i. Cayley Hamilton Theorem

The Cayley-Hamilton Theorem states that every square matrix satisfies its own characteristic equation. This means if you substitute the matrix itself into its characteristic polynomial, the result will be the zero matrix.

ii. Eigenvalues

Eigenvalues are special scalar values that represent factors by which eigenvectors are stretched or shrunk by a linear transformation. For a square matrix A, an eigenvalue λ satisfies the equation $Av = \lambda v$, where v is a non-zero eigenvector.

iii. Real and imaginary parts of Log Z

For a complex number $Z = x + iy = re^{i\theta}$, its natural logarithm is given by $LogZ = Log(re^{i\theta}) = Logr + i\theta$. Here, $r = |Z| = \sqrt{x^2 + y^2}$ is the modulus and $\theta = arg(Z) = tan^{-1}(y/x)$ is the principal argument. Thus, the real part is $Logr$ and the imaginary part is θ.

iv. Expand sin z by Taylor's theorem

The Taylor series expansion of $sinz$ around $z = 0$ (Maclaurin series) is given by:

$sinz = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ... = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$

v. Real and imaginary parts of sinh(x+iy)

We know that $sinh(A + B) = sinhAcosB + coshAsinB$.

So, $sinh(x + iy) = sinhxcos(iy) + coshxsin(iy)$.

Using the identities $cos(iy) = coshy$ and $sin(iy) = isinhy$:

$sinh(x + iy) = sinhxcoshy + coshx(isinhy) = sinhxcoshy + icoshxsinhy$.

Therefore, the real part is $sinhxcoshy$ and the imaginary part is $coshxsinhy$.

vi. Evaluate cos nπ

The value of $cosn\pi$ depends on whether 'n' is an even or an odd integer.
If 'n' is an even integer, $cosn\pi = 1$ (e.g., $cos0 = 1, cos2\pi = 1$).
If 'n' is an odd integer, $cosn\pi = -1$ (e.g., $cos\pi = -1, cos3\pi = -1$).
This can be summarized as $cosn\pi = (-1)^n$.

vii. Ordinary differential equation

An ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and its derivatives[cite: 1]. For example, $\frac{dy}{dx} + P(x)y = Q(x)$.

viii. $e^{i\pi}$ =.....

According to Euler's formula, $e^{i\theta} = cos\theta + isin\theta$.
Substituting $\theta = \pi$:
$e^{i\pi} = cos\pi + isin\pi = -1 + i(0) = -1$.

ix. Bessel's Differential Equation

Bessel's differential equation is a second-order linear ordinary differential equation that arises in solving various problems in physics and engineering, particularly those involving cylindrical symmetry. The standard form is $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \nu^2)y = 0$, where ν is an arbitrary constant (order of the Bessel function).

x. Exact Differential equation.

A first-order differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ is called an exact differential equation if there exists a function $\phi(x, y)$ such that $d\phi = Mdx + Ndy$. A necessary and sufficient condition for exactness is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

xi. Residue

In complex analysis, the residue of a complex function $f(z)$ at an isolated singularity z_0 is a coefficient of the $(z - z_0)^{-1}$ term in the Laurent series expansion of $f(z)$ around z_0. It plays a crucial role in the residue theorem for evaluating contour integrals.

xii. Perfect number

A perfect number is a positive integer that is equal to the sum of its proper positive divisors (divisors excluding the number itself). For example, 6 is a perfect number because its proper divisors are 1, 2, and 3, and $1+2+3=6$.

xiii. Poles

In complex analysis, a pole is a type of isolated singularity of a complex function where the function's value becomes infinite. More formally, z_0 is a pole of order 'm' if $lim_{z \to z_0}(z - z_0)^m f(z) = L \neq 0$ and L is finite.

xiv. Singularities

In complex analysis, a singularity of a complex function is a point where the function is not analytic (i.e., not differentiable in a complex sense). Singularities can be isolated (like poles or essential singularities) or non-isolated (like branch points).

xv. Sin(ix)=....

We know that $sinz = \frac{e^{iz}-e^{-iz}}{2i}$.
Substituting $z = ix$:
$sin(ix) = \frac{e^{i(ix)}-e^{-i(ix)}}{2i} = \frac{e^{-x}-e^{x}}{2i}$.
To simplify, multiply numerator and denominator by 'i':
$sin(ix) = \frac{i(e^{-x}-e^{x})}{2i^2} = \frac{i(e^{-x}-e^{x})}{-2} = \frac{i(e^{x}-e^{-x})}{2} = i\left(\frac{e^{x}-e^{-x}}{2}\right)$.
Since $sinhx = \frac{e^{x}-e^{-x}}{2}$, we have:
$sin(ix) = isinhx$.

Okay, let's move on to **Section B.**

As per the instructions, Section B questions are short answer type, carrying 5 marks each. You need to attempt any 3 questions. I will provide answers long enough to understand what is going on in them.

Here are the solutions for Section B:

Section B (Attempt any 3 questions - 5 marks each)

1) **Reduce to normal form and find rank of matrix** $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$

To reduce the matrix to normal form, we use elementary row and column operations to transform it into the form $\begin{pmatrix} I_r & O \\ O & O \end{pmatrix}$, where I_r is the identity matrix of order r, and 'r' is the rank of the matrix.

