

WHERE SYNTAX MEETS SEMANTICS

A3.1 – Ambiguous Grammar [10 minutes]

Given the grammar

$G1 : \langle \textit{subexp} \rangle ::= \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \langle \textit{subexp} \rangle - \langle \textit{subexp} \rangle$

Create all possible parse trees for the following input:

$a - b - c$

$a - b - a - c$

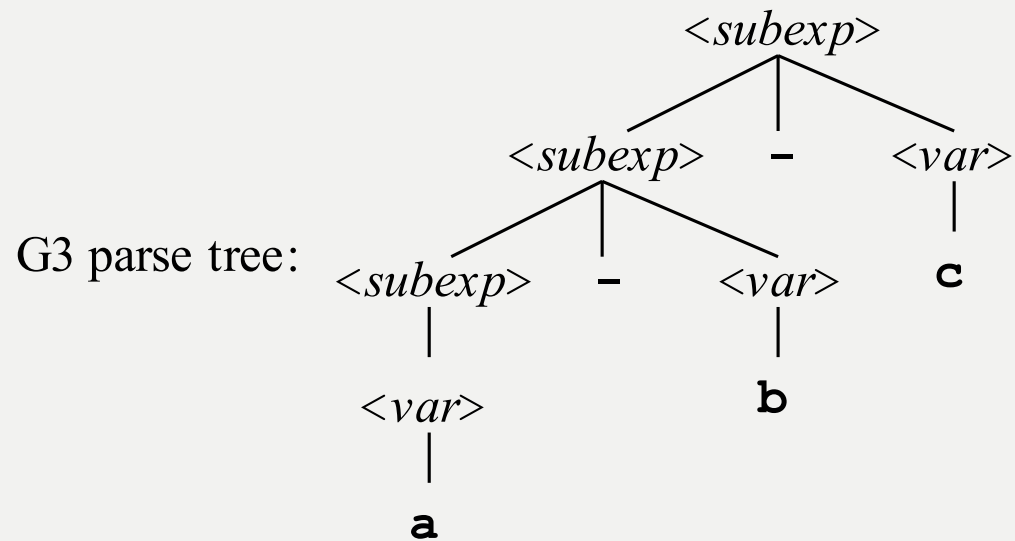
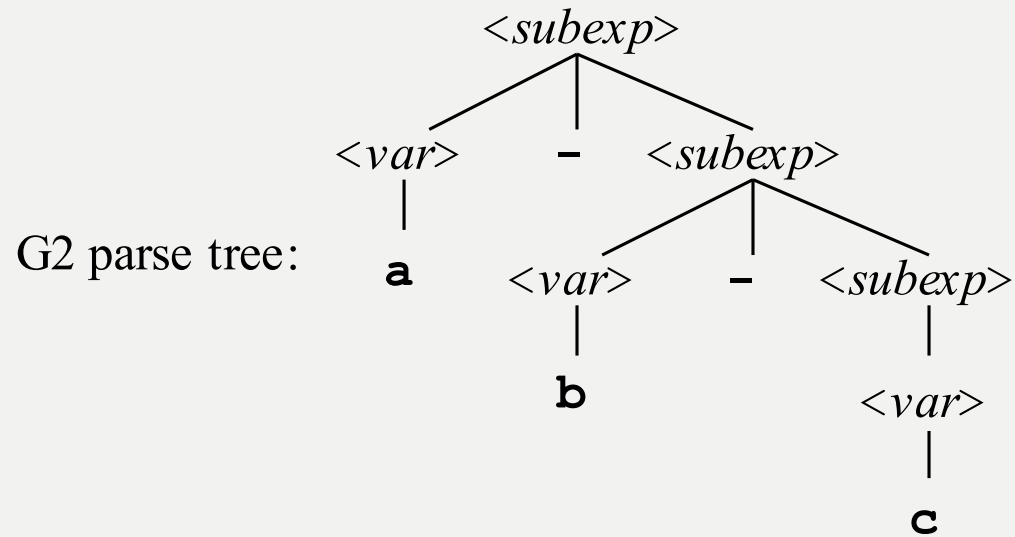
Three “Equivalent” Grammars

G1: $\langle \text{subexp} \rangle ::= \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \mid \langle \text{subexp} \rangle - \langle \text{subexp} \rangle$

G2: $\langle \text{subexp} \rangle ::= \langle \text{var} \rangle - \langle \text{subexp} \rangle \mid \langle \text{var} \rangle$
 $\langle \text{var} \rangle ::= \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

G3: $\langle \text{subexp} \rangle ::= \langle \text{subexp} \rangle - \langle \text{var} \rangle \mid \langle \text{var} \rangle$
 $\langle \text{var} \rangle ::= \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

These grammars all define the same language: the language of strings that contain one or more **as**, **bs** or **cs** separated by minus signs. But...



