Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2225280

How Close to Be a Millionaire?

Summary

In the pursuit of the maximum economic returns, market traders buy and sell volatile assets on a frequent basis with the unavoidable commission for each transaction. Two representative assets are gold and bitcoin. In the market where bitcoin and gold co-exist, maximizing the returns requires traders' well-designed and comprehensively-considered investment portfolio. We establish the ITRS Model, consisting of four sub-models, **Index Model, Trend Prediction Model, Risk Control Model and Subject Determination Model** to account for the trading schedule.

Firstly, We preprocess the original data and establish the **Index Model**, including **historical indicators** and **technical indicators**. The existing market information can be entirely extracted by our three-level index system using more than 30 specific indicators. We also use **PCA** (Principal Component Analysis) to reduce the dimensionality of them.

Secondly, we develop the **Trend Prediction Model** which can accurately forecast the price. Based on **SVR** and **LSTM** of the advanced artificial intelligence models, our Trend Prediction Model is much better than the traditional prediction through the simple technical indicators.

Thirdly, learning form the **Kelly Criterion** and its extension, we propose the **Risk Control Model**. Taking the **volatility**, **trend of market** and **risk from our prediction model** into account, our Dynamic Position Management Model can minimize the potential risk and maximize the potential returns from the risk asset towards total asset.

Fourthly, we form the **Subject Determination Model**, including **Dual-subject Optimization** and **Single-subject Optimization**. Based on the Markowitz Mean-Variance Model, taking into account both return and risk, **Dual-subject Optimization** obtain the results of assets allocation and specific trading behavior through **maximizing the Sharpte Ratio**. **Single-subject Optimization** still does a good job when Dual-subject Optimization cannot work due to closed gold market.

Then, we compare our model to other three strategies to confirm that our model offers the unrivaled strategy. Using our model and strategy, the initial \$1,000 investment worth on 9/10/2021 will turn out to be **\$402,287**. Five perspectives for comparison include, **Cumulative Rate of Return**, **Annualized Rate of Return**, **Standard Deviation of Daily Returns**, **Sharpe Ratio** and **Max Drawdown**.

Finally, we conducted the **sensitivity analysis**. Considering that the commission for each purchase or sale costs $\alpha\%$ of the amount traded, we demonstrated the sensitivity of our strategy to cost and clarify its influence mechanism in terms of **proportion of gold, number of transactions, commission, and final value**.

In addition, we discuss the strength and weakness of the models and a memorandum is also presented in the paper.

Keywords: PCA; SVR; LSTM; Kelly Criterion; Sharpe Ratio; Non-linear; Optimiation

Contents

1 Introduction

1.1 Background

In the pursuit of the maximum economic returns, market traders buy and sell volatile assets on a frequent basis with the unavoidable commission for each transaction. Two representative assets are gold and bitcoin. Many people are acquaintances to gold, but strangers to bitcoin. Bitcoin is a digital currency created for utilization in peer-to-peer online trading behavior, operating with no central authority or banks.

Nowadays, global traders are increasingly moving into Bitcoin. Christopher Wood, global head of equity strategy at independent investment bank Jeffries Financial, has reportedly cut his gold exposure to buy more Bitcoin. Undoubtedly, we all have witnessed Bitcoin's parabolic surging in contemporary society. Does this mean that Bitcoin's ongoing rally could steal the whole demand from gold traders? One of the world's biggest investment banks, Goldman Sachs reportedly sent a note to investors, reassuring its clients that Bitcoin does not pose an existential threat to gold. "We do not see evidence that Bitcoin's rally is cannibalizing gold's bull market and believe the two can coexist," the company wrote.

In the market where bitcoin and gold co-exist, maximizing the returns requires traders' well-designed and comprehensively-considered investment portfolio, under the models basing on the past stream of daily prices.

1.2 Literature Review

The current regulatory attitude of governments towards bitcoin varies, but most publicly or tacitly acknowledge bitcoin as a legitimate financial investment tool, and no government has yet determined that bitcoin has monetary properties. Prior to 2017, bitcoin transactions were primarily conducted through the block-chain, when exchange trading was around 30% of the volume. By May 2018, exchange trading had accounted for about 90% of the volume. This has actually defeated the original purpose of bitcoin as an anonymous cryptocurrency, as exchange transactions require real names. It suggests that bitcoin is gradually changing into a speculative asset.