Given corrected matrix:

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$$

Step 1: Make the (1,1) element 1 (it already is).

Step 2: Use R1 to make other elements in C1 zero.

$R_2 \to R_2 - R_1$

$$A \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

Step 3: Use C1 to make other elements in R1 zero.

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - 2C_1$

$$A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix}$$

Step 4: Make the (2,2) element 1 (it already is).

Step 5: Use R2 to make other elements in C2 zero.

$R_3 \to R_3 + R_2$

$$A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Step 6: Use C2 to make other elements in R2 zero.

$C_3 \to C_3 - C_2$

$$A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

This is the normal form $\begin{pmatrix} I_2 & O \\ O & O \end{pmatrix}$.

The number of non-zero rows in the normal form is 2.

Therefore, the rank of matrix A is **2**.

Section B (Attempt any 3 questions - 5 marks each)

2) Prove that $log_i(i) = \frac{4m+1}{4n+1}$, here m and n are integers.

Let $log_i(i) = x$.
By the definition of a logarithm, this means $i^x = i$.

The principal value of x is clearly $x = 1$.
Thus, $log_i(i) = 1$.

Now, we need to prove that $1 = \frac{4m+1}{4n+1}$.
This equality holds if and only if:

$$4n + 1 = 4m + 1$$

$$4n = 4m$$

$$n = m$$

Therefore, the statement "$log_i(i) = \frac{4m+1}{4n+1}$" is true **if and only if the integers m and n are equal ($m = n$).** It is not a universally true statement for all integers m and n, but rather a conditional one. If the problem implies that such an equality always holds, then the problem statement itself has a constraint that $m = n$.

3) Solve the differential equation $(y\cos x + 1)dx + \sin x dy = 0$

This is a first-order differential equation. Let's check if it's exact.
The equation is in the form $M(x, y)dx + N(x, y)dy = 0$.
Here, $M(x, y) = y\cos x + 1$ and $N(x, y) = \sin x$.

To check for exactness, we need to verify if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Calculate $\frac{\partial M}{\partial y}$:
$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y\cos x + 1) = \cos x$

Calculate $\frac{\partial N}{\partial x}$:
$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(\sin x) = \cos x$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the differential equation is **exact**.

For an exact differential equation, the solution is given by $\int M dx + \int (N - \frac{\partial}{\partial y}\int M dx)dy = C$.
Alternatively, we can find a function $\Phi(x, y)$ such that $\frac{\partial \Phi}{\partial x} = M$ and $\frac{\partial \Phi}{\partial y} = N$.

From $\frac{\partial \Phi}{\partial x} = M$:
$\frac{\partial \Phi}{\partial x} = y \cos x + 1$
Integrate with respect to x (treating y as a constant):
$\Phi(x, y) = \int (y \cos x + 1) dx = y \sin x + x + h(y)$
Here, $h(y)$ is an arbitrary function of y.

Now, differentiate this $\Phi(x, y)$ with respect to y and equate it to N:
$\frac{\partial \Phi}{\partial y} = \frac{\partial}{\partial y}(y \sin x + x + h(y)) = \sin x + h'(y)$

We know that $\frac{\partial \Phi}{\partial y} = N = \sin x$.
So, $\sin x + h'(y) = \sin x$
This implies $h'(y) = 0$.
Integrating with respect to y, $h(y) = C_1$, where C_1 is an arbitrary constant.

Substitute $h(y)$ back into the expression for $\Phi(x, y)$:
$\Phi(x, y) = y \sin x + x + C_1$

The general solution of the exact differential equation is $\Phi(x, y) = C_2$, where C_2 is a constant.
So, $y \sin x + x + C_1 = C_2$
Let $C = C_2 - C_1$.

The general solution is:
$y \sin x + x = C$

4) State and prove polar form of CR Equations.

The Cauchy-Riemann (CR) equations are fundamental conditions for a complex function $f(z) = u(x, y) + iv(x, y)$ to be analytic (holomorphic) at a point.

Statement of CR Equations in Polar Form:

If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function, where $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$, then the Cauchy-Riemann equations in polar form are:

1. $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$
2. $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

Proof:

We start with the CR equations in Cartesian coordinates:

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ (1)

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ (2)

We use the transformation equations between Cartesian and polar coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

Now, we need to express the partial derivatives with respect to x and y in terms of partial derivatives with respect to r and θ using the chain rule.

$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}$
$\frac{\partial x}{\partial r} = \cos\theta$
$\frac{\partial y}{\partial r} = \sin\theta$
So, $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta$ (3)

$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta}$
$\frac{\partial x}{\partial \theta} = -r\sin\theta$
$\frac{\partial y}{\partial \theta} = r\cos\theta$
So, $\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}(-r\sin\theta) + \frac{\partial u}{\partial y}(r\cos\theta)$
$\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta$ (4)

Similarly for v:
$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\sin\theta$ (5)
$\frac{1}{r}\frac{\partial v}{\partial \theta} = -\frac{\partial v}{\partial x}\sin\theta + \frac{\partial v}{\partial y}\cos\theta$ (6)

Now, substitute (1) and (2) into (3) and (4):

From (3): $\frac{\partial u}{\partial r} = \frac{\partial v}{\partial y}\cos\theta - \frac{\partial v}{\partial x}\sin\theta$ (using (1) and (2)) (7)

From (4): $\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial y}\sin\theta - \frac{\partial v}{\partial x}\cos\theta$ (using (1) and (2)) (8)

Comparing (7) with (6):
$\frac{\partial u}{\partial r} = \frac{\partial v}{\partial y}\cos\theta - \frac{\partial v}{\partial x}\sin\theta$
And $\frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial y}\cos\theta - \frac{\partial v}{\partial x}\sin\theta$ (from (6) by rearranging terms for comparison)
Thus, we get the first polar CR equation: $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$.

