Theoretical Mechanics, Lab 8: DYN PART [NON]INERT

Intro to Dynamics

Particle Dynamics for inertial systems

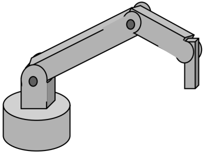
Particle Dynamics for non-inertial systems

Forward Dynamics (2nd dynamics problem)

We know applied forces and want to find equation of motion.

- direct relation

$$u(t) = \begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix}$$



$$q(t) = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix}$$

input for $t \in [0, T]$ + $q(0), \dot{q}(0)$

initial state at $t = 0$

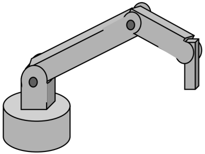
resulting motion

Inverse Dynamics (1st dynamics problem)

We know equation of motion (X, $\dot{X}$, $\ddot{X}$) and we want to find applied forces respect to time.

- inverse relation

$$q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \rightarrow$$



$$\rightarrow u_d(t)$$

desired motion
for $t \in [0, T]$

required input
for $t \in [0, T]$

Method of solving forward dynamics

1. Analyze the system
   - Show the frame of reference
   - Show the particle in an common position
   - Show the initial, final position of the particle
   - Write the initial, final conditions
2. Analyze forces
   - Make a list of all the forces
   - Specify the formula for calculating the forces
   - Show all forces on the scheme
3. Solve the problem
   - Write the differential equations of motion
   - Find a common solution
   - Substitute the initial conditions to find the constants of integration
   - Substitute other conditions to find the answer

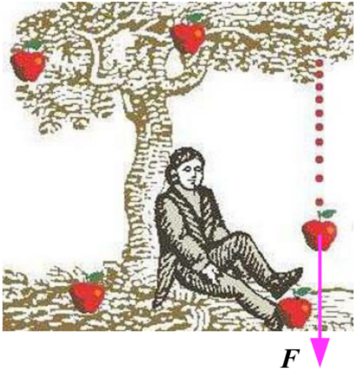
Force types

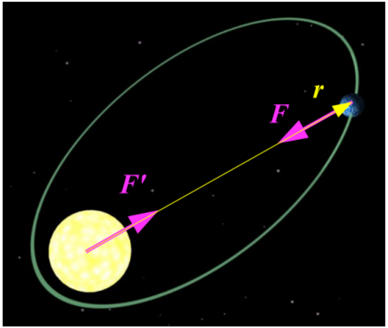
In our course we will use only:

- $\Rightarrow$ Constant force $\vec{F} = const$
- $\Rightarrow$ Time dependent force $\vec{F} = \vec{F}(t)$
- $\Rightarrow$ Velocity dependent force $\vec{F} = \vec{F}(\dot{\vec{r}})$
- $\Rightarrow$ Position dependent force $\vec{F} = \vec{F}(t, \vec{r}, \dot{\vec{r}}) \Rightarrow \begin{cases} F_x = F_x(t, x, y, \dot{x}, \dot{y}) \\ F_y = F_y(t, x, y, \dot{x}, \dot{y}) \end{cases}$

Force types

Constant force



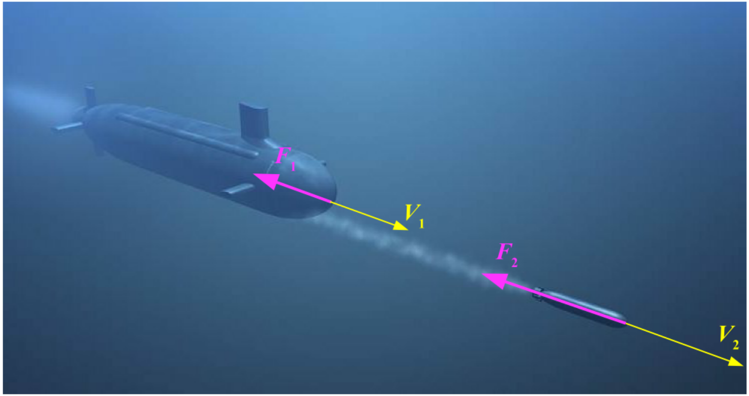


$$\boldsymbol{F} = const$$

$$\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{r}) = -G\frac{m_1 m_2}{r^3}\boldsymbol{r}$$