Why Parse Trees Matter

- We want the structure of the parse tree to correspond to the semantics of the string it generates
- This makes grammar design much harder: we're interested in the structure of each parse tree, not just in the generated string
- Parse trees are where syntax meets semantics

Outline

- Operators
- Precedence
- Associativity
- Other ambiguities: dangling else
- Cluttered grammars
- Parse trees and EBNF
- Abstract syntax trees

Operators

- Special syntax for frequently-used simple operations like addition, subtraction, multiplication and division
- The word *operator* refers both to the token used to specify the operation (like $+$ and $*$) and to the operation itself
- Usually predefined, but not always
- Usually a single token, but not always

Operator Terminology

- *Operands* are the inputs to an operator, like **1** and **2** in the expression **1+2**
- *Unary* operators take one operand: **-1**
- *Binary* operators take two: **1+2**
- *Ternary* operators take three: **a?b:c**

More Operator Terminology

- In most programming languages, binary operators use an *infix* notation: **a + b**
- Sometimes you see *prefix* notation: **+ a b**
- Sometimes *postfix* notation: **a b +**
- Unary operators, similarly:
 - (Can't be *infix*, of course)
 - Can be *prefix*, as in **-1**
 - Can be *postfix*, as in **a++**

A3.2 – Prefix/Postfix Notations [5 minutes]

Convert the following expressions into prefix and postfix notations:

$$5 - 3 - 7$$

$$6 + 3 * 4 / 2$$

$$9 * (6 + 7 * 2) / 3$$

PRECEDENCE



A3.3 – Precedence in Grammar [10 minutes]

Given the grammar

G4: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle$
 $| \langle \text{exp} \rangle * \langle \text{exp} \rangle$
 $| (\langle \text{exp} \rangle)$
 $| \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

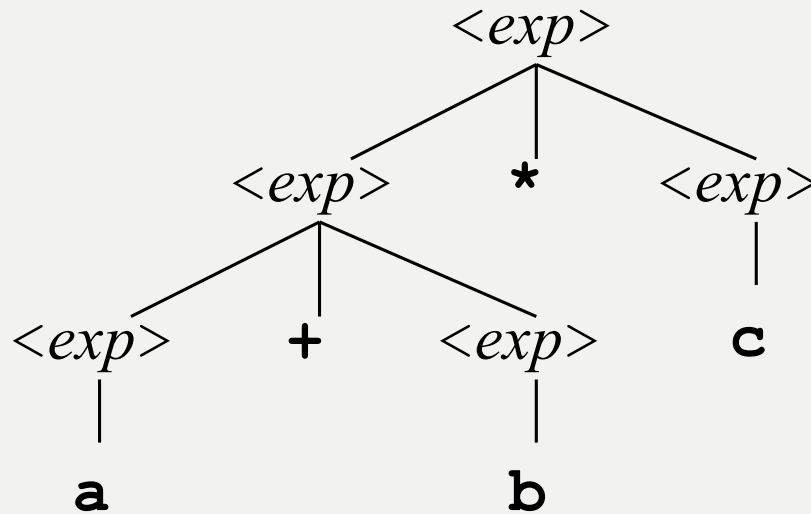
Revise the grammar so that $*$ will be evaluated before $+$

Working Grammar

$$\begin{aligned} \text{G4: } \langle \text{exp} \rangle &::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \\ &| \langle \text{exp} \rangle * \langle \text{exp} \rangle \\ &| (\langle \text{exp} \rangle) \\ &| \mathbf{a} \mid \mathbf{b} \mid \mathbf{c} \end{aligned}$$

This generates a language of arithmetic expressions using parentheses, the operators **+** and *****, and the variables **a**, **b** and **c**

Issue #1: Precedence



Our grammar generates this tree for **a+b*c**. In this tree, the addition is performed before the multiplication, which is not the usual convention for operator *precedence*.

Operator Precedence

- Applies when the order of evaluation is not completely decided by parentheses
- Each operator has a *precedence level*, and those with higher precedence are performed before those with lower precedence, as if parenthesized
- Most languages put ***** at a higher precedence level than **+**, so that

$$\mathbf{a+b*c = a+(b*c)}$$

Precedence Examples

- C (15 levels of precedence—too many?)

`a = b < c ? * p + b * c : 1 << d ()`

- Pascal (5 levels—not enough?)

`a <= 0 or 100 <= a` Error!