Comparing (7) with (6):

$\frac{\partial u}{\partial r} = \frac{\partial v}{\partial y}\cos\theta - \frac{\partial v}{\partial x}\sin\theta$

And $\frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial y}\cos\theta - \frac{\partial v}{\partial x}\sin\theta$ (from (6) by rearranging terms for comparison)

Thus, we get the first polar CR equation: $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$.

Now, let's derive the second equation.

From (5): $\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\cos\theta + \frac{\partial v}{\partial y}\sin\theta$

Substitute (1) and (2) into (5):

$\frac{\partial v}{\partial r} = -\frac{\partial u}{\partial y}\cos\theta + \frac{\partial u}{\partial x}\sin\theta$ (9)

From (4): $\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta$

Multiply (4) by -1:

$-\frac{1}{r}\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\sin\theta - \frac{\partial u}{\partial y}\cos\theta$ (10)

Comparing (9) and (10), we see they are identical.

Thus, we get the second polar CR equation: $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.

These two equations are the polar form of the Cauchy-Riemann equations.

5) Find the solution of differential equation $(D^3 - 6D^2 + 11D - 6)y = e^x$

This is a linear non-homogeneous differential equation with constant coefficients.
The general solution y is the sum of the complementary function (y_c) and the particular integral (y_p).
$y = y_c + y_p$

Step 1: Find the Complementary Function (y_c)
The auxiliary equation is obtained by replacing D with m:
$m^3 - 6m^2 + 11m - 6 = 0$

We can find the roots by trial and error.
If $m = 1$: $1 - 6 + 11 - 6 = 0$. So, $m = 1$ is a root.
This means $(m-1)$ is a factor. We can perform polynomial division or synthetic division.

Using synthetic division with root 1:

```
1 | 1  -6  11  -6
  |     1  -5   6
  -----------------
    1  -5   6   0
```

The depressed equation is $m^2 - 5m + 6 = 0$.
Factoring this quadratic: $(m-2)(m-3) = 0$.
So, the roots are $m = 1, m = 2, m = 3$.

Since all roots are real and distinct, the complementary function is:

$y_c = C_1e^x + C_2e^{2x} + C_3e^{3x}$

Step 2: Find the Particular Integral (y_p)

The equation is $(D^3 - 6D^2 + 11D - 6)y = e^x$.

Here, $f(D) = D^3 - 6D^2 + 11D - 6$ and $X = e^x$.

The particular integral is given by $y_p = \frac{1}{f(D)}X = \frac{1}{D^3-6D^2+11D-6}e^x$.

For $X = e^{ax}$, we replace D with a, provided $f(a) \neq 0$. Here $a = 1$.

Let's evaluate $f(1)$:

$f(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$.

Since $f(1) = 0$, this is a case of failure. This means e^x is a part of the complementary function (as $m = 1$ is a root).

When $f(a) = 0$, we use the rule:

$y_p = x\frac{1}{f'(D)}e^{ax}$

First, find $f'(D)$:

$f(D) = D^3 - 6D^2 + 11D - 6$

$f'(D) = 3D^2 - 12D + 11$

Now, substitute $D = a = 1$ into $f'(D)$:

$f'(1) = 3(1)^2 - 12(1) + 11 = 3 - 12 + 11 = 2$.

Since $f'(1) \neq 0$, we can proceed.

$y_p = x\frac{1}{f'(1)}e^x = x\frac{1}{2}e^x = \frac{1}{2}xe^x$

Step 3: General Solution

The general solution is $y = y_c + y_p$:

$y = C_1e^x + C_2e^{2x} + C_3e^{3x} + \frac{1}{2}xe^x$

6) Expand log z using taylor's series.

To expand $log z$ using Taylor's series, we need to choose a point a around which to expand. The Taylor series expansion of a function $f(z)$ around a point a is given by:

$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(z-a)^n = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + ...$

For $f(z) = log z$:

$f(z) = log z$

$f'(z) = \frac{1}{z}$

$f''(z) = -\frac{1}{z^2}$

$f'''(z) = \frac{2}{z^3}$

$f^{(n)}(z) = (-1)^{n-1}\frac{(n-1)!}{z^n}$ for $n \geq 1$.

We cannot expand around $z = 0$ because $log z$ is undefined at $z = 0$. A common choice is to expand around $a = 1$.

Evaluating the derivatives at $a = 1$:

$f(1) = log 1 = 0$

$f'(1) = \frac{1}{1} = 1$

$f''(1) = -\frac{1}{1^2} = -1$

$f'''(1) = \frac{2}{1^3} = 2$

$f^{(n)}(1) = (-1)^{n-1}(n-1)!$ for $n \geq 1$.

6) Expand log z using taylor's series.