Force types

Resistance force

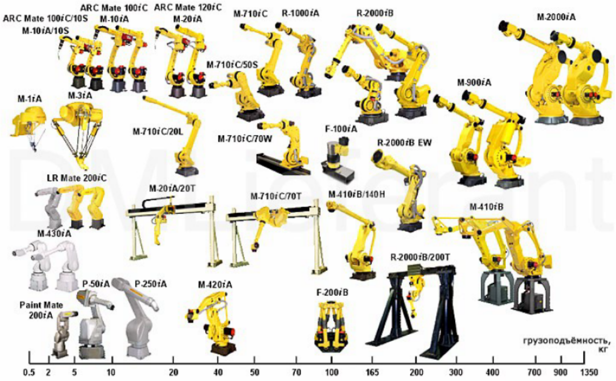


$$\boldsymbol{F}_1=\boldsymbol{F}_1(\boldsymbol{V}_1)=-\mu\,\boldsymbol{V}_1$$

$$\boldsymbol{F}_2=\boldsymbol{F}_2(\boldsymbol{V}_2)=-k\,V_2\boldsymbol{V}_2$$

Force types

Controlled force

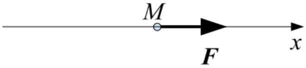


$$\boldsymbol{F} = \boldsymbol{F}(t) \qquad \boldsymbol{F} = \boldsymbol{F}(\ddot{\boldsymbol{r}}) \qquad \boldsymbol{F} = \boldsymbol{F}(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, \ddot{\boldsymbol{r}}, \dddot{\boldsymbol{r}}, \dots)$$

Typical differential equations

Constant and time dependent forces

Let the particle make a rectilinear motion

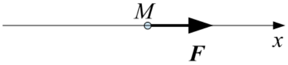


$\boldsymbol{F} = const$	$\boldsymbol{F} = \boldsymbol{F}(t)$
$m\ddot{x} = F$	$m\ddot{x} = F(t) = F_0 e^{-kt}$
$\dot{x} = \dfrac{F}{m} t + C_1$	$\dot{x} = -\dfrac{F_0}{km} e^{-kt} + C_1$
$x = \dfrac{F}{m}\dfrac{t^2}{2} + C_1 t + C_2$	$x = \dfrac{F_0}{k^2 m} e^{-kt} + C_1 t + C_2$

Typical differential equations

Resistance force: 1D case

Let the particle make a rectilinear motion



$$\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{V})=-\mu\boldsymbol{V}$$

$$m\ddot{x}=-\mu\dot{x}$$

$$\dot{x}=e^{-\frac{\mu}{m}t+C_1}$$

$$x=-\frac{m}{\mu}e^{-\frac{\mu}{m}t+C_1}+C_2$$

$$\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{V})=-k\,V\,\boldsymbol{V}$$

$$m\ddot{x}=-k\dot{x}^2$$

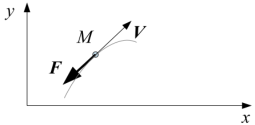
$$\dot{x}=\frac{1}{\frac{k}{m}t-C_1}$$

$$x=\frac{m}{k}\ln\left(\frac{k}{m}t-C_1\right)+C_2$$

Typical differential equations

Resistance force: 2D case

Let the particle make a curvilinear motion



$$\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{V})=-\mu\boldsymbol{V}$$

$$m\ddot{x}=-\mu\dot{x}$$

$$m\ddot{y}=-\mu\dot{y}$$

$$x=-\frac{m}{\mu}e^{-\frac{\mu}{m}t+C_1}+C_2$$

$$y=-\frac{m}{\mu}e^{-\frac{\mu}{m}t+C_3}+C_4$$

$$\boldsymbol{F}=\boldsymbol{F}(\boldsymbol{V})=-kV\boldsymbol{V}$$

$$m\ddot{x}=-k\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{x}$$

$$m\ddot{y}=-k\sqrt{\dot{x}^2+\dot{y}^2}\,\dot{y}$$

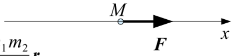
$$x=x(t,C_1,C_2,C_3,C_4)$$

$$y=y(t,C_1,C_2,C_3,C_4)$$

Typical differential equations

Position dependent force

Let the particle make a rectilinear motion



$$\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{r}) = -G\frac{m_1 m_2}{r^3}\boldsymbol{r}$$

$$m\ddot{x} = -\frac{k}{x^2}$$

$$\ddot{x} = \frac{\mathrm{d}\dot{x}}{\mathrm{d}t} = \frac{\mathrm{d}\dot{x}}{\mathrm{d}t}\frac{\mathrm{d}x}{\mathrm{d}x} = \frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}\dot{x}}{\mathrm{d}x} = \dot{x}\frac{\mathrm{d}\dot{x}}{\mathrm{d}x}$$

$$m\frac{\dot{x}^2}{2} = \frac{k}{x} + C_1$$

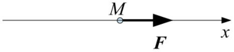
Solution depends on values of parameters. One of possible solutions for $C_1 < 0$ is

$$\frac{1}{C_1}\left(\sqrt{C_1 x^2 + kx} + \frac{k}{2}\frac{1}{\sqrt{-C_1}}\arcsin\frac{2C_1 x + k}{k}\right) = \sqrt{\frac{2}{m}}t + C_2$$

