Theoretical Mechanics, Lab 1: KIN PART

Intro

Linear Algebra recap

Particle kinematics

Innopolis University

About me

Education

- Bachelor — Bauman University (BMSTU) (honor diploma)

  Topic: Aim control development of mobile vehicle «Plastun»

- Master — Innopolis University (IU), Robotics

  Topic: Development of biomimetic centipede robot «StriRus»

- PhD — Innopolis University (IU), Robotics. *Defense*: Volgograd State Technical University (VSTU)

  Topic: Tactile perception method development for a mobile walking robot

Current jobs

- Senior lecturer (AGLA 1,2; Mechanics and Machines; Theoretical Mechanics)
- Coach (RAGE club channel: Hiking, Folk Games, HEMA)

Agreements among the group (for now)

Books for labs

- Reference books
  - Eng
    - Yablonski
      - Solving Practical Engineering Mechanics Problems DYNAMICS.pdf
      - Solving Practical Engineering Mechanics Problems KINEMATICS.pdf
      - Solving Practical Engineering Mechanics Problems STATICS.pdf
    - Dynamics of Multibody Systems_J Wittenburg.pdf
    - Herbert Goldstein, Charles P. Poole, John L. Safko - Classical mechanics-Addison Wesley (2001).djvu
    - meshchersky-collection-of-problems-in-theoretical-mechanics.pdf
    - targ_s_theoretical_mechanics_a_short_course.pdf
  - Rus
    - Mejerski.djvu M (RUS)
    - Vittenburg1980ru.djvu
    - Тарг_Краткий_курс_теоретической.pdf
    - Яблонский.pdf

Tasks goal

1. **Quizzes** — to check the understanding of the previous material.
   Starts in the beginning of the labs. Basic questions. *Style guide formal criteria*.
2. **Hometasks** — to depress you :) *Style guide formal criteria + coding*.
   - 6 weekly HWs. Will be given on **Wednesday**.
     Deadline — Thursday, 9:00.
   - 2 Big HWs. 2-3 weeks for solving.
     Deadline — the day before midterm/final, 9:00.

Formal criteria

If you don’t follow formal criteria, you are losing 40% of task grade score

For now:

1. Despite the paper sheet or digital, the answer should be highlighted

Grading criteria

Qz: Quizzes: 10%: 5 best (2 % each)

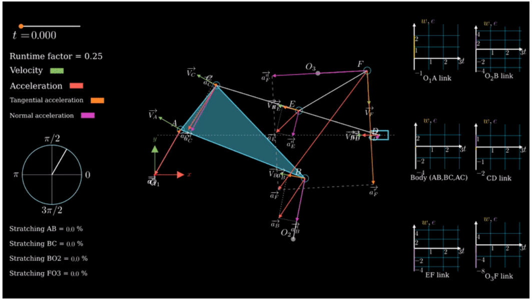
WHW: Weekly HWs: 30 % (5 % each)

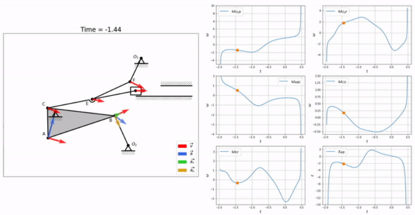
BHW: Big HWs: 20% (10 % each)

Late policy:

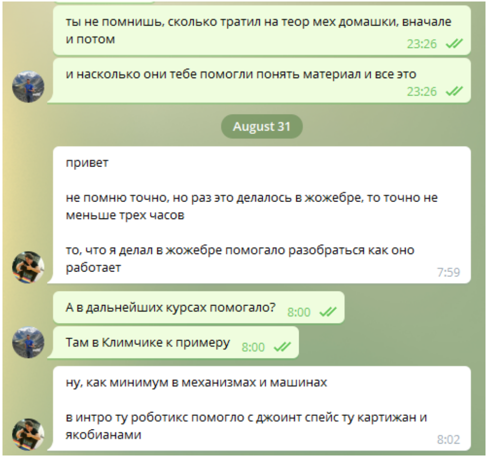
- WHWs — no late policy. 0 score, if no data in Moodle. You can put a text file (**PDF**) with a link to Github, I wouldn't check the submission time, but if there are no solution — 0 score.
- BHW 1 — -50% on max grade for a task, BHW 2 — no late policy.