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For $f(z) = logz$:

$f(z) = logz$

$f'(z) = \frac{1}{z}$

$f''(z) = -\frac{1}{z^2}$

$f'''(z) = \frac{2}{z^3}$

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Evaluating the derivatives at $a = 1$:

$f(1) = log1 = 0$

$f'(1) = \frac{1}{1} = 1$

$f''(1) = -\frac{1}{1^2} = -1$

$f'''(1) = \frac{2}{1^3} = 2$

$f^{(n)}(1) = (-1)^{n-1}(n-1)!$ for $n \geq 1$.

Now, substitute these into the Taylor series formula around $a = 1$:

$$logz = f(1) + f'(1)(z-1) + \frac{f''(1)}{2!}(z-1)^2 + \frac{f'''(1)}{3!}(z-1)^3 + ...$$

$$logz = 0 + 1(z-1) + \frac{-1}{2}(z-1)^2 + \frac{2}{6}(z-1)^3 + \frac{-6}{24}(z-1)^4 + ...$$

$$logz = (z-1) - \frac{(z-1)^2}{2} + \frac{(z-1)^3}{3} - \frac{(z-1)^4}{4} + ...$$

In summation notation, for $n \geq 1$:

$$logz = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(z-1)^n$$

This expansion is valid for $|z-1| < 1$, which means $0 < z < 2$ along the real axis. This is the Taylor series (or Maclaurin series for $log(1+x)$ with $x = z - 1$).

7) Construct an analytic function f(z) of which real part is $e^x(x\cos y - y\sin y)$.

Let the analytic function be $f(z) = u(x,y) + iv(x,y)$, where $u(x,y) = e^x(x\cos y - y\sin y)$.

We need to find $v(x,y)$ such that $f(z)$ is analytic, by using the Cauchy-Riemann equations:

1. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
2. $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

First, find the partial derivatives of $u(x,y)$:
$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}[e^x(x\cos y - y\sin y)]$
Using product rule on e^x and $(x\cos y - y\sin y)$:
$\frac{\partial u}{\partial x} = e^x(x\cos y - y\sin y) + e^x(\cos y)$
$\frac{\partial u}{\partial x} = e^x(x\cos y - y\sin y + \cos y)$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}[e^x(x\cos y - y\sin y)]$
$\frac{\partial u}{\partial y} = e^x(-x\sin y - (\sin y + y\cos y))$
$\frac{\partial u}{\partial y} = e^x(-x\sin y - \sin y - y\cos y)$
$\frac{\partial u}{\partial y} = -e^x(x\sin y + \sin y + y\cos y)$

Now, use CR equations to find $v(x,y)$:

From CR equation 1: $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$
$\frac{\partial v}{\partial y} = e^x(x\cos y - y\sin y + \cos y)$

From CR equation 1: $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$
$\frac{\partial v}{\partial y} = e^x(x\cos y - y\sin y + \cos y)$
Integrate with respect to y (treating x as constant):
$v(x, y) = \int e^x(x\cos y - y\sin y + \cos y)dy$
$v(x, y) = e^x \int(x\cos y - y\sin y + \cos y)dy$
$v(x, y) = e^x\left[x\sin y - (\sin y - y\cos y) + \sin y\right] + g(x)$ (using integration by parts for $\int y\sin y\, dy = -y\cos y - \int(-\cos y)dy = -y\cos y + \sin y$)
$v(x, y) = e^x\left[x\sin y - \sin y + y\cos y + \sin y\right] + g(x)$
$v(x, y) = e^x(x\sin y + y\cos y) + g(x)$

Now, use CR equation 2: $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$
We already found $-\frac{\partial u}{\partial y} = e^x(x\sin y + \sin y + y\cos y)$.
Now, differentiate our derived $v(x, y)$ with respect to x:
$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}[e^x(x\sin y + y\cos y) + g(x)]$
$\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y) + e^x(\sin y) + g'(x)$
$\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y + \sin y) + g'(x)$

Equating the two expressions for $\frac{\partial v}{\partial x}$:
$e^x(x\sin y + y\cos y + \sin y) + g'(x) = e^x(x\sin y + \sin y + y\cos y)$
This implies $g'(x) = 0$, so $g(x) = C$, where C is a constant.

Thus, $v(x, y) = e^x(x\sin y + y\cos y) + C$.

The analytic function $f(z)$ is $u(x, y) + iv(x, y)$:
$f(z) = e^x(x\cos y - y\sin y) + i[e^x(x\sin y + y\cos y) + C]$
$f(z) = e^x(x\cos y - y\sin y + ix\sin y + iy\cos y) + iC$
$f(z) = e^x(x(\cos y + i\sin y) + y(-\sin y + i\cos y)) + iC$
Recall that $\cos y + i\sin y = e^{iy}$ and $-\sin y + i\cos y = i(\cos y + i\sin y) = ie^{iy}$.
$f(z) = e^x(xe^{iy} + yie^{iy}) + iC$
$f(z) = e^x e^{iy}(x + iy) + iC$
Since $e^x e^{iy} = e^{x+iy} = e^z$ and $x + iy = z$:
$f(z) = e^z \cdot z + iC$

Therefore, the analytic function is $f(z) = ze^z + C'$, where C' is an arbitrary complex constant.

