07-131 GPI Homework 1

Firstname Lastname (andrewid)

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what do you want for dinner?
Gates 6th?
ok.

Question 1. For any $n \in \mathbb{N}$, define $S_n = 2\left(\sum_{i=1}^{n} i\right)$. Prove that $\forall n \in \mathbb{N}$, $S_n = n(n+1)$

Proof. Let $P(n) \iff S_n = n(n+1)$. We will prove $P(n)$ by induction on $n \in \mathbb{N}$.

> Base case:

When $n = 0$,

$$P(0) \iff 0(0+1) = 0 = S_0. \text{ Since } S_0 = 2\left(\sum_{i=1}^{0} i\right) = 0,\quad P(n) \text{ is true.}$$

✓

When $n = 1$,

$$P(1) \iff 1(1+1) = 2 = S_1 \text{ since } S_1 = 2\left(\sum_{i=1}^{1} i\right) = 2. \quad P(n) \text{ is true.}$$

✓

Wahooo~

> Induction Hypothesis:

Assume $P(k)$ (is true) for some $k \in \mathbb{N}$.

> Induction step:

Note that $S_k = (k)(k+1)$ and that

$$S_{k+1} = 2\left(\sum_{i=1}^{k+1} i\right) = 2\left(\sum_{i=1}^{k} i\right) + 2(k+1) = S_k + 2(k+1)$$

too much maaaath

Therefore, we can substitute and rewrite the expression as follows:

$$\begin{aligned} S_{k+1} &= S_k + 2(k+1) \\ &= k(k+1) + 2(k+1) \\ &= (k+2)(k+1) \end{aligned}$$

BOOM

Thus, we can conclude that $P(k+1)$ is true.

Therefore, because the base case and the induction step holds, $P(n)$ is true for all $n \in \mathbb{N}$ by induction.

~~mic drop~~ ∎

~ underground block market ~

Question 2. Assume that free food appears every day with probability p, and that you must eat every day. For any given number of days k, derive the probability that you can subsist for at least k days on free food.

proof. We will proceed with weak induction.

Let $g(n)$ represent the probability that we can subsist for at least n days on free food.

Let $P(n) \iff g(n) = p^n$. We will prove $P(n)$ by induction on $n \in \mathbb{N}$.

> Base case:

When $n=0$, $P(0) \iff g(n) = p^0 = 1$. But $g(n) = 1$, since we can always subsist for at least zero days, so $P(0)$ is true.

> Induction Hypothesis:

Assume $P(k)$ for some $k \in \mathbb{N}$, and then prove $P(k+1)$.

> Induction Step:

Note that each day food appears with probability p, so we can subsist one day more with probability p. So then:

$$\begin{aligned} g(k+1) &= g(k) \cdot p \\ &= p^k \cdot p && \text{(IH)} \\ &= p^{k+1} \end{aligned}$$

So then $P(k+1)$ is true.

By weak induction, $P(n)$ for all $n \geq 0$. So the probability of subsisting on free food for at least n days is p^n.

$\blacksquare$

ughhhh I'm hungryyyyyy