According to Kaponda (2018), bitcoin has the following five main characteristics, decentralization, scarcity, high divisibility, high anonymity and irreversibility, compared to traditional currencies. He strongly approved bitcoin as the new "digital gold". Caldararo (2018) took a different view that its value is extremely volatile. The current mania for bitcoin is very similar to many bubble events in history. In empirical studies, Klein (2018) found that the time series of bitcoin's price exhibits fat-tailed, long-memory, and asymmetric properties. Baur (2017) conducted a detailed analysis of the transaction data of bitcoin users and concluded that it is currently used primarily for speculative purposes rather than as a medium of exchange. The particularities of the bitcoin market, such as the 24-hour-a-day, seven-day-a-week trading and the existence of multiple trading platforms for arbitrage opportunities, make high-frequency trading easier to achieve. However, the charging of transaction fees, the highly volatility and unpredictable price have raised the cost of bitcoin speculation.

Baur (2017) found that the correlation between bitcoin and traditional assets such as gold, stocks, and bonds is weak, whether the market is in a period of financial turmoil or not. Accordingly, he argued that bitcoin as an investment vehicle should have a risk diversification function. However, Klein (2018) challenged the conjecture that "Bitcoin is the new gold". Using conditional variance and BEKK-GARCH model, he found that there are significant differences between bitcoin and gold in terms of their risk hedging functions.

Bystrom (2018) published the first bitcoin volatility genesis article. Based on correlation analysis, regression analysis and vector autoregression (VAR) analysis, the author investigated the factors influencing daily, weekly and volatility of bitcoin during 2011-2017, respectively. Meanwhile, Conrad (2018) extracted the long-term volatility in bitcoin price using GARCH-MIDAS mixing model and found that there is a significant negative correlation between bitcoin trading volume and long-term volatility.

1.3 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems,

- **Problem 1**. Model establishment. With data sources only available for prices as of September 10, 2021, we need to propose the optimal strategies for daily trading. Especially, our model are supposed to take the trading schedule into account, since bitcoin can be traded daily, but gold is only traded on market-opening days.
- **Problem 2**. Value estimation. With the initial state [1000, 0, 0] (the portfolio consisting of cash, gold, and bitcoin [C, G, B] in U.S. dollars, troy ounces, and bitcoins, respectively), we need to apply our model and strategy to our daily trading behavior in the next five-year trading period, from 9/11/2016 to 9/10/2021. Eventually, we are supposed to derive the terminal value of the original investment.
- **Problem 3**. Optimality Verification. A large amount of classical trading strategies prevailing in markets, we need to adduce powerful evidence to confirm that our model offers the unrivaled strategy.
- **Problem 4**. Sensitivity Analysis. Considering that the commission for each purchase or sale costs $\alpha\%$ (αgold = 1%, αbitcoin = 2%) of the amount traded and there is no cost to hold an asset during the transaction, we need to identify the sensitivity of our strategy to cost and clarify its influence mechanism.

2 Assumptions

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **1**. Investors are risk-averse, which means that for different investment projects with the same rate of return, they first choose the less risky investment project. In order to attract investors to invest in the risky investment project, it is necessary to ensure that the investment project has a high expected return to pay them a risk remuneration.

- **2**. In the Position Management Model, we assume that the investor's objective is to maximize the long-term growth rate of assets and that each period the investment is chosen only between risky assets and cash.
- **3**. We assume that the price movement of the risky asset follows a Geometric Brownian motion, which implies that short-term returns follow an approximate normal distribution and that returns are independently identically distributed each period.
- **4**. The implicit assumption in risk-adjusting returns by standard deviation is that the assets calculated constitute the entirety of the investor's investment.
- **5**. The returns between multiple assets are correlated, namely, if the correlation coefficient between each asset is known, it is possible to select the portfolio with the lowest risk.
- **6**. The risk-free rate of return is zero, which means cash will get no interest.

