Theoretical Mechanics, Lab 2: KIN ROT PLANE1

Rotational Motion

Plane Motion: Intro

Formulas

$y = y(x)$ – trajectory in geometry space (can be called as equation of the path)

Forms

1. Radius vector $\vec{r} = \vec{r}(t)$
2. Coordinate $\begin{matrix} x = x(t) \\ y = y(t) \\ z = z(t) \end{matrix}$
3. Natural (arc length) $\sigma = \sigma(t)$

Transformations (general)

- $2 \to 1; \vec{r} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = x\vec{i} + y\vec{j} + z\vec{k}$
- $1 \to 2; \begin{matrix} x \\ y \\ z \end{matrix} = \begin{bmatrix} \cos(\alpha_{rx})\vec{r} \\ \cos(\alpha_{ry})\vec{r} \\ \cos(\alpha_{rz})\vec{r} \end{bmatrix}$
- $2 \to 3; \sigma(t) = \int_0^t \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$ – useless without knowing trajectory. Also, you are losing signs. More Info.

Transformations (planar)

- $2 \to 1; \vec{r} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = x\vec{i} + y\vec{j}$
- $1 \to 2; \begin{matrix} x \\ y \end{matrix} = \begin{bmatrix} cos(\alpha_{rx})\vec{r} \\ cos(\alpha_{ry})\vec{r} \end{bmatrix}$
- $2 \to 3; \sigma(t) = \int_0^t \sqrt{\dot{x}^2 + \dot{y}^2} dt$ – in practice, useless without knowing trajectory.
- $1(2) \to y(x) \to \sigma(x); \sigma(x) = \int_a^b \sqrt{1 + (y'(x))^2} dx$ works if y is unique for each x
- $\sigma(x) \to y(x) \to 1(2); \sigma_{cur} - \sigma(x) = 0.$
  Can be solved, using numerical optimization or brute force. Info.

Linear and angular components of rigid body motion

Linear part

Position type – 1 = $\vec{r}$

Velocity type – 1 = $\vec{V}$, Speed = $|\vec{V}|$

$\vec{V} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = \dot{\sigma}\vec{\tau}$

Velocity is always tangent to the trajectory function

Path function is f

$y = f'(x)(x - x_{cur}) + f(x_{cur})$ — easy to convert to $\vec{\tau}$

Acceleration types – 2: tangent and normal = $\vec{a}_\tau,\ \vec{a}_n$

$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} = \vec{a}_\tau + \vec{a}_n$

$\vec{a}_\tau = \ddot{\sigma}\vec{\tau} = \frac{\vec{a}\cdot\vec{V}}{V}\vec{\tau}$

$\vec{a}_n = \frac{\dot{\sigma}^2}{\rho}\vec{n} = \frac{|\vec{a}\times\vec{V}|}{V}\vec{n}$

$\rho = \frac{1}{\kappa}$, where κ is curvature

$\kappa(x) = \frac{|f''|}{(\sqrt{1+f'^2})^3}$

Angular part

All of these guys are pseudovectors. We can put them wherever we want.

Angle type – 1 = $\vec{\phi}$

Angular velocity type – 1 = $\vec{\omega}$

Angular acceleration – 1 (on a plane) = $\vec{\varepsilon}$

Linear ⟷ Angular

$\vec{v} = \vec{\omega} \times \vec{r}$

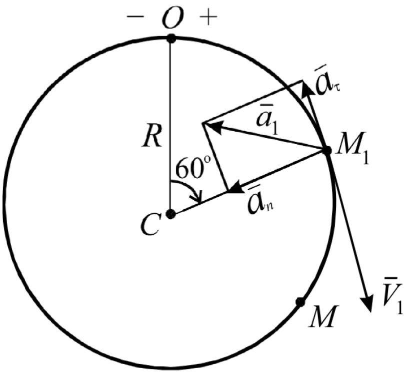
$\vec{a}_\tau = \vec{\varepsilon} \times \vec{r}$

$\vec{a}_n = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Task 1 (yours)

Point M moves on a circle $R = 60$. Motion law is given in natural form $s = \smile OM = \frac{\pi R}{6}(3t - t^2)$. You should find velocity and acceleration, when t = 1.

