Theoretical Mechanics, Lab 13: EULER LAGRANGE

Euler-Lagrange equation

INNOPOLIS UNIVERSITY

Generalized coordinates

Let a system have N particles connected by l constraints:

$$f_s(t, x_k, y_k, z_k) = 0 \qquad s = 1, 2, \ldots, l \qquad k = 1, 2, \ldots, N$$

Then the independent coordinates are $n = 3N - l$

Instead of Cartesian coordinates, we can specify n any other independent parameters

$$q_i = q_i(x_k, y_k, z_k) \qquad i = 1, 2, \ldots, n \qquad k = 1, 2, \ldots, N$$

$$x_k = x_k(q_i, i)$$
$$y_k = y_k(q_i, i)$$
$$z_k = z_k(q_i, i)$$

$$\vec{r}_k = \vec{r}_k(q_i, i) \qquad \delta \vec{r}_k = \sum_{i=1}^{n} \frac{\partial \vec{r}_k}{\partial q_i} \delta q_i$$

Generalized forces

1) $$Q_i=\sum_{k=1}^{N}\vec F_k\cdot\frac{\partial \vec r_k}{\partial q_i}=\sum_{k=1}^{N}\left(F_{kx}\frac{\partial x_k}{\partial q_i}+F_{ky}\frac{\partial y_k}{\partial q_i}+F_{kz}\frac{\partial z_k}{\partial q_i}\right)$$

2) $$Q_i=\frac{\left(\sum_{k=1}^{N}\vec R_k\cdot\delta\vec r_k\right)_{q_i}}{\delta q_i}$$

For conservative forces:

$$\boxed{F_{kx}=-\frac{\partial \Pi}{\partial x_k}\qquad F_{ky}=-\frac{\partial \Pi}{\partial y_k}\qquad F_{kz}=-\frac{\partial \Pi}{\partial z_k}}$$

3) $$Q_i=-\frac{\partial \Pi}{\partial q_i}$$

$$\boxed{\delta A=\sum_{i=1}^{n}Q_i\,\delta q_i}$$

$$\boxed{Q_i=\sum_{k=1}^{N}\vec F_k\cdot\frac{\partial \vec r_k}{\partial q_i}}\qquad Q_m=\frac{\delta A_m}{\delta q_m}.$$

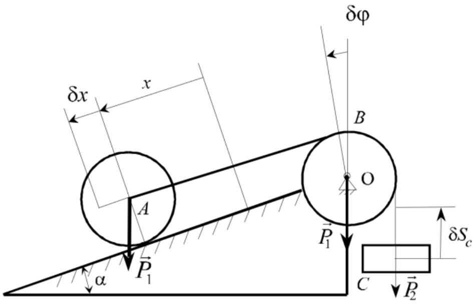
Euler-Lagrange equation

R. O.	Eqn #	Equations	Applications	Extra Info
Body, System	s, s – d.o.f of system	$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i$ or, $\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^D$ $L = T - \Pi$	To find accelerations, forces, motion equations.	• It is the most classical and flexible method for multi-body systems, concluding only holonomic constraints. • q – generalized coordinates. • Q – all generalized forces. • Q^D – non-potential general forces.

Task 1 (mine)

The system consists of body A, mass m_1 goes down without slippering. It is connected with body C (mass m_2), using body B. Block B rotates along the center of mass. The angle of the slope is α.

It is needed to find the acceleration of CoM A.



Task 2 (yours): Solution subfolder

There is a mechanical system. It moves because of gravity force. The generalized coordinates are x and ξ. No slipping. Body 1 is a particle. Body 3 is a homogeneous disk;

The goals are following:

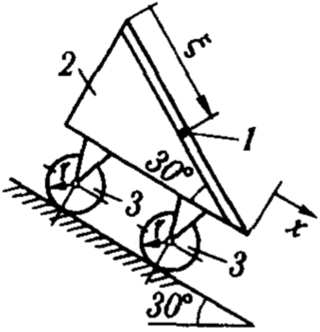
1. find an equation of motion for this system;
2. simulate this system (obtain all positions) or make plots for each body $x(t)$, $y(t)$.

Needed variables:

$m_1 = 1,\ m_2 = 3,\ m_3 = 2;$

$b = 0.001$, where b is viscous drag coefficient;

Initial conditions: $x_0 = 0,\ \dot{x}_0 = 0,\ \xi_0 = 0,\ \dot{\xi}_0 = 3.$



Task 2
(Yablonskii (rus) D21)

Deserve “A” grade!

– Oleg Bulichev

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Room 105 (Underground robotics lab)