Theoretical Mechanics, Lab 11: KIN ENERGY

Theorem on the:

Change of Kinetic Energy of a System

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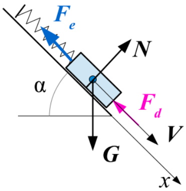
Change of Kinetic Energy of a System

R. O.	Eqn #	Equations	Applications	Extra Info
Body, System	1	$\mathrm{d}T = \delta A$, or $T_2 - T_1 = A_{12}$ $T_{lin} = \frac{mv^2}{2}$; $T_{rot} = \frac{J\omega^2}{2}$ $A_{12} = \int_1^2 \delta A$; $A_{12} = \sum A_i$ $\delta A = \vec{F} \cdot \delta\vec{r} = \vec{F}\delta\vec{r}cos(\vec{F}\hat{\ }\delta\vec{r}) = \lvert M\rvert\delta\vec{\phi}$ $\delta A = \Pi_1 - \Pi_2 = F\Delta h$ potential force	To find a correlation between displacement and velocity. Helpful if you need to find *one* force.	• Work of internal forces may be **not equal to zero!** • $\delta\vec{r}$ can be changed on $\mathrm{d}\vec{r}$ because it is usually independent of time (scleronomic). This is done in order to be able to integrate by coordinates rather than by time.

How to calculate work of reaction forces



$$G = mg = const$$

$$N = G\cos\alpha = const$$

$$F_e = c(f_{st} + x)$$

$$F_d = \mu V = \mu \dot{x}$$

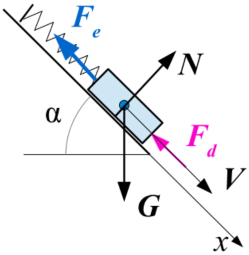
$$x \in [x_1, x_2] \qquad S = x_2 - x_1$$

$$A_{12} = A_{12}^{G} + A_{12}^{N} + A_{12}^{Fe} + A_{12}^{Fd}$$

$$A_{12}^{G} = \int_{x_1}^{x_2} G\cos(\pi/2 - \alpha)\,\mathrm{d}x = G\cos(\pi/2 - \alpha)(x_2 - x_1) = G\cos(\pi/2 - \alpha)S$$

$$A_{12}^{N} = \int_{x_1}^{x_2} N\cos(\pi/2)\,\mathrm{d}x = 0$$

How to calculate work of reaction forces (2)



$$F_e = c(f_{st} + x) \qquad x \in [x_1, x_2]$$

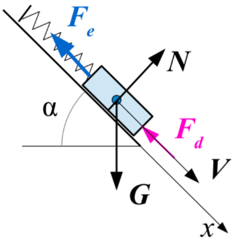
$$A_{12}^{Fe} = \int_{x_1}^{x_2} F_e \cos(\pi)\, \mathrm{d}x = \int_{x_1}^{x_2} c(f_{st} + x)\cos(\pi)\, \mathrm{d}x$$

$$= -c f_{st}(x_2 - x_1) - c\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right)$$

$$x^* = f_{st} + x$$

$$A_{12}^{Fe} = -c\left(\frac{(x_2^*)^2}{2} - \frac{(x_1^*)^2}{2}\right)$$

How to calculate work of reaction forces (3)



$$F_d = \mu V = \mu \dot{x}$$

$$A_{12}^{Fd} = \int_{x_1}^{x_2} F_d \cos(\pi)\, dx = -\mu \int_{x_1}^{x_2} \dot{x}\, dx$$

$$x = x(t)$$

$$\dot{x} = \dot{x}(t)$$

$$dx = \dot{x}\, dt$$

$$A_{12}^{Fe} = -\mu \int_{t_1}^{t_2} \dot{x}^2(t)\, dt$$

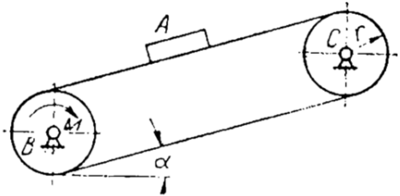
How to calculate work of reaction forces

Questions: Why do we need to find work of N (reaction force) here (when we have slippering)

Answer: Despite that it is internal force, it can have work.