Typical differential equations

Position and velocity dependent force

Let the particle make a rectilinear motion



$$m\ddot{x} = -\mu\dot{x} - cx$$

$$\ddot{x} + 2n\dot{x} + k^2x = 0 \qquad 2n = \frac{\mu}{m} \qquad k^2 = \frac{c}{m}$$

$$k > n \qquad k_1 = \sqrt{k^2 - n^2} \qquad x = C_1e^{-nt}\cos(k_1t) + C_2e^{-nt}\sin(k_1t)$$

$$k < n \qquad k_2 = \sqrt{n^2 - k^2} \qquad x = C_1e^{-nt}e^{k_2t} + C_2e^{-nt}e^{-k_2t}$$

$$k = n \qquad x = (C_1t + C_2)e^{-kt}$$

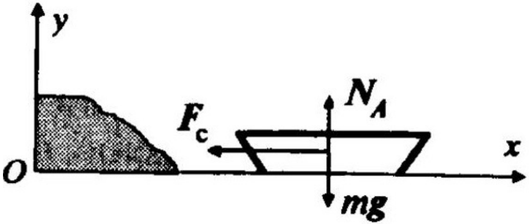
2nd Newton Law for Inertial systmes

Research object	Amount of equations (for one research object)	Equation	The best applications	Additional comments
Particle	1–3	$m\vec{a} = \sum \vec{F}$, where m — mass, $\vec{a}$ — acceleration, $\vec{F}$ — forces	For everything, if you can represent your system or body as a particle	

Task 1 (mine)

The boat has an initial velocity v_o and a mass m. Resistance force $F_c(v)$ also affects on the boat. You should:

1. Find an equation of motion of this boat.
2. Find the time when the boat speed will be reduced twice.



Task 1

Task 2 (yours)

The body D has a mass m. It moves up because of F force. There is no friction between D and ground.

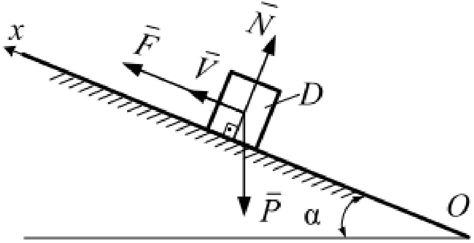
The task is to find an equation of D motion. Needed variables:

$m = 2,\ F_x = 12\cos(3t);$

$\alpha = 30,\ v_0 = 4.$

Answer:

$x = 4t - 2.45t^2 - 0.67(\cos(3t) - 1)$



Task 2

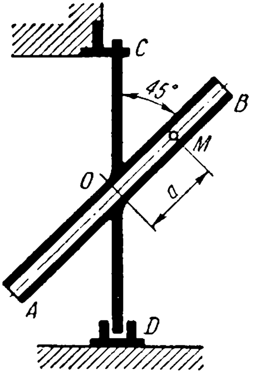
2nd Newton Law for Non Inertial systmes

Research object	Amount of equations (for one research object)	Equation	The best applications	Additional comments
Particle	1–3	$m\vec{a}_r = \sum \vec{F} + \vec{\Phi}_{cor} + \vec{\Phi}_{trans}$, where Φ is inertia force	For everything, if you can represent your system or body as a particle	• Inertial force is not real force, it is needed for compensating non-inertial nature of a system. • Not obligatory that the particle should be on a body, like turning bus and a man. It can be Earth and satellite on Jupiter.

Task 3 (mine)

The figure shows a pipe *AB*, which rotates about a vertical axis *CD* with a constant angular velocity ω. The angle between *AB* and *CD* is always 45°.
A small heavy ball is placed in the pipe. Determine the motion of the ball, assuming that its initial velocity is 0 and the initial distance between the ball and a point *O* equals *a*. Neglect friction.

Answer: $OM = \frac{1}{2}\left(a - \frac{g\sqrt{2}}{\omega^2}\right)\left(e^{0.5\omega t\sqrt{2}} + e^{-0.5\omega t\sqrt{2}}\right) + \frac{g\sqrt{2}}{\omega^2}$



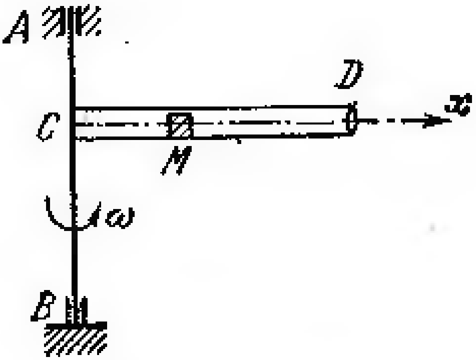
Task 3

Task 4 (yours): M (rus) 33.10

The horizontal tube CD rotates with the constant angular velocity $\omega = const$ along AB axis. There is a body M inside the tube. The task is to find the velocity of M related to the tube in a moment, when the body leaves the tube. No friction between M and the tube.

We know that the length of the tube is equal to L, the initial velocity is equal to zero ($v_0 = 0$) and initial position was x_0.

Answer: $v = \sqrt{L^2 - x_0^2}\,\omega$



Task 4

Deserve "A" grade!

– Oleg Bulichev

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Room 105 (Underground robotics lab)