- Smalltalk (1 level for all binary operators)

`a + b * c`

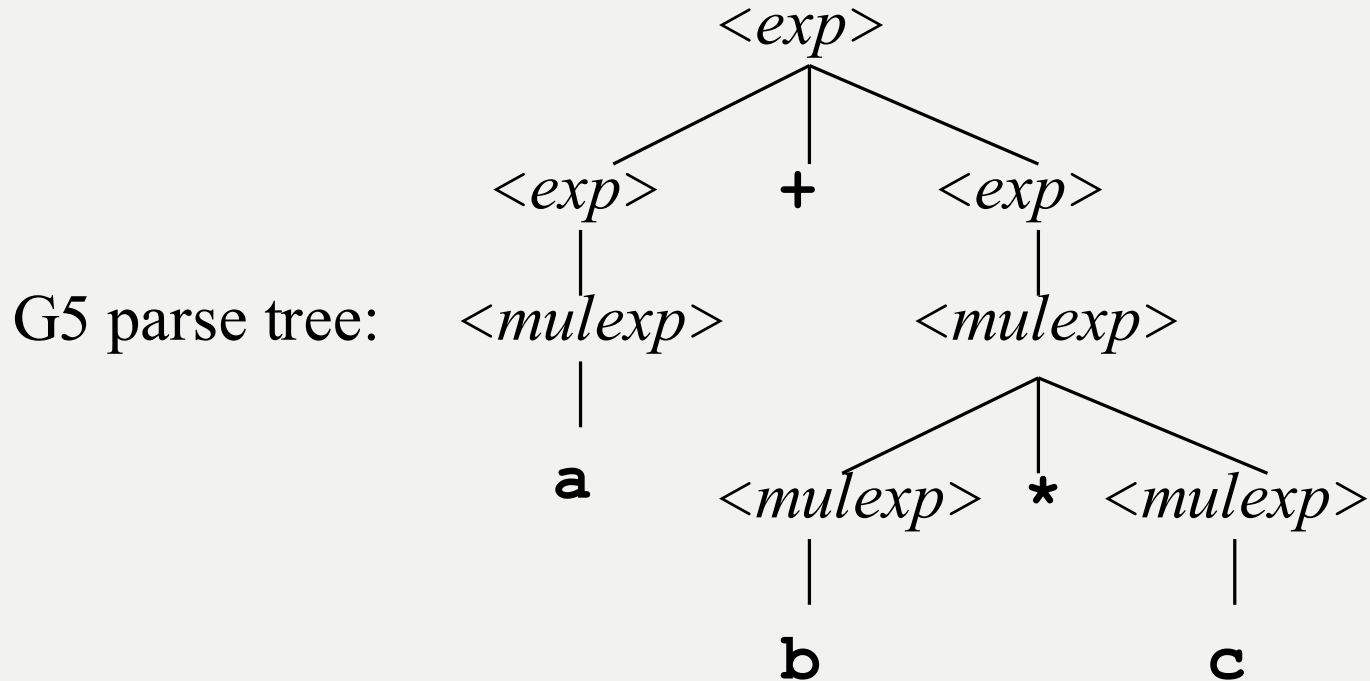
Precedence In The Grammar

G4: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle$
 | $\langle \text{exp} \rangle * \langle \text{exp} \rangle$
 | $(\langle \text{exp} \rangle)$
 | **a** | **b** | **c**

To fix the precedence problem, we modify the grammar so that it is forced to put ***** below **+** in the parse tree.

G5: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{mulexp} \rangle$
 | $(\langle \text{exp} \rangle)$
 | **a** | **b** | **c**

Correct Precedence

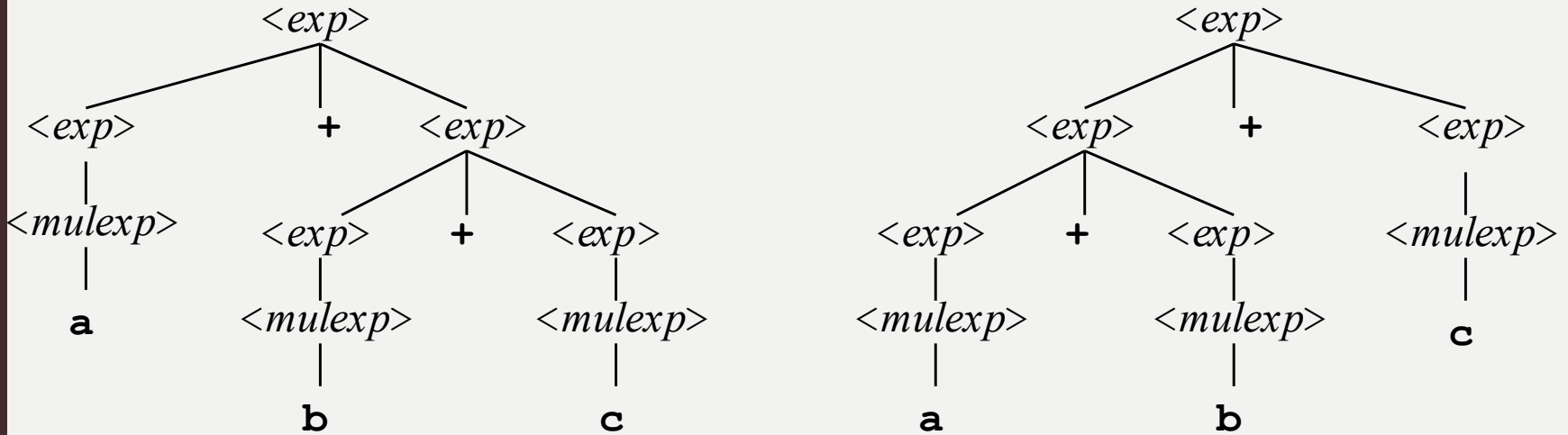


Our new grammar generates this tree for **a+b*c**. It generates the same language as before, but no longer generates parse trees with incorrect precedence.

ASSOCIATIVITY



Issue #2: Associativity



Our grammar G5 generates both these trees for **a+b+c**.
The first one is not the usual convention for operator *associativity*.

Operator Associativity

- Applies when the order of evaluation is not decided by parentheses or by precedence
- *Left-associative* operators group left to right: **$a+b+c+d = ((a+b)+c)+d$**
- *Right-associative* operators group right to left: **$a+b+c+d = a+(b+(c+d))$**
- Most operators in most languages are left-associative, but there are exceptions

Associativity Examples

- C
- ML
- Fortran

a<<b<<c — most operators are left-associative
a=b=0 — right-associative (assignment)

3-2-1 — most operators are left-associative
1::2::nil — right-associative (list builder)

a/b*c — most operators are left-associative
ab**c** — right-associative (exponentiation)

A3.4 – Associativity in Grammar [10 minutes]

Given the grammar

G5: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{mulexp} \rangle$
 $\mid (\langle \text{exp} \rangle)$
 $\mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

Revise the grammar so that + and * are left associative

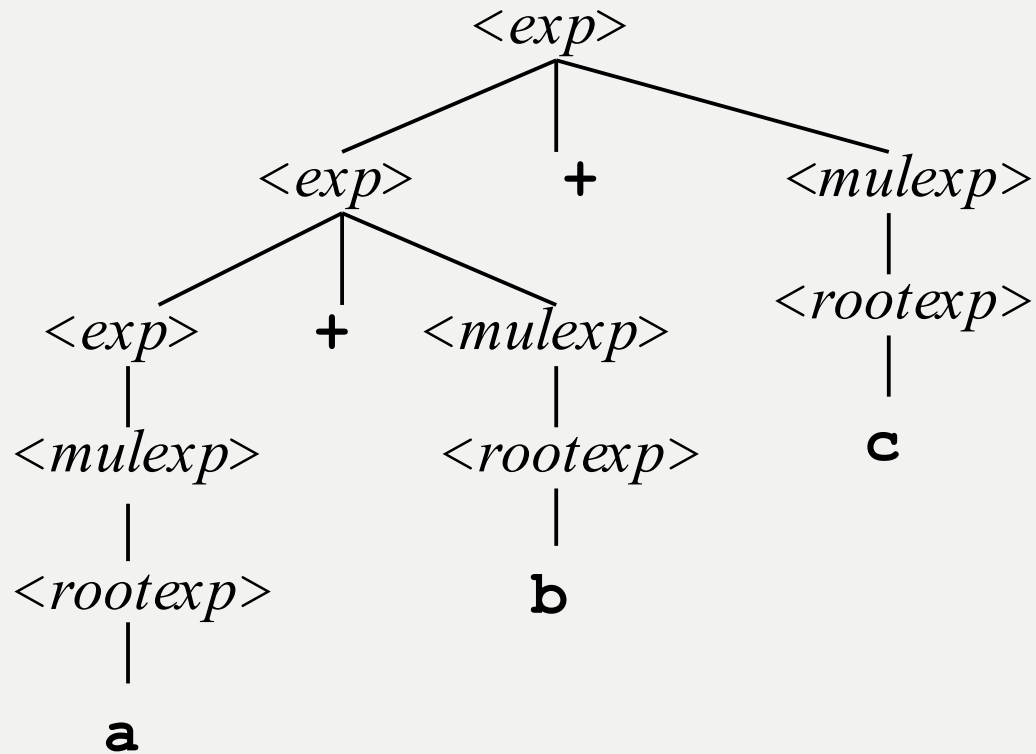
Associativity In The Grammar

G5: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{mulexp} \rangle$
 $\quad \mid (\langle \text{exp} \rangle)$
 $\quad \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

To fix the associativity problem, we modify the grammar to make trees of **+**s grow down to the left (and likewise for *****s)

G6: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{mulexp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{rootexp} \rangle \mid \langle \text{rootexp} \rangle$
 $\langle \text{rootexp} \rangle ::= (\langle \text{exp} \rangle)$
 $\quad \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

Correct Associativity



Our new grammar generates this tree for **a+b+c**. It generates the same language as before, but no longer generates trees with incorrect associativity.

A3.5 – Associativity in Grammar [10 minutes]

Starting with this grammar:

G6: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{mulexp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{rootexp} \rangle \mid \langle \text{rootexp} \rangle$
 $\langle \text{rootexp} \rangle ::= (\langle \text{exp} \rangle)$
 $\mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

- 1.) Add a left-associative **&** operator, at lower precedence than any of the others
- 2.) Then add a right-associative ****** operator, at higher precedence than any of the others

G6: $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{mulexp} \rangle \mid \langle \text{mulexp} \rangle$
 $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{rootexp} \rangle \mid \langle \text{rootexp} \rangle$
 $\langle \text{rootexp} \rangle ::= (\langle \text{exp} \rangle)$
 $\mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{c}$

DANGLING ELSE

Other Ambiguities

Issue #3: Ambiguity

- G4 was *ambiguous*: it generated more than one parse tree for the same string
- Fixing the associativity and precedence problems eliminated all the ambiguity
- This is usually a good thing: the parse tree corresponds to the meaning of the program, and we don't want ambiguity about that
- Not all ambiguity stems from confusion about precedence and associativity...

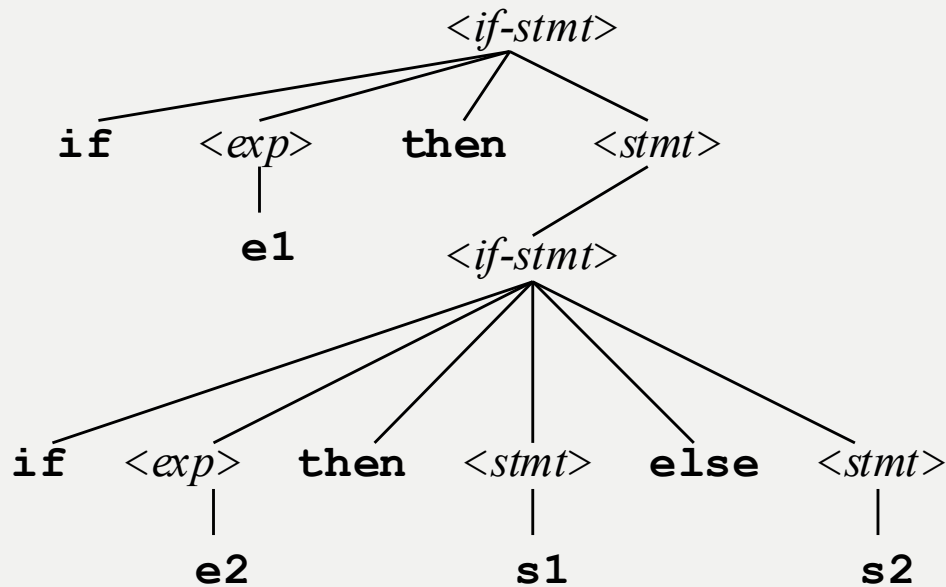
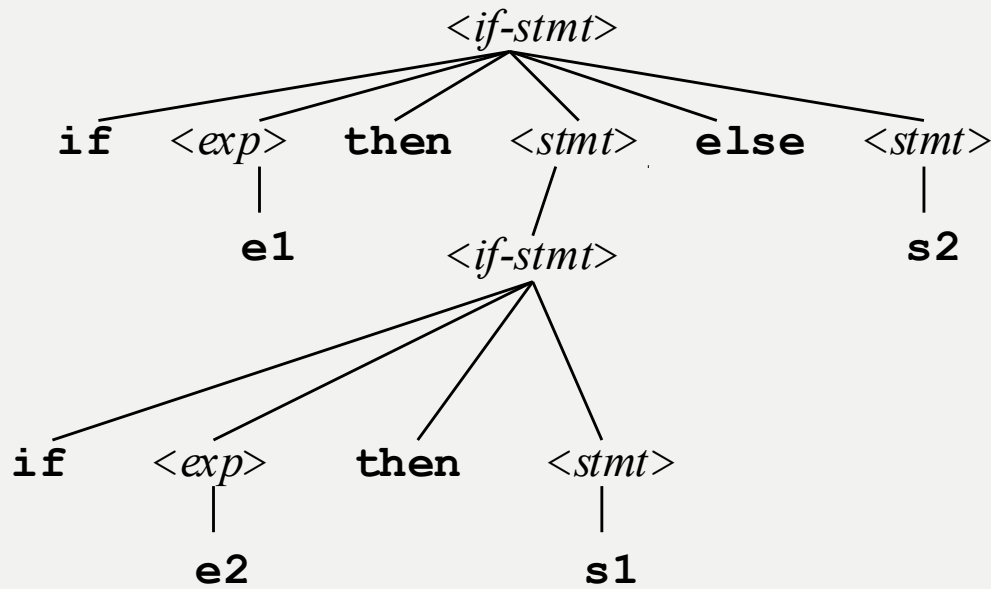
Dangling Else In Grammars

```
<stmt> ::= <if-stmt> | s1 | s2  
<if-stmt> ::= if <expr> then <stmt> else <stmt>  
           | if <expr> then <stmt>  
<expr> ::= e1 | e2
```

This grammar has a classic “dangling-else ambiguity.” The statement we want derive is

```
if e1 then if e2 then s1 else s2
```

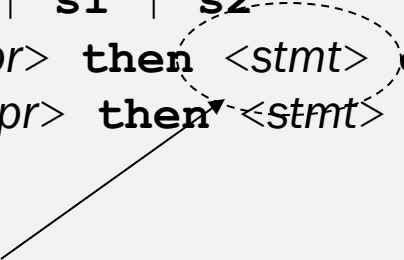
and the next slide shows two different parse trees for it...



Most languages that have this problem choose this parse tree: **else** goes with nearest unmatched **then**

Eliminating The Ambiguity

```
<stmt> ::= <if-stmt> | s1 | s2  
<if-stmt> ::= if <expr> then <stmt> else <stmt>  
           | if <expr> then <stmt>  
<expr> ::= e1 | e2
```

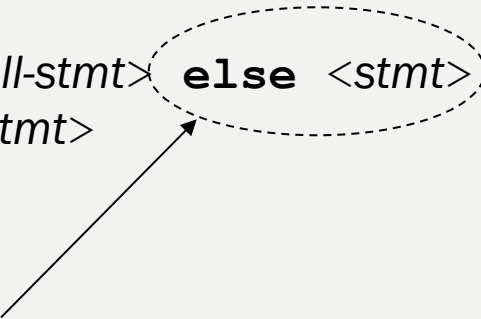


We want to insist that if this expands into an **if**, that **if** must already have its own **else**. First, we make a new non-terminal *<full-stmt>* that generates everything *<stmt>* generates, except that it can not generate **if** statements with no **else**:

```
<full-stmt> ::= <full-if> | s1 | s2  
<full-if> ::= if <expr> then <full-stmt> else <full-stmt>
```


Eliminating The Ambiguity

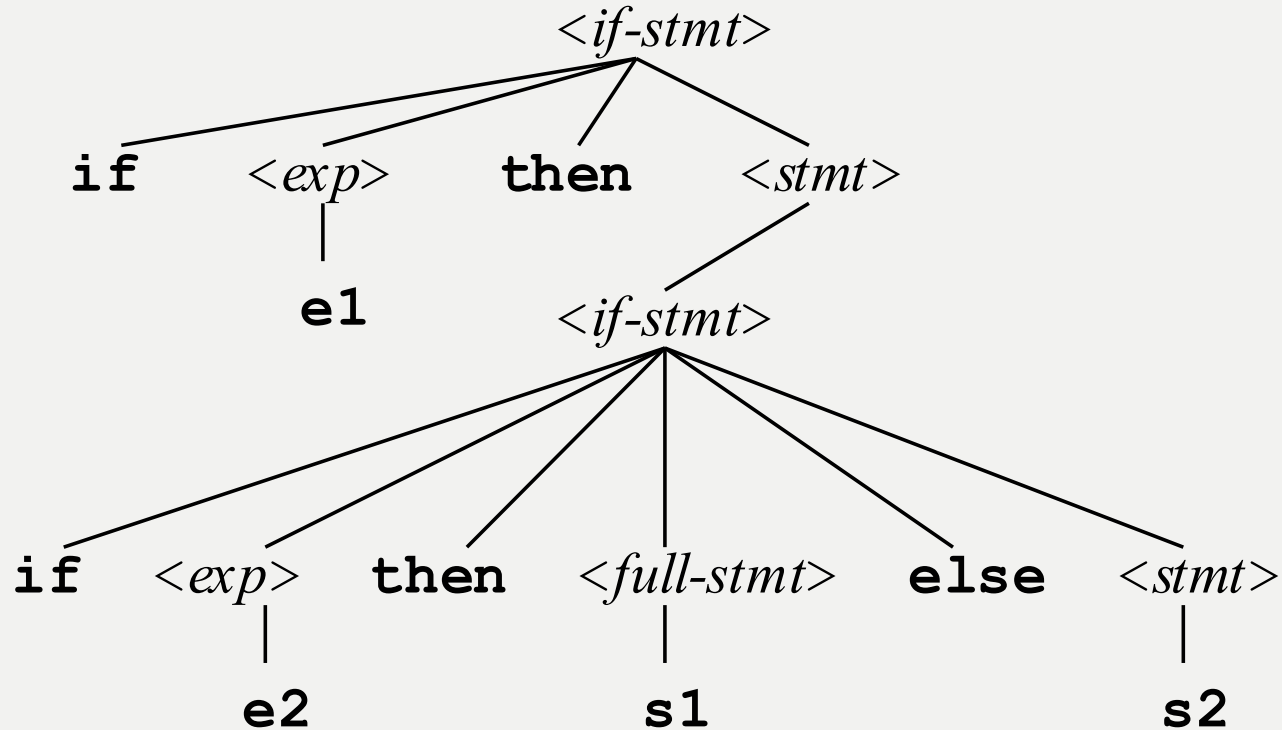
```
<stmt> ::= <if-stmt> | s1 | s2  
<if-stmt> ::= if <expr> then <full-stmt> else <stmt>  
           | if <expr> then <stmt>  
<expr> ::= e1 | e2
```



