Chapter 2: Evaluative Feedback

- **Evaluating** actions vs. **instructing** by giving correct actions
- Pure evaluative feedback depends totally on the action taken. Pure instructive feedback depends not at all on the action taken.
- Supervised learning is instructive; optimization is evaluative
- **Associative** vs. **Nonassociative**:
  - Associative: inputs mapped to outputs; learn the best output **for each** input
  - Nonassociative: “learn” (find) one best output
- *n*-armed bandit (at least how we treat it) is:
  - Nonassociative
  - Evaluative feedback

The n-Armed Bandit Problem

- Choose repeatedly from one of n actions; each choice is called a **play**
- After each play a_t, you get a reward r_t, where

$$E\langle r_t \mid a_t \rangle = Q^*(a_t)$$

These are unknown **action values**
Distribution of r_t depends only on a_t

- Objective is to maximize the reward in the long term, e.g., over 1000 plays

To solve the n-armed bandit problem,
you must **explore** a variety of actions
and the **exploit** the best of them

The Exploration/Exploitation Dilemma

- Suppose you form estimates

$$Q_t(a) \approx Q^*(a) \qquad \textbf{action value estimates}$$

- The **greedy** action at *t* is

$$a_t^* = \arg\max_a Q_t(a)$$

$$a_t = a_t^* \Rightarrow \text{exploitation}$$

$$a_t \neq a_t^* \Rightarrow \text{exploration}$$

- You can't exploit all the time; you can't explore all the time
- You can never stop exploring; but you should always reduce exploring

Action-Value Methods

- Methods that adapt action-value estimates and nothing else, e.g.: suppose by the t-th play, action a had been chosen k_a times, producing rewards $r_1, r_2, \ldots, r_{k_a}$, then

$$Q_t(a) = \frac{r_1 + r_2 + \cdots r_{k_a}}{k_a}$$

"sample average"

- $\lim_{k_a \to \infty} Q_t(a) = Q^*(a)$

ε-Greedy Action Selection

- Greedy action selection:

$$a_t = a_t^* = \arg\max_a Q_t(a)$$

- ε-Greedy:

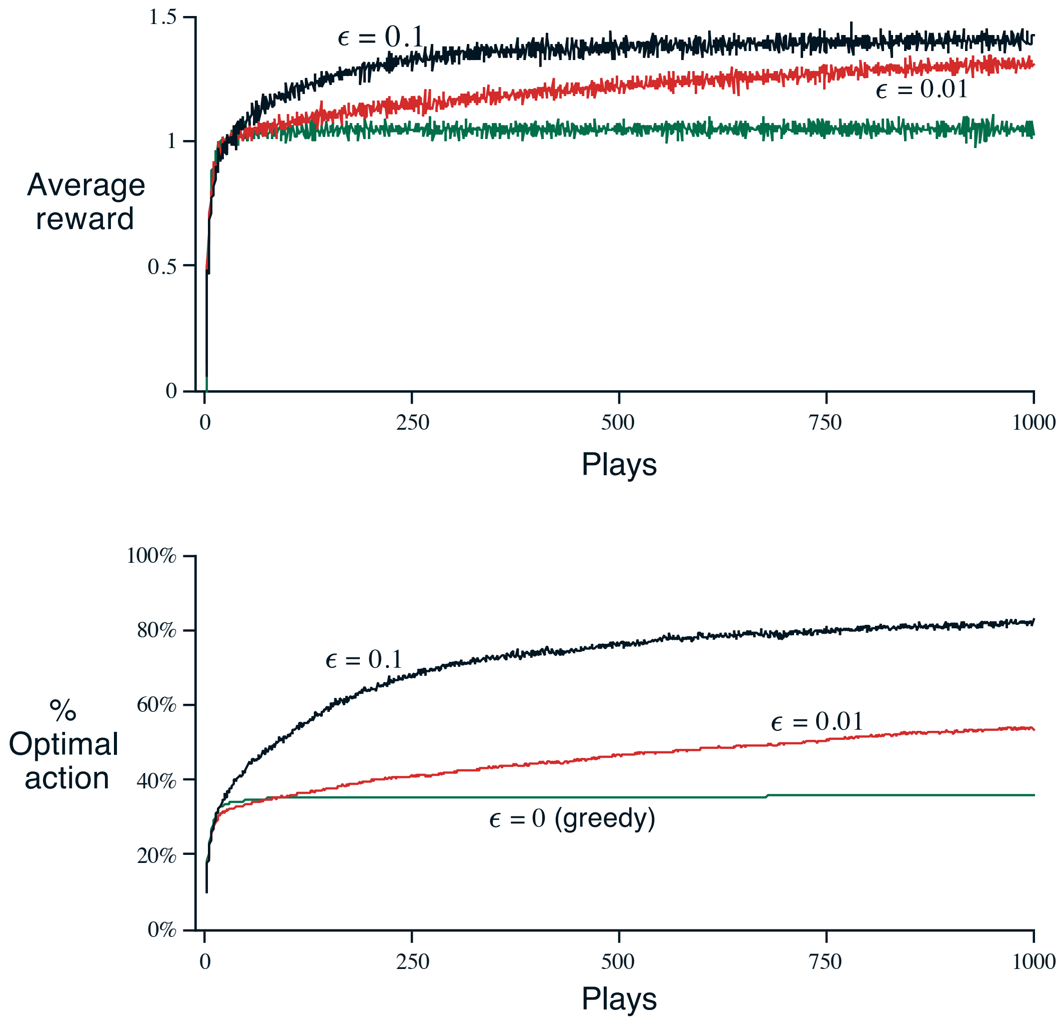
$$a_t = \begin{cases} a_t^* & \text{with probability } 1 - \varepsilon \\ \text{random action with probability } \varepsilon \end{cases}$$

. . . the simplest way to try to balance exploration and exploitation

10-Armed Testbed

- $n = 10$ possible actions
- Each $Q^*(a)$ is chosen randomly from a normal distribution: $\eta(0,1)$
- each r_t is also normal: $\eta(Q^*(a_t),1)$
- 1000 plays
- repeat the whole thing 2000 times and average the results

ε-Greedy Methods on the 10-Armed Testbed



Softmax Action Selection

- Softmax action selection methods grade action probs. by estimated values.
- The most common softmax uses a Gibbs, or Boltzmann, distribution:

Choose action a on play t with probability

$$\frac{e^{Q_t(a)/\tau}}{\sum_{b=1}^{n} e^{Q_t(b)/\tau}},$$

where τ is the
"computational temperature"

Binary Bandit Tasks

Suppose you have just **two** actions: $a_t = 1$ or $a_t = 2$

and just **two** rewards: $r_t = \textit{success}$ or $r_t = \textit{failure}$

Then you might infer a **target** or **desired action**:

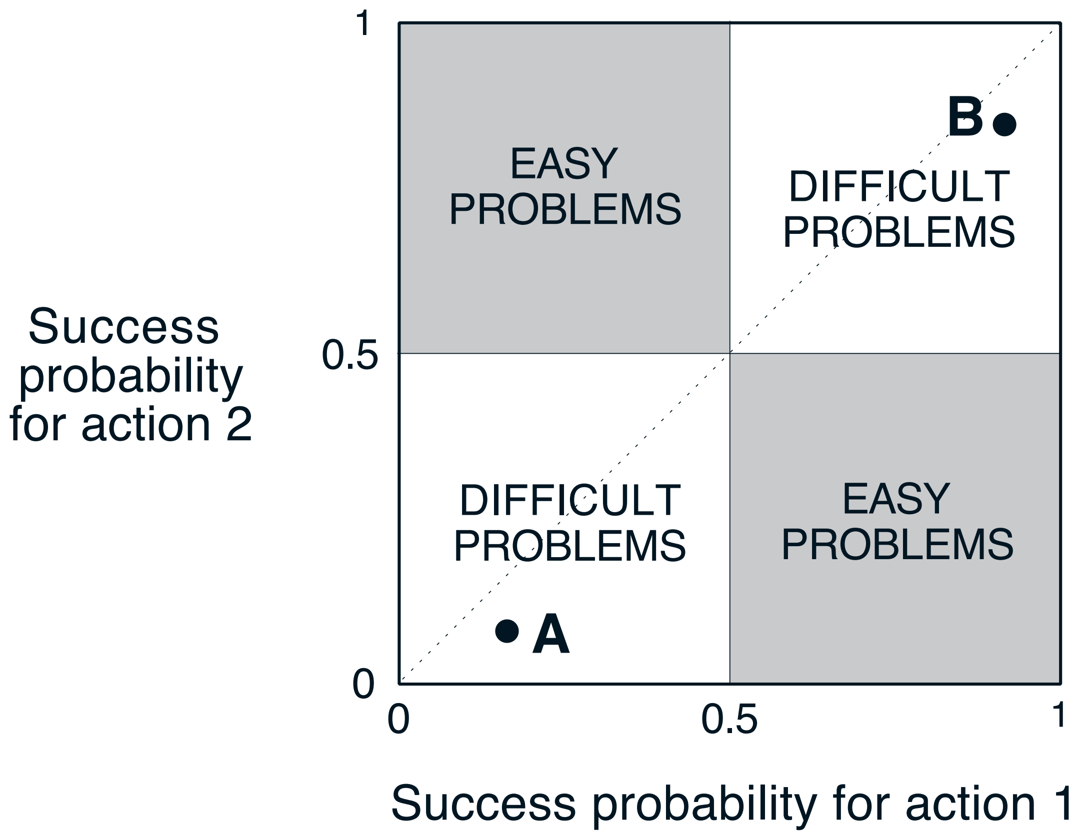
$$d_t = \begin{cases} a_t & \text{if } \textit{success} \\ \text{the other action} & \text{if } \textit{failure} \end{cases}$$

and then always play the action that was most often the target

Call this the **supervised algorithm**
It works fine on deterministic tasks…

Contingency Space

The space of all possible binary bandit tasks:



Linear Learning Automata

Let $\pi_t(a) = \Pr\{a_t = a\}$ be the only adapted parameter

$\mathrm{L}_{\mathrm{R-I}}$ (Linear, reward - inaction)

On *success*: $\pi_{t+1}(a_t) = \pi_t(a_t) + \alpha(1 - \pi_t(a_t)) \qquad 0 < \alpha < 1$

(the other action probs. are adjusted to still sum to 1)

On *failure*: no change

$\mathrm{L}_{\mathrm{R\text{-}P}}$ (Linear, reward - penalty)

