Statistics II | seminar 1

Work with data frames, descriptive statistics, linear regression model, t-test

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Example (1)

1. Load data from file `people.csv` into a table. Notice column separator and encoding (UTF-8). Work with this table for the rest of the seminar.
2. Explore data types of variables in the table.
3. Check the data types and change them if needed. Data type `factor` is used for categorical variables and `numeric` or `integer` for numerical variables. Correct data types or/and values of variables *Weight*, *Sex* and *BMI*.
4. Recall exploratory data analysis (EDA). Numerical variables: compute sample means, standard deviations, variances and medians, draw boxplots and histograms. Categorical variables: compute frequency tables, draw frequency bar charts.
5. Draw scatter plots of Height (x), Weight (y) and Height (x), BMI (y) with coloring based on the variable Sex.
6. Variable *BMI* shows some missing (`NA`) or mistaken observations. Find and return these rows of the table. Using the BMI formula and variables *Weight* and *Height* add new variable *BMI.new* with correctly computed values.

Example (2)

7. Use linear regression model with regression line to analyze the dependence of *Weight* on *Height* and *Sex*. Which regression coefficients are statistically significant? (Use significance level $\alpha = 0{,}05$.) Interpret the meaning of each coefficient. Data and model represent graphically, including 95% prediction intervals.
8. Recall (two-sample) t-test. Test a hypothesis about differences in mean BMI of men and women. Formulate null and alternative hypothesis and decide the result of the test.
9. Test previous hypothesis for another binary variables. E.g., test a hypothesis about differences in mean BMI when grouped by *iPhoneOwner* variable and explain the result.

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Statistics II | seminar 2

One-way analysis of variance (ANOVA)

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Example (1)

Total weight of potatoes grown from one plant was measured for four potato varieties (denoted 1–4).

Variety	Weight (in kg)				
1	0,9	0,8	0,6	0,9	
2	1,3	1,0	1,3		
3	1,3	1,5	1,6	1,1	1,5
4	1,1	1,2	1,0		

Data file `potatoes.csv`: At a significance level of 5 %, test that the expected value of weight is not distinct across the varieties. Count effects and arithmetic means of the potato varieties. Recall ANOVA table and the meaning of the variables in it. Determine which pairs of varieties differ in case of rejecting the null hypothesis. Use both Tukey and Scheffe test.

Overall: Remember to use descriptive statistics and visualisation to get to know your data. Make sure that all of the conditions related to the chosen testing method are satisfied.

Example (2)

Data file `salesman.csv` contains monthly sales (in thousands Kč) for three salesmen in half-year period.

At a significance level of 5 %, test null hypothesis that the expected values of the monthly sales for all salesmen are equal. Determine pairs of salesmen exhibiting different sales in case of rejecting the null hypothesis.

Example (3)

Data file `coagulation.csv`: Clinical study observed relationship between a diet and time needed for blood coagulation. The research was performed on 24 animals fed with four different diets (A–D).

At a significance level of 0.05, test null hypothesis that the expected values of the coagulation time for all diets are equal. In case of rejecting the null hypothesis, determine pairs of diets that have significantly different coagulation time.

Example (4, manual calculation, R only as calculator and quantile table)

It is given $a = 5$ independent random samples of sizes $n_i = 5; 7; 6; 8; 5$, where i-th sample is of a distribution $N(\mu_i, \sigma^2)$, $i = 1, \ldots, 5$.
The total sum of squares $S_T = 15$ and the sum of squares of the residual error $S_e = 3$ are known.

Compute the treatment sum of squares S_A and at a significance level of 5 %, test hypothesis that expected values are equal in all five considered samples.

Example (5)

Three species of mouse were tested on aggression based on their behavior in a maze. At the start each mouse was placed to the center of the square maze with sides equal to 1 m and divided to 49 identical squares. While the mouse was trying to escape the maze for 5 minutes, the number of crossed squares was observed.
Data file `mice.csv` contains the number of crossed squares for each species of mouse noted in separated columns. There are some missing values (`NA`) for the third species.
At a significance level of 0,05, test hypothesis that the expected values of the number of crossed squares for all three species of mouse are equal. Determine which pairs of species significantly differ in case of rejecting the null hypothesis.

Example (6)

Consider linear regression models

$$\begin{aligned} M_1 : &\quad y = \beta_0 + \beta_1 x, \\ M_2 : &\quad y = \beta_0 + \beta_1 x + \beta_2 x^2, \\ M_3 : &\quad y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \end{aligned}$$

for values from file `data01.csv`. Using the analysis of variance compare these models and select the most suitable one for the data.

Example (7)

Annual depth data (ft) for lake Huron between the years 1875 and 1972 are stored in variable `LakeHuron` in *R* environment. Using the linear regression model fit the polynomial function of degree 8 to the data. Then using the analysis of variance look into the possibility of reducing the degree of the polynomial.

Example (8)

Sugar beet yield (in hundredweight per hectare) was monitored for 126 factories in Czechia in relation to the used amount of fertilizer K_2O (in kilograms per hectare).

Interval data are in file `sugarbeet.csv` in 4 columns:

1. lower limit of an interval K_2O (kg/ha),
2. upper limit of an interval K_2O (kg/ha),
3. number of factories that use K_2O in given interval,
4. average yield for sugar beet (q/ha).

