



# Theoretical Mechanics, Lab 4: KIN PLANE3 SPHER

Plane motion 3

Spherical motion

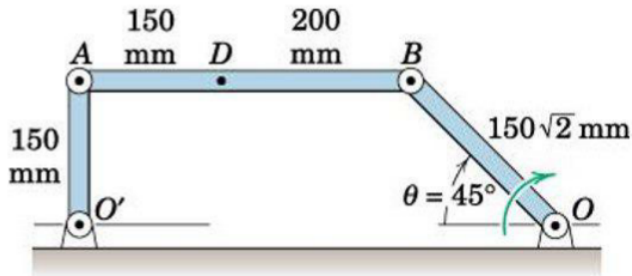
# Questions from the class



*No questions this time*

## Task 1 (yours)

Arm  $OB$  of the linkage has a clockwise angular velocity of  $10 \text{ rad/sec}$  in the position shown where  $\phi = 45^\circ$ . Determine the velocity of  $A$ , the velocity of  $D$ , and the angular velocity of link  $AB$  for the shown position.



Task 1

# Task 1 (yours): solution

Directions of vel of  $A$  and  $B$  are tangent to their circular paths @ fixed centers  $O$  and  $O'$

- Intersection of the two perpendiculars to the vel from  $A$  and  $B \rightarrow$  IC  $C$  for the link  $AB$

$$AC = AB \tan 45 = 350 \tan 45 = 350 \text{ mm}$$

$$CD^2 = 350^2 + 150^2 \rightarrow CD = 381 \text{ mm}$$

$$CB^2 = 350^2 + 350^2 \rightarrow CB = 350\sqrt{2} \text{ mm}$$

$$\text{Angular vel of } OB: \omega_{OB} = 10 \text{ rad/s}$$

$$v_B = r \omega = 150\sqrt{2} \times 10 = 2121.3 \text{ mm/s}$$

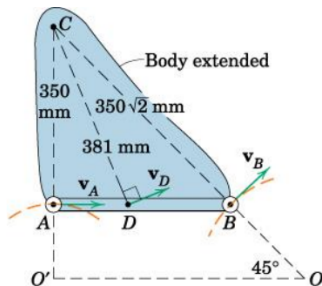
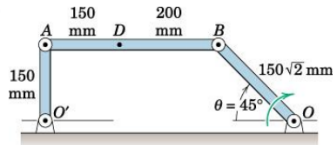
$$\text{Ang vel of } BC: \omega_{BC} = 2121.3 / 350\sqrt{2} = 4.29 \text{ rad/s}$$

$BC$  is a line considered on the extended rigid body  $\rightarrow \omega_{BC} = \omega_{AC} = \omega_{DC} = \omega_{AB}$  (clockwise)

$$v_A = r \omega = 350 \times 4.29 = 1.5 \text{ m/s}$$

$$v_D = r \omega = 381 \times 4.29 = 1.632 \text{ m/s}$$

Directions of velocities are shown.



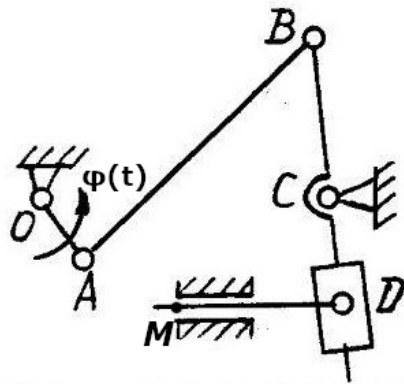
## Task 2 (mine)



The task to find kinematics for the whole mechanism and velocities for  $D$ .

You know all lengths ( $OA$ ,  $AB$ ,  $BC$ ),  $\varphi(t)$ .

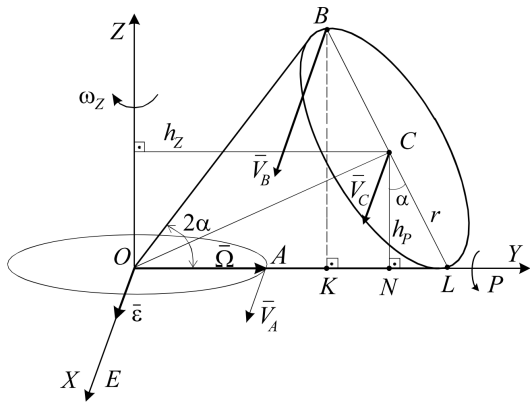
Coordinates of all bases are known. The basis near to  $M$  point is horizontal respect to the ground.



Task 2

## Task 3 (mine)

The cone (angle —  $2\alpha$ ,  $\alpha = 30^\circ$ ,  $r = 20$  — base) is rolling on a ground without friction.  $\vec{V}_C = \text{const} = 60$ .  
It is needed to find  $\Omega$ ,  $\varepsilon$ ,  $\vec{V}_B$ ,  $\vec{a}_B$ .



Task 3

# Task 3 (hints)



## Угловая скорость и угловое ускорение

Вектор **углового ускорения** равен скорости движения конца вектора мгновенной угловой скорости по его годографу

$$\vec{\varepsilon} = \frac{d\vec{\Omega}}{dt}$$

Скорость точки конца вектора мгновенной угловой скорости  $\vec{\Omega}$

$$\vec{V}_A = \frac{d\vec{\Omega}}{dt}$$

Следовательно:

$$\vec{\varepsilon} = \vec{V}_A$$



При сферическом движении тела направления векторов  $\vec{\Omega}$  и  $\vec{\varepsilon}$  и не совпадают.

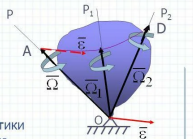
г) угловое ускорение тела:

Векторная величина, характеризующая изменение с течением времени угловой скорости по модулю и по направлению – **мгновенное угловое ускорение тела**

$$\vec{\varepsilon} = d\vec{\Omega}/dt$$

AD – годограф вектора  $\vec{\Omega}$   
Направление  $\varepsilon$  совпадает с касательной к кривой AD в соответствующей точке

Векторы  $\vec{\Omega}$  и  $\vec{\varepsilon}$  – основные кинематические характеристики сферического движения тела



MyShared

## Task 4 (yours): M (rus) 19.9

**321.** A disk  $OA$  of radius  $R=4\sqrt{3}$  cm rotating about a fixed point  $O$  rolls round a stationary cone of semi-angle  $30^\circ$  at a vertex (Fig. 239). Find the angular velocity of rotation of the disk about its axis of symmetry if the acceleration  $w_A$  of a point  $A$  on the disk has a constant magnitude and equals  $48 \text{ cm/sec}^2$ .

*Ans.*  $\omega = 2 \text{ sec}^{-1}$ .

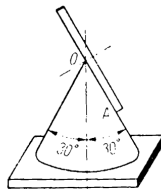


Fig. 239



# Deserve "A" grade!

– Oleg Bulichev

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📍 @Lupasic

🏢 Room 105 (Underground robotics lab)