Daniel Lupercio - STAT 724 HW 1

In [2]:

```python
# %load ../standard_import.txt
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import axes3d
import seaborn as sns

from sklearn.preprocessing import scale
import sklearn.linear_model as skl_lm
from sklearn.metrics import mean_squared_error, r2_score
import statsmodels.api as sm
import statsmodels.formula.api as smf

%matplotlib inline
plt.style.use('seaborn-white')
```

Chapter 3, Exercise 10

In [3]:

```python
import os
UP_DIR = '/Users/daniel421/Desktop/STAT_724/ISLR_data'
csv_file = os.path.join(UP_DIR,'Carseats.csv')
car_seats = pd.read_csv(csv_file)
car_seats.head()
```

Out[3]:

	Unnamed: 0	Sales	CompPrice	Income	Advertising	Population	Price	ShelveLoc	Age	Education	Urban	US
0	1	9.50	138	73	11	276	120	Bad	42	17	Yes	Yes
1	2	11.22	111	48	16	260	83	Good	65	10	Yes	Yes
2	3	10.06	113	35	10	269	80	Medium	59	12	Yes	Yes
3	4	7.40	117	100	4	466	97	Medium	55	14	Yes	Yes
4	5	4.15	141	64	3	340	128	Bad	38	13	Yes	No

(a) Fit a multiple regression model to predict $Sales$ using $Price$, $Urban$, and US.

In [4]:

```python
model_fit = smf.ols("Sales ~ Price + Urban + US", car_seats).fit()
```

(b) Provide an interpretation of each coefficient in the model. Be careful—some of the variables in the model are qualitative!

In [5]:

```python
model_fit.summary().tables[1]
```

Out[5]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	13.0435	0.651	20.036	0.000	11.764	14.323
Urban[T.Yes]	-0.0219	0.272	-0.081	0.936	-0.556	0.512
US[T.Yes]	1.2006	0.259	4.635	0.000	0.691	1.710
Price	-0.0545	0.005	-10.389	0.000	-0.065	-0.044

For fixed values of $Urban$ and US, a 1-unit increase in $\mathbf{Price}$ results in a change of $Sales$ of -0.0545 units (54 sales).

For fixed values of $Price$ and $Urban$, the effect of the store being located in the $\mathbf{US}$ is a change of $Sales$ of 1.2006 units (1,200 sales).

For fixed values of $Price$ and US, the effect of the store being located in an $\mathbf{Urban}$ location is a change of $Sales$ of -.0219 units (decrease of 22 sales).

(c) Write out the model in equation form, being careful to handle the qualitative variables properly.

$$\hat{y} = 13.0435 - 0.0219 * Urban + 1.2006 * US - 0.0545 * Price$$

- $Urban$ = 1 for a store in an urban location, 0 elsewhere
- US = 1 for a store in the US, 0 elsewhere

(d) For which of the predictors can you reject the null hypothesis $H_0 : \beta_j = 0$?

$$H_0 : \beta_{2,3} = 0 \; \& \; H_A : \beta_{2,3} \neq 0$$

Based on the p-values of US and $Price$, we can reject $H_0 : \beta_{2,3} = 0$

(e) On the basis of your response to the previous question, fit a smaller model that only uses the predictors for which there is evidence of association with the outcome.

In [6]:

```python
model_fit2 = smf.ols("Sales ~ Price + US", car_seats).fit()
```

In [7]:

```python
model_fit2.summary().tables[0]
```

Out[7]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.239
Model:	OLS	**Adj. R-squared:**	0.235
Method:	Least Squares	**F-statistic:**	62.43
Date:	Sat, 25 Sep 2021	**Prob (F-statistic):**	2.66e-24
Time:	13:54:09	**Log-Likelihood:**	-927.66
No. Observations:	400	**AIC:**	1861.
Df Residuals:	397	**BIC:**	1873.
Df Model:	2		
Covariance Type:	nonrobust		

(f) How well do the models in (a) and (e) fit the data?

