The Point M motion is given in following form:
$$\begin{cases} x = 2t \\ y = t^2 \end{cases}$$
When $t = 1$ sec, the goal is to find:

- $y(x)$ – trajectory,
- $\vec{v}$ – velocity (magnitude and direction),
- $\vec{a}$ – acceleration (magnitude and direction),
- a_n, a_τ – normal and tangent acceleration,
- ρ – curvature.

Answer: $y(x) \to y = \frac{x^2}{4}$, $v = 2\vec{i} + 2\vec{j}$, $a = 2\vec{j}$, $a_n = \sqrt{2}$, $a_\tau = \sqrt{2}$, $\rho = 5.64$.

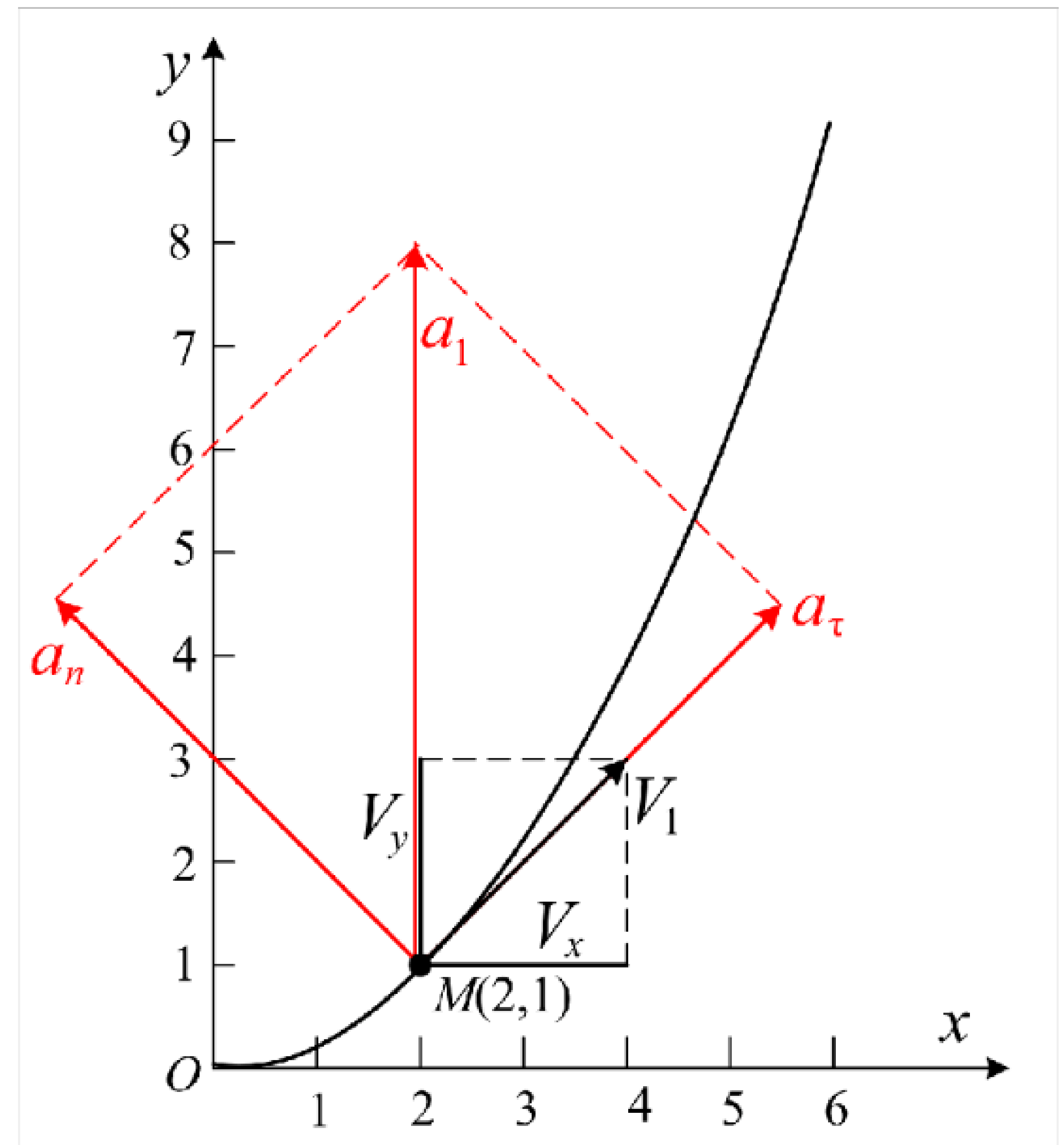

Figure 1: Lab 1, task 1

Let's solve this task in 2 ways:
a) When we know the parametric form
b) When we know x(t), y(t)

TIPS
1) (b) approach is overkill for this task, but it can be useful, when you don't know a parametric form or you have y(x) and several x(t)/y(t)
2) In our case
y(x) - unique, x(y) -> not.
We cannot use x(y) to find path.

a) 1) $\begin{cases} x = 2t \\ y = t^2 \end{cases} \Rightarrow t = 1 \quad \begin{matrix} x = 2 \\ y = 1 \end{matrix}$

2) Find velocity and acceleration by taking derivatives by "t"

$\dot{x} = \dot{V}_x = 2 \qquad \ddot{x} = a_x = 0$

$\dot{y} = V_y = 2t \qquad \ddot{y} = a_y = 2$

$\Downarrow$

$2t\big|_{t=1} = 2$

3) Find whole $\bar{v}$, $\bar{a}$

$$\bar{v} = \bar{v}_x + \bar{v}_y = 2i + 2j = \begin{bmatrix} 2 \\ 2 \end{bmatrix}; \quad \bar{a} = \bar{a}_x + \bar{a}_y = 0i + 2j = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$|\bar{v}| = \sqrt{v_x^2 + v_y^2} = 2\sqrt{2}; \quad |\bar{a}| = 2$$

4) Find a_τ, a_n, ρ = ?

$$a_\tau = \frac{\bar{a}\bar{v}}{|\bar{v}|} = \frac{2 \cdot 0 + 2 \cdot 2}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41$$

$$a_n = \frac{|\bar{a} \times \bar{v}|}{|\bar{v}|} = \sqrt{a^2 - a_t^2} = 1,41$$

$$\rho = \frac{|\bar{v}|^2}{a_n} = 5,64$$

b)

1) $\begin{matrix} x(t) \\ y(t) \end{matrix} \Rightarrow y(x) \Rightarrow y = \frac{x^2}{4};\ t(x) \Rightarrow t = \frac{x}{2};\ t = 1$

2) $y(x) \to \sigma(x) = \int \sqrt{1 + (y'(x))^2}\, dx$

Partial derivative

$y'(x) = \frac{x}{2};\ \sigma(x) = \int \sqrt{1 + \frac{x^2}{4}}\, dx = \frac{x\sqrt{x^2+4}}{4} + \operatorname{arcsinh}\left(\frac{x}{2}\right)$

$+ C;\ t = 0 \to x = 0 \to 0$

3) $\sigma(x) \to \sigma(t);\ \sigma(t) = t\sqrt{t^2+1} + \operatorname{arcsinh}(t)$

$|\bar{v}| = \dot{\sigma}(t) = \frac{1}{\sqrt{t^2+1}} + \sqrt{t^2+1} + \frac{t^2}{\sqrt{t^2+1}} = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 2\sqrt{2}$