Consider linear regression models

$$\begin{aligned} M_1: &\quad y = \beta_0 + \beta_1 x, \\ M_2: &\quad y = \beta_0 + \beta_1 x + \beta_2 x^2. \end{aligned}$$

Choose the midpoint of the interval for the values of x.

Using the analysis of variance select the most suitable model for the data.

Results

1. ANOVA rejects H_0. Distinct pairs: Tukey: 2-1, 3-1. Scheffe: 3-1.
2. ANOVA rejects H_0. Distinct pairs: 3-1, 3-2.
3. ANOVA rejects H_0. Distinct pairs: A-B, A-C, B-D, C-D.
4. ANOVA rejects H_0. $S_A = 12$, $F_A = 26$.
5. ANOVA rejects H_0. Distinct pairs: all.
6. M_2
7. Polynomial function of degree 7.
8. M_2

Statistics II | seminar 3

Two-way analysis of variance (ANOVA)

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Example (1)

Data file `hay.csv`:
Analyze the hay yields (in tons per hectare) in relation to the soil type (neutral / acidic) and fertilizer (none / dung / calcium fertilizer). Each combination was realized four times and independently of each other. Perform two-way analysis of variance with and without interactions and one-way analysis of variance. Test null hypothesis at a significance level of 0.05:

- soil type and fertilizer are independent, i.e. they do not interact.
- soil type has no effect on yields;
- fertilizer has no effect on yields;

Determine distinct groups in case of rejecting the null hypothesis. Which factor causes the difference in expected value of the yields?

Example (2)

Work with the tabel in variable `ToothGrowth` (in *R* environment) that contains the results of the study on the effect of the vitamin C on guinea-pig's teeth growth:
`len` = length of odontoblasts (cells of the outer surface of the tooth);
`supp` = administration of vitamin C (`OJ` = in orange juice, `VC` = in ascorbic acid);
`dose` = dose of vitamin C (3 groups: 0.5; 1.0 a 2.0 mg/day).

Perform two-way analysis of variance with interactions in relation to the administration and dose of vitamin C. Test corresponding null hypotheses at a significance level of 0.05. Determine distinct groups in case of rejecting the null hypothesis that the expected values of odontoblasts length are equal. Which factors and their interaction cause the difference in expected value of odontoblasts length?

Example (3)

Data file `goods.csv`:

Certain goods sales were observed over the same period of time in relation to two factors. The goods were sold packed in a bag or box and with the support of advertising campaign only in the press or in the press and also on television or without any advertising at all. File contains profit data in millions Kč from the goods sales under the stated conditions. Perform two-way analysis of variance with and without the interactions. Test null hypotheses at a significance level of 0.05:

- there is no interaction between the type of the packaging and the type of the advertisement;
- expected values of the profit do not depend on the type of the packaging;
- expected values of the profit do not depend on the type of the advertisement.

Determine distinct groups in case of rejecting the null hypothesis that the expected values are equal. Which factor causes the difference in expected value of the profit?

Example (4)

Data file `corn.csv`:
When determining the yield of corn, measurements were made on three different types of seeds and five different methods of fertilization. Two measurments were made for each combination. Perform two-way analysis of variance with and without interactions.. Test null hypotheses at a significance level of 0.05:

- there is no interaction between the type of seed and fertilization method;
- expected values of the yields are equal for all types of seeds;
- expected values of the yields are equal for all fertilization methods.

Determine distinct groups in case of rejecting the null hypothesis that the expected values are equal. Which factor causes the difference in expected value of the yields?

Results

1. Two-way ANOVA does not reject H_0 for Soil, but it does for Fertilizer, therefore Fertilizer has effect on Yield. Distinct pairs: none-calcium, none-dung.
   Two-way ANOVA with interactions moreover rejects H_0 for interactions. Distinct pairs for Fertilizer: all. Distinct pairs for interactions: 8 (Tukey), 7 (Sheffe).
2. Two-way ANOVA rejects H_0 for both factors. Distinct pairs: all.
   Two-way ANOVA with interactions moreover rejects H_0 for interactions. Distinct pairs for interactions: 11.
3. Two-way ANOVA rejects H_0 for both factors. Distinct pairs for Advert: PressTV-None, PressTV-Press.
   Two-way ANOVA with interactions cannot be performed due to the low number of observations.
4. Two-way ANOVA rejects H_0 for both factors. Distinct pairs for Variety: C-A, C-B. Distinct pairs for Fertilizer: 2-1, 4-2, 5-2.
   Two-way ANOVA with interactions does not reject H_0 for interactions, therefore two-way ANOVA without interactions is sufficient.

Statistics II | seminar 4

Rank-based tests and methods

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Example (1)

Data file minute.csv. Ten research participants had to guess independently of each other and without prior training when a minute has passed after the sound signal. Test hypothesis that half of the participants had a period of one minute underestimated and the second half had it overestimated. Check functions `sort` and `rank`. Perform the sign test and one-sample Wilcoxon signed-rank test.

Example (2)

Data file field.csv. Two fertilization methods were tested on 13 fields of the same soil quality. The first method A was tested on 8 fields and the second method B was tested on the remaining fields. Data file contains wheat yields (in tons per hectare) of tested fields. Determine if the fertilization method has effect on the wheat yields. Perform two-sample Wilcoxon rank-sum test (Mann-Whitney test).