3 Our Work & Model Overview

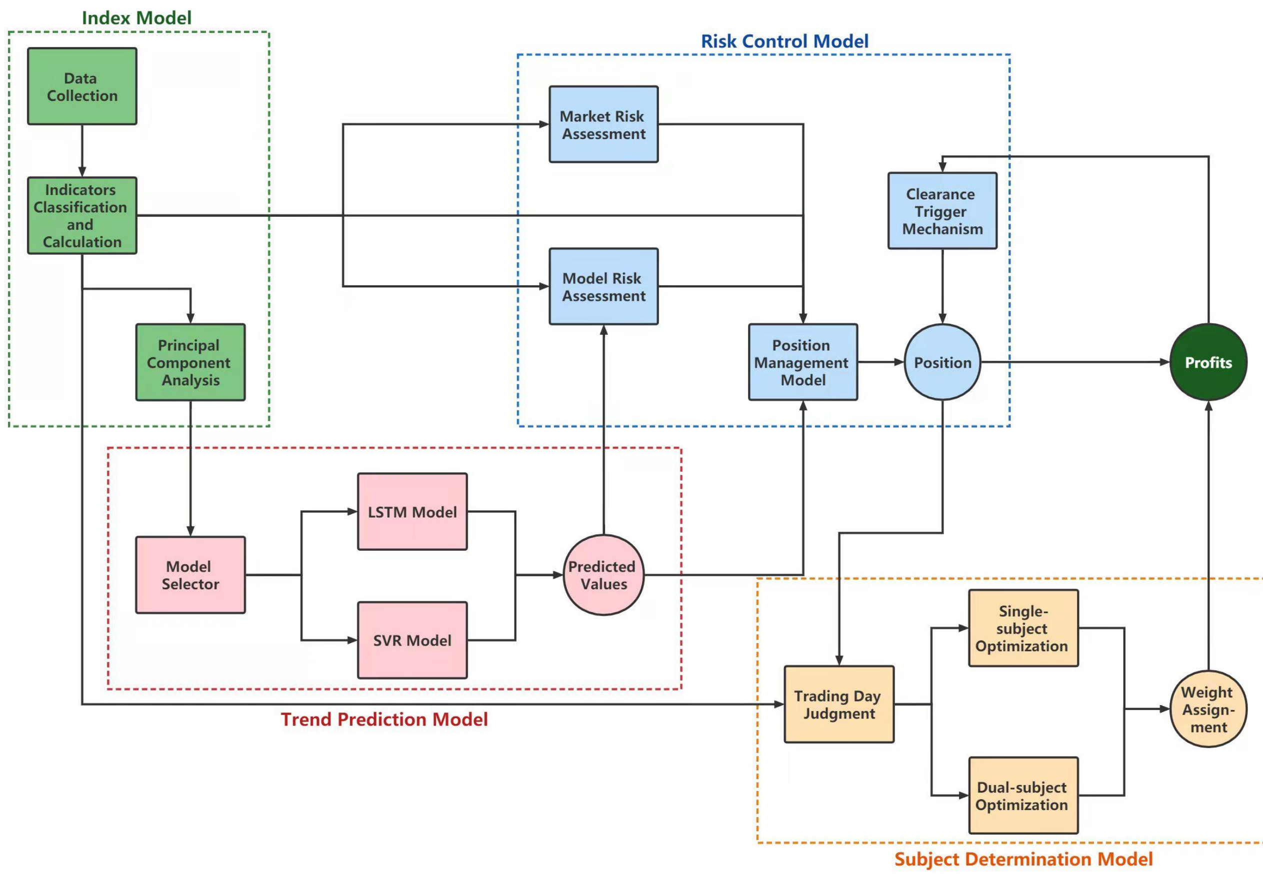

Figure 1: Our Work & Model Overview

4 Sub-model I: Index Model

We preprocess the data from “LBMA-GOLD.csv” and “BCHAIN-MKPRU.csv”, normalize the numeric format of date, and establish the Index Model.

4.1 Historical Indicators

Historical Indicators	Price	Price of the day
		Price of the yesterday
		Average price for the previous 5 days
		Average price for the previous 20 days
	Range of the Advance-Decline	Range of the day
		Range of the yesterday
		Average range for the previous 5 days
		Average range for the previous 20 days

Table 1: Historical Indicators

4.2 Technical Indicators

Technical Indicators	Trend Indicators	MACD (Moving Average Convergence / Divergence)
		PRICEOSC (Price Oscillator)
		MTM (Momentum Index)
		BBI (Bull and Bear Index)
		MA (Moving Average)
		EXPMA (Exponential Moving Average)
	Energy Indicators	PSY (Psychological Line)
	Counter-trend Indicators	BIAS
		RSI (Relative Strength Index)
		DPO (Direct Public Offering)
		DBCD (Differential Bias of Commonalities and Differences)
	Pressure Support Indicators	BOLL (Bollinger Bands)
		BBIBOLL (Superposition of BBI and BOLL)

Table 2: Technical Indicators

- Trend Indicators

  Trend indicators are used to determine the trend of price movements of assets.

  - MACD (Moving Average Convergence / Divergence)

    Developed from the double exponential moving average, it uses the convergence and divergence between the short-term (often 12-day) exponential moving average and the long-term (often 26-day) exponential moving average of the closing price to make a judgment on the timing of buying and selling.

