Neural Networks

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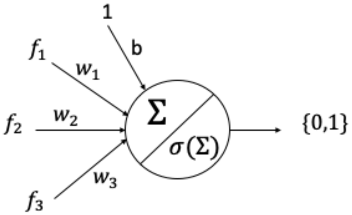
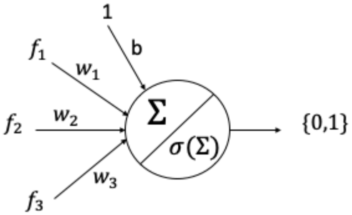
Content

A neuron for animal classification (Elephant or Dog)



A neuron for animal classification (Elephant or Dog)

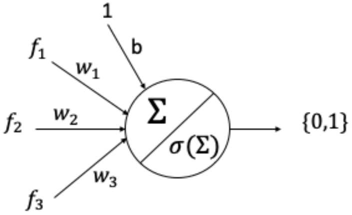


$$\Sigma = b + w_1 f_1 + w_2 f_2 + w_3 f_3$$

where:

f_1 : Weight of the animal
f_2 : Height of the animal
f_3 : Color of the animal

A neuron for animal classification (Elephant or Dog)



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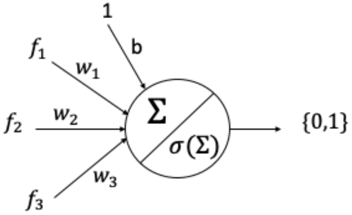
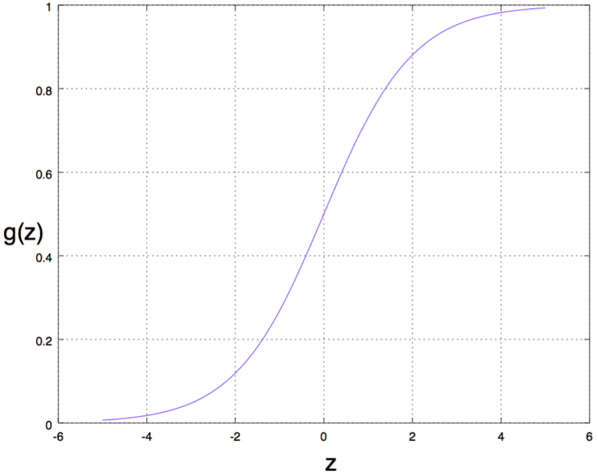


$$\Sigma = b + w_1 f_1 + w_2 f_2 + w_3 f_3$$

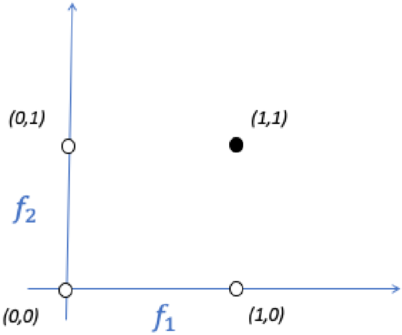
$$\sigma(\Sigma) = \frac{1}{1+e^{-\Sigma}} \qquad \sigma : \mathbb{R} \to (0,1)$$

Sigmoid function

$$g(z) = \frac{1}{1+e^{-z}} \qquad g : \mathbb{R} \to (0,1)$$



A neuron for the AND function



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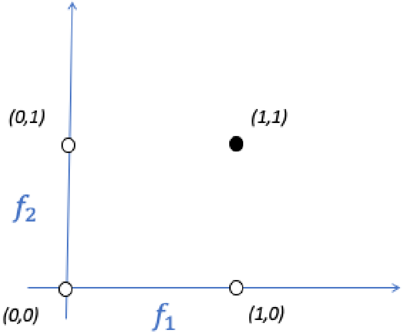
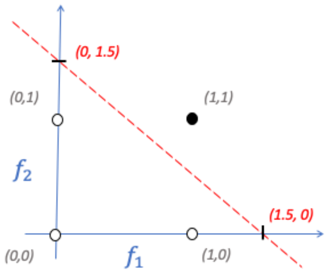