Example (3)

Use data file potatoes.csv from the previous seminar. Test hypothesis using the Kruskal-Wallis and the median test that the expected value of weight of the potatoes grown from one plant does not depend on the variety. Furthermore, determine significantly different pairs of varieties.

Example (4)

Data file `octane.csv`. Octane rating was observed for 10 samples of gasoline. Test $H_0 : \widetilde{x} = 98$ using the sign test and the Wilcoxon signed-rank test.

Example (5)

Data file `bloodpressure.csv`. Systolic blood pressure was measured for 8 randomly chosen people before and after a medical procedure. Test hypothesis that the medians of the systolic blood pressure before and after the medical procedure are equal.

Example (6)

Data file `machines.csv`. There are 6 machines from 3 different manufacturers in the factory. Perform the Kruskal-Wallis test for the hypothesis that the expected value of machine efficiency does not depend on the manufacturer.

Example (7)

Data file `activesubstance.csv`. The amount of active substance was observed for products of two suppliers. Test hypothesis that the medians of the amount of active substance are equal for both suppliers.

Example (8)

Data file minute2.csv. Thirty research participants had to guess independently of each other and without prior training when a minute has passed after the sound signal. Data file contains measured times in seconds. Test hypothesis that half of the participants had a period of one minute underestimated and the second half had it overestimated. Use the sign test and one-sample Wilcoxon signed-rank test with one-sided and two-sided H_1.

Example (9)

Data file nickel.csv. Four laborants measured the containment of nickel in steel. Perform non-parametric one-way analysis using the Kruskal-Wallis test and the median test. Determine the pairs of laborants whose results significantly differ.

Example (10)

Data file potatoes2.csv. Potato yields of 4 potato varieties were observed. The yields in tons per hectare are recorded in the file. Perform the Kruskal-Wallis test amd the median test. Determine the pairs of varieties whose expected yields significantly differ.

Example (11)

Data file `paper.csv`. Four labs measured the smoothness of the paper. Perform non-parametric one-way analysis using the Kruskal-Wallis test and the median test. Determine the pairs of labs whose results significantly differ.

Example (12)

Data file `IQvitaminB.csv`. The effect of vitamin B on IQ was observed in 24 pairs of children. One child out of a pair was administered the vitamin B and the other one was administered the placebo. Compute the difference between the IQ in pairs. Test hypothesis that the medians of the differences in IQ between both groups of children are equal. (Try also one-sided alternatives.)

Example (13)

Data file `rats.csv`. One of the methods of wound sealing was chosen after surgery for twenty rats – the stitches or a compression bandage. Data on tension in the wound edges were collected. Test hypothesis that the medians of tension for both methods of wound sealing are equal.

Example (14)

Data file `sales.csv`: numbers of sold pieces of 5 different products in 7 different stores. Use Friedman test to test the hypothesis that the product type has no effect on the sales in particular stores (blocks).

Results

1. The sign test does not reject H_0 and the Wilcoxon signed-rank test does.
2. Wilcoxon rank-sum test rejects H_0.
3. Kruskal-Wallis test rejects H_0 and median test does not. Distinct pairs of varieties: 3-4, 3-1, 2-1, 4-1.
4. Both test do not reject H_0.
5. Paired Wilcoxon signed-rank test does not reject H_0.
6. H_0 that medians are equal is not rejected.
7. Wilcoxon rank-sum test does not reject H_0.
8. Both tests reject H_0. After repeating the test for one-sided H_1 we show that median is < 60.
9. Both methods reject H_0. Distinct pairs: D-A, D-C, B-C, (B-A).
10. Both methods reject H_0. Distinct pairs: B-A, B-D, C-D, A-D, (B-C).
11. Both methods reject H_0. Distinct pairs: all pairs except B-C, (A-B).
12. Paired Wilcoxon signed-rank test does not reject H_0.
13. Wilcoxon test does not reject H_0.
14. Friedman test does not reject H_0.

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Statistics II | seminar 5

Goodness-of-fit tests

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Example (1)

Data file `families.csv`. It was randomly selected 84 families from a set of families of 5 children and the number of boys was detected for each family. At a significance level of 0.05, test hypothesis that number of boys in families of 5 children has binomial distribution $\mathrm{Bi}(5;\ 0.5)$.

Example (2)

Data file `line.csv`. Waiting time (in minutes) was observed for 70 clients of a certain company that they spent waiting for service (from the moment of taking their ticket). At a significance level of 0.05, test hypothesis that waiting time has exponential distribution using Pearson's chi-squared test and test for exponential distribution.

Example (3)

Data file `trains.csv`. The number of arriving trains during 1 hour was observed at the station for 5 days (i.e. 360 hours). At a significance level of 0.05, test hypothesis that the number of arriving trains during 1 hour follows Poisson distribution using Pearson's chi-squared test and test for Poisson distribution.

Example (4)

Data file `Brno.csv` contains a number of residents of Brno (data from the year 2001) sorted by birth month. At a significance level of 0.05, test hypothesis that the number of births in each month corresponds to the number of days in the month (consider a non-leap year).

Example (5)

Data file `dice.csv`. The absolute frequencies of the numbers thrown for 60 dice rolls were observed. At a significance level of 0.05, test hypothesis that the dice is homogeneous (fair).

Example (6)

Data file `emergency.csv` contains a number of patients in the emergency room during 8 hour shift in a total of 75 days. Test hypothesis that the number of patients has Poisson distribution using Pearson's chi-squared test and test for Poisson distribution.

Example (7)

Data file dice2.csv. The number of sixes getting in 12 dice was observed for 4096 rolls with 12 dice. At a significance level of 0.05, test hypothesis that the number of sixes in one roll has binomial distribution Bi$(12; \frac{1}{6})$.

Example (8)

Data file pond.csv. Five traps was set into a pond glowing with white, yellow, blue, green and red light. The number of fish caught in each trap is contained in the data file. At a significance level of 0.05, test hypothesis that the color of light has no effect on the number of fish caught.

Example (9)

Data file supermarkets.csv. 300 customers of a supermarket chain were questioned which day of the week they shop the most. Data file contains these numbers. At a significance level of 0.05, test hypothesis that the customers shop evenly every day of the week.

Example (10)

Data file football.csv. Total number of goals scored in a total match time was observed for 84 football matches. Data file contains these numbers. At a significance level of 0.05, test hypothesis that the number of goals scored has Poisson distribution using Pearson's chi-squared test.

Example (11)

Variables JANT and JULT in data file pollution.csv contain January and July temperatures in $^\circ$F. Convert them to $^\circ$C and use the Lilliefors test (also the Kolmogorov-Smirnov test) and Pearson's chi-squared test at a significance level of 0.05 to test that these variables have normal distribution. Plot an empirical and a teoretical distribution functions.

Example (12)

Variable Rainfall in data file Hawaii.csv contains amount of precipitation in Hawaii. Use the Kolmogorov-Smirnov test and Pearson's chi-squared test at a significance level of 0.05 to test that the amount of precipitation has chi-squared distribution with 18 degrees of freedom. Plot an empirical and a teoretical distribution functions.

Results

1. $K = 3.1$, $\chi^2_{0.95}(5) = 11.1$. Assumptions $\Rightarrow$ merge outer pairs of categories $\Rightarrow K = 2.4$, $\chi^2_{0.95}(3) = 7.8$. We do not reject the null hypothesis.
2. $K = 4.8$, $\chi^2_{0.95}(6) = 12.6 \Rightarrow$ Pearson's chi-squared test does not reject the null hypothesis.
   $Q = 35.7 \rightarrow$ Simple test rejects the null hypothesis.
3. $K = 9.7$, $\chi^2_{0.95}(6) = 12.6$; $Q = 331.1$. We do not reject the null hypothesis.
4. $K = 1506.2$, $\chi^2_{0.95}(11) = 19.7$. We reject the null hypothesis, month of birth distribution is not uniform distribution.
5. $K = 2.8$, $\chi^2_{0.95}(5) = 11.1$. We do not reject the null hypothesis.
6. Assumptions $\Rightarrow$ merge the last 2 categories $\Rightarrow K = 8.7$, $\chi^2_{0.95}(8) = 15.5$; $Q = 1158.6 \rightarrow$ Both tests do not reject the null hypothesis.
7. $K = 5.5$, $\chi^2_{0.95}(7) = 14.1$. We do not reject the null hypothesis.
8. $K = 14.1$, $\chi^2_{0.95}(4) = 9.5$. We reject the null hypothesis, color has effect on the number of fish caught.
9. $K = 78$, $\chi^2_{0.95}(6) = 12.6$. We reject the null hypothesis, preferences of the shopping days are not uniform.
10. $K = 2.1$, $\chi^2_{0.95}(3) = 7.8$. We do not reject the null hypothesis.

Statistics II | seminar 6

Correlation analysis

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Perform correlation analysis using the Pearson correlation coefficients, partial correlation coefficients, their significance tests and visualizations (scatterplots and correlograms). Interpret the results. Calculate also Spearman's and Kendall rank correlation coefficients.

Example (1)

Data file `households.csv`. The following variables were observed in 7 households: number of members, income and food and drink expenses (in thousands Kč per 3 months). Calculate sample multiple correlation between `expense` and remaining variables.

Example (2)

20 children in different ages underwent pedagogical-psychological research within which among others they wrote a dictation. Data file `children.csv` contains children's weight, age and points earned in dictation. Calculate sample multiple correlation between `Points` and remaining variables.

Example (3)

Data file `children2.csv`. Memory capacity, IQ and speed reading ability were observed for 20 children of different ages. Calculate sample multiple correlation between `IQ` and remaining variables.

Example (4)

Data file enrollment.csv. The number of filed university applications (ROLL) was observed in one of the states of USA for 29 years (YEAR) in relation to the average unemployment rate (UNEM, in %), the average wage (INC, in USD) and the number of students (HGRAD) who successfullly graduated from high school in a given year. Focus on the dependence of the number of university applications (the coefficient of multiple correlation) on the rest of the variables. Calculate sample multiple correlation between HRAD and remaining variables.

Example (5)

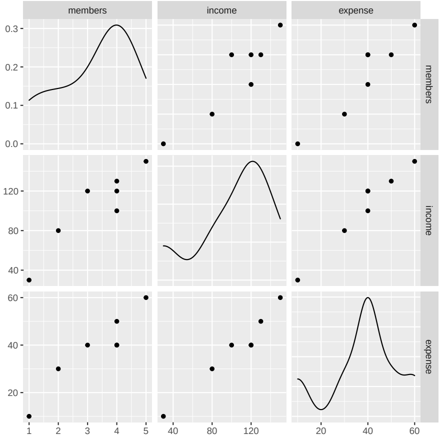
Data file heptathlon.csv contains heptathletes' results from a competition. Perform correlation analysis of given variables of results of 7 disciplines and column score. Examine their associations and interpret the results. Calculate sample multiple correlation between score and disciplines' results.