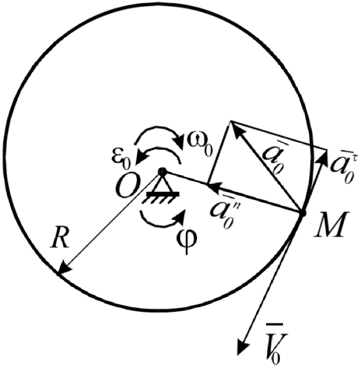
Answer: $a_n = 16.4,\ a_\tau = -62.8,\ a = 64.9,\ v = 31.4$



Task 1

Task 2 (mine)

Disk $R = 2$ rotates around O. Its motion is $\phi = \phi(t) = 2e^{-2t}$. It is needed to find angular velocity and angular acceleration for the body. Also, you need to find v_M, a_M for $t = 0$.

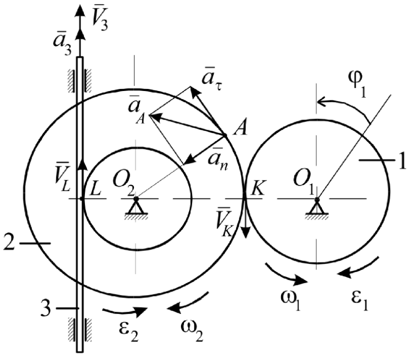


Task 2

Task 3 (mine)

The mechanism contains 2 wheels: **1**, $R_1 = 4$, and **2**, $R_2 = 2,\ r_2 = 1$, which are connected with a toothed bar **3**. We also know the motion law of **1**, $\phi(t) = 4t - t^2$. Tasks:

1. For t = 1, find acceleration and velocity for **3**
2. Find all types of acceleration for A.



Task 3

How to find velocities and acc of a rigid body

Velocity

Approaches: 1) Analytical 2) Instantaneous centre of zero velocity 3) Geometrically

We need to think about direction, length.

Notation: if know 1, both

$\vec{v}_b = \vec{v}_a + \vec{v}_{ba} = \vec{v}_a + \vec{\omega} \times \vec{r}_{ba}$

Accelerations

Approaches: 1) Analytical

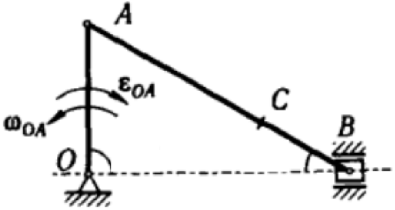
$\vec{a} = \vec{a}_a + \vec{a}_{ba}^{\tau} + \vec{a}_{ba}^{n} = \vec{a}_a + \vec{\varepsilon} \times \vec{r}_{ba} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{ba})$

Task 4 (mine)

You should simulate this mechanism (obtain all positions) $(x_i(t),\ y_i(t))$, where i are $A,\ B,\ C$ points) If $\omega_{OA} = const = 1$;

t — 1 cycle

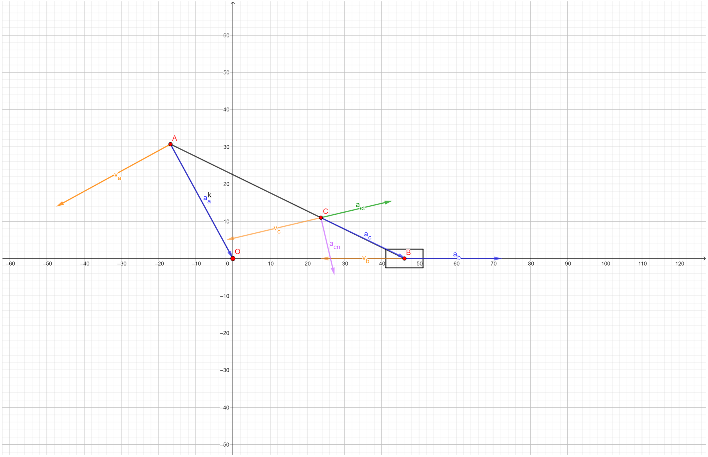
$OA = 35,\ AB = 70,\ AC = 45.$



Task 4

Task 4 (mine)

Simulation in Geogebra



3 typical approaches for kinematics

Triangular

Consider a mechanics as a set of triangles.

Solution based on

Sin and Cosine rules mainly.

+: fast, easy to code.

-: applicable only for simple mechanisms

Geometrical

Represent a mechanism as a set of figures (mainly circles and lines)

Solution based on

Finding intersection b/w figures (line-line,circle-line,circle-circle,sphere-line). Need nonlinear solver!

+: Solve most of mech., choosing roots are intuitive

-: difficult to imagine

Vector-based

Represent a mechanism as a set of vectors

Solution based on

Writing a system of nonlinear equations and put it in nonlinear solver!

+: Best for tough mech., easy to imagine

-: Need to prepare a solver for finding roots

Deserve “A” grade!

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Room 105 (Underground robotics lab)