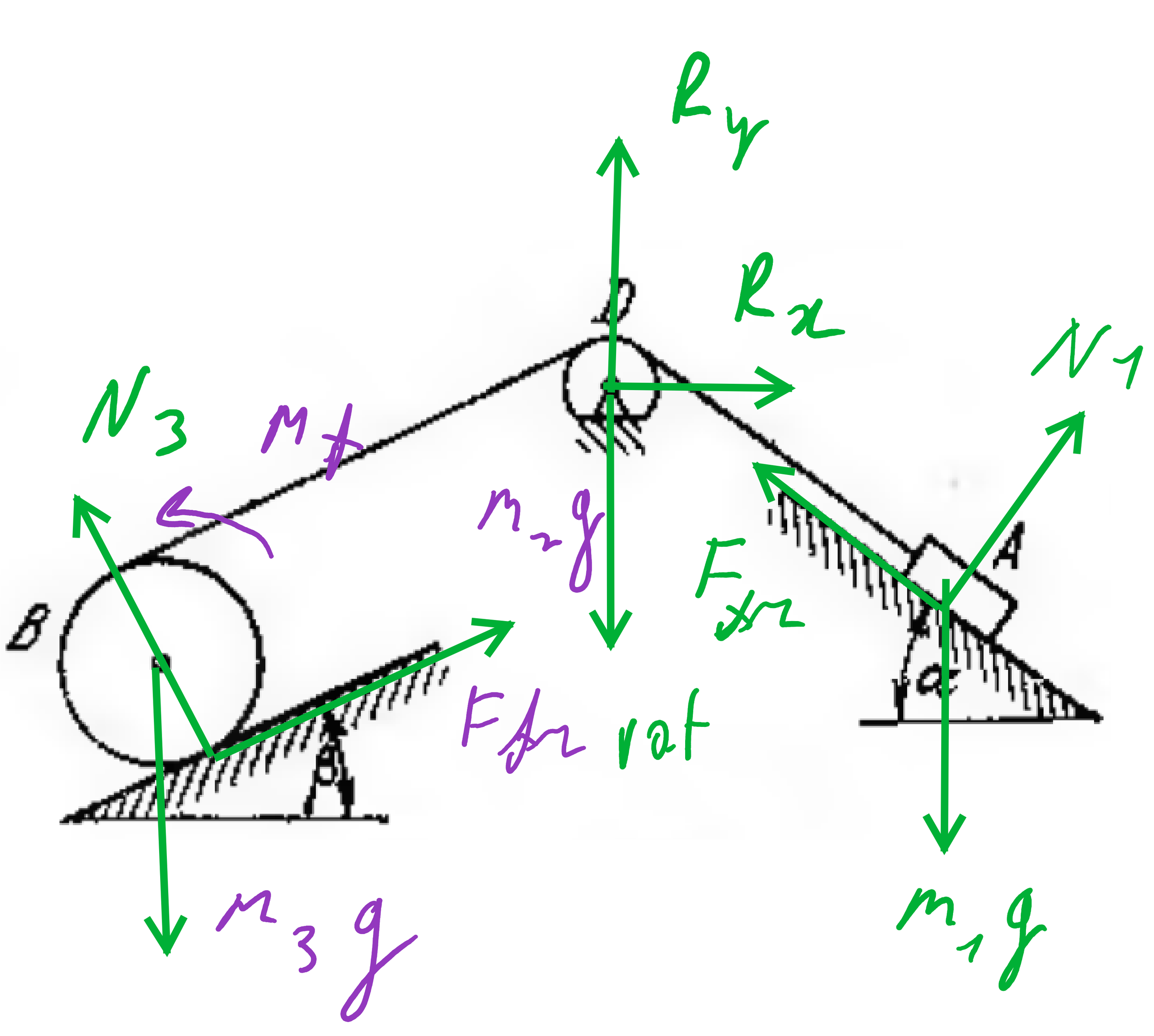
A load A of mass M_1 has an ideal string attached to it, thrown over the block D of mass M_2 and wound on the side surface of the cylindrical roller B of mass M_3. When load A moves down an inclined plane located at an angle α to the horizon, block D rotates, and the roller B rolls without slippering up the inclined plane, forming an angle β.