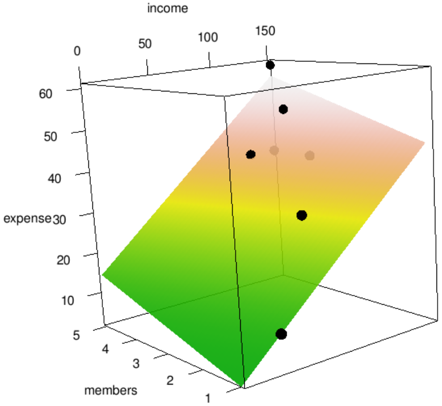
Example (6)

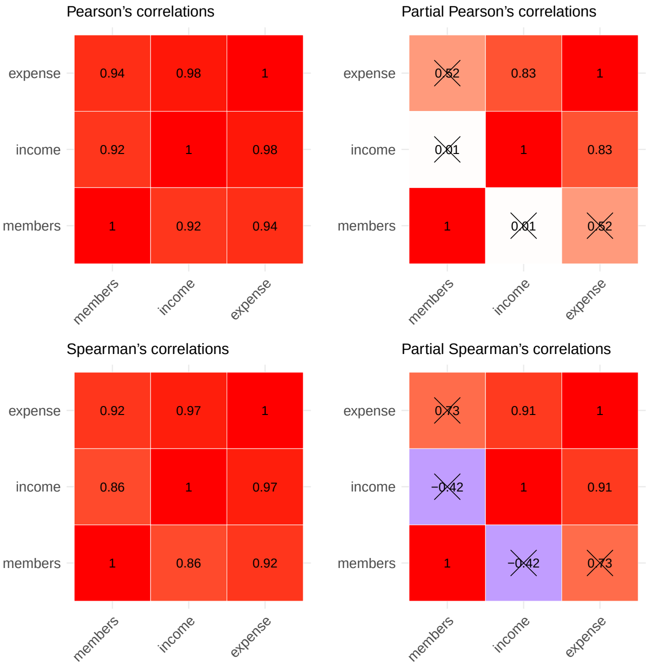
Data file decathlon.csv contains decathlets' results from competitions. Perform correlation analysis of given variables of results of 10 disciplines and column Points. Examine their associations and interpret the results. Calculate sample multiple correlation between Points and disciplines' results.

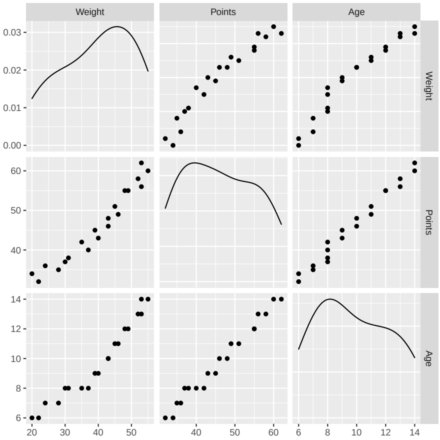
Results

Sample multiple correlations:
1. 0.983; **2.** 0.990; **3.** 0.731; **4.** 0.959; **5.** 0.997; **6.** 1.






Pearson's correlations
expense 0.94 0.98 1
income 0.92 1 0.98
members 1 0.92 0.94
members income expense
Partial Pearson's correlations
expense 0.52 0.83 1
income 0.01 1 0.83
members 1 0.01 0.52
members income expense
Spearman's correlations
expense 0.92 0.97 1
income 0.86 1 0.97
members 1 0.86 0.92
members income expense
Partial Spearman's correlations
expense 0.73 0.91 1
income −0.42 1 0.91
members 1 −0.42 0.73
members income expense


Weight
Points
Age
0.03
0.02
0.01
0.00
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50
40
14
12
10
8
6
20
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40
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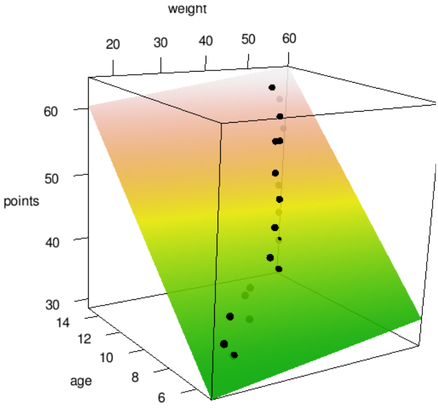
60
6
8
10
12
14


weight
20
30
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50
60
points
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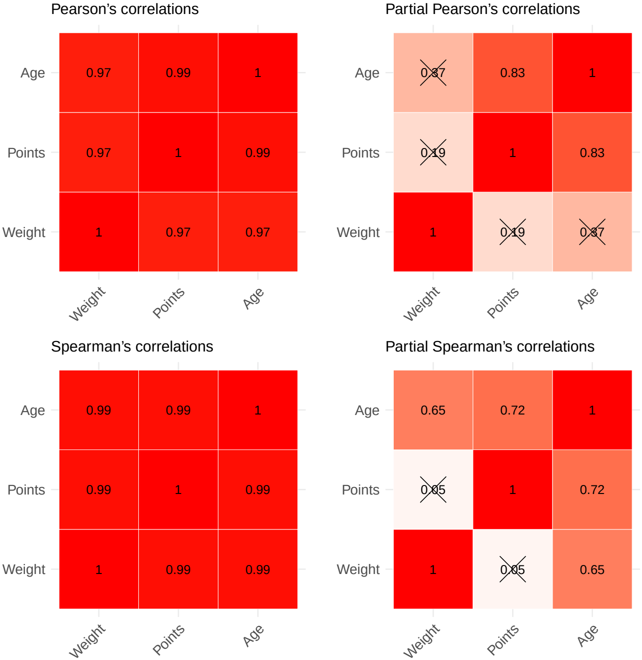
50
40
30
age
14
12
10
8
6