8) Evaluate $\oint \frac{e^z}{(z-1)(z-4)} dz$ here c is the circle $|z| = 2$ using Residue theorem.

The integral is $\oint_C \frac{e^z}{(z-1)(z-4)} dz$, where C is the circle $|z| = 2$.
The function is $f(z) = \frac{e^z}{(z-1)(z-4)}$.
The singularities of $f(z)$ are the points where the denominator is zero, i.e., $z - 1 = 0 \implies z = 1$ and $z - 4 = 0 \implies z = 4$.

Now, we check which of these singularities lie inside the given contour $C : |z| = 2$.
For $z = 1$: $|1| = 1$. Since $1 < 2$, $z = 1$ lies inside the circle $|z| = 2$.
For $z = 4$: $|4| = 4$. Since $4 > 2$, $z = 4$ lies outside the circle $|z| = 2$.

So, only $z = 1$ is a pole inside the contour. It is a simple pole.

According to the Residue Theorem, $\oint_C f(z)dz = 2\pi i \times (\text{sum of residues of } f(z) \text{ inside } C)$.
Here, we only need to calculate the residue at $z = 1$.

For a simple pole at $z = a$, the residue is given by $Res_{z=a} f(z) = \lim_{z \to a}(z - a)f(z)$.

$$Res_{z=1} \frac{e^z}{(z-1)(z-4)} = \lim_{z \to 1}(z - 1) \frac{e^z}{(z-1)(z-4)}$$
$$= \lim_{z \to 1} \frac{e^z}{z-4}$$
$$= \frac{e^1}{1-4} = \frac{e}{-3} = -\frac{e}{3}$$

Now, apply the Residue Theorem:

$$\oint_C \frac{e^z}{(z-1)(z-4)} dz = 2\pi i \times \left(-\frac{e}{3}\right)$$
$$= -\frac{2\pi i e}{3}$$

Section C (Attempt any 2 questions - 15 marks each)

1) Solve the following equations with the help of matrix:

$x + y + z = 9$

$2x - 3y + 4z = 13$

$3x + 4y + 5z = 40$

This is a system of linear equations. We can solve this using matrix methods, such as Gaussian elimination (row echelon form) or Cramer's rule, or by finding the inverse of the coefficient matrix. Given the context of "help of matrix," Gaussian elimination or Cramer's rule are common and detailed approaches. I'll use Gaussian elimination to reduce the augmented matrix to row echelon form and then back-substitute.

Step 1: Write the system of equations in augmented matrix form.

The coefficients of x, y, z form the coefficient matrix, and the constants form the augmented part.

$$A|B = \begin{pmatrix} 1 & 1 & 1 & | & 9 \\ 2 & -3 & 4 & | & 13 \\ 3 & 4 & 5 & | & 40 \end{pmatrix}$$

Step 2: Apply elementary row operations to transform the augmented matrix into row echelon form.

- **Make the (2,1) element zero:** $R_2 \to R_2 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2-2(1) & -3-2(1) & 4-2(1) & 13-2(9) \\ 3 & 4 & 5 & 40 \end{array}\right)$$
$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 3 & 4 & 5 & 40 \end{array}\right)$$

- **Make the (3,1) element zero:** $R_3 \to R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 3-3(1) & 4-3(1) & 5-3(1) & 40-3(9) \end{array}\right)$$
$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right)$$

- **Make the (2,2) element 1 (optional, can do later, or swap R2 and R3):** Let's swap R_2 and R_3 to get a 1 in the (2,2) position, which is usually simpler for further steps.

$R_2 \leftrightarrow R_3$
$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & -5 & 2 & -5 \end{array}\right)$$

- **Make the (3,2) element zero:** $R_3 \to R_3 + 5R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0+5(0) & -5+5(1) & 2+5(2) & -5+5(13) \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 12 & 60 \end{array}\right)$$

- **Make the (3,3) element 1:** $R_3 \to \frac{1}{12} R_3$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

Step 3: Convert the row echelon form back into a system of linear equations and solve by back-substitution.

From the last row:

$0x + 0y + 1z = 5 \implies z = 5$

From the second row:

$0x + 1y + 2z = 13$

$y + 2(5) = 13$

$y + 10 = 13$

$y = 13 - 10 \implies y = 3$

From the first row:

$1x + 1y + 1z = 9$

$x + 3 + 5 = 9$

$x + 8 = 9$

$x = 9 - 8 \implies x = 1$

Step 4: Verify the solution.

Substitute $x = 1, y = 3, z = 5$ into the original equations:

1. $1 + 3 + 5 = 9 \implies 9 = 9$ (True)
2. $2(1) - 3(3) + 4(5) = 2 - 9 + 20 = -7 + 20 = 13 \implies 13 = 13$ (True)
3. $3(1) + 4(3) + 5(5) = 3 + 12 + 25 = 15 + 25 = 40 \implies 40 = 40$ (True)

All equations are satisfied.

The solution to the system of equations is $x = 1, y = 3, z = 5$.

Section C (Attempt any 2 questions - 15 marks each)

2) Find the residues of $\frac{z}{(z-1)^4(z-2)(z-3)}$ at its Pole.

The function is $f(z) = \frac{z}{(z-1)^4(z-2)(z-3)}$.