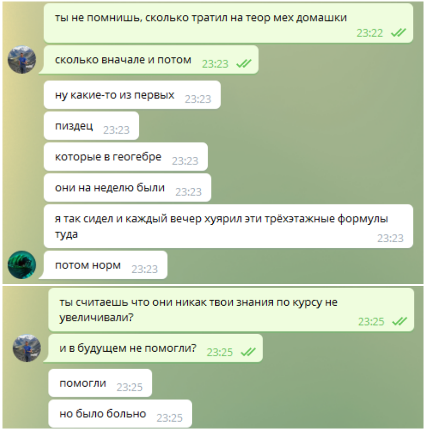
HW example: 2nd week



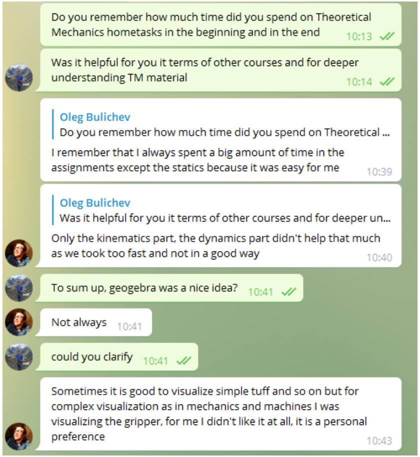


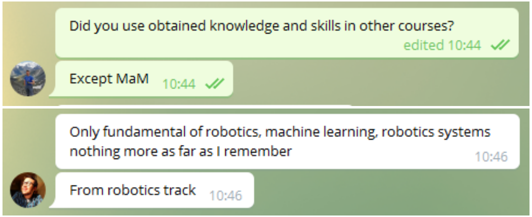
Feedback from previous batch (1)





Feedback from previous batch (2)





Feedback from previous batch (3)

Свое пожелание будущим поколениям (можно на любом из языков), добавлю это в первук
лабу)

13 ответов

Домашки не удары, можно и пропустить парочку)

Курс супер полезный. Нужно лишь всегда посещать лекции и лабы по возможности, тогда не придётся сильно напрягаться на следующих квизов и домашках. И в принципе знать этот курс нужно

Ресерч 2 нужно начинать заранее, если работать в команде, то в целом можно за одну ночь сделать

Будьте готовы относится к предмету, как к девушке, ТеорМех требует очень много внимания и времени, любое свободное время тратье на него, даже если все кажется понятным. ТеорМех – сложный квест, который пройти в одиночку не получится, не бойтесь объединятся и задавать вопросы.

Быть готовым к боли и страданиям, но не падать духом

Может показаться, что квизы каждую неделю это пытка, созданная чтобы вас дропнуть, но на деле чем больше квизов - тем больше баллов. В конце курса придётся упрашивать Олега провести квиз, так что дерзайте уже прямо сейчас.

ЪПосос неизбеженЪ

The best course in the university yet. When I went to Inno, I regret that I would lack such a courses, I'm glad that I was wrong

Tools for HWs and reports

Modeling — Python (Collab or Docker) / Matlab (Live Script) / Geogebra

Report — any tool (markdown, latex, word)

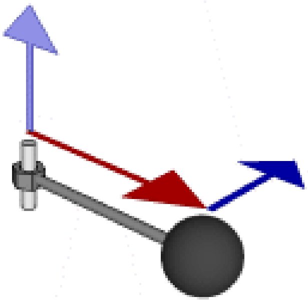
Report template will be given before the 1st HW.

Cross product

Where it can be used

Relationship between force (F), torque(τ), momentum (p), and angular momentum (L) vectors in a rotating system. r is the position vector

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$



Cross product

How to calculate it: classical approach

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$$\mathbf{X} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(\mathbf{X}) = a * d - b * c$$

Cross product

How to calculate it: skew-symmetric matrix

$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \qquad c = a \times b \Rightarrow c = \hat{a} b$$

vectors $\Rightarrow$ matrices

$a \times \Rightarrow \hat{a}$: a skew-symmetric matrix

$$c = \hat{a} b = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \qquad \boxed{c = \hat{a} b}$$

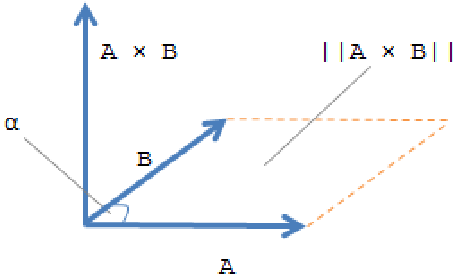
Cross product

Geometrical representation

$$||A \times B|| = ||A||||B||sin\alpha$$

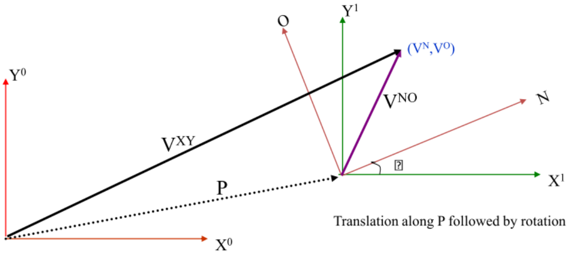
$||A \times B||$ - area

$||A||$ - length of the vector



Change the coordinate frame

Classic way



Translation along P followed by rotation by θ

$$\mathbf{V}^{XY} = \begin{bmatrix} \mathbf{V}^X \\ \mathbf{V}^Y \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{V}^N \\ \mathbf{V}^O \end{bmatrix}$$

(Note : P_x, P_y are relative to the original coordinate frame. Translation followed by rotation is different than rotation followed by translation.)

In other words, knowing the coordinates of a point (V^N,V^O) in some coordinate frame (NO) you can find the position of that point relative to your original coordinate frame (X^0Y^0).

Change the coordinate frame

Homogeneous representation

$$\mathbf{V}^{XY} = \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \end{bmatrix}$$

What we found by doing a translation and a rotation

$$= \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \\ 1 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \\ 1 \end{bmatrix}$$

Padding with 0's and 1's

$$= \begin{bmatrix} \mathbf{V}^{X} \\ \mathbf{V}^{Y} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{P}_x \\ \sin\theta & \cos\theta & \mathbf{P}_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}^{N} \\ \mathbf{V}^{O} \\ 1 \end{bmatrix}$$

Simplifying into a matrix form

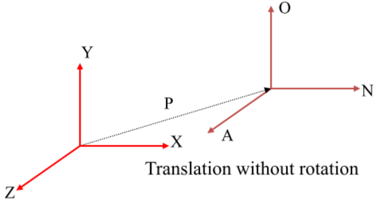
$$\mathbf{H} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{P}_x \\ \sin\theta & \cos\theta & \mathbf{P}_y \\ 0 & 0 & 1 \end{bmatrix}$$

Homogenous Matrix for a Translation in XY plane, followed by a Rotation around the z-axis

Change the coordinate frame

Case studies

H is a 4x4 matrix that can describe a translation, rotation, or both in one matrix



Translation without rotation

$$\mathbf{H} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{P_x} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{P_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{P_z} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

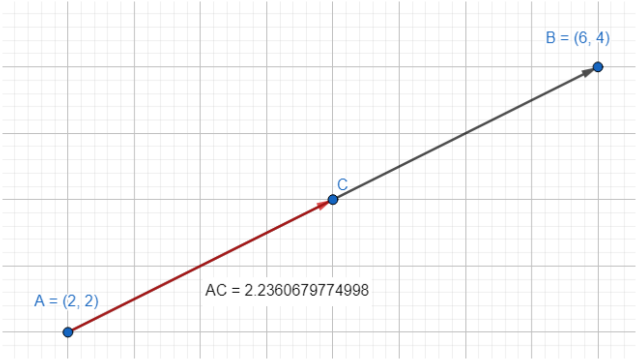
$$\mathrm{Trans}_{y,b} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad Rot_{y,\beta} = \begin{bmatrix} c_\beta & 0 & s_\beta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\beta & 0 & c_\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find a particular coordinate on the line

A, B coordinates and $|AC|$ are given:

$\vec{AC}$ we want to find.

$$\vec{AC} = \vec{A} + |AC| \frac{\vec{AB}}{||AB||}$$



Formulas

$y = y(x)$ – trajectory in geometry space (can be called as equation of the path)

Forms

1. Radius vector $\vec{r} = \vec{r}(t)$
2. Coordinate $\begin{matrix} x = x(t) \\ y = y(t) \\ z = z(t) \end{matrix}$
3. Natural (arc length) $\sigma = \sigma(t)$

Transformations (general)

- $2 \to 1; \vec{r} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = x\vec{i} + y\vec{j} + z\vec{k}$
- $1 \to 2; \begin{matrix} x \\ y \\ z \end{matrix} = \begin{bmatrix} \cos(\alpha_{rx})\vec{r} \\ \cos(\alpha_{ry})\vec{r} \\ \cos(\alpha_{rz})\vec{r} \end{bmatrix}$
- $2 \to 3; \sigma(t) = \int\limits_0^t \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$ – useless without knowing trajectory. Also, you are losing signs. More Info.

