Collision Events

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1 Collision event in Ball Game

The total energy for a system of N balls before any collision takes place is given by

$$E_b = \sum_{i=1}^{N} \frac{1}{2} m_i (\boldsymbol{v}_i \cdot \boldsymbol{v}_i) \tag{1}$$

Assume two balls are colliding. Introduce the vector $\boldsymbol{N}_i$ defined as the vector from the center of the ball i to the collision point. Further on, we introduce a quantity k defined from

$$\boldsymbol{N_i} = k_i \frac{\boldsymbol{N_i}}{|\boldsymbol{N_i}|} = k_i \bar{\boldsymbol{N}}_i \tag{2}$$

Note that $\bar{\boldsymbol{N}}_1 = -\bar{\boldsymbol{N}}_2$. Assume two balls 1 and 2 are colliding. During collision, it is assumed that body 1 new velocity will get a contribution from $\boldsymbol{N}_2$ and similarly body 2 will get a contribution from $\boldsymbol{N}_1$. Conservation of linear momentum dictates that following relation must hold before and after the collision (where $\boldsymbol{v}_i$ denote velocity vector before collision),

$$m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2 = m_1(\boldsymbol{v}_1 + k_2 \bar{\boldsymbol{N}}_2) + m_2(\boldsymbol{v}_2 + k_1 \bar{\boldsymbol{N}}_1) \tag{3}$$

E.g that

$$m_1 k_2 \bar{\boldsymbol{N}}_2 = -m_2 k_1 \bar{\boldsymbol{N}}_1 \tag{4}$$

And since the vectors are of exactly opposite direction, we have that $m_1 k_2 = m_2 k_1$.

The total energy before,E_b and after the collision, E_a of two bodies is assumed to satisfy

$$E_a \leq E_b \tag{5}$$

To ensure this, a dissipation quantity, defined as

$$d = \tilde{d}(m_1 + m_2)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot (\boldsymbol{v}_2 - \boldsymbol{v}_1) \tag{6}$$

is introduced. According to (6), the higher the relative velocity between the two bodies, the higher the dissipation will become. Also the higher the total mass of the two bodies, the higher the dissipation. A dissipation constant $\tilde{d} \geq 0$ is also introduced. If $\tilde{d} = 0$, the energy will conserved. The inequality (5) will now be satisfied by solving

$$\begin{aligned} &\frac{1}{2}m_1\left(\boldsymbol{v}_1 \cdot \boldsymbol{v}_1\right) + \frac{1}{2}m_2\left(\boldsymbol{v}_2 \cdot \boldsymbol{v}_2\right) - d = \\ &\frac{1}{2}m_1\big((\boldsymbol{v}_1 + k_2\bar{\boldsymbol{N}}_2)\cdot(\boldsymbol{v}_1 + k_2\bar{\boldsymbol{N}}_2)\big) + \frac{1}{2}m_2\big((\boldsymbol{v}_2 + k_1\bar{\boldsymbol{N}}_1)\cdot(\boldsymbol{v}_2 + k_1\bar{\boldsymbol{N}}_1)\big) \end{aligned} \tag{7}$$

Simplification of the expression above results in the following equation .

$$\frac{(m_2k_1^2 + m_1k_2^2)}{2} + \big(k_2m_1(\boldsymbol{v}_1 \cdot \bar{\boldsymbol{N}}_2) + k_1m_2(\boldsymbol{v}_2 \cdot \bar{\boldsymbol{N}}_1)\big) + d = 0 \tag{8}$$

Utilizing that $k_2 = \dfrac{m_2}{m_1}k_1$ we can rewrite the above into a second degree equation with only one unknown

$$\left(\frac{m_1m_2 + m_2^2}{2m_1}\right)k_1^2 + k_1m_2\big((\boldsymbol{v}_1 \cdot \bar{\boldsymbol{N}}_2) + (\boldsymbol{v}_2 \cdot \bar{\boldsymbol{N}}_1)\big) + d = 0 \tag{9}$$

The solutions are given by

$$k_1 = -\frac{p}{2} \pm \frac{1}{2}\sqrt{p^2 - \frac{4d(2m_1)}{m_1m_2 + m_2^2}} \tag{10}$$

where $p = \left(\dfrac{2m_1}{m_1 + m_2}\right)((\boldsymbol{v}_1 \cdot \boldsymbol{N}_2) + (\boldsymbol{v}_2 \cdot \boldsymbol{N}_1))$.

There will be two solutions to the equation. The plus sign is used if $p < 0$ and the minus sign is used if $p > 0$ in the solution above (corresponding to the physical relevant solutions if $d = 0$. If $p^2 < \dfrac{4d(2m_1)}{m_1m_2 + m_2^2}$ complex roots will occur and then d is changed in the program to satisfy $p^2 > \dfrac{4d(2m_1)}{m_1m_2 + m_2^2}$ with a relative tolerance of 10^{-6}.