Pearson's correlations
Partial Pearson's correlations
Spearman's correlations
Partial Spearman's correlations
Age
Points
Weight
Weight
Points
Age
0.97
0.99
1
0.97
1
0.99
1
0.97
0.97
0.37
0.83
1
0.19
1
0.83
1
0.19
0.37
0.99
0.99
1
0.99
1
0.99
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0.99
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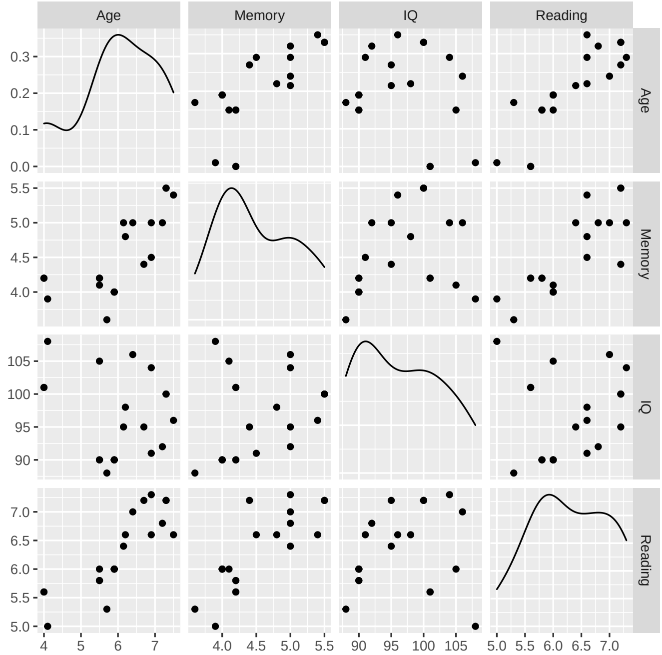
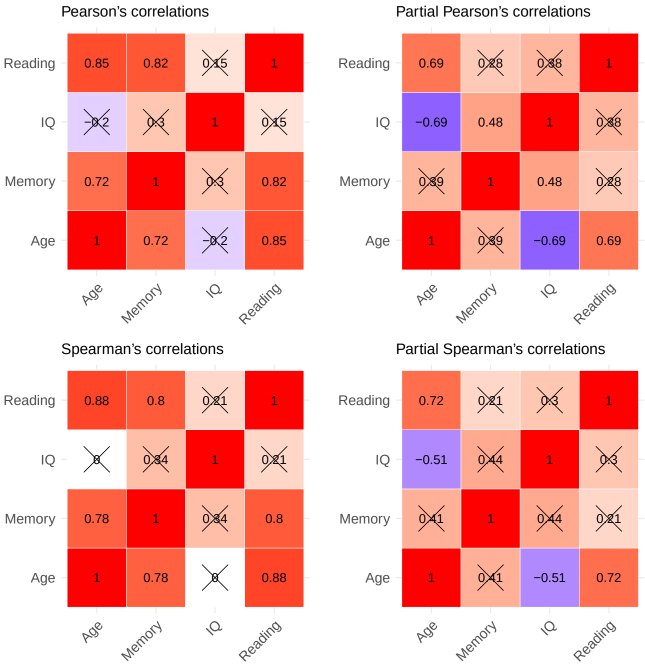
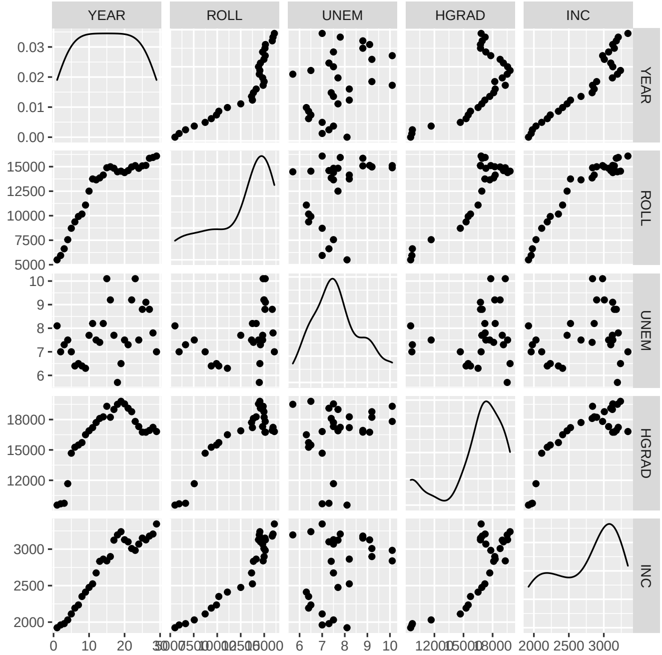
1
0.05
1
0.72
1
0.05
0.65