Transformations (planar)

- $2 \to 1; \vec{r} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = x\vec{i} + y\vec{j}$
- $1 \to 2; \begin{matrix} x \\ y \end{matrix} = \begin{bmatrix} cos(\alpha_{rx})\vec{r} \\ cos(\alpha_{ry})\vec{r} \end{bmatrix}$
- $2 \to 3; \sigma(t) = \int\limits_0^t \sqrt{\dot{x}^2 + \dot{y}^2} dt$ – in practice, useless without knowing trajectory.
- $1(2) \to y(x) \to \sigma(x); \sigma(x) = \int\limits_a^b \sqrt{1 + (y'(x))^2} dx$ works if y is unique for each x
- $\sigma(x) \to y(x) \to 1(2); \sigma_{cur} - \sigma(x) = 0.$ Can be solved, using numerical optimization or brute force. Info.

Linear component of rigid body motion

Linear part

Position type – 1 = $\vec{r}$

Velocity type – 1 = $\vec{V}$, Speed = $|\vec{V}|$

$\vec{V} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k} = \dot{\sigma}\vec{\tau}$

Velocity is always tangent to the trajectory function

Path function is f

$y = f'(x)(x - x_{cur}) + f(x_{cur})$ — easy to convert to $\vec{\tau}$

Acceleration types – 2: tangent and normal = $\vec{a}_\tau,\ \vec{a}_n$

$\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \ddot{z}\vec{k} = \vec{a}_\tau + \vec{a}_n$

$\vec{a}_\tau = \ddot{\sigma}\vec{\tau} = \frac{\vec{a}\cdot\vec{V}}{V}\vec{\tau}$

$\vec{a}_n = \frac{\dot{\sigma}^2}{\rho}\vec{n} = \frac{|\vec{a}\times\vec{V}|}{V}\vec{n}$

$\rho = \frac{1}{\kappa}$, where κ is curvature

$\kappa(x) = \frac{|f''|}{(\sqrt{1+f'^2})^3}$

Task 1 (mine): solution in subfolder "solution"

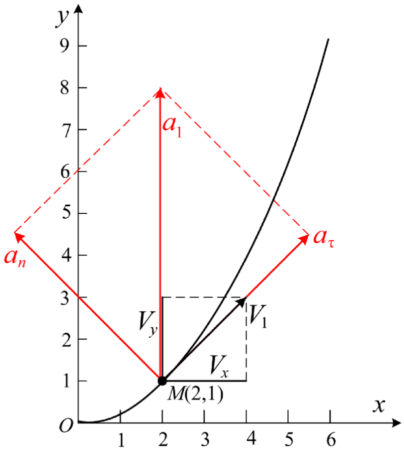
The point M motion is given in the following form:

$$\begin{cases} x = 2t \\ y = t^2 \end{cases}$$

When $t = 1$ sec, the goal is to find:

1. $y(x)$ — trajectory;
2. $\vec{v}$ — velocity (magnitude and direction);
3. $\vec{a}$ — acceleration (magnitude and direction);
4. $a_n,\ a_\tau$ — normal and tangent acceleration;
5. ρ — radius of curvature.

Answer: $y(x) \rightarrow y = \frac{x^2}{4},\ \vec{v} = 2\vec{i} + 2\vec{j},\ \vec{a} = 2\vec{j},\ a_n = \sqrt{2},\ a_\tau = \sqrt{2},\ \rho = 5.64$



Task 2 (yours): M (rus) 12.15

184. The motion of a particle is defined by the equations:

$$x = a(e^{kt} + e^{-kt}), \quad y = a(e^{kt} - e^{-kt}),$$

where a and k are constants.

Find the equation of the path of the particle, and its velocity and acceleration as a function of radius-vector $r = \sqrt{x^2 + y^2}$.

Ans. The hyperbola: $x^2 - y^2 = 4a^2$; $v = kr$; $w = k^2 r$.

Hint: you should eliminate *t*, for it, take *x*, *y* in power of 2 and think

Task 3 (yours): M (rus) 12.23

192. A coastal gun is placed at a height of $h=30$ m above sea level (Fig. 148). A projectile is fired from the gun at the angle of elevation $\alpha_0=45°$, and initial velocity is $v_0=$ $=1000$ m/sec. Neglecting air friction, find how far from the gun the projectile will hit a target located at sea level.

Ans. 102 km.

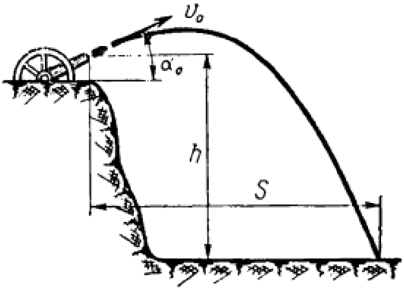


Fig. 148

Deserve “A” grade!

– Oleg Bulichev

o.bulichev@innopolis.ru

@Lupasic

Room 105 (Underground robotics lab)