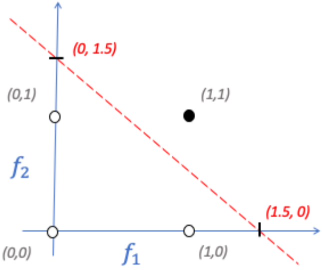
f_1	f_2	Output
0	0	0
0	1	0
1	0	0
1	1	1

A neuron for the AND function



A neuron for the AND function

$$\Sigma = b + w_1 f_1 + w_2 f_2$$



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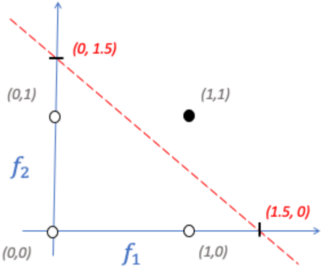
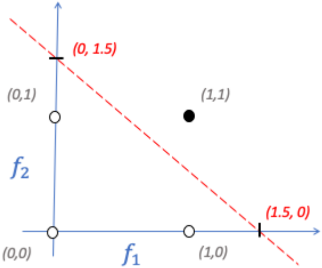


$$\Sigma = b + w_1 f_1 + w_2 f_2$$

$$w_1 f_1 + w_2 f_2 + b = 0$$

A neuron for the AND function



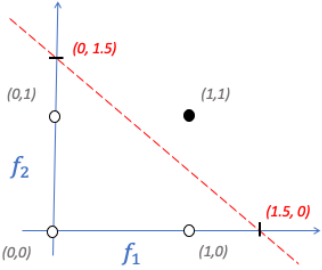
$\Sigma = b + w_1 f_1 + w_2 f_2$

$w_1 f_1 + w_2 f_2 + b = 0$

Solving for $(1.5, 0)$, we have

$w_1 = -1$

A neuron for the AND function



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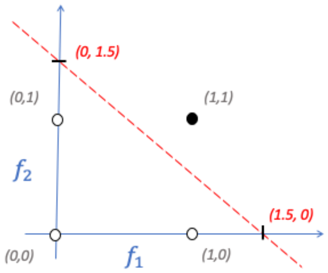
A neuron for the AND function



$\Sigma = b + w_1 f_1 + w_2 f_2$

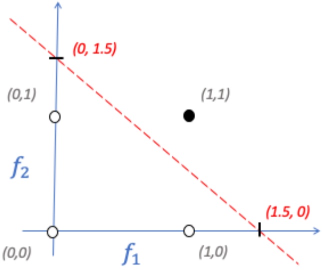
$w_1 f_1 + w_2 f_2 + b = 0$

Solving for $(1.5, 0)$, we have $w_1 = -1$

Solving for $(0, 1.5)$, we have $w_2 = -1$

Using the values of $w_1 = -1$ and $w_2 = -1$ we know that $b = 1.5$

A neuron for the AND function



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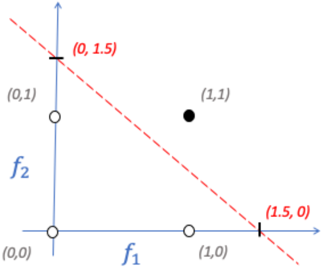
Therefore, our equation is $-f_1 - f_2 + 1.5 = 0$

A neuron for the AND function



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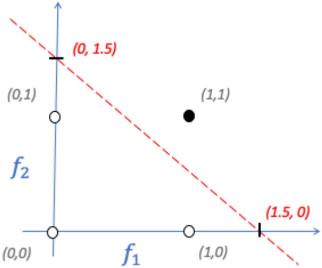
Therefore, our equation is $-f_1 - f_2 + 1.5 = 0$

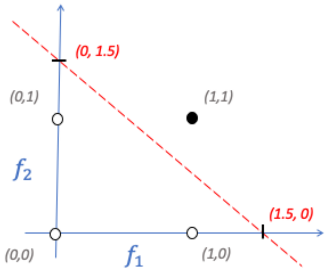
Multiplying by -1: $f_1 + f_2 - 1.5 = 0, \;\; w_1 = 1, \; w_2 = 1, \; b = -1.5$

A neuron for the AND function



Classifying new points with
$\Sigma = f_1 + f_2 - 1.5$

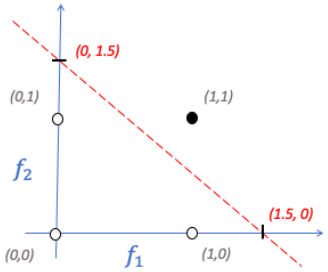
A neuron for the AND function



Classifying new points with
$\Sigma = f_1 + f_2 - 1.5$

Evaluating $(0, 0)$, we have $\Sigma = -1.5$
Since $\Sigma < 0$, the label is 0 (Neg.)