Then we use the new non-terminal here.

The effect is that the new grammar can match an **else** part with an **if** part only if all the nearer **if** parts are already matched.

Correct Parse Tree



Dangling Else

- We fixed the grammar, but...
- The grammar trouble reflects a problem with the language, which we did not change
- A chain of if-then-else constructs can be very hard for people to read
- Especially true if some but not all of the else parts are present

A3.6 – Dangling Else [3 minutes]

```
int a=0;  
if (0==0)  
    if (0==1) a=1;  
else a=2;
```

What is the value of **a** after this fragment executes?

Clearer Styles

```
int a=0;  
if (0==0)  
    if (0==1) a=1;  
    else a=2;
```

Better: correct indentation

```
int a=0;  
if (0==0) {  
    if (0==1) a=1;  
    else a=2;  
}
```

Even better: use of a block
reinforces the structure

Languages That Don't Dangle

- Some languages define if-then-else in a way that forces the programmer to be more clear
 - *Algol does not allow the **then** part to be another **if** statement – though it can be a block containing an **if** statement*
 - *Ada requires each **if** statement to be terminated with an **end if***
 - *Python requires nested **if** statement to be indented*

CLUTTERED GRAMMARS



Clutter

- The new if-then-else grammar is harder for people to read than the old one
- It has a lot of clutter: more productions and more non-terminals
- Same with G4, G5 and G6: we eliminated the ambiguity but made the grammar harder for people to read
- This is not always the right trade-off

Reminder: Multiple Audiences

- In Chapter 2 we saw that grammars have multiple audiences:
 - *Novices want to find out what legal programs look like*
 - *Experts—advanced users and language system implementers—want an exact, detailed definition*
 - *Tools—parser and scanner generators—want an exact, detailed definition in a particular, machine-readable form*
- Tools often need ambiguity eliminated, while people often prefer a more readable grammar

Options

- Rewrite grammar to eliminate ambiguity
- Leave ambiguity but explain in accompanying text how things like associativity, precedence, and the dangling else should be parsed
- Do both in separate grammars

PARSE TREES AND EBNF



EBNF and Parse Trees

- You know that $\{x\}$ means "zero or more repetitions of x " in EBNF
- So $\langle \text{exp} \rangle ::= \langle \text{mulexp} \rangle \{+ \langle \text{mulexp} \rangle\}$ should mean a $\langle \text{mulexp} \rangle$ followed by zero or more repetitions of $+ \langle \text{mulexp} \rangle$
- But what then is the associativity of that $+$ operator? What kind of parse tree would be generated for **$a+a+a$** ?

EBNF and Associativity

- One approach:

- *Use {} anywhere it helps*
- *Add a paragraph of text dealing with ambiguities, associativity of operators, etc.*

- Another approach:

- *Define a convention: for example, that the form $\langle \text{exp} \rangle ::= \langle \text{mulexp} \rangle \{ + \langle \text{mulexp} \rangle \}$ will be used only for left-associative operators*
- *Use explicitly recursive rules for anything unconventional:*

$\langle \text{expa} \rangle ::= \langle \text{expb} \rangle [= \langle \text{expa} \rangle]$

About Syntax Diagrams

- Similar problem: what parse tree is generated?
- As in EBNF applications, add a paragraph of text dealing with ambiguities, associativity, precedence, and so on

