# Theoretical Mechanics, Lab 10: DYN ANGULAR

Theorem on the:

Change of Principal Angular momentum of a system

Innopolis University

# Difference between torque and moment

In Theoretical mechanics (we are operating with absolute rigid bodies) — it’s the same.

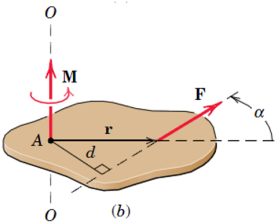
In Strength of materials — it different.  Torque - twisting moment, moment - Bending moment

# Moment (torque)

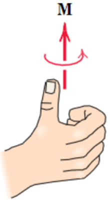
The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$\boxed{M = Fd} \qquad \mathbf{(2/5)}$$

The moment is a vector **M** perpendicular to the plane of the body. The sense of **M** depends on the direction in which **F** tends to rotate the body. The right-hand rule, Fig. 2/8*c*, is used to identify this sense. We represent the moment of **F** about *O-O* as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.



(*b*)



(*c*)

# Torque and couple - intuitive explanation

**Torque** is a value that shows the possibility of force to rotate the body

- It's not real thing (force is real), it's math technique. It is the reason why torque value depends on point which we are using to calculate.

**Couple** — special case of torque, when it's appears because of two parallel forces (more info on next slide). It has some unique properties (doesn't depend on the point of calculation).

# Couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

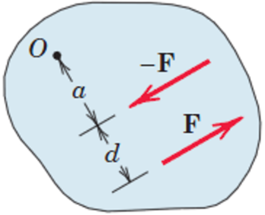
Consider the action of two equal and opposite forces **F** and −**F** a distance $d$ apart, as shown in Fig. 2/10$a$. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as $O$ in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance $a$ which locates the forces with respect to the moment center $O$. It follows that the moment of a couple has the same value for all moment centers.




($a$)

# Force-Couple System

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The combination of the force and couple in the right-hand part of Fig. 2/12 is referred to as *a force–couple system*.

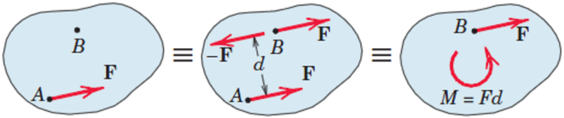



Figure 2/12

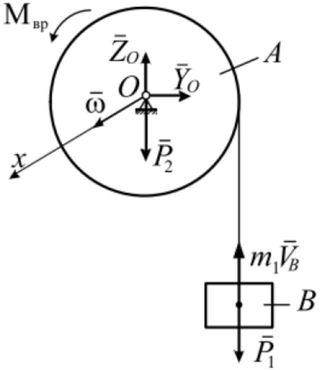
# Change of Principal Angular momentum of a system

| R. O. | Eqn # | Equations | Applications | Extra Info |
|---|---|---|---|---|
| System | 1 | $\frac{\mathrm{d}\vec{L}_c}{\mathrm{d}t} = \sum \vec{M}_c$<br>$c$ – point of calculation<br>$\vec{L}_c = \sum \vec{L}_i,\ \vec{L}_i = J\vec{\omega} = \vec{Q} \times R$ | We are interested in angular motion. Easy to find a angular velocities for bodies. | The choice of a point depends of the motion. If rotation – more convenient to put it in the center of rotation, if planar – in the center of mass. |

# Task 1 (mine)

Body $A$ has a radius $r$, mass $m_2$ and a torque $M_r = \alpha t$, where $\alpha = const$. $B$ body has a mass $m_1$. It goes up.
We need to find an angular velocity if $A$ is just a uniform cylinder.
Initial conditions: $t = 0,\ \omega_0 = 0$.

*Answer*: $\omega = \dfrac{t(\alpha t - 2m_1 g r)}{r^2(m_2 + 2m_1)}$



# Task 2 (yours): M (rus) 37.13

**599.** To adjust the hairspring of a watch, special balance wheels are used, such as shown in Fig. 378. The balance wheel $A$ is free to rotate about the axis perpendicular to its plane which passes through the centre of gravity $O$ and has the moment of

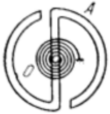



Fig. 378

inertia $J$ with respect to this axis. The balance wheel is set in motion by a helical spring, one end of which is attached to the wheel while the other end is fixed to the watch frame. When the balance wheel starts to rotate, the moment of the elastic forces of the spring is not zero, it is proportional to the angle of twist. To coil the spring for one radian a moment $c$ is required. Initially, when the moment of the elastic forces was zero, the balance wheel was given an initial angular velocity $\omega_0$. State the law of motion of the balance wheel.

*Ans.* $\varphi = \omega_0 \sqrt{\frac{J}{c}} \sin \sqrt{\frac{c}{J}}\, t.$

# Deserve “A” grade!

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Room 105 (Underground robotics lab)