  - PRICEOSC (Price Oscillator)

    The Price Oscillator can give warning signals that the current price trend may be about to come to a halt. It can also be used to measure the strength of a market movement.

  - MTM (Momentum Index)

    In the stock market, there is a phenomenon similar to the constant velocity principle in physics. If the upward (downward) trend of the price is continuing, the upward (downward) velocity will remain roughly the same. MTM is precisely an indicator that examines the upward (downward) velocity of the stock price from the theory above. It analyzes the trend of price based on the velocity change.

  - BBI (Bull and Bear Index)

    It is a composite indicator after weighting and averaging different daily moving averages. When using moving averages, investors often have different preferences for the choice of parameter values, and BBI solves the problem of the reasonableness of the period of the short- and medium-term moving averages. The four average stock prices (or indices) of 3, 6, 12 and 24 days are generally used as parameters for the calculation.

  - MA (Moving Average)

    It applies statistical analysis to average the prices over a certain period of time and connect the averages to form a line, which is utilized to observe the trend of price movements. The commonly used parameters are 5, 10, 30, 60, 120 and 240 days.

  - EXPMA (Exponential Moving Average)

    From a statistical point of view, only by plotting the moving average (MA) at the midpoint of the price time span can the price movement trend be correctly reflected, but this will make the signal lag in time, while the EXPMA indicator is a compensation for the moving average. Its calculation formula focuses on the weight of the price for the day (current period).

- Energy Indicators

  Energy Indicator is a long-term technical analysis tool to examine the contrast in long-short forces of markets and to grasp the time of purchasing-in and selling-out.

  - PSY (Psychological Line)

    It is the parameter of stock trading based on the study of investors' psychological tendency. It digitizes investors' preference for purchasing-side or selling-side.

- Counter-trend Indicators

  Both counter-trend indicators and trend indicators are designed to determine price trends, but the difference is that counter-trend indicators use reverse thinking.

  - BIAS

    By calculating the percentage difference between the market index or closing price and a certain moving average, it is an indicator that reflects the degree of price deviation from its MA over a certain period of time. It shows the likelihood of a price pullback or rebound due to deviation from the moving average trend in times of sharp fluctuations, as well as the plausibility that the price will move within the normal range of fluctuations and form a continuation of the original trend.

  - RSI (Relative Strength Index)

    It is a technical curve according to the ratio of rising points summation to falling points summation, reflecting the prosperity of market. Relative Strength Index, suitable for measuring and analyzing the short-term investment, is designed to reflect the strength of the price trend with three lines, providing the operation basis for investors.

- DPO (Direct Public Offering)

  Security issuers publishes listing information and transmits offering documents on the Internet without the help of underwriters or investment banking firms, thus directly offering the company's shares publicly. Unlike IPO (Initial Public Offering), which has cumbersome filing and registration procedures and strict disclosure requirements, it can take full advantage of the cross-space provided by the Internet to connect listed companies directly with investors. It has five major advantages, high issuance efficiency, low issuance cost, few restrictions, wide audience and convenient information exchange.

- DBCD (Differential Bias of Commonalities and Differences)

  It is the probability of being able to keep the indicators closely synchronized and with smooth lines and clear signals that can effectively filter out false signals.

• Pressure Support Indicators

Graphical and intuitive, it is mainly used to anticipate dense trading areas formed by historical transactions and to pre-judge possible pressure and support levels as a result.

- BOLL (Bollinger Bands)

  In general, the movement of the stock price is always around a certain value pivot (such as the average, cost line, etc.) in a certain range of movement. Based on the above conditions, Bollinger Bands indicator introduces the concept of "stock channel", which shows that the width of the stock channel changes with the price fluctuations. Besides, the stock channel has variability, adjusting itself automatically. Because of its flexibility, intuitiveness and trend characteristics, the BOLL indicator has been popular utilized by investors in transaction. While most technical analysis indicators are constructed by quantitative methods, the BOLL indicator is inextricably linked to the pattern and trend of the stock price.

- BBIBOLL (Superposition of BBI and BOLL)

  It takes BBI as the center line and the standard deviation of BBI as the bandwidth. In other words, BBIBOLL is the superposition of BBI and BOLL. While UPR is the pressure line, having a suppressive effect on price, and DWN line is the support line, having a supportive effect , BBIBOLL line is the central axis between them.