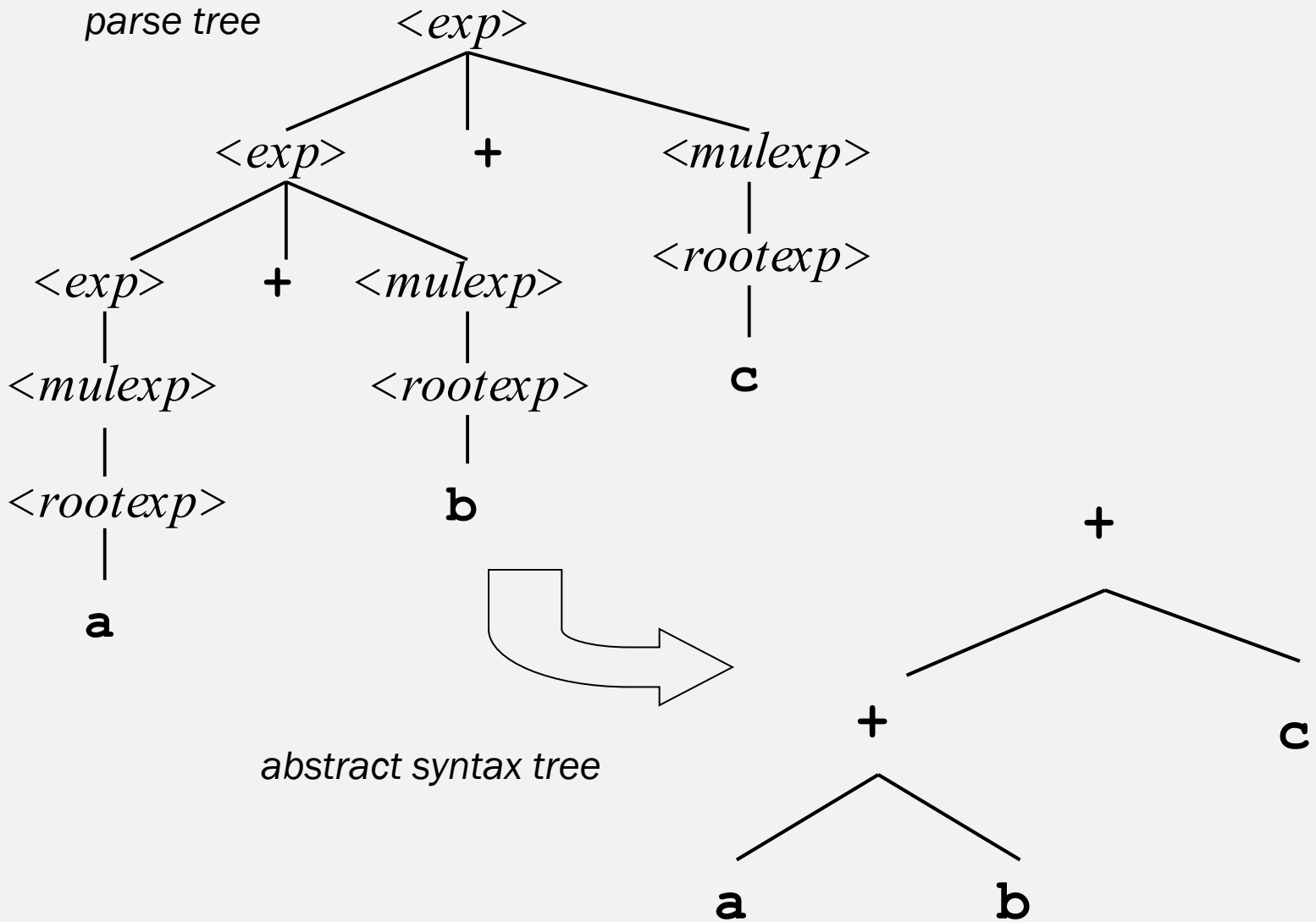
ABSTRACT SYNTAX TREE

Full-Size Grammars

- In any realistically large language, there are many non-terminals
- Especially true when in the cluttered but unambiguous form needed by parsing tools
- Extra non-terminals guide construction of unique parse tree
- Once parse tree is found, such non-terminals are no longer of interest

Abstract Syntax Tree

- Language systems usually store an abbreviated version of the parse tree called the *abstract syntax tree*
- Details are implementation-dependent
- Usually, there is a node for every operation, with a subtree for every operand



Parsing, Revisited

- When a language system parses a program, it goes through all the steps necessary to find the parse tree
- But it usually does not construct an explicit representation of the parse tree in memory
- Most systems construct an AST instead
- We will see ASTs again in Chapter 23

Conclusion

- Grammars define syntax, *and more*
- They define not just a set of legal programs, but a parse tree for each program
- The structure of a parse tree corresponds to the order in which different parts of the program are to be executed
- Thus, grammars contribute (a little) to the definition of semantics