# Journal Club: Mean Flows for One-step Generative Modeling

#### **WU SHENGYE**

Institute of Science Tokyo

June 4, 2025

#### Notation

- Here are some notations used in today's slides.
- ullet The instantaneous velocity field is denoted as  $v\left(z_t,t
  ight)$
- The average velocity field is denoted as  $u(z_t, t)$
- In the MeanFlow part, we will define that our time range t is in [0,1]. When t=0,  $z_0=x$  means the target data; When t=1,  $z_1=\epsilon$  means the noise.
- Thus our sampling process will be started from t = 1 to t = 0.

#### Content

- Background of Flow matching
- Proposed Method: Mean Flow
- Experiment
- Conclusion

- Background of Flow matching
- Proposed Method: Mean Flow
- Experiment
- Conclusion

# Flow matching

- Given access to a training dataset of samples from some target distribution q over  $\mathbb{R}^d$ , our goal is to build a model capable of generating new samples from it.
- Formally, we want to generate a novel sample  $X_1 \sim q$  from the source distribution  $X_0 \sim p$ .
- There are several ways of generating the samples, such as Flow matching and Diffusion.
- Flow matching is a stimulation-free methods, which leads to the faster generation.

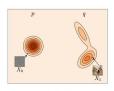
- FM builds a probability path  $(p_t)_{0 \le t \le 1}$ , from a known source distribution  $p_0 = p$  to the data target distribution  $p_1 = q$ , where each  $p_t$  is a distribution over  $\mathbb{R}^d$ .
- FM is a regression objective to train the velocity field neural network which convert  $p_0$  into  $p_1$  along the probability path  $p_t$ .
- This velocity field determines a time-dependent flow  $\psi: [0,1] \times \mathbb{R}^d \to \mathbb{R}^d$ , defined as

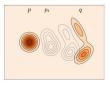
$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_t(x) = v_t\left(\psi_t(x)\right)$$

where  $\psi_t := \psi(t, x)$  and  $\psi_0(x) = x$ . The velocity field  $v_t$  generates the probability path  $p_t$  if its flow  $\psi_t$  satisfies

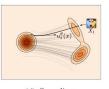
$$X_t := \psi_t(X_0) \sim p_t \text{ for } X_0 \sim p_0$$

4□ > 4□ > 4□ > 4 = > = 900









(a) Data.

(b) Path design.

(c) Training.

(d) Sampling.

- Tips: Please notice that here we use  $v_t$  instead of  $u_t$  since in next part two velocity field will have different meanings.
- Our training objective Flow Matching Loss is as follows:

$$\mathcal{L}_{ ext{FM}}( heta) = \mathbb{E}_{t,X_t} \left\| v_t^{ heta}\left(X_t
ight) - v_t\left(X_t
ight) 
ight\|^2, ext{ where } t \sim \mathcal{U}[0,1] ext{ and } X_t \sim p_t$$

During the sampling process, we will try to deal with:

$$X_1 = X_0 + \int_0^1 v(X_\tau, \tau) d\tau$$

ㅁㅏ ◀♬ㅏ◀ㅌㅏ◀ㅌㅏ - ㅌ - 쒸٩@

- The ground truth  $v_t$  is intractable, but we can use a conditional velocity field instead of it.
- e.g. If we define random variables  $X_t$  as follows:

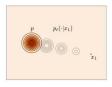
$$X_t = tX_1 + (1-t)X_0 \sim p_t$$

Then given the instance  $X_1 = x_1$  we can write the conditional random variables:

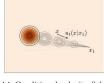
$$X_{t|1} = tx_1 + (1-t)X_0 \sim p_{t|1}(\cdot \mid x_1) = \mathcal{N}(\cdot \mid tx_1, (1-t)^2 I)$$

In that case, the conditional velocity field will be defined as:

$$v_t\left(x\mid x_1\right) = \frac{x_1-x}{1-t}$$









path  $p_t(x|x_1)$ .

path  $p_t(x)$ .

 $u_t(x|x_1)$ .

(a) Conditional probability (b) (Marginal) Probability (c) Conditional velocity field (d) (Marginal) Velocity field  $u_t(x)$ .

- Given a fixed target sample  $X = x_1$ , its conditional velocity field  $v_t(x \mid x_1)$  generates the conditional probability path  $p_t(x \mid x_1)$ .
- The conditional Flow Matching Loss:

$$\mathcal{L}_{ ext{CFM}}( heta) = \mathbb{E}_{t,X_t,X_1} \left\| v_t^{ heta}\left(X_t
ight) - v_t\left(X_t \mid X_1
ight) 
ight\|^2$$

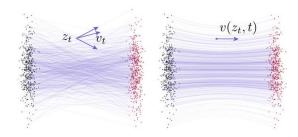
where  $t \sim U[0,1], X_0 \sim p, X_1 \sim q$ 

 It can be proved that the FM loss and CFM loss provide the same gradients to learn  $v_t^{\theta}$ :

$$abla_{ heta} \mathcal{L}_{ ext{FM}}( heta) = 
abla_{ heta} \mathcal{L}_{ ext{CFM}}( heta)$$

### The limitations

- For the integration  $X_1 = X_0 + \int_0^1 v(X_\tau, \tau) d\tau$ , we should use some numerical method to approximate it in discrete time steps.
- ullet e.g. Euler Method:  $X_{t_{i+1}} = X_{t_i} + (t_{i+1} t_i) \, v \left( X_{t_i}, t_i 
  ight)$
- Some higher-order solvers might be adopted.
- It will lead to some inaccurate result while applying coarse dicertizations over curved trajectories.



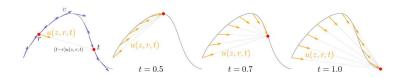
- Recently, the most popular way of defining the conditional velocity field is to use a linear interpolation assumption.
- However, even when the conditional flows are designed to be straight("rectified"), the marginal velocity field typically induces a curved trajectory.
- To improve the accuracy, higher NFE (Number of Function Evaluations) is needed.

- Background of Flow matching
- Proposed Method: Mean Flow
- Experiment
- Conclusion

# Proposed Method: MeanFlow

- Motivation: We want to implement some algorithms which needs less steps of generating.
- We may refer to some physics process.
- E.g. Consider that we have two different velocity  $v_1$  and  $v_2$  for two time intervals [a, b] and [b, c]. We would like to get the average velocity  $\bar{v}$  of the total interval [a, c]
- Then we can easily calculate:  $\bar{v} = \frac{(b-a)v_1 + (c-b)v_2}{c-a}$
- We can use the average velocity to describe the velocity over a certain period of time

## MeanFlow



 We define the average velocity field u<sub>t</sub> between two time steps r and t as follows:

$$u(z_t, r, t) \triangleq \frac{1}{t-r} \int_r^t v(z_\tau, \tau) d\tau$$

- Obviously, the definition of  $u_t$  can satisfy the certain boundary conditions and "consistency" constraints.
- But in this definition, we still need to evaluate the function v for a lot of steps, can we do better?



## MeanFlow

#### MeanFlow Indentity

From the definition of average velocity field:

$$(t-r)u(z_t,r,t)=\int_r^t v(z_\tau,\tau)\,d\tau$$

We differentiate both sides with respect to t, treating r as independent of t.

$$u(z_t,r,t) = v(z_t,t) - (t-r)\frac{d}{dt}u(z_t,r,t)$$

- Now we can see that we don't need the instantaneous velocity field  $v_t$  at every time step t.
- During sampling process, we can use:  $z_0 = z_1 u_\theta(z_1, 0, 1)$

4 D > 4 D > 4 D > 4 D > 3 D 9 Q

# Training Process

- For the training objective, it is nature to consider about using  $u_{\theta}$  to approximate the average velocity  $u_t$
- However, for  $u(z_t, r, t) = v(z_t, t) (t r) \frac{d}{dt} u(z_t, r, t)$ , we still need to know the marginal velocity field v. Besides, we need to calculate the derivative of average velocity field u.
- To deal with this, we define our target field  $u_{tgt}$  as:

$$u_{\text{tgt}} = v_t - (t - r) \left( v_t \frac{\partial u_\theta}{\partial_z} + \frac{\partial u_\theta}{\partial_t} \right)$$

where  $v_t = a_t'x + b_t'\epsilon$  is the conditional velocity, and by default,  $v_t = \epsilon - x$ . And we use the parameters of the network  $t_\theta$  itself to get the derivative, which is called Bootstrapping strategy.

# Training Process

• The training object is:

$$L( heta) = \mathbb{E} \|u_{ heta}(z_t, r, t) - \operatorname{sg}(u_{\operatorname{tgt}})\|_2^2$$
 where  $u_{\operatorname{tgt}} = v_t - (t - r) \left( v_t rac{\partial u_{ heta}}{\partial_z} + rac{\partial u_{ heta}}{\partial_t} 
ight)$ 

- The sg  $(u_{\mathrm{tgt}})$  means the stop gradient operation.
- $u_{\rm tgt}$  has already contained  $\frac{\partial u_{\theta}}{\partial z}$  and  $\frac{\partial u_{\theta}}{\partial t}$ . If we continue calculating its derivative during the process of minimizing the  $L(\theta)$ , it will lead to double backpropagation.
- Thus we just use  $u_{\rm tgt}$  as a constant.

# Training process

- The training process of the proposed method comprises the following steps:
- **1** Sample x from data distribution and sample  $\epsilon$  from noise distribution.
- ② According to the predefined trajectory (e.g. linear interpolation), calculate  $z_t$  and condition velocity  $v_t$
- **3** Sample the paired time steps (r, t), where  $0 \le r \le t \le 1$
- Evaluate  $u_{\theta}(z_t, r, t)$  by the network
- **Solution** Calculate  $\frac{d}{d\theta}u_{\theta}\left(z_{t},r,t\right)$  throughout Jacobian-Vector Product.
- **3** Construct the target field  $u_{\mathrm{tgt}} = v_t (t-r) \left( v_t \frac{\partial u_{\theta}}{\partial_z} + \frac{\partial u_{\theta}}{\partial_t} \right)$
- **1** Minimize the loss function  $L(\theta)$  and update the parameters  $\theta$



- Background of Flow matching
- Proposed Method: Mean Flow
- Experiment
- Conclusion

# **Experiment**













Figure 1: One-step generation on ImageNet 256×256 from scratch. Our *MeanFlow* (MF) model achieves significantly better generation quality than previous state-of-the-art one-step diffusion/flow methods. Here, iCT [43], Shortcut [13], and our MF are all 1-NFE generation, while IMM's 1-step result [52] involves 2-NFE guidance. Detailed numbers are in Tab. 2. Images shown are generated by our 1-NFE model.

# Experiment

method	params	NFE	FID
1-NFE diffusion/flow from scratch			
iCT-XL/2 [43] <sup>†</sup>	675M	1	34.24
Shortcut-XL/2 [13]	675M	1	10.60
MeanFlow-B/2	131M	1	6.17
MeanFlow-M/2	308M	1	5.01
MeanFlow-L/2	459M	1	3.84
MeanFlow-XL/2	676M	1	3.43
2-NFE diffusion/flow from scratch			
iCT-XL/2 [43] <sup>†</sup>	675M	2	20.30
iMM-XL/2 [52]	675M	$1\times2$	7.77
MeanFlow-XL/2	676M	2	2.93
MeanFlow-XL/2+	676M	2	2.20

- Background of Flow matching
- Proposed Method: Mean Flow
- Experiment
- Conclusion

#### Conclusion

- Proposed a novel perspective on average velocity modeling: Shifted the core of generative flow modeling from instantaneous velocity to average velocity, providing a new theoretical foundation for few-step or one-step generation.
- Derived and utilized the MeanFlow identity: Starting from the definition of average velocity, rigorously derived a mathematical identity and adopted it as a principled training objective, avoiding reliance on heuristic constraints.
- Achieved state-of-the-art one-step generation performance: MeanFlow demonstrated leading generation quality under 1-NFE and 2-NFE settings across multiple standard datasets, particularly excelling in high-resolution generation on ImageNet 256×256.