4.3 Principal Component Analysis

Some of the indicators' characteristics have similar meanings. To reduce the influence of col-linearity when calculating similarity, we use PCA (Principal Component Analysis) to avert such troubles while preserving as much of the data's variation as possible. After calculation, the cumulative variance contribution rate is shown in Table 2&3.

Table 3: Cumulative Variance Contribution Rate of Bitcoin

Number of Principal Component	1	2	3	4	5
Cumulative Variance Contribution Rate	0.39	0.70	0.81	0.87	0.91

Table 4: Cumulative Variance Contribution Rate of Gold

Number of Principal Component	1	2	3	4	5
Cumulative Variance Contribution Rate	0.39	0.60	0.73	0.81	0.87

Based on the result of PCA, we choose several suitable principal components and ignore the rest. These variables maintain more than 80% information of the original data.

5 Sub-model II: Trend Prediction Model

To establish the stable foundation of the total model, it's critical to develop a reliable prediction model which can accurately forecast the tendency and the price limit. Considering the ever-changing characteristics of the bitcoin and gold, we have tried 3 kind of approaches that act excellently in analysis of time series, which are Time-Series Analysis, Support Vector Regression (SVR) and Long-Short Term Memory (LSTM). However, because the Time-Series Analysis is based on single indicator, it's hard to predict the fine structure of the future prices. Moreover, price of the bitcoin and gold is influenced by diversified extraneous factors, timeliness must be included. Timeliness will be reflected in the accuracy of the prediction model, on which we based to establish a selector to decide which method to choose in a particular period.

The prediction will be operated by model training once a day.

5.1 Model Selector

Even though technical analysis is the only method in our model, we have to admit that the price of bitcoin and gold has always changed with the external environment. This can cause some troubles for our model, which means that the indicators we build the model for now may not be applicable to the next phase. So in a particular period, it is necessary to use a model that is more credible. So we built a index of confidence that includes both the accuracy of the trend forecast a_1 (whether prices will go up or down tomorrow) and the degree of deviation from the percentage of increase forecast a_2. a_1 is the percentage of predicted success days over the past 20 days. And a_2 is the average relative error of predicted values for each day over the past 20 days:

$$a_2 = \frac{1}{20} \cdots \sum^{20} \frac{|price'_i - price_i|}{price_i}$$

And we build the index Cost of Risk CoR:

$$CoR = (1 - a_1) + (1 - a_2) + Cov * (1 - a_1) * (1 - a_2)$$

In the equation, Cov is the covariance between a_1 and a_1. So CoR can evaluate the risk we make in an operation. The model with lower CoR will be selected.

5.2 Long Short-Term Memory Model (LSTM)

LSTM is the descendant of the Recurrent Neural Network (RNN), which can commendably predict the time-series data. However, because of the characteristics of short-term memory, when training data, earlier important information may be omitted. Worse still, in the case of backpropagation, CNN may terminate learning because of the disappearance of the gradient.To solve this problem, we chose to use LSTM, which has an internal structure called gate. The gate automatically identifies the important data that needs to be retained during the training process.

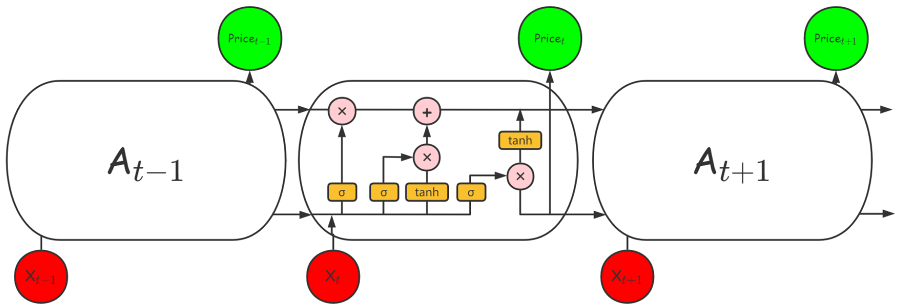

Figure 2: LSTM Model

LSTM model is connected by cells, in which we need to input a series of time-continuous data. To train the model smoothly, we use a string of seven bitcoin data when using a string of five bitcoin data, each of which has five indicators based on principal component analysis and the price of bitcoin and gold that day. The structure of each data can be formulated as:

$$X_t = [x_1, x_2, x_3, x_4, x_5, price]_t$$

In each round, the cell needs to perform the calculations below:

$$\begin{cases} i = \sigma(W_{ii}x + b_{ii} + W_{hi}h + b_{hi}) \\ f = \sigma(W_{if}x + b_{if} + W_{hf}f + b_{hf}) \\ g = \tