Lesson 5
Inferential Statistics

Mathematics and Statistics for Data Science

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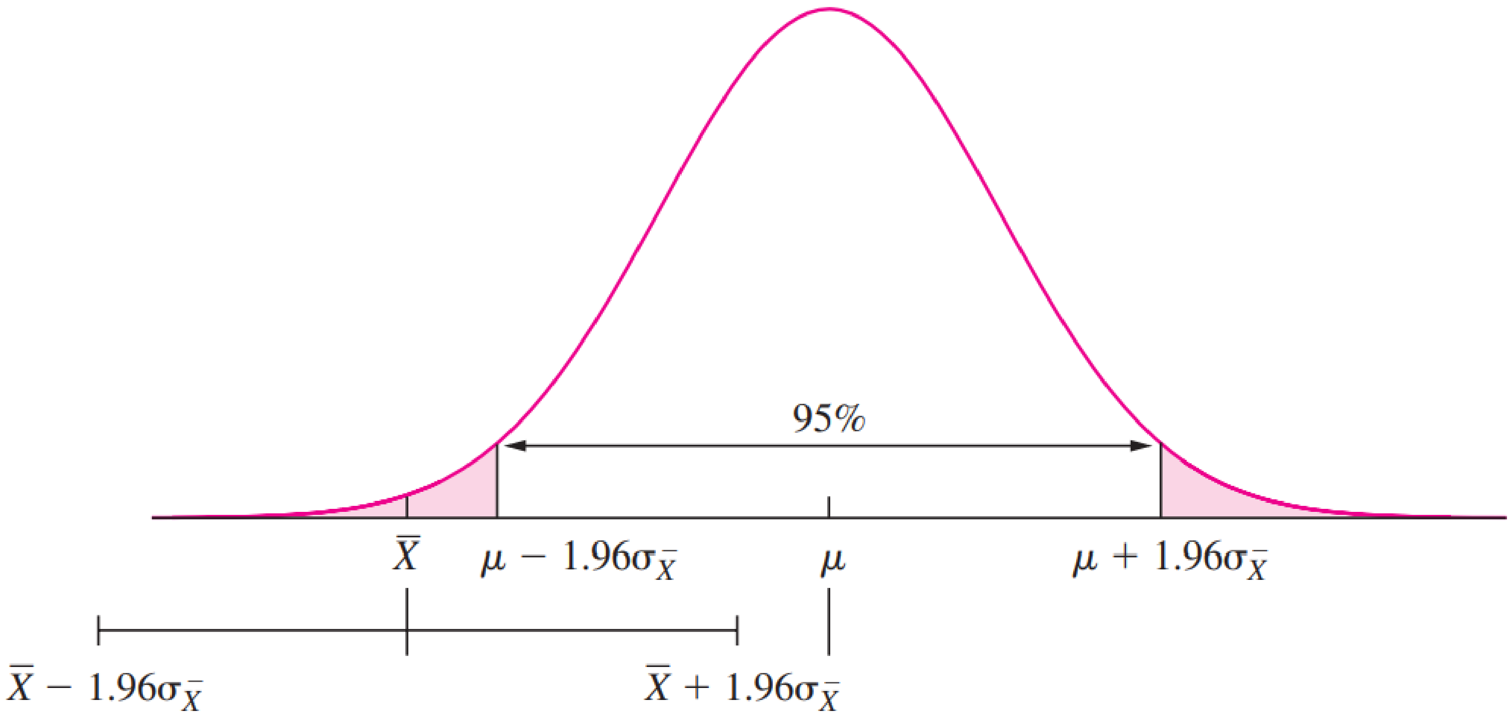
Content

- Confidence intervals
- Hypothesis testing & statistical significance

Point Estimators

- In general, a quantity calculated from data is called a statistic.
- A statistic that is used to estimate an unknown constant, or parameter, is called a point estimator or point estimate.
- For example, if $X_1, \ldots, X_n$ is a random sample from a population,
  - The sample mean X is often used to estimate the population mean μ, and
  - The sample variance s^2 is often used to estimate the population variance σ^2
- Big questions:
  - How to decide whether an estimator is good?
  - How to construct a good point estimator?