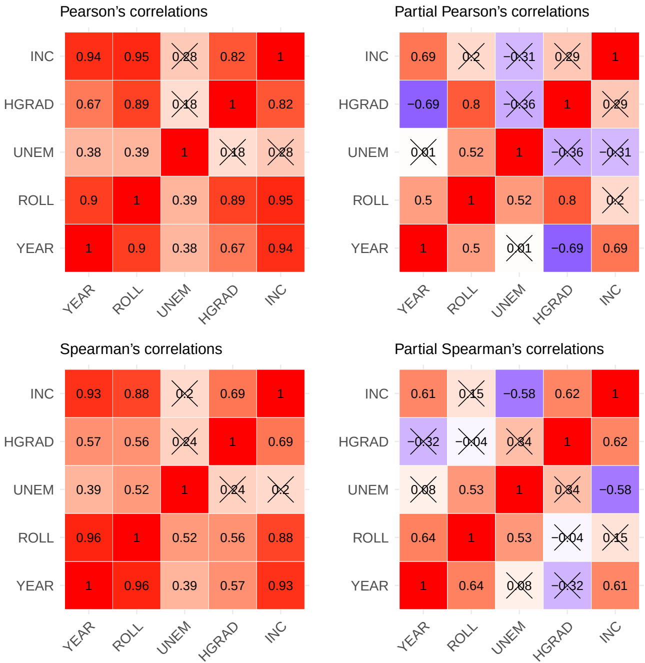


Age
Memory
IQ
Reading
0.3
0.2
0.1
0.0
5.5
5.0
4.5
4.0
105
100
95
90
7.0
6.5
6.0
5.5
5.0
4
5
6
7
4.0
4.5
5.0
5.5
90
95
100
105
5.0
5.5
6.0
6.5
7.0

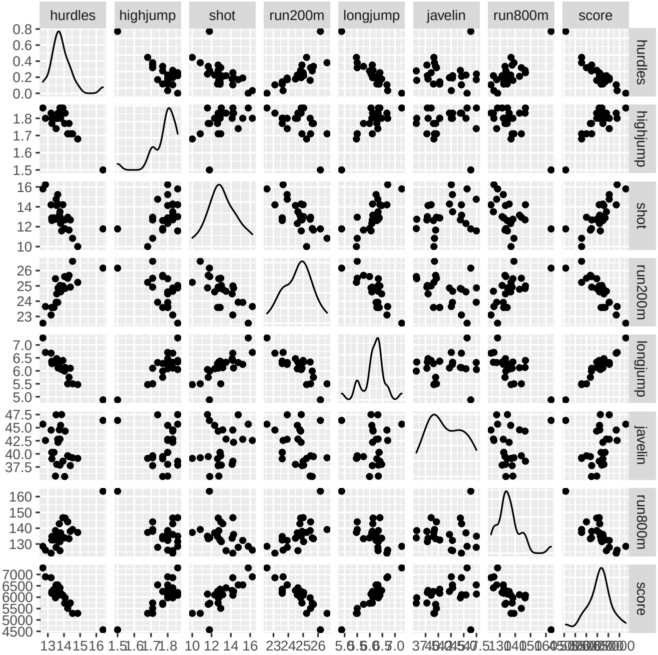

Pearson’s correlations
Partial Pearson’s correlations
Spearman’s correlations
Partial Spearman’s correlations
Reading
IQ
Memory
Age
Age
Memory
IQ
Reading



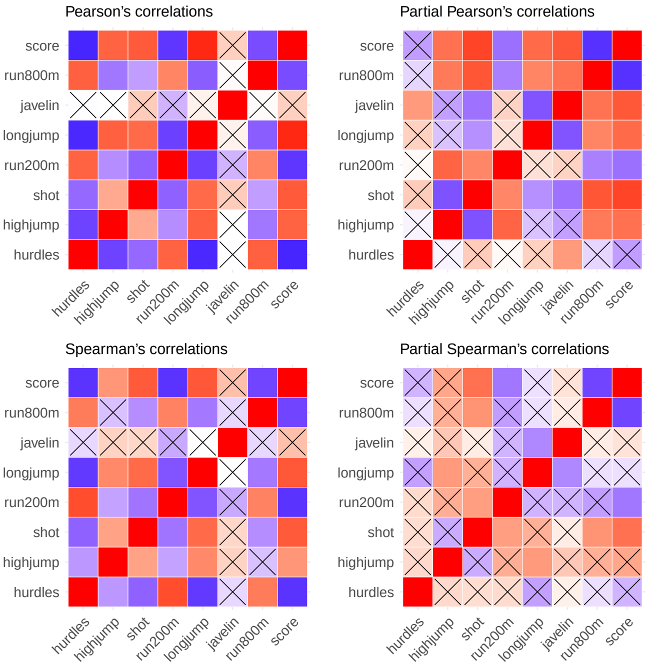
YEAR
ROLL
UNEM
HGRAD
INC
0.03
0.02
0.01
0.00
15000
12500
10000
7500
5000
10
9
8
7
6
18000
15000
12000
3000
2500
2000
0
10
20
30
5000
7500
10000
12500
15000
6
7
8
9
10
12000
15000
18000
2000
2500
3000

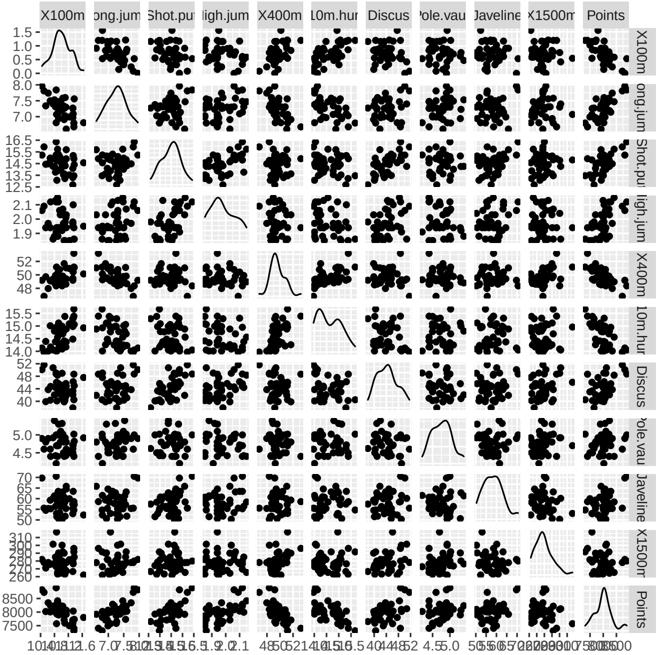


Pearson's correlations
INC 0.94 0.95 0.28 0.82 1
HGRAD 0.67 0.89 0.18 1 0.82
UNEM 0.38 0.39 1 0.18 0.28
ROLL 0.9 1 0.39 0.89 0.95
YEAR 1 0.9 0.38 0.67 0.94
YEAR ROLL UNEM HGRAD INC
Partial Pearson's correlations
INC 0.69 0.2 −0.31 0.29 1
HGRAD −0.69 0.8 −0.36 1 0.29
UNEM 0.01 0.52 1 −0.36 −0.31
ROLL 0.5 1 0.52 0.8 0.2
YEAR 1 0.5 0.01 −0.69 0.69
YEAR ROLL UNEM HGRAD INC
Spearman's correlations
INC 0.93 0.88 0.2 0.69 1
HGRAD 0.57 0.56 0.24 1 0.69
UNEM 0.39 0.52 1 0.24 0.2
ROLL 0.96 1 0.52 0.56 0.88
YEAR 1 0.96 0.39 0.57 0.93
YEAR ROLL UNEM HGRAD INC
Partial Spearman's correlations
INC 0.61 0.15 −0.58 0.62 1
HGRAD −0.32 −0.04 0.34 1 0.62
UNEM 0.08 0.53 1 0.34 −0.58
ROLL 0.64 1 0.53 −0.04 0.15
YEAR 1 0.64 0.08 −0.32 0.61
YEAR ROLL UNEM HGRAD INC

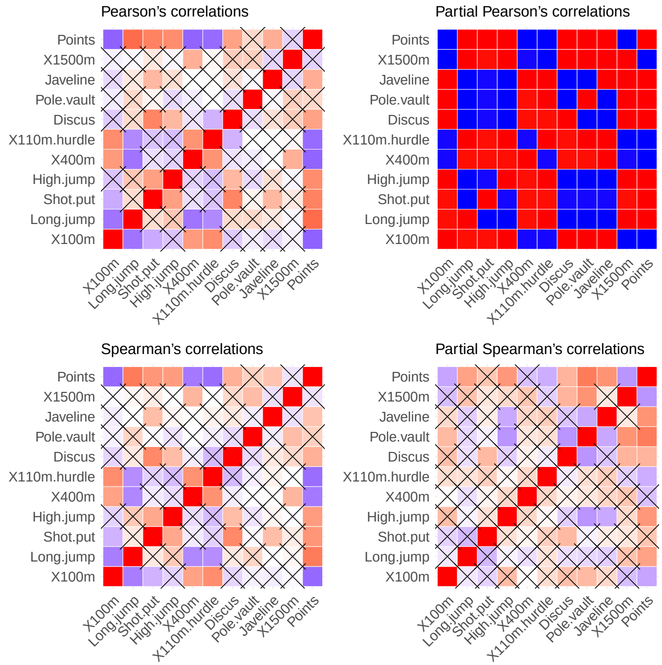


hurdles
highjump
shot
run200m
longjump
javelin
run800m
score
0.8
0.6
0.4
0.2
0.0
1.8
1.7
1.6
1.5
16
14
12
10
26
25
24
23
7.0
6.5
6.0
5.5
5.0
47.5
45.0
42.5
40.0
37.5
160
150
140
130
7000
6500
6000
5500
5000
4500
13 14 15 16
1.5 1.6 1.7 1.8
10 12 14 16
23 24 25 26


Pearson's correlations
Partial Pearson's correlations
Spearman's correlations
Partial Spearman's correlations
score
run800m
javelin
longjump
run200m
shot
highjump
hurdles



X100m
Shot.put
X400m
Discus
Javeline
X1500m
Points
Points


Pearson's correlations
Partial Pearson's correlations
Spearman's correlations
Partial Spearman's correlations
Points
X1500m
Javeline
Pole.vault
Discus
X110m.hurdle
X400m
High.jump
Shot.put
Long.jump
X100m