific case of the previous ones.
  - Equality restrictions: They are conditions that have to be met strictly in order for the design to be acceptable.

Function Generation Synthesis About fitness function and errors



We will use Freudenstein's equation (Eq. 10.8) and the method developed in Appendix B of this book to obtain the relationship between the output angle, ψ, and input one, φ (Eq. 10.38):

$$\psi = 2\arctan\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \psi(a, b, c, d, \varphi) \tag{10.38}$$

$$\left.\begin{array}{l} A = \cos\varphi - R_1 - R_2\cos\varphi + R_3 \\ B = -2\sin\varphi \\ C = R_1 - (R_2+1)\cos\varphi + R_3 \end{array}\right\}$$

where R_1, R_2 and R_3 are known functions of a, b, c, and d.

$$\min \sum_{i=1}^{N} \left(\psi_i(X, \varphi_i^d) - \psi_i^d\right)^2$$

Subject to:

$$x_i \in [li_i, ls_i] \quad \forall \ x_i \in X = [a, b, c, d]$$

REF

This is a collection of applets on the Synthesis of Mechanisms in the context of the "Mechanism and Machine Theory" subject in the Mechanical Engineering Degree at the Public University of Navarra.
MMT: Synthesis of Mechanisms – GeoGebra

Fundamentals of Machine Theory and Mechanisms" Fundamentals of Machine Theory and Mechanisms | SpringerLink . We are interested in Chapter 10. Link is here. Synthesis of planar mechanisms.pdf — Яндекс.Диск

Mechanics of Machinery (MOM) Module 6 Synthesis of Mechanisms - YouTube