The poles are the values of z that make the denominator zero:

- $z-1=0 \implies z=1$. This factor is raised to the power of 4, so $z=1$ is a pole of order **4**.
- $z-2=0 \implies z=2$. This is a simple pole (order 1).
- $z-3=0 \implies z=3$. This is a simple pole (order 1).

Now, we calculate the residue at each pole.

Residue at $z=1$ (Pole of order 4):

For a pole of order m at $z=a$, the residue is given by the formula:

$$Res_{z=a}f(z) = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}[(z-a)^m f(z)]$$

Here, $a=1$ and $m=4$. So we need to calculate:

$$Res_{z=1}f(z) = \frac{1}{(4-1)!}\lim_{z\to 1}\frac{d^{4-1}}{dz^{4-1}}[(z-1)^4\frac{z}{(z-1)^4(z-2)(z-3)}]$$

$$= \frac{1}{3!}\lim_{z\to 1}\frac{d^3}{dz^3}\left[\frac{z}{(z-2)(z-3)}\right]$$

$$= \frac{1}{6}\lim_{z\to 1}\frac{d^3}{dz^3}\left[\frac{z}{z^2-5z+6}\right]$$

Let $g(z) = \frac{z}{z^2-5z+6}$. We need to find its third derivative.
We previously found the first derivative:
$g'(z) = \frac{-z^2+6}{(z^2-5z+6)^2}$

Now for the second derivative $g''(z)$:
Using the quotient rule: $\frac{u'v-uv'}{v^2}$
$u = -z^2+6 \implies u' = -2z$
$v = (z^2-5z+6)^2 \implies v' = 2(z^2-5z+6)(2z-5)$

$g''(z) = \frac{(-2z)(z^2-5z+6)^2-(-z^2+6)[2(z^2-5z+6)(2z-5)]}{((z^2-5z+6)^2)^2}$
$g''(z) = \frac{-2z(z^2-5z+6)-2(-z^2+6)(2z-5)}{(z^2-5z+6)^3}$ (Canceled one term of (z^2-5z+6) from numerator and denominator)
$g''(z) = \frac{-2z^3+10z^2-12z-2(-2z^3+5z^2+12z-30)}{(z^2-5z+6)^3}$
$g''(z) = \frac{-2z^3+10z^2-12z+4z^3-10z^2-24z+60}{(z^2-5z+6)^3}$
$g''(z) = \frac{2z^3-36z+60}{(z^2-5z+6)^3}$
$g''(z) = \frac{2(z^3-18z+30)}{(z^2-5z+6)^3}$

Now for the third derivative $g'''(z)$:
Let $h(z) = z^3-18z+30 \implies h'(z) = 3z^2-18$
Let $k(z) = (z^2-5z+6)^3 \implies k'(z) = 3(z^2-5z+6)^2(2z-5)$

$g'''(z) = 2\left[\frac{(3z^2-18)(z^2-5z+6)^3-(z^3-18z+30)[3(z^2-5z+6)^2(2z-5)]}{((z^2-5z+6)^3)^2}\right]$
$g'''(z) = 2\left[\frac{(3z^2-18)(z^2-5z+6)-3(z^3-18z+30)(2z-5)}{(z^2-5z+6)^4}\right]$ (Canceled one term of $(z^2-5z+6)^2$)

This expression is quite complex. Let's try evaluating $g'''(z)$ at $z = 1$ directly rather than fully expanding the numerator.
Let $D(z) = z^2 - 5z + 6$.
At $z = 1$, $D(1) = 1 - 5 + 6 = 2$.

The expression for $Res_{z=1} f(z)$ is $\frac{1}{6} \lim_{z \to 1} \frac{d^3}{dz^3} \left[\frac{z}{D(z)} \right]$.
Consider Taylor series expansion of $\frac{z}{(z-2)(z-3)}$ around $z = 1$.
Let $f(z) = \frac{z}{(z-2)(z-3)}$.
$f(z) = \frac{z}{(z-1-1)(z-1-2)} = \frac{z}{(w-1)(w-2)}$ where $w = z - 1$.
This approach might be more calculation-intensive.

Let's stick with the derivative.

$$g'''(1) = 2 \left[\frac{(3(1)^2-18)((1)^2-5(1)+6)-3((1)^3-18(1)+30)(2(1)-5)}{((1)^2-5(1)+6)^4} \right]$$
$$g'''(1) = 2 \left[\frac{(3-18)(1-5+6)-3(1-18+30)(2-5)}{(1-5+6)^4} \right]$$
$$g'''(1) = 2 \left[\frac{(-15)(2)-3(13)(-3)}{(2)^4} \right]$$
$$g'''(1) = 2 \left[\frac{-30-3(-39)}{16} \right]$$
$$g'''(1) = 2 \left[\frac{-30+117}{16} \right]$$
$$g'''(1) = 2 \left[\frac{87}{16} \right]$$
$$g'''(1) = \frac{87}{8}$$

So, $Res_{z=1}f(z) = \frac{1}{6} \cdot \frac{87}{8}$
$= \frac{87}{48}$
Divide by 3: $87/3 = 29, 48/3 = 16.$
$Res_{z=1}f(z) = \frac{29}{16}$

Residue at $z = 2$ (Simple pole):
$Res_{z=2}f(z) = \lim_{z \to 2}(z-2)\frac{z}{(z-1)^4(z-2)(z-3)}$
$= \lim_{z \to 2} \frac{z}{(z-1)^4(z-3)}$
Substitute $z = 2$:
$= \frac{2}{(2-1)^4(2-3)}$
$= \frac{2}{(1)^4(-1)}$
$= \frac{2}{-1} = -2$

Residue at $z = 3$ (Simple pole):
$Res_{z=3}f(z) = \lim_{z \to 3}(z-3)\frac{z}{(z-1)^4(z-2)(z-3)}$
$= \lim_{z \to 3} \frac{z}{(z-1)^4(z-2)}$
Substitute $z = 3$:
$= \frac{3}{(3-1)^4(3-2)}$
$= \frac{3}{(2)^4(1)}$
$= \frac{3}{16}$

Summary of Residues (Corrected):

- Residue at $z = 1$ is $\frac{29}{16}$.
- Residue at $z = 2$ is -2.
- Residue at $z = 3$ is $\frac{3}{16}$.

The given differential equation is:
$(2x-1)^3\frac{d^3y}{dx^3}+(2x-1)\frac{dy}{dx}-2y=\log(2x-1)$

This is a Cauchy-Euler type differential equation. We can transform it into a linear differential equation with constant coefficients by using a suitable substitution.

Let $2x-1=e^t$.
Then $t=\log(2x-1)$.

Now, we need to express the derivatives in terms of t.

First, let's find $\frac{dy}{dx}$:
$\frac{dy}{dx}=\frac{dy}{dt}\frac{dt}{dx}$
$\frac{dt}{dx}=\frac{d}{dx}(\log(2x-1))=\frac{1}{2x-1}\cdot 2=\frac{2}{2x-1}$
So, $\frac{dy}{dx}=\frac{2}{2x-1}\frac{dy}{dt}$
$(2x-1)\frac{dy}{dx}=2\frac{dy}{dt}$

Now, let's find $\frac{d^2y}{dx^2}$:
$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{2}{2x-1}\frac{dy}{dt}\right)$
Using the product rule:
$\frac{d^2y}{dx^2}=2\left[\frac{d}{dx}\left(\frac{1}{2x-1}\right)\frac{dy}{dt}+\frac{1}{2x-1}\frac{d}{dx}\left(\frac{dy}{dt}\right)\right]$
$\frac{d}{dx}\left(\frac{1}{2x-1}\right)=\frac{-1}{(2x-1)^2}\cdot 2=\frac{-2}{(2x-1)^2}$
$\frac{d}{dx}\left(\frac{dy}{dt}\right)=\frac{d}{dt}\left(\frac{dy}{dt}\right)\frac{dt}{dx}=\frac{d^2y}{dt^2}\frac{2}{2x-1}$
So, $\frac{d^2y}{dx^2}=2\left[\frac{-2}{(2x-1)^2}\frac{dy}{dt}+\frac{1}{2x-1}\left(\frac{d^2y}{dt^2}\frac{2}{2x-1}\right)\right]$

So, $\frac{d^2y}{dx^2} = 2\left[\frac{-2}{(2x-1)^2}\frac{dy}{dt} + \frac{1}{2x-1}\left(\frac{d^2y}{dt^2}\frac{2}{2x-1}\right)\right]$

$$\frac{d^2y}{dx^2} = 2\left[\frac{-2}{(2x-1)^2}\frac{dy}{dt} + \frac{2}{(2x-1)^2}\frac{d^2y}{dt^2}\right]$$
$$\frac{d^2y}{dx^2} = \frac{4}{(2x-1)^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$
$$(2x-1)^2\frac{d^2y}{dx^2} = 4\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

Now, let's find $\frac{d^3y}{dx^3}$:

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{4}{(2x-1)^2}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right]$$
$$\frac{d^3y}{dx^3} = 4\left[\frac{d}{dx}\left(\frac{1}{(2x-1)^2}\right)\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \frac{1}{(2x-1)^2}\frac{d}{dx}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right]$$
$$\frac{d}{dx}\left(\frac{1}{(2x-1)^2}\right) = \frac{-2}{(2x-1)^3}\cdot 2 = \frac{-4}{(2x-1)^3}$$
$$\frac{d}{dx}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) = \left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\frac{dt}{dx} = \left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\frac{2}{2x-1}$$

So, $\frac{d^3y}{dx^3} = 4\left[\frac{-4}{(2x-1)^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \frac{1}{(2x-1)^2}\left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\frac{2}{2x-1}\right]$

$$\frac{d^3y}{dx^3} = 4\left[\frac{-4}{(2x-1)^3}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + \frac{2}{(2x-1)^3}\left(\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2}\right)\right]$$
$$\frac{d^3y}{dx^3} = \frac{8}{(2x-1)^3}\left[\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} - 2\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\right]$$
$$\frac{d^3y}{dx^3} = \frac{8}{(2x-1)^3}\left[\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right]$$
$$(2x-1)^3\frac{d^3y}{dx^3} = 8\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right)$$

Now, substitute these back into the original differential equation:

$$8\left(\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right) + 2\frac{dy}{dt} - 2y = t$$
$$8\frac{d^3y}{dt^3} - 24\frac{d^2y}{dt^2} + 16\frac{dy}{dt} + 2\frac{dy}{dt} - 2y = t$$
$$8\frac{d^3y}{dt^3} - 24\frac{d^2y}{dt^2} + 18\frac{dy}{dt} - 2y = t$$

Let $D = \frac{d}{dt}$. The equation becomes:
$(8D^3 - 24D^2 + 18D - 2)y = t$

We need to find the complementary function (y_c) and the particular integral (y_p).

