1 Implementation

Define

$$A[g(\omega)] = \int_{E_0}^{\infty} dE\, e^{\alpha E} \left| \sum_{t=0}^{t_{\max}} g_\tau e^{(-t-1)E} - \Delta_\sigma(E,\omega) \right|^2 , \tag{1}$$

$$B[g(\omega)] = \vec{g}(\omega)^T \cdot \mathrm{Cov}_d \cdot \vec{g}(\omega) , \tag{2}$$

$$\Delta_\sigma(E,\omega) = \frac{e^{\frac{-(E-\omega)^2}{2\sigma^2}}}{Z(\omega)} , \tag{3}$$

$$Z(\omega) = \int_0^{\infty} dE\, e^{\frac{-(E-\omega)^2}{2\sigma^2}} = \sqrt{\frac{\pi}{2}}\sigma \left[1 + \mathrm{Erf}\left(\frac{\omega}{\sqrt{2}\sigma}\right)\right] , \tag{4}$$

$$A_0(\omega) \equiv A[0](\omega) = \int_{E_0}^{\infty} dE\, e^{\alpha E} \Delta_\sigma(E,\omega)^2 = \frac{\mathrm{Erf}\left(\frac{\omega - E_0 + \alpha\sigma^2/2}{\sigma}\right) + 1}{\sigma\sqrt{\pi}\left(\mathrm{Erf}\left(\frac{\omega}{\sqrt{2}\sigma}\right) + 1\right)^2} e^{\alpha\omega + \alpha^2\sigma^2/4} . \tag{5}$$

At each energy, we minimise

$$W[g] = (1-\lambda)\frac{A[g]}{A_0} + \lambda \frac{B[g]}{B_{\mathrm{norm}}} . \tag{6}$$

B_{norm} can be a function of ω. In order to nicely express the solution it is useful to define

$$\begin{aligned} f_t(\omega) &= \int_{E_0}^{\infty} dE\; \Delta_\sigma(E,\omega)\, e^{(-t+\alpha)E} \\ &= \frac{e^{\frac{\sigma^2}{2}(\alpha-t)^2} e^{(\alpha-t)\omega} \left\{1 - \mathrm{Erf}\left[\frac{E_0 - \omega + \sigma^2(t-\alpha)}{\sqrt{2}\sigma}\right]\right\}}{1 + \mathrm{Erf}\left[\frac{\omega}{\sigma\sqrt{2}}\right]} \end{aligned} \tag{7}$$

$$\mathtt{S}_{tr} = \frac{e^{-(2+t+r-\alpha)E_0}}{2+t+r-\alpha} , \qquad \mathtt{B}_{tr} = \mathrm{Cov}_{t+1,\,r+1} , \quad \mathtt{f}_t = f_{t+1} \tag{8}$$

The minimisation then amounts to solve the following linear system

$$\vec{g} = \left(\mathtt{S} + \frac{\lambda\, A_0(\omega)}{(1-\lambda)B_{\mathrm{norm}}(\omega)}\, \mathtt{B}\right)^{-1} \vec{\mathtt{f}} \tag{9}$$

.

In practice, W is rescaled

$$W[g] = (1-\lambda)\left\{\frac{A[g]}{A_0} + \frac{\lambda}{(1-\lambda)}\frac{B[g]}{B_{\mathrm{norm}}}\right\}, \tag{10}$$

and we redefine $\lambda' \in (0,\infty)$. We use three norms, with $\alpha = 0, -1, -1.99$.

1.1 Input parameters

- `datapath`: path to data file.
- `outdir`: location where output is saved.
- `prec`: numerical precision for the arithmetic, approximatively given in the number of digits.
- `tmax`: number of correlators that will be used for the reconstruction. By default, this is $T-1$ for open and $T/2$ for periodic boundaries, T being the time extent of the lattice. In this way, one uses $c(t)$ with $t \in [1, T-1]$ for open and $[1, T/2]$ for periodic boundaries. $t = 0$ is always excluded.
- `sigma`: smearing radius of the Gaussian kernel.
- `mpi`: a unit of mass, e.g. m_π if known.
- `emin`, `emax`, `ne`: The reconstruction is performed at `ne` points in energy, spacing from `emin` to `emax`. The points are equally spaced. `emin` and `emax` are given in unit of `mpi`.
- `nboot`: number of bootstrap samples
- `e0`: value of E_0 used in the equations above.