Collision Events

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1 Collision event in Ball Game

The total energy for a system of N balls before any collision takes place is given by

$$E_b = \sum_{i=1}^{N} \frac{1}{2} m_i (\boldsymbol{v}_i \cdot \boldsymbol{v}_i) \tag{1}$$

Assume two balls are colliding. Introduce the vector $\boldsymbol{N}_1$ defined as the vector from the center of the ball 1 to the collision point, or equivalently the vector between the center points of the balls times half. Further on, we introduce a quantity k defined from

$$\boldsymbol{N_i} = k_i \frac{\boldsymbol{N_i}}{|\boldsymbol{N_i}|} = k_i \bar{\boldsymbol{N}}_i \tag{2}$$

Since we do not want to perform any time integration, lets assume that the velocity change in ball 1 is proportional to the mass of ball 2 and vice verce. E.g. we have that the velocity after collisions of ball 1 is equal to

$$\boldsymbol{v}_1^{(a)} = \boldsymbol{v}_1 + m_2 k_2 \bar{\boldsymbol{N}}_2 \tag{3}$$

where $\boldsymbol{v}_i$ denote velocity vector before collision. And a similar formula applies for ball 2. Note that $\bar{\boldsymbol{N}}_1 = -\bar{\boldsymbol{N}}_2$. Conservation of linear momentum dictates that now following relation must hold before and after the collision

$$m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2 = m_1 (\boldsymbol{v}_1 + m_2 k_2 \bar{\boldsymbol{N}}_2) + m_2 (\boldsymbol{v}_2 + m_1 k_1 \bar{\boldsymbol{N}}_1) \tag{4}$$

E.g that

$$m_1 m_2 k_2 \bar{\boldsymbol{N}}_2 = -m_2 m_1 k_1 \bar{\boldsymbol{N}}_1 \tag{5}$$

And since the vectors are of exactly opposite direction, we have that $k_2 = k_1$. The total energy before,E_b and after the collision, E_a of two bodies is assumed to satisfy

$$E_a \leq E_b \tag{6}$$

To ensure this, a dissipation quantity, defined as

$$d = \tilde{d}(m_1 + m_2)(\boldsymbol{v}_2 - \boldsymbol{v}_1) \cdot (\boldsymbol{v}_2 - \boldsymbol{v}_1) \tag{7}$$

is introduced. According to (7), the higher the relative velocity between the two bodies, the higher the dissipation will become. Also the higher the total mass of the two bodies, the higher the dissipation. A dissipation constant $\tilde{d} \geq 0$ is also introduced. If $\tilde{d} = 0$, the energy will conserved. The inequality (6) will now be satisfied by solving

$$\frac{1}{2}m_1 (\boldsymbol{v}_1 \cdot \boldsymbol{v}_1) + \frac{1}{2}m_2 (\boldsymbol{v}_2 \cdot \boldsymbol{v}_2) - d =$$
$$\frac{1}{2}m_1 ((\boldsymbol{v}_1 + km_2\bar{\boldsymbol{N}}_2) \cdot (\boldsymbol{v}_1 + km_2\bar{\boldsymbol{N}}_2)) + \frac{1}{2}m_2 ((\boldsymbol{v}_2 + km_1\bar{\boldsymbol{N}}_1) \cdot (\boldsymbol{v}_2 + km_1\bar{\boldsymbol{N}}_1)) \tag{8}$$

where it is utilized that $k = k_1 = k_2$ above. Simplification of the expression above results in the following equation .

$$\frac{m_1 m_2 (m_1 + m_2)}{2}k^2 + km_1 m_2 \left((\boldsymbol{v}_1 \cdot \bar{\boldsymbol{N}}_2) + (\boldsymbol{v}_2 \cdot \bar{\boldsymbol{N}}_1)\right) + d = 0 \tag{9}$$

The solutions are given by

$$k = -\frac{p}{2} \pm \frac{1}{2}\sqrt{p^2 - \frac{8d}{(m_1 + m_2)m_1 m_2}} \tag{10}$$

where $p = \left(\dfrac{2}{m_1 + m_2}\right) ((\boldsymbol{v}_1 \cdot \boldsymbol{N}_2) + (\boldsymbol{v}_2 \cdot \boldsymbol{N}_1))$. There will be two solutions to the equation. The plus sign is used if $p < 0$ and the minus sign is used if $p > 0$ in the solution above (corresponding to the physical relevant solutions if $d = 0$). If $p^2 < \dfrac{8d}{m_1 m_2 (m_1 + m_2)}$ complex roots will occur and then d is changed in the program to satisfy $p^2 > \dfrac{8d}{m_1 m_2 (m_1 + m_2)}$ with a relative tolerance of 10^{-6}.