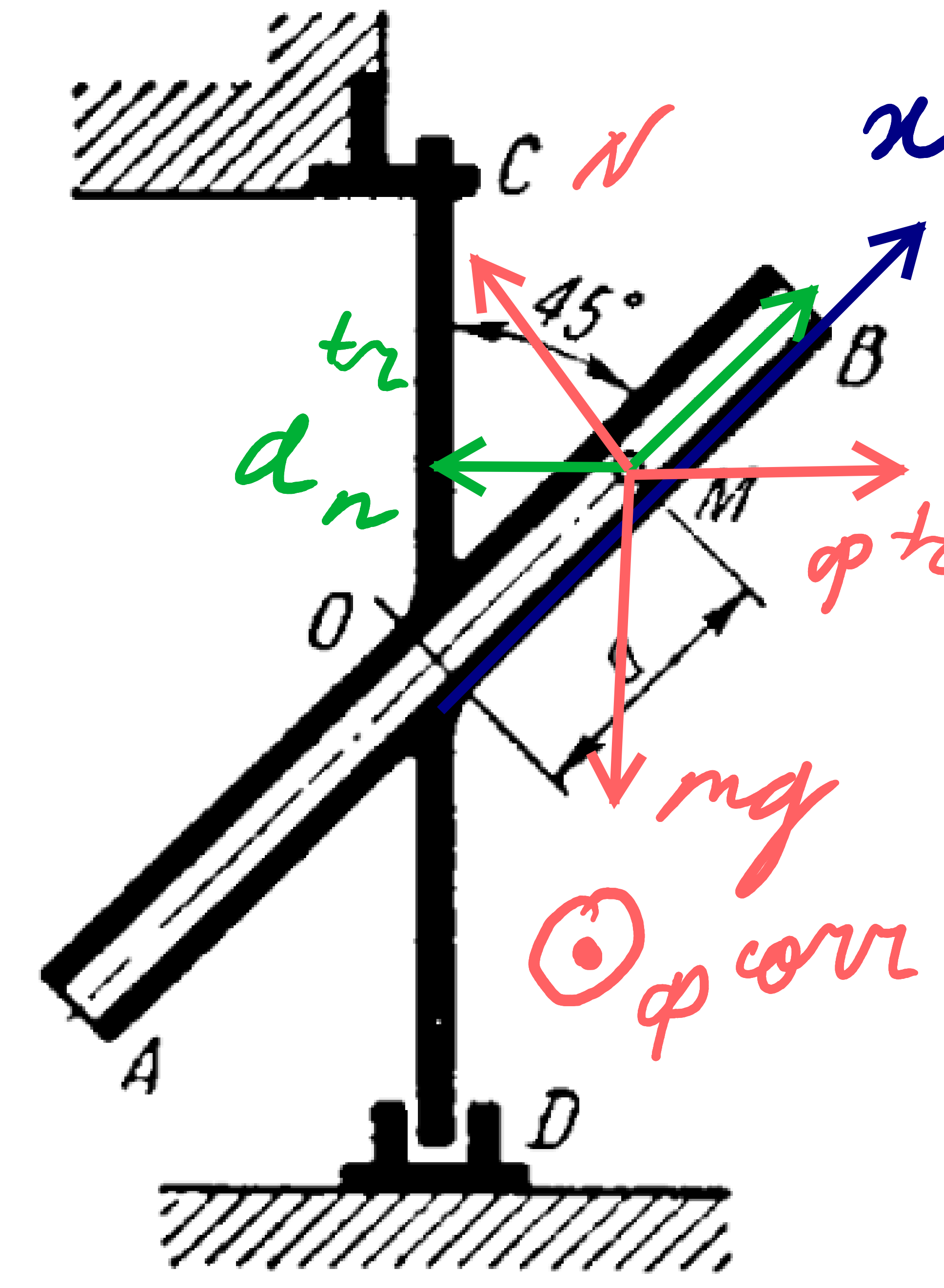
The figure shows a pipe AB, which rotates about a vertical axis CD with a constant angular velocity ω. The angle between AB and CD is always 45°.

A small heavy ball is placed in the pipe. Determine the motion of the ball, assuming that its initial velocity is 0 and the initial distance between the ball and a point O equals a. Neglect friction.



$$Ans:\ OM = \frac{1}{2}\left(a - \frac{g\sqrt{2}}{\omega^2}\right)\left(e^{0.5\omega t\sqrt{2}} + e^{-0.5\omega t\sqrt{2}}\right) + \frac{g\sqrt{2}}{\omega^2}$$

Lab 9, Task 2

2nd Newton's Law for non inertial systems

$$\left.\begin{aligned} &m\bar{a} = \Sigma \bar{F} \\ &\bar{a}^{rel} + \bar{a}^{tr} + \bar{a}^{cor} \end{aligned}\right\} \Rightarrow m\bar{a}^{rel} = \Sigma \bar{F} \underbrace{- m\bar{a}^{tr}}_{+\bar{\varphi}^{tr}} \underbrace{- m\bar{a}^{corr}}_{+\bar{\varphi}^{cor}}$$

Inertia forces have the opposit sign related to acceleration. It's not a physical force, we need it to compencate a non inertiality of a system.

Reasearch Object: a system, consists of a) particle M (ball), b) tube AB

Motion: M - rectlinear, AB - rotational along CD

Conditions:

"0" - Initial "1" - Final

$x_0 = a$ $\quad x_1 - ?$

$\dot{x}_0 = 0$ $\quad \dot{x}_1 - ?$

$t_0 = 0$ $\quad t_1 - ?$

Kinematic Analysis:

We need to know directions of all accelerations (they are Inertia Forces components)

$a_n^{tr} = \omega^2 R \quad \bar{a}_\tau^{tr} = \varepsilon R \quad \bar{a}^{corr} = 2\omega \times v^{rel}$

Force Analysis:

$\varphi^{tr} = -m a_n^{tr} = m\omega^2 r$, $r = \sin\alpha\, x$; N

Solution:

$m\bar{a}^{rel} = \bar{N} + m\bar{g} + \bar{\varphi}^{tr} + \varphi^{corr}$. $\quad x: m\ddot{x} = -mg\cos\alpha + m\omega^2 \sin\alpha\, x$

$\ddot{x} = \omega^2 \frac{\sqrt{2}}{2} x - g\frac{\sqrt{2}}{2} \Rightarrow \dot{x} = \omega\frac{\sqrt{2}}{2}\left(C_1 e^{\omega\frac{\sqrt{2}}{2}t} - C_2 e^{-\omega\frac{\sqrt{2}}{2}t}\right)$

$x = C_1 e^{\omega\frac{\sqrt{2}}{2}t} + C_2 e^{-\omega\frac{\sqrt{2}}{2}t} + g\frac{\sqrt{2}}{\omega^2}$