Determine the velocity v of the load A as a function of its path s, if the system is at rest at the initial moment. Consider the block and the roller as homogeneous circular cylinders. The forces of friction are neglected.



HINTS:

1) The method is very nice for find a correlation between velocities and forces.

2) We need kinematics to represent all velocities using only one variable

$$dT = \delta A \qquad T_2 - T_1 = A_{12}$$

$$\frac{dT}{dt} = N \qquad T_2 - T_1 = \Pi_1 - \Pi_2 + \tilde{A}_{12}$$

Work depends on a nature of force

$$A = FS\cos(F\hat{\ }S) = \int \bar{F}\cdot d\bar{s} = \int_{\bar{r}_0}^{\bar{r}_1} \bar{F}(\bar{r})\,d\bar{r}$$

$A = |\bar{M}|\theta$ For rotation motion

Research Object:

System consists of 3 bodies: brick "1", disk "2", cylinder "3"

Motion:

"1" - translatory motion, "2" - rotational motion, "3" - plane motion

Know:

$m_1, m_2, m_3, R_3, R_2, \rho_1, \rho_2, \alpha, \beta, \cancel{f}, \gamma$

Friction, friction coefficient

Kinematics (common kinematics from the 1st part of the course):

$$V_1 = \omega_2 r_2 \Rightarrow \omega_2 = \frac{V_1}{r_2}; \quad V_2 = V_1$$

$$V_2 = \omega_3 2R_3 \Rightarrow \frac{V_2}{2R_3} = \omega_3 \Rightarrow V_3 = \frac{V_2}{2}$$

$$\omega_2 = \frac{V_1}{R_2}; \quad \omega_3 = \frac{V_1}{2R_3}; \quad V_3 = \frac{V_1}{2}$$

$$\varepsilon_2 = \frac{a_1}{R_2} \qquad \varepsilon_3 = \frac{R_2}{2R_2R_3}a_1 \qquad \varphi_2 = \frac{x_1}{R_2} + \varphi_{02}$$

$$a_3 = \frac{a_1}{2} \qquad \varphi_3 = \frac{x_1}{2R_3} + \varphi_{03} \qquad x_3 = \frac{x_1}{2}$$

Force Analysis:

$$(m_1, m_2, m_3)g;\ N_1 = mg\,s\alpha;\ N_3 = m_2 g\,c\beta$$

$$R_y, R_x;\ F_{fr} = \gamma N_1;\ F_{fr\,rot};\ M_{fr} = \delta N_2;\ \rho = \sqrt{\frac{J}{m}} = ?$$

Solution:

$$T_{II} - T_I = A_{I-II}$$

$T_1 = 0$

along axis of rotation   center of mass

$$T_{II} = \underbrace{\frac{m_1 v_1^2}{2}}_{\text{1st body}} + \underbrace{\frac{J_{02}\omega_2^2}{2}}_{\text{2nd body}} + \underbrace{\frac{J_{03}\omega_3^2}{2} + \frac{m_3 V_3^2}{2}}_{\text{3rd body}} =$$

$$= \frac{V_1^2}{2}\left(m_1 + \frac{J_{02}}{R_2^2} + \frac{J_{03}}{4R_3^2} + \frac{1\,m_3}{4R_3^2}\right)$$

$$A_{I-II} = \left\{\begin{array}{l} m_1 g h\, c(90-\alpha);\ N_1 \cdot h \cdot c90 = 0 \\ m_2 g \cdot 0 = 0;\quad N_3 x_3\, c90 = 0 \\ m_3 g x_3\, c(90+\beta);\ R_y = R_x = 0 \\ F_{fr} h \cos(\pi);\ -M_f \varphi_3;\ F_{fr\,rot} = 0 \end{array}\right. +$$

Important to choose the sign correctly   in classical -> cosine do this job by itself

$$T_{II} = A_{I-II} \Rightarrow V_1$$