1. Complementary Function (y_c)
The characteristic equation is:
$8m^3 - 24m^2 + 18m - 2 = 0$
Divide by 2:
$4m^3 - 12m^2 + 9m - 1 = 0$

Let's test for rational roots using the Rational Root Theorem. Possible rational roots are $\pm 1, \pm 1/2, \pm 1/4$.
Test $m = 1$: $4(1)^3 - 12(1)^2 + 9(1) - 1 = 4 - 12 + 9 - 1 = 0$. So $m = 1$ is a root.
This means $(m - 1)$ is a factor. We can perform polynomial division or synthetic division.

Using synthetic division with root 1:

```
1 | 4  -12   9  -1
  |     4  -8   1
  ----------------
    4  -8   1   0
```

The remaining quadratic equation is $4m^2 - 8m + 1 = 0$.
Use the quadratic formula: $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$
$$m = \frac{8 \pm \sqrt{64 - 16}}{8}$$
$$m = \frac{8 \pm \sqrt{48}}{8}$$
$$m = \frac{8 \pm 4\sqrt{3}}{8}$$
$$m = 1 \pm \frac{\sqrt{3}}{2}$$

So the roots are $m_1 = 1, m_2 = 1 + \frac{\sqrt{3}}{2}, m_3 = 1 - \frac{\sqrt{3}}{2}$.

The complementary function is:

$$y_c = C_1 e^t + C_2 e^{(1+\frac{\sqrt{3}}{2})t} + C_3 e^{(1-\frac{\sqrt{3}}{2})t}$$
$$y_c = C_1 e^t + C_2 e^t e^{\frac{\sqrt{3}}{2}t} + C_3 e^t e^{-\frac{\sqrt{3}}{2}t}$$
$$y_c = e^t \left(C_1 + C_2 e^{\frac{\sqrt{3}}{2}t} + C_3 e^{-\frac{\sqrt{3}}{2}t} \right)$$

2. Particular Integral (y_p)

$$y_p = \frac{1}{8D^3 - 24D^2 + 18D - 2} t$$

Since the right-hand side is a polynomial in t, we can use the method of undetermined coefficients or series expansion.

Let's try to expand the operator.

$$y_p = \frac{1}{-2 + 18D - 24D^2 + 8D^3} t$$
$$y_p = \frac{1}{-2(1 - 9D + 12D^2 - 4D^3)} t$$
$$y_p = -\frac{1}{2}(1 - (9D - 12D^2 + 4D^3))^{-1} t$$

$y_p = -\frac{1}{2}(1 - (9D - 12D^2 + 4D^3))^{-1}t$

Using the binomial expansion $(1-x)^{-1} = 1 + x + x^2 + ...$

Here $x = 9D - 12D^2 + 4D^3$.

Since we are operating on t (a polynomial of degree 1), we only need terms up to D^1. Higher order derivatives of t will be zero.

$y_p = -\frac{1}{2}(1 + (9D - 12D^2 + 4D^3) + ...)t$

$y_p = -\frac{1}{2}(t + 9D(t) - 12D^2(t) + 4D^3(t) + ...)$

$D(t) = 1$

$D^2(t) = 0$

$D^3(t) = 0$

So, $y_p = -\frac{1}{2}(t + 9(1) - 0 + 0)$

$y_p = -\frac{1}{2}(t + 9)$

3. General Solution

The general solution is $y = y_c + y_p$.

$$y = e^t \left(C_1 + C_2 e^{\frac{\sqrt{3}}{2}t} + C_3 e^{-\frac{\sqrt{3}}{2}t}\right) - \frac{1}{2}(t + 9)$$

Now, substitute back $t = \log(2x - 1)$ and $e^t = 2x - 1$.

$e^{\frac{\sqrt{3}}{2}t} = e^{t\frac{\sqrt{3}}{2}} = (e^t)^{\frac{\sqrt{3}}{2}} = (2x-1)^{\frac{\sqrt{3}}{2}}$

$e^{-\frac{\sqrt{3}}{2}t} = (2x-1)^{-\frac{\sqrt{3}}{2}}$

So, the solution in terms of x is:

$$y = (2x-1)\left(C_1 + C_2(2x-1)^{\frac{\sqrt{3}}{2}} + C_3(2x-1)^{-\frac{\sqrt{3}}{2}}\right) - \frac{1}{2}(\log(2x-1) + 9)$$

So, the solution in terms of x is:

$$y = (2x - 1)\left(C_1 + C_2(2x-1)^{\frac{\sqrt{3}}{2}} + C_3(2x-1)^{-\frac{\sqrt{3}}{2}}\right) - \frac{1}{2}(\log(2x-1) + 9)$$

This can be further simplified as:

$$y = C_1(2x-1) + C_2(2x-1)^{1+\frac{\sqrt{3}}{2}} + C_3(2x-1)^{1-\frac{\sqrt{3}}{2}} - \frac{1}{2}\log(2x-1) - \frac{9}{2}$$

This is the final solution to the given differential equation.