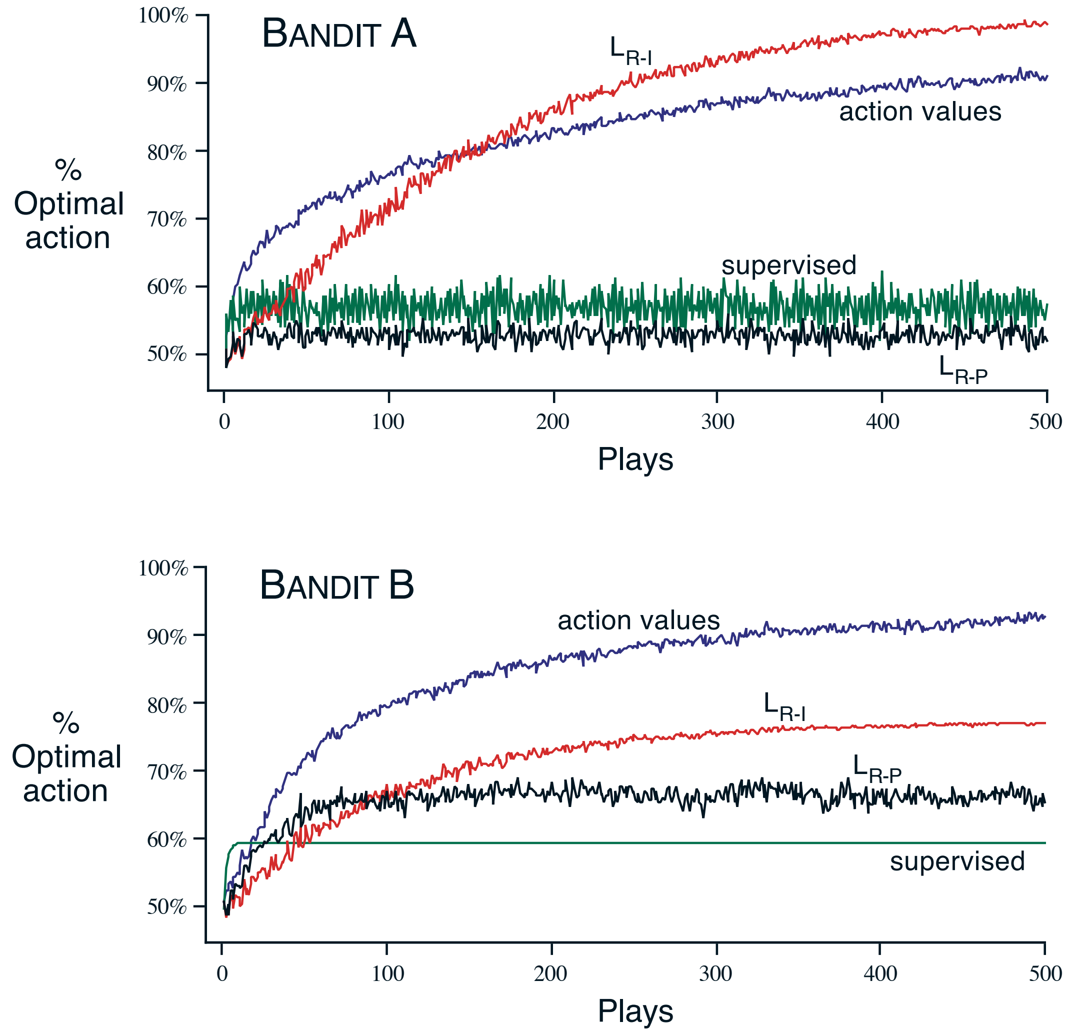
On *success*: $\pi_{t+1}(a_t) = \pi_t(a_t) + \alpha(1 - \pi_t(a_t)) \qquad 0 < \alpha < 1$

(the other action probs. are adjusted to still sum to 1)

On *failure*: $\pi_{t+1}(a_t) = \pi_t(a_t) + \alpha(0 - \pi_t(a_t)) \qquad 0 < \alpha < 1$

For two actions, a stochastic, incremental version of the supervised algorithm

Performance on Binary Bandit Tasks A and B



Incremental Implementation

Recall the sample average estimation method:

The average of the first k rewards is (dropping the dependence on a):

$$Q_k = \frac{r_1 + r_2 + \cdots r_k}{k}$$

Can we do this incrementally (without storing all the rewards)?

We could keep a running sum and count, or, equivalently:

$$Q_{k+1} = Q_k + \frac{1}{k+1}\left[r_{k+1} - Q_k\right]$$

This is a common form for update rules:

$$NewEstimate = OldEstimate + StepSize[Target - OldEstimate]$$

Tracking a Nonstationary Problem

Choosing Q_k to be a sample average is appropriate in a stationary problem,

i.e., when none of the $Q^*(a)$ change over time,

But not in a nonstationary problem.