INTERNAL AND EXTERNAL FORCES - DEFINITION

External forces are forces caused by external agent present outside of the system. External non-zero net force imparts an acceleration to the center of mass of the system regardless of point of application.
Internal forces are forces exchanged by the objects in the system. Internal forces may cause acceleration in different parts of the system but does not cause any acceleration in the center of mass of the entire system.
Example:
Friction is an external force if the body experiencing friction in the system.
If both the bodies involved in friction are considered as a system, then it acts as an internal force.

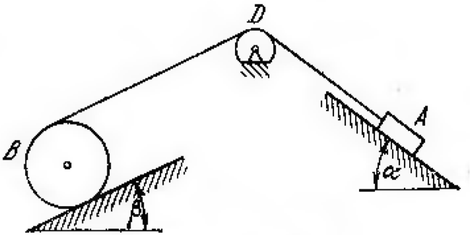


- We cannot find find work for precise time, only for transfer between two points
- $A = F \cdot s$, it is displacement between two bodies who interacts in this force (displacement related from one body to another)

Task 1 (Mine)

A load *A* of mass M_1 has an ideal string attached to it, thrown over the block *D* of mass M_2 and wound on the side surface of the cylindrical roller *B* of mass M_3.
When load *A* moves down an inclined plane located at an angle α to the horizon, block *D* rotates, and the roller *B* rolls without slippering up the inclined plane, forming an angle β.

Determine the velocity *v* of the load *A* as a function of its path *s*, if the system is at rest at the initial moment. Consider the block and the roller as homogeneous circular cylinders. The forces of friction are neglected.



Task 2 (yours): M (rus) 38.5

621. Compute the value of the kinetic energy of the slider-crank mechanism, shown in Fig. 391, if the following data are given: the mass of the crank is m_1, its length is r, the mass of the slider is m_2 and the length of the connecting rod is l. The crank is considered to be a uniform rod, and its angular velocity is ω. Neglect the mass of the connecting rod.

Ans. $$T = \frac{1}{2}\left\{\frac{1}{3}m_1 + m_2\left[\sin\varphi + \frac{r}{2l}\frac{\sin 2\varphi}{\sqrt{1-\left(\frac{r}{l}\right)^2\sin^2\varphi}}\right]^2\right\}r^2\omega^2.$$

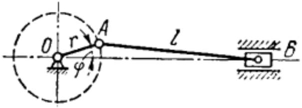
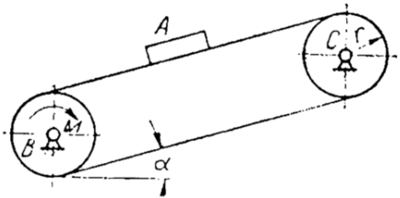


Fig. 391

Task 3 (yours): M (rus) 38.20

A conveyor belt is set in motion from rest by a drive connected to the lower pulley *B*. The drive imparts a constant torque *M* to this pulley. The drive gives this pulley a constant torque *M*.

Determine the velocity of the conveyor belt *v* as a function of its displacement *s*, if the mass of the lifted load *A* is equal to m_1, and the pulleys *B* and *C* of radius *r* and mass m_2 each are uniform circular cylinders.



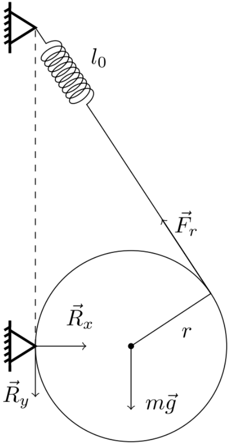
The conveyor belt, the mass of which should be neglected, forms an angle α with the horizon. There is no sliding of the belt on the pulleys.

Answer: $v = \sqrt{\frac{2(M - m_1 g r \sin(\alpha))}{r(m_1 + m_2)} s}$

Task 4 (mine)

Find an angular velocity, when the mechanism reaches —90° angle.

The main goal of the task is to show how to find work in different ways.



Deserve “A” grade!

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Room 105 (Underground robotics lab)