$R^2 = 0.239$ and $\bar{R}^2 = 0.234$ for the (a) model
$R^2 = 0.239$ and $\bar{R}^2 = 0.235$ for the (e) model

Both models can explain approximately 23.9% of the variance in $Sales$. However, the $\bar{R}^2$ for model (e) has a slight increase. This can be attributed in part, to the removal of the $Urban$ variable. Although we have very limiting information, it would be best to use the model in (e).

(g) Using the model from (e), obtain 95% confidence intervals for the coefficient(s).

In [8]:

```python
model_fit2.conf_int(alpha=0.05, cols=None)
```

Out[8]:

	0	1
Intercept	11.79032	14.271265
US[T.Yes]	0.69152	1.707766
Price	-0.06476	-0.044195

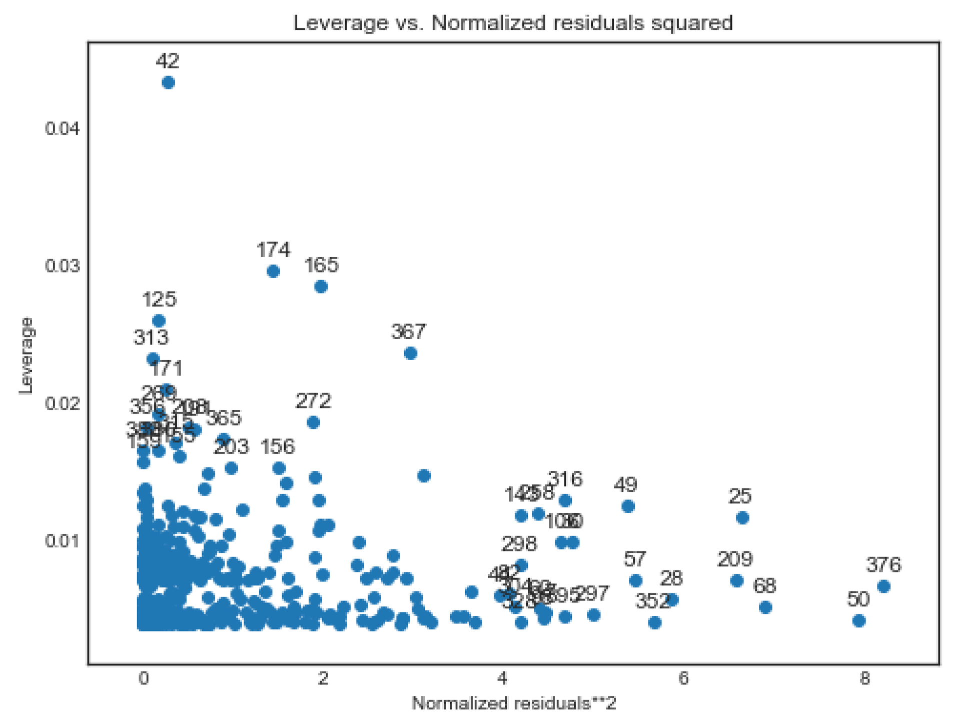
We can say that there is a 95% probability that, on average, the true parameter for $Price$ (β_2) falls within (-0.0648, -0.0442).
We can say that there is a 95% probability that, on average, the true parameter for US (β_1) falls within (0.692, 1.708).

(h) Is there evidence of outliers or high leverage observations in the model from (e)?

In [9]:

```python
from statsmodels.graphics.regressionplots import plot_leverage_resid2

fig, ax = plt.subplots(figsize=(8, 6))
fig = plot_leverage_resid2(model_fit2, ax=ax)
```



In [10]:

```python
from statsmodels.stats.outliers_influence import OLSInfluence as influence
# Compute the influence to get Cook's distance
inf = influence(model_fit2)

# cooks_distance is an attribute of incluence, here C, not sure about P (p-value maybe?)
C, P = inf.cooks_distance
```