Better in the nonstationary case is:

$$Q_{k+1} = Q_k + \alpha\left[r_{k+1} - Q_k\right]$$

for *constant* α, $0 < \alpha \leq 1$

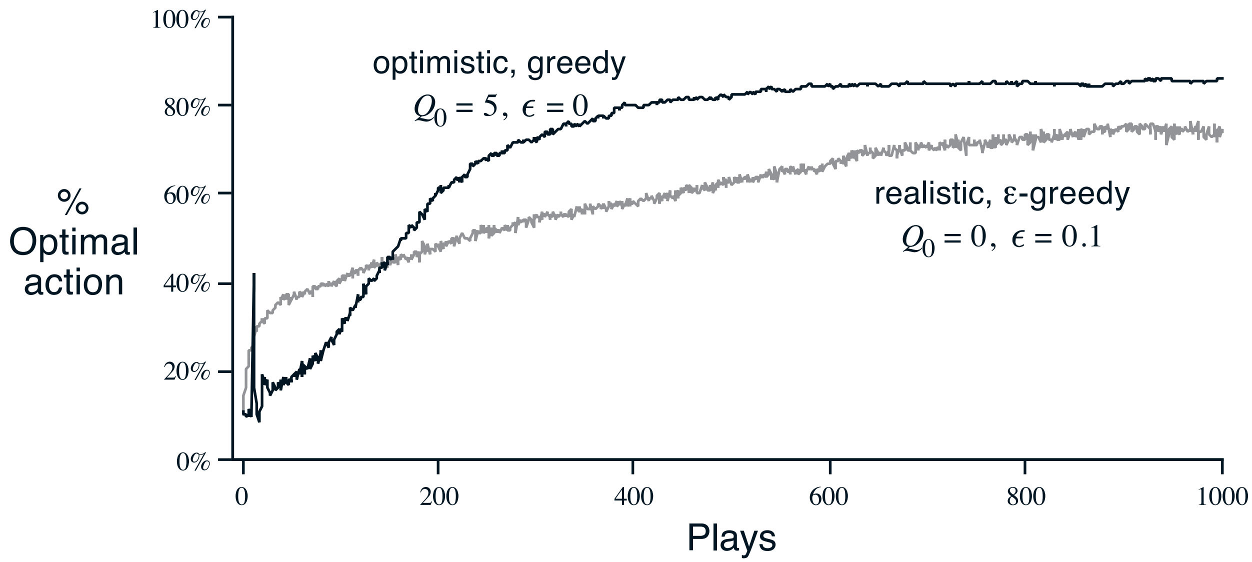
$$= (1-\alpha)^k Q_0 + \sum_{i=1}^{k} \alpha(1-\alpha)^{k-i} r_i$$

exponential, recency-weighted average

Optimistic Initial Values

- All methods so far depend on $Q_0(a)$, i.e., they are **biased.**
- Suppose instead we initialize the action values **optimistically,**

  i.e., on the 10-armed testbed, use $\quad Q_0(a) = 5 \quad$ for all a



Reinforcement Comparison

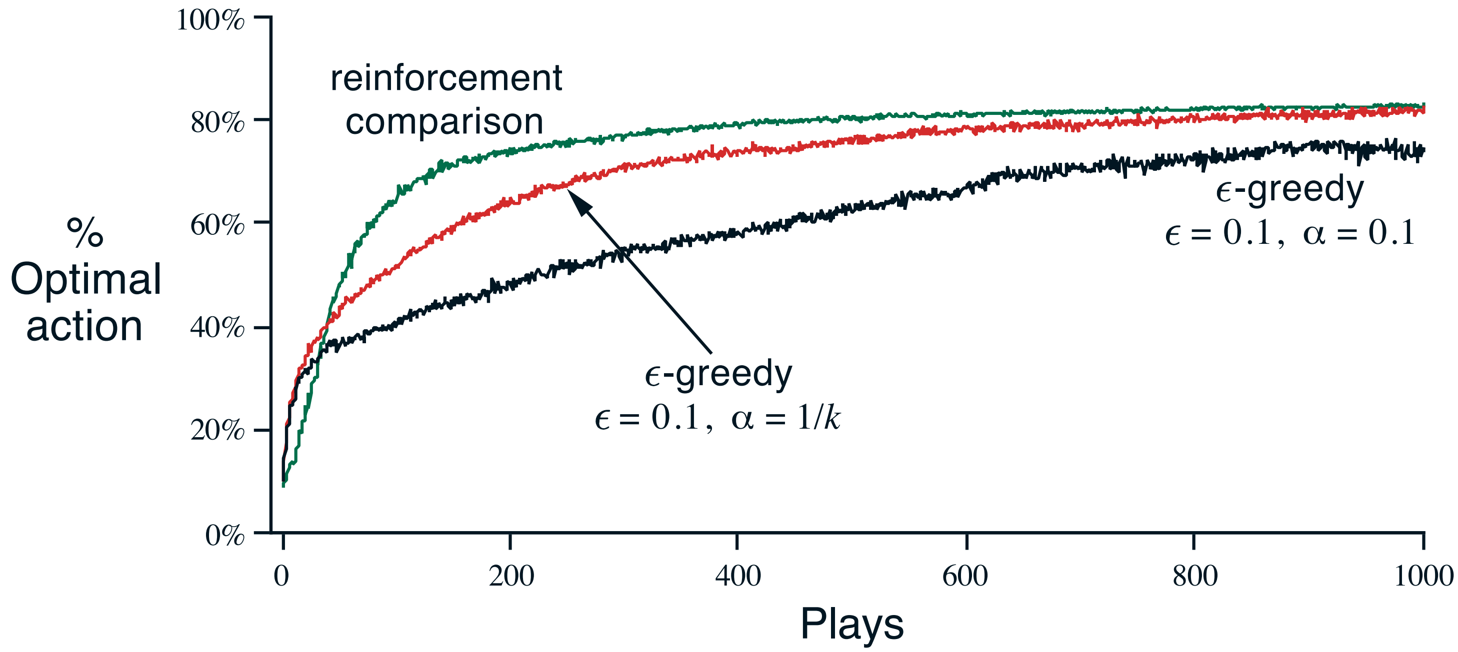
- Compare rewards to a reference reward, $\bar{r}_t$, e.g., an average of observed rewards
- Strengthen or weaken the action taken depending on $r_t - \bar{r}_t$
- Let $p_t(a)$ denote the **preference** for action a
- Preferences determine action probabilities, e.g., by Gibbs distribution:

$$\pi_t(a) = \Pr\left\{a_t = a\right\} = \frac{e^{p_t(a)}}{\sum_{b=1}^{n} e^{p_t(b)}}$$

- Then:

$$p_{t+1}(a_t) = p_t(a) + \left[r_t - \bar{r}_t\right] \quad \text{and} \quad \bar{r}_{t+1} = \bar{r}_t + \alpha\left[r_t - \bar{r}_t\right]$$

Performance of a Reinforcement Comparison Method



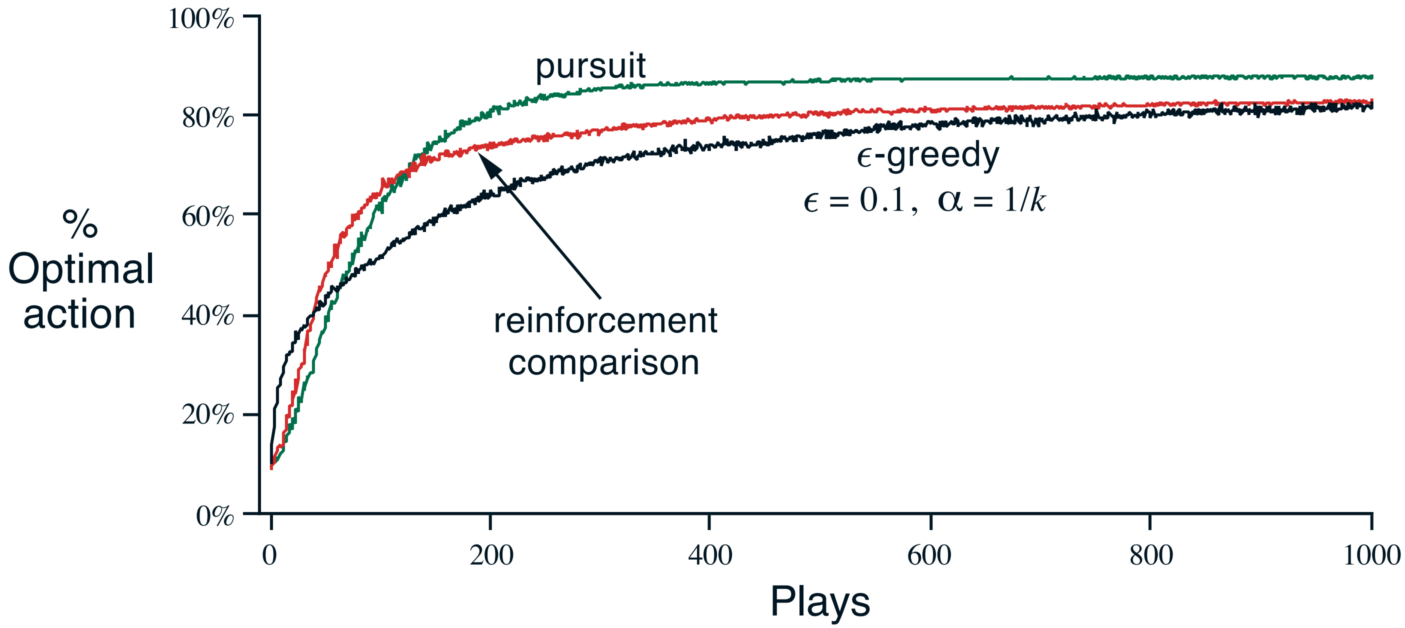
Pursuit Methods

- Maintain both action-value estimates and action preferences
- Always "pursue" the greedy action, i.e., make the greedy action more likely to be selected
- After the *t*-th play, update the action values to get Q_{t+1}
- The new greedy action is $a^*_{t+1} = \arg\max_a Q_{t+1}(a)$
- Then:

$$\pi_{t+1}(a^*_{t+1}) = \pi_t(a^*_{t+1}) + \beta\left[1 - \pi_t(a^*_{t+1})\right]$$

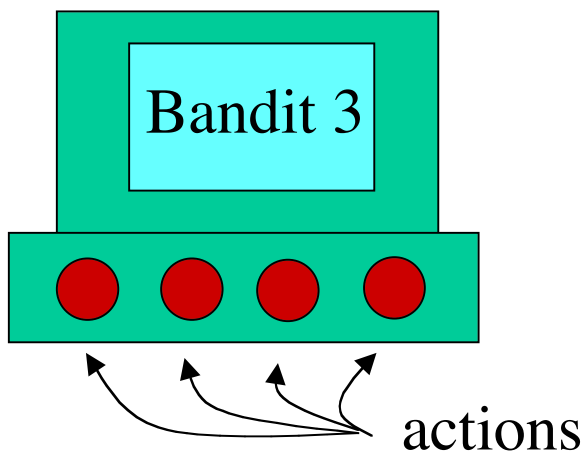
and the probs. of the other actions decremented to maintain the sum of 1

Performance of a Pursuit Method



Associative Search

Imagine switching bandits at each play



Conclusions

- These are all very simple methods
  - but they are complicated enough—we will build on them
- Ideas for improvements:
  - estimating uncertainties . . . interval estimation
  - approximating Bayes optimal solutions
  - Gittens indices
- The full RL problem offers some ideas for solution . . .