Principal Component Analysis

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Content

Principal Component Analysis

- We will develop a method that tries to identify the subspace in which the data approximately lies: Principal Component Analysis (PCA).

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- PCA requires only an eigenvector calculation and does not need to resort to EM.

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- Since patterns in data can be hard to find in data of high dimension, where a graphical representation is not available, PCA is a powerful tool for analysing data.

Principal Component Analysis

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- This technique is used in image compression, for example.

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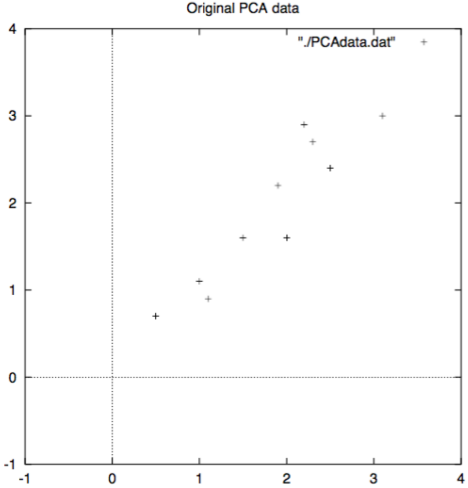
6. Derive the new data set.

1. Get Some Data.

Data =

x	y
2.5	2.4
0.5	0.7
2.2	2.9
1.9	2.2
3.1	3.0
2.3	2.7
2	1.6
1	1.1
1.5	1.6
1.1	0.9



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- So, all the x_1 values have μ_1 (the mean of the x_1 values of all the data points) subtracted, and all the values x_2 have μ_2 subtracted from them.

2. Substract the Mean

- This produces a data set whose mean is zero.

DataAdjust =

x	y
.69	.49
-1.31	-1.21
.39	.99
.09	.29
1.29	1.09
.49	.79
.19	-.31
-.81	-.81
-.31	-.31
-.71	-1.01

3. Calculate the Covariance Matrix

- The covariance matrix for this example is

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- So, since the non-diagonal elements in this covariance matrix are positive, we should expect that both the x_1 and x_2 variable increase together.

4. Calculate the Eigenvectors and Eigenvalues of the Covariance Matrix

- Since the covariance matrix is square, we can calculate the eigenvectors and eigenvalues for this matrix. These are rather important, as they tell us useful information about our data.

$$eigenvalues = \begin{pmatrix} .0490833989 \\ 1.28402771 \end{pmatrix}$$

$$eigenvectors = \begin{pmatrix} -.735178656 & -.677873399 \\ .677873399 & -.735178656 \end{pmatrix}$$

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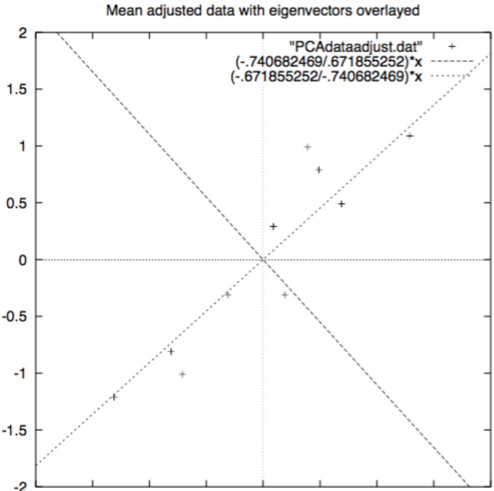
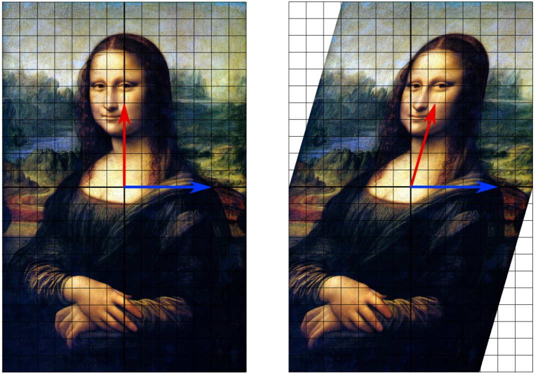


4. Calculate the Eigenvectors and Eigenvalues of the Covariance Matrix

A (non-zero) vector v of dimension N is an eigenvector of a square $N \times N$ matrix A if it satisfies the linear equation $A\mathrm{v} = \lambda\mathrm{v}$, where λ is a scalar and the eigenvalue corresponding to v.



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- In our example, the eigenvector with the largest eigenvalue was the one that pointed down the middle of the data.
- It is the most significant relationship between the data dimensions.

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- This gives you the components in order of significance.
- If you like, you can decide to ignore the components of lesser significance. You do lose some information, but if the eigenvalues are small, you don't lose much.

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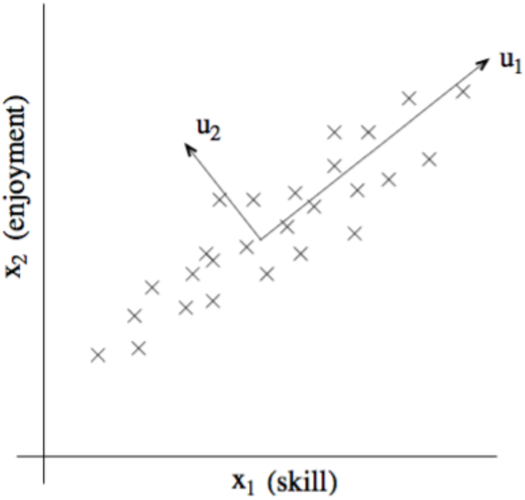
- Once we have chosen the components (eigenvectors) that we wish to keep in our data and formed a feature vector, we simply take the transpose of the vector and multiply it on the left of the original data set, transposed.
- It will give us the original data solely in terms of the vectors we chose.

Principal Component Analysis

Prior to running PCA, we first pre-process the data to normalize its mean and variance as follows:

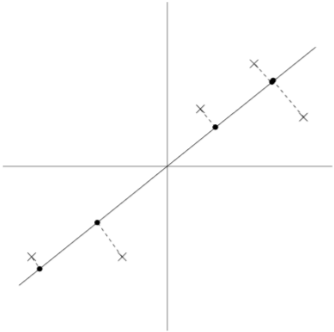
1. Let $\mu = \frac{1}{m}\sum_{i=1}^{m} x^{(i)}$.
2. Replace each $x^{(i)}$ with $x^{(i)} - \mu$.
3. Let $\sigma_j^2 = \frac{1}{m}\sum_i (x_j^{(i)})^2$
4. Replace each $x_j^{(i)}$ with $x_j^{(i)}/\sigma_j$.

Principal Component Analysis



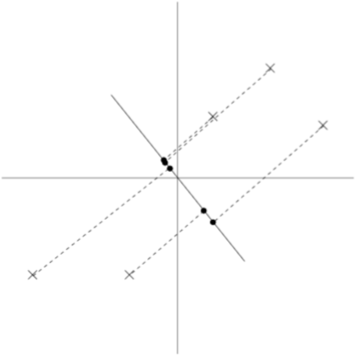
Principal Component Analysis

First principal component



Principal Component Analysis

Second principal component



Principal Component Analysis

	England	Wales	Scotland	N Ireland
Cheese	105	103	103	66
Carcass meat	245	227	242	267
Other meat	685	803	750	586
Fish	147	160	122	93
Fats and oils	193	235	184	209
Sugars	156	175	147	139
Fresh potatoes	720	874	566	1033
Fresh Veg	253	265	171	143
Other Veg	488	570	418	355
Processed potatoes	198	203	220	187
Processed Veg	360	365	337	334
Fresh fruit	1102	1137	957	674
Cereals	1472	1582	1462	1494
Beverages	57	73	53	47
Soft drinks	1374	1256	1572	1506
Alcoholic drinks	375	475	458	135
Confectionery	54	64	62	41

Table 1: UK food consumption in 1997 (g/person/week). Source: DEFRA website

Principal Component Analysis

Figure 1: Projections onto first principal component (1-D space)

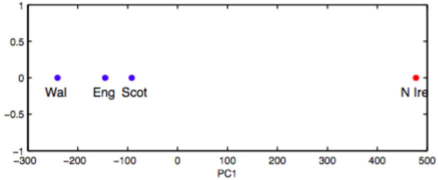
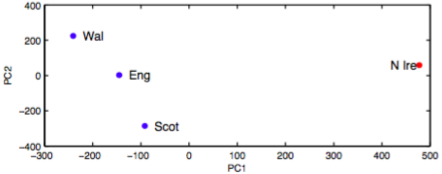


Figure 2: Projections onto first 2 principal components (2-D space)



Face Recognition

Matthew Turk and Alex Pentland

Face Recognition

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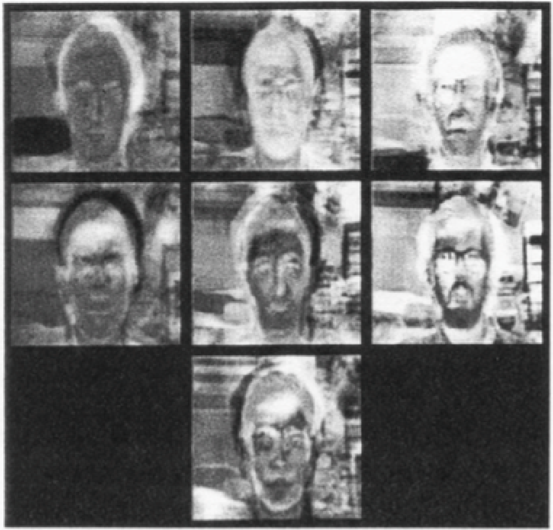

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3. Calculate the corresponding distribution in M-dimensional weight space for each known individual, by projecting their face images onto the *face space*.

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5. (Optional) If the same unknown face is seen several times, calculate its characteristic weight pattern and incorporate it into the known faces.

Reference

- Matthew Turk, Alex Pentland. **Eigenfaces for recognition**. Journal of Cognitive Neuroscience. 3 (1): 71-86. 1991.
- Andrew Ng. **Machine Learning Course Notes**. 2003.

Thank you!

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