A neuron for the AND function



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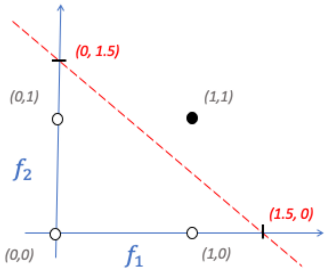

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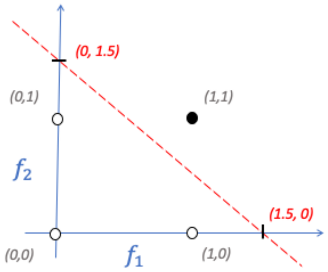

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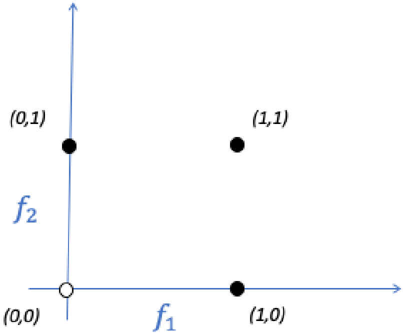
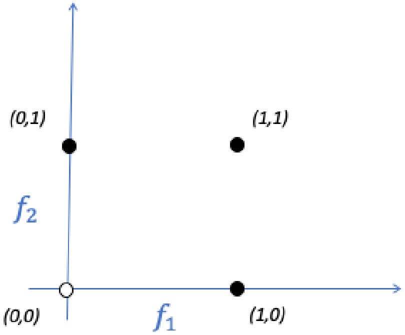
Evaluating $(1,1)$, we have $\Sigma = 0.5$
Since $\Sigma > 0$, the label is 1 (Pos.)

A neuron for the OR function



A neuron for the OR function



f_1	f_2	Output
0	0	0
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1	0	1
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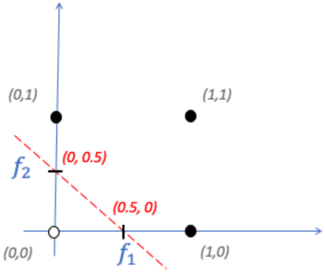
A neuron for the OR function



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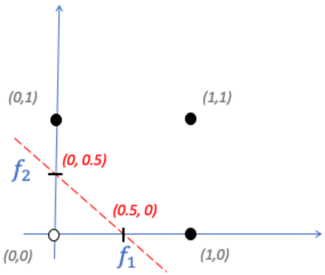

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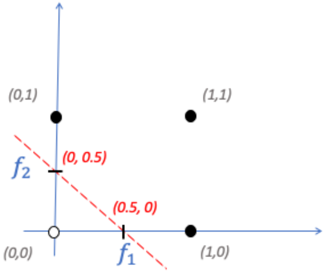

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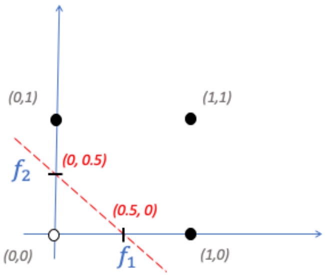

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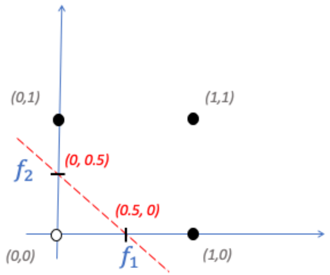

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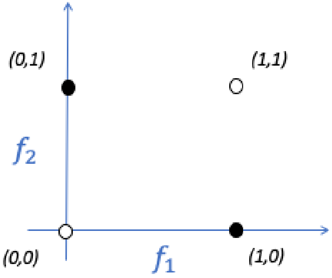
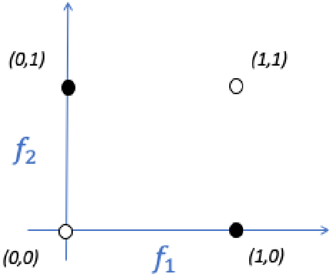
A neuron for the XOR function