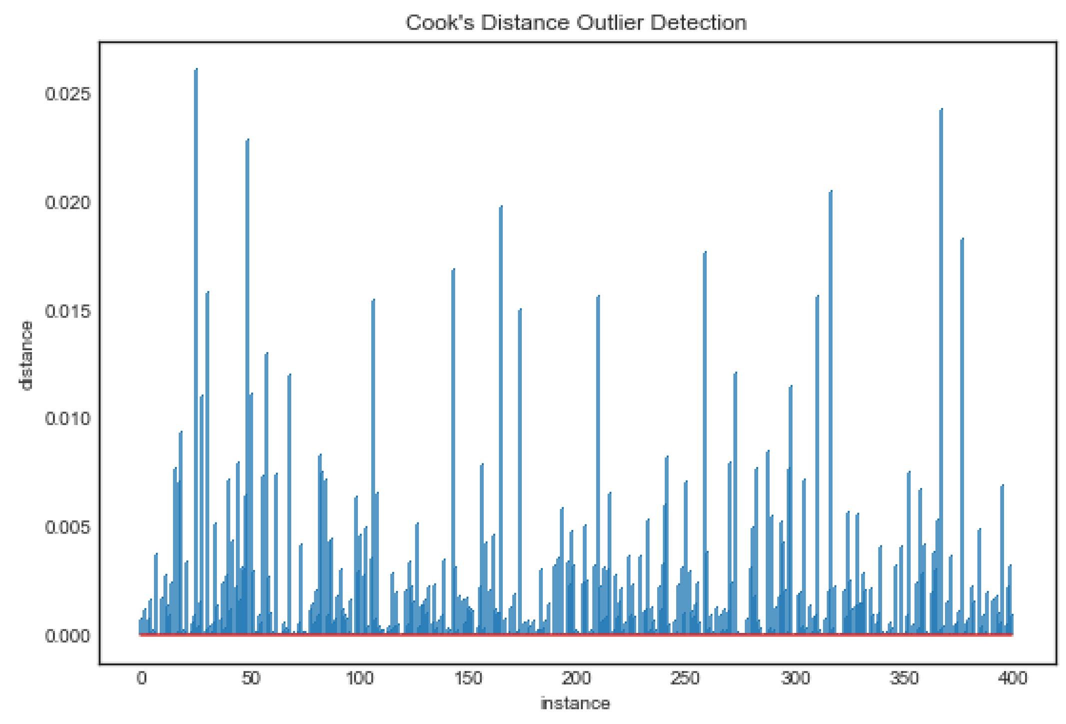
In [11]:

```python
def plot_cooks_distance(c):
    _, ax = plt.subplots(figsize=(9,6))
    ax.stem(c, markerfmt=",")
    ax.set_xlabel("instance")
    ax.set_ylabel("distance")
    ax.set_title("Cook's Distance Outlier Detection")
    return ax


plot_cooks_distance(C)
```

Out[11]:

```
<AxesSubplot:title={'center':"Cook's Distance Outlier Detection"}, xlabel='instance', ylabel='distance'>
```



There are instances of high leverage obserations, however, there is no indication of strong outliers. This is based off of low cook's distances.

Homework_1

Thursday, September 16, 2021 11:03 AM

Chapter 3 Exercises

5) $$\hat{y}_i = x_i \hat{\beta},$$

where
$$\hat{\beta} = \left(\sum_{i=1}^{n} x_i y_i\right) / \left(\sum_{i'=1}^{n} x_{i'}^2\right) \qquad (3.38)$$

Show that we can write

$$\hat{y}_i = \sum_{i'=1}^{n} a_{i'} y_{i'}$$

From (3.38)

$$\hat{y}_i = \frac{x_i \left(\sum_{i=1}^{n} x_i y_i\right)}{\sum_{i'=1}^{n} x_{i'}^2}$$

Let $d_i = \dfrac{x_i}{\sum_{i'=1}^{n} x_i^2}$, as d_i will be a constant for some $i \in \mathbb{Z}$.

Thus,

$$\hat{y}_i = d_i \sum_{i=1}^{n} x_i y_i$$

$$\hat{y}_i = \sum_{i=1}^{n} (d_i x_i) y_i$$ algebra laws

Let $a_{i'} = d_i x_i$

$$\hat{y}_i = \sum_{i=1}^{n} a_{i'} y_i$$

The fitted values are linear combinations of weigthed response values. With $a_{i'}$ being the weight of each y_i, $i \in \mathbb{Z}$ $\blacksquare$