Dynamics methods overview
Theoretical mechanics

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Table 1: Methods overview

Method	Research object	Amount of equations (for one research object)	Equation	The best applications	Additional comments
2-nd Newtons law for inertial systems	Particle	1–3	$m\vec{a} = \sum \vec{F}$, where m — mass, $\vec{a}$ — acceleration, $\vec{F}$ — forces	For everything, if you can represent your system or body as a particle	
2-nd Newtons law for non-inertial systems	Particle	1–3	$m\vec{a}_r = \sum \vec{F} + \vec{\Phi}_{cor} + \vec{\Phi}_{trans}$, where Φ is inertia force	For everything, if you can represent your system or body as a particle	• Inertial force is not real force, it is needed for compensating non-inertial nature of a system. • Not obligatory that the particle should be on a body, like turning bus and a man. It can be Earth and satellite on Jupiter.
Theorem on: 1. Motion of the centre of mass of a system 2. Change of linear momentum of a system	System	1–3	1. $m\vec{a}_c = \sum \vec{F};\ \vec{x}_c = \dfrac{\sum m_i \vec{x}_i}{\sum m_i}$ 2. $\dfrac{\mathrm{d}\vec{Q}_c}{\mathrm{d}t} = \sum \vec{F};\ Q_c = \sum m\vec{v}_i$	We are interested in linear motion. 1. Easy to find a displacement for a body of a system, motion equation for system, external forces. 2. Easy to find a velocities for bodies.	
Theorem on change of angular momentum of a system	System	1	$\dfrac{\mathrm{d}\vec{L}_c}{\mathrm{d}t} = \sum \vec{M}_c$ c – point of calculation $\vec{L}_c = \sum \vec{L}_i,\ \vec{L}_i = J\vec{\omega} = \vec{Q} \times R$	We are interested in angular motion. Easy to find a angular velocities for bodies.	The choice of a point depends of the motion. If rotation – more convenient to put it in the center of rotation, if planar – in the center of mass.
d'Alembert principle (kinetostatics)	Body, system	1–6	$\begin{cases} \sum \vec{F} + \vec{\Phi} = 0 \\ \sum \vec{M} + \vec{M}_\Phi = 0 \end{cases}$	To find reaction forces, if you know the motion equations of the system.	• You can use it, when it is applicable to imagine in each time that the system is static. • In contrast of the Coriolis or Transport forces of inertia, the d'Alambert force of inertia has not a special physical meaning. It's a math trick.

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Theorem on the change of kinetic energy of a system	Body, System	1	$\mathrm{d}T = \delta A$, or $T_2 - T_1 = A_{12}$ $T_{lin} = \frac{mv^2}{2};\ T_{rot} = \frac{J\omega^2}{2}$ $A_{12} = \int_1^2 \delta A;\ A_{12} = \sum A_i$ $\delta A = \vec{F} \cdot \delta\vec{r} = \vec{F}\delta\vec{r}cos(\vec{F}\hat{\ }\delta\vec{r}) = \vec{M}\delta\vec{\phi}$ $\delta A = \Pi_1 - \Pi_2 = F\Delta h$ potential force	To find a correlation between displacement and velocity. Helpful if you need to find *one* force.	• Work of internal forces may be **not equal to zero!** • $\delta\vec{r}$ can be changed on $\mathrm{d}\vec{r}$ because it is usually independent of time (scleronomic). This is done in order to be able to integrate by coordinates rather than by time.
Principle of virtual: 1. Displacements (work) 2. Velocities	System	1	1. $\sum \delta A = 0$, where A is a virtual work 2. $\sum W = 0;\ W = \vec{F} \cdot \vec{v} = \vec{F}\vec{v}cos(\vec{F}\hat{\ }\vec{v})$, where W is power, v – virtual velocity	To find *one* force or reaction force.	• Virtual work has infinitesimal displacements. • System must be in static each time.
Lagrange-d'Alambert principle (General Equation of Dynamics)	System	1 in cartesian, n in generalized coordinates	• $\sum \delta A + \sum \delta A^{\Phi} = 0$, where A is a virtual work • $\sum W + \sum W^{\Phi} = 0$, where W is a power	To find accelerations, motion equations	
Newton-Euler equations	System	$6k + \sum_0^k (6 - m_i)$, k – amount of bodies, m_i – particular joint d.o.f	$\begin{cases} f_i = F_i + f_{i+1} \\ m_i = M_i + m_{i+1} + \vec{p}_{c_i} \times F_i + \vec{p}_{i+1} \times f_{i+1} \\ \tau_i = \begin{cases} m_i \cdot Z_i, \text{ if revolute} \\ f_i \cdot Z_i, \text{ if prismatic} \end{cases} \end{cases}$ Where $F_i = m\dot{\vec{v}}_{C_i}$ $M_i = I_{C_i}\dot{\omega}_i + \omega_i \times I_{C_i}\omega_i$	To find accelerations, forces, motion equations.	We divide a system into separate bodies, write equations for rotations and translations, plus constraints. At the end we have a dozens of equations, which should be solved numerically.
Euler-Lagrange equations	Body, System	s, s – d.o.f of system	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i$ or, $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^D$ $L = T - \Pi$	To find accelerations, forces, motion equations.	• It is the most classical and flexible method for multi-body systems, concluding only holonomic constraints. • q – generalized coordinates. • Q – all generalized forces. • Q^D – non-potential general forces.