Interval Estimators



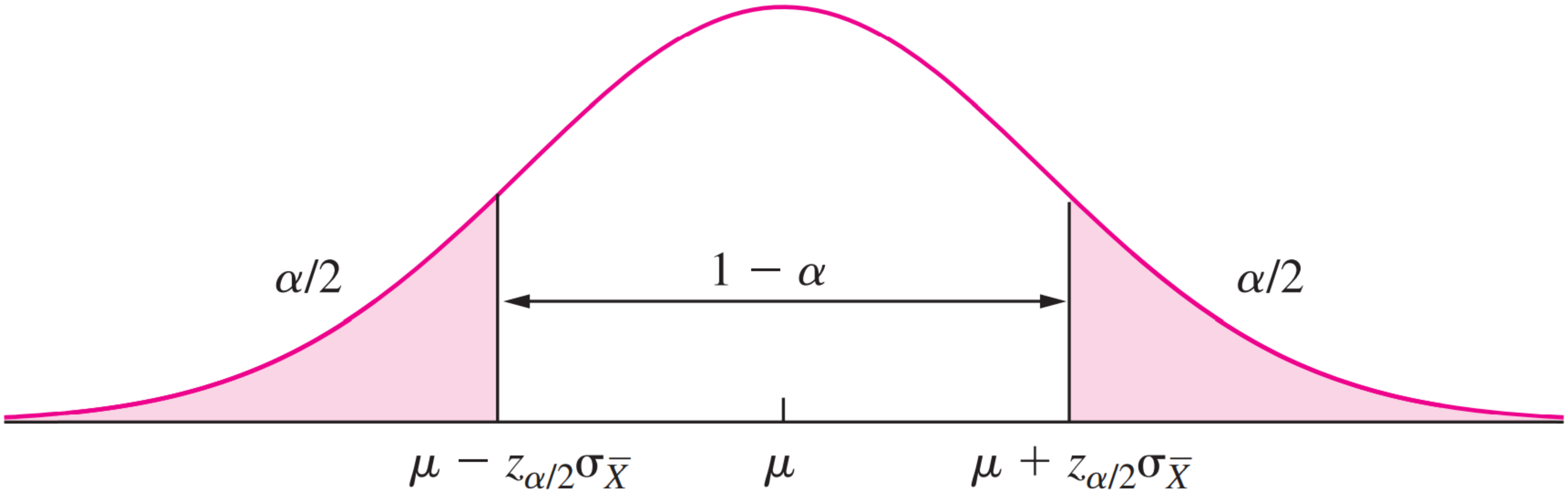
Confidence Interval

- Let $X_1, \ldots, X_n$ be a large ($n > 30$) random sample from a population with mean μ and standard deviation σ, so that X is approximately normal. Then a level $100(1 - \alpha)\%$ confidence interval for μ is

$$\overline{X} \pm z_{\alpha/2}\sigma_{\overline{X}}$$

  - where $\sigma_X = \sigma/\sqrt{n}$. When the value of σ is unknown, it can be replaced with the sample standard deviation s.

Confidence Interval - Illustration



Levels & Significances

$$\overline{X} \pm \frac{s}{\sqrt{n}} \text{ is a 68\% confidence interval for } \mu.$$

$$\overline{X} \pm 1.645\frac{s}{\sqrt{n}} \text{ is a 90\% confidence interval for } \mu.$$

$$\overline{X} \pm 1.96\frac{s}{\sqrt{n}} \text{ is a 95\% confidence interval for } \mu.$$

$$\overline{X} \pm 2.58\frac{s}{\sqrt{n}} \text{ is a 99\% confidence interval for } \mu.$$

$$\overline{X} \pm 3\frac{s}{\sqrt{n}} \text{ is a 99.7\% confidence interval for } \mu.$$

Exercise 1

- The sample mean and standard deviation for the fill weights of 100 boxes are X = 12.05 and s = 0.1. Find an 95% confidence interval for the mean fill weight of the boxes.