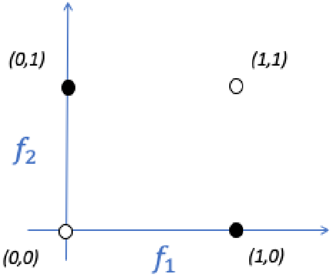


A neuron for the XOR function



f_1	f_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

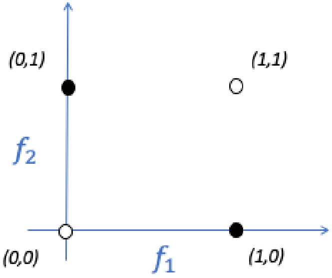
A neuron for the XOR function



f_1	f_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

- XOR problem with perceptron (Minsky and Papert, 1969).

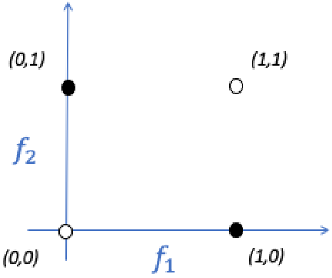
A neuron for the XOR function



f_1	f_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

- XOR problem with perceptron (Minsky and Papert, 1969).
- Proposal of multilayer perceptrons (MLP) (Werbos,1982).

A neuron for the XOR function

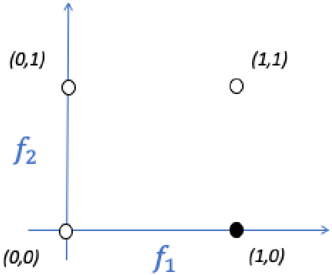


f_1	f_2	Output
0	0	0
0	1	1
1	0	1
1	1	0

- XOR problem with perceptron (Minsky and Papert, 1969).
- Proposal of multilayer perceptrons (MLP) (Werbos,1982).
- MLPs are popularized (Hinton et al., 1986).

Solving the XOR problem, with 3 neurons

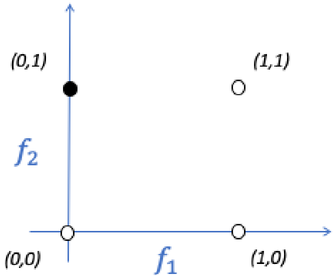
Neuron 1



f_1	f_2	Output
0	0	0
0	1	0
1	0	1
1	1	0

Solving the XOR problem, with 3 neurons

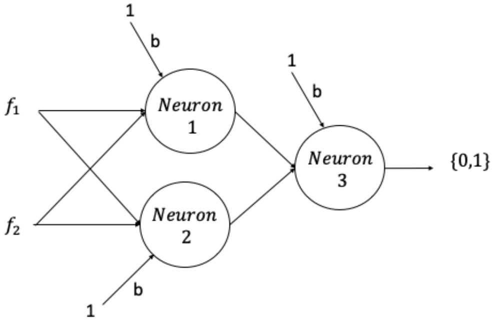
Neuron 2



f_1	f_2	Output
0	0	0
0	1	1
1	0	0
1	1	0

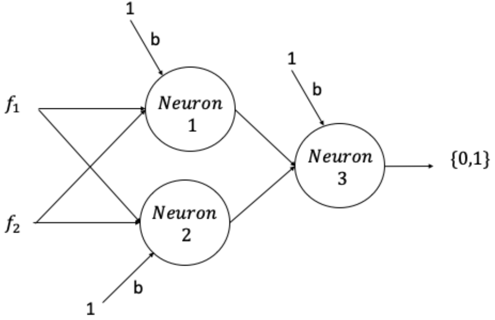
Solving the XOR problem, with 3 neurons

Neuron 3



Solving the XOR problem, with 3 neurons

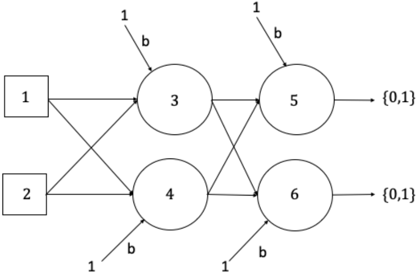
Neuron 3



Neuron 1	Neuron 2	Neuron 3 (XOR)
0	0	0
0	1	1
1	0	1
1	1	0

Backpropagation algorithm

MLP with topology 2 x 2 x 2:



Backpropagation algorithm

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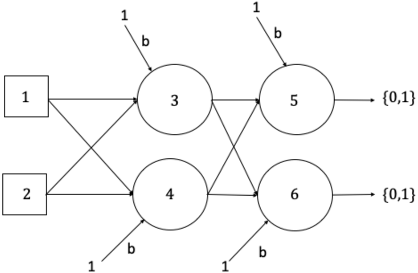
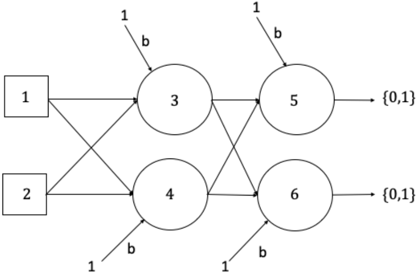


Repeat for every example in the training set:

1. Take an example and propagate the input signal, going forward: from the input neurons to the output neurons, passing through all the hidden neurons.

Backpropagation algorithm

MLP with topology 2 x 2 x 2:

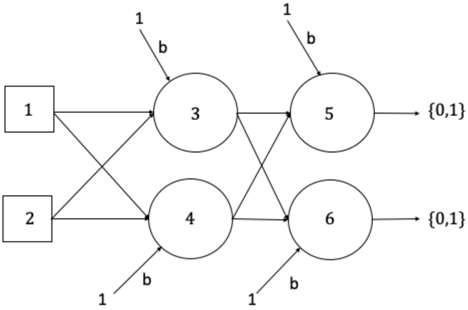


Repeat for every example in the training set:

1. Take an example and propagate the input signal, going forward: from the input neurons to the output neurons, passing through all the hidden neurons.
2. Backpropagate the error of the network with respect to the current example, going backward: from the output neurons to the input neurons, passing through all the hidden neurons.

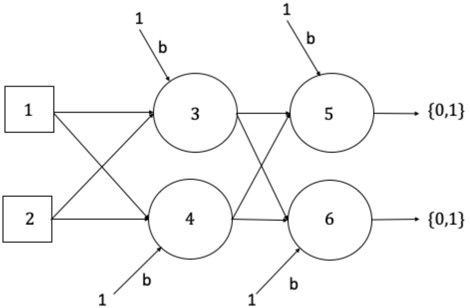
Backpropagation algorithm

MLP with topology 2 x 2 x 2:



Backpropagation algorithm

MLP with topology 2 x 2 x 2:

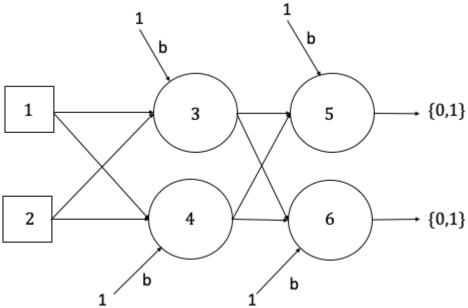


Learning rule for an output neuron:

$$\delta_5 := o_5(1 - o_5)(y_5 - o_5)$$

Backpropagation algorithm

MLP with topology 2 x 2 x 2:

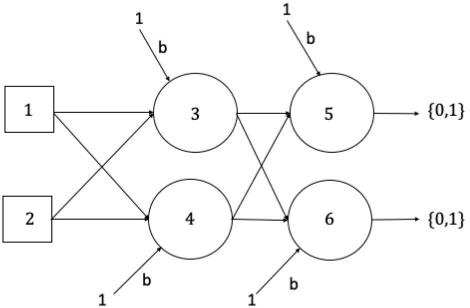


Learning rule for an output neuron:

$$\delta_5 := o_5(1 - o_5)(y_5 - o_5)$$
$$w_{5,3} := w_{5,3} + \alpha\delta_5 o_3$$

Backpropagation algorithm

MLP with topology 2 x 2 x 2:



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$$w_{5,4} := w_{5,4} + \alpha\delta_5 o_4$$

Backpropagation algorithm

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$$w_{5,3} := w_{5,3} + \alpha \delta_5 o_3$$

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Backpropagation algorithm

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δ_k : error of neuron k

o_k : output of neuron k, for instance $o_k = \sigma(\Sigma_k)$

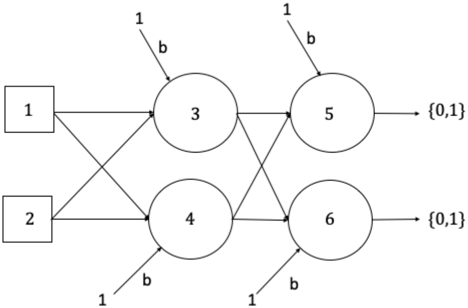
y_k : desired output of neuron k

$w_{i,j}$: weight of connection going from j to i

α : learning rate

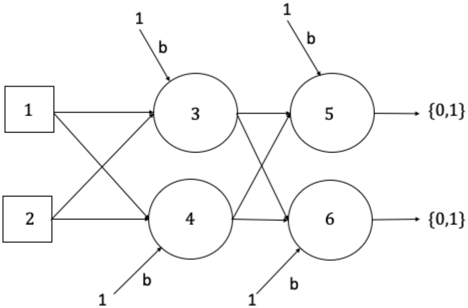
Backpropagation algorithm

MLP with topology 2 x 2 x 2:



Backpropagation algorithm

MLP with topology 2 x 2 x 2:

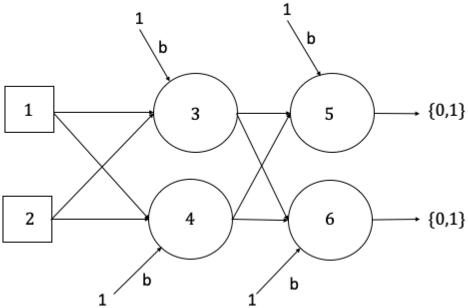


Learning rule for a hidden neuron:

$$\delta_3 := o_3(1 - o_3)(w_{5,3}\delta_5 + w_{6,3}\delta_6)$$

Backpropagation algorithm

MLP with topology 2 x 2 x 2:



Learning rule for a hidden neuron:

$$\delta_3 := o_3(1 - o_3)(w_{5,3}\delta_5 + w_{6,3}\delta_6)$$
$$w_{3,1} := w_{3,1} + \alpha\delta_3 o_1$$

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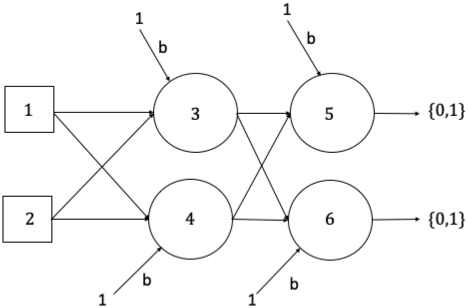

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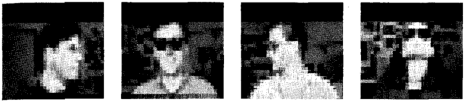
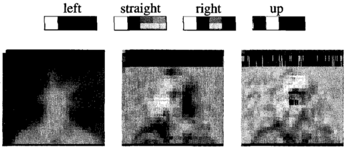
$$w_{3,2} := w_{3,2} + \alpha\delta_3 o_2$$

MLP example

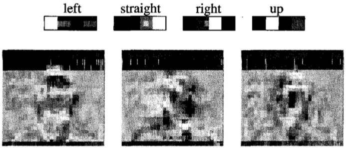
Topology 960 x 3 x 4, trained on 260 grey-level images, it achieves 90% performance. (Taken from Tom Mitchell's book)



30 × 32 resolution input images



Network weights after 1 iteration through each training example



Network weights after 100 iterations through each training example

Deep neural networks

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- One important kind of network specially powerful for learning patterns on images and video is the convolutional neural network (CNN). Some of the most popular are: LeNet, AlexNet and GoogleNet.
- Another neural network specially tailored for memorizing sequences of data such as audio and text is the recurrent neural network (RNN). Among these networks, the long-short term memory (LSTM) and the gated recurrent unit (GRU) are by far the most popular.

Thank you!

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