

Research object: a system consists of:

A - plane motion

B - rot motion

C - trans. motion

1 dof

Force analysis: mg_1, mg_2, R_x, R_y, B

Generalized coords:

x - ~~static~~ transl motion of body A (center of mass)

$$V_A = \dot{x}$$

$$\omega_A = \frac{V_A}{R} = \frac{\dot{x}}{R}$$

$$V_A = \omega_A \times R = \frac{\dot{x}}{R}$$

$$\omega_B = \frac{V_A}{R} = \frac{\dot{x}}{R}$$

$$V_C = V_A = \dot{x}$$

Kinetic energy: $T = T_A + T_B + T_C$

$$T_A = \frac{1}{2} m_1 V_A^2 + \frac{1}{2} J_A \omega_A^2$$

plane motion is
rot + trans

$$T_B = \frac{1}{2} J_0 \omega_B^2$$

rot

$$T_C = \frac{1}{2} m_2 V_C^2$$

trans

$$J_A = J_0 = \frac{m_1 R^2}{2}$$

from the task

$$T = \frac{1}{2} (2m_1 + m_2) \dot{x}^2$$

Partial derivatives:

$$\frac{\partial T}{\partial \dot{x}} = (2m_1 + m_2) \dot{x} \quad \frac{\partial T}{\partial x} = 0$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) = (2m_1 + m_2) \ddot{x}$$

Generalized forces

$$\delta A = m_1 g \sin \alpha \delta x - m_2 g \delta s_C = g(m_1 \sin \alpha - m_2) \delta x$$

$$Q_1 = \frac{\delta A_1}{\delta x} = g(m_1 \sin \alpha - m_2)$$

Solution

$$(2m_1 + m_2) \ddot{x} = g(m_1 \sin \alpha - m_2)$$