Exercise 2

- In a random sample of 53 concrete specimens, the average porosity was 21.6% and the standard deviation was 3.2%.
  - Find a 90% confidence interval for the mean porosity of specimens of this type of concrete.
  - Find a 95% confidence interval for the mean porosity of specimens of this type of concrete.
  - What is the confidence level of the interval(21.0,22.2)?
  - How many specimens must be sampled so that a 95% confidence interval specifies the mean to within ±0.3?

Hypothesis Testing

- A test to determine how certain we can be about a *hypothesis*.
- A hypothesis can be about:
  - A population mean μ
  - A population proportion p
  - A difference between two means or proportions
  - Paired data
  - Chi-square test
  - Variances

Exercise 3

- A certain type of automobile engine emits a mean of 100 mg of NOx per second. A modification to the engine design has been proposed that may reduce NOx emissions.
- The new design will be put into production if it can be demonstrated that its mean emission rate is less than 100 mg/s.
- A sample of 50 modified engines are tested. The sample mean NOx emission is 92 mg/s, and the sample standard deviation is 21 mg/s.

- Are the modified engine design really reducing the NOx emission?

Null vs. Alternative Hypothesis

- Two possibilities:
  - The population mean is actually greater than or equal to 100, and the sample mean is lower only because of *random variation* from the population mean.
  - The population mean is *actually* less than 100, and the sample mean reflects this fact. (i.e. emission really reduced)
- Null hypothesis: The effect indicated by the sample is due only to random variation between the sample and the population.
- Alternative hypothesis: The effect indicated by the sample is real, in that it accurately represents the whole population.

Steps of Hypothesis Testing

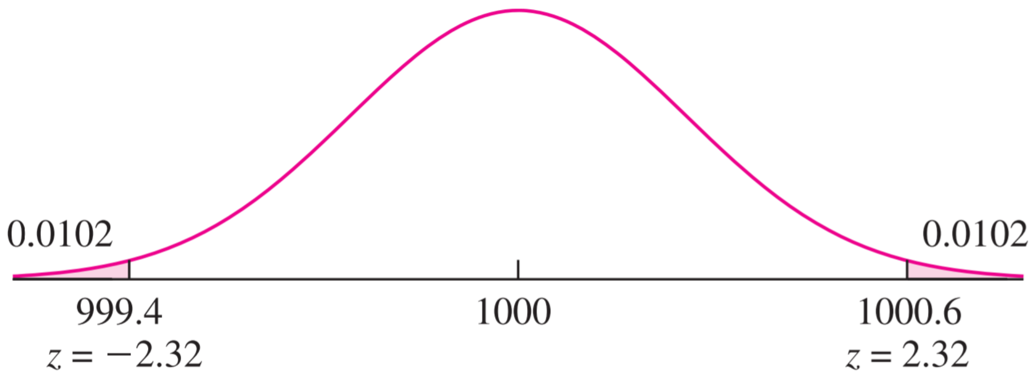
- Define null and alternative hypotheses, HO and H1.
- Assume HO to be true.
- Compute a test statistic. A test statistic is a statistic that is used to assess the strength of the evidence against HO.
- Compute the P-value of the test statistic. The P-value is the probability that the test statistic would have a value whose disagreement with HO is as great as or greater than that actually observed.
- Conclude about the strength of the evidence against HO.

Exercise 4

- A scale is to be calibrated by weighing a 1000 g test weight 60 times. The 60 scale readings have mean 1000.6 g and standard deviation 2 g.
- Find the P-value for testing $H0 : \mu = 1000$ versus $H1 : \mu \neq 1000$.

Exercise 4 - Solution

- A scale is to be calibrated by weighing a 1000 g test weight 60 times. The 60 scale readings have mean 1000.6 g and standard deviation 2 g.



$$\boxed{z = \frac{1000.6 - 1000}{0.258} = 2.32}$$

Exercise 5

- A certain type of stainless steel powder is supposed to have a mean particle diameter of μ=15 μm. A random sample of 87 particles had a mean diameter of 15.2 μm, with a standard deviation of 1.8 μm.
- Do you believe it is plausible that the mean diameter is 15 μm, or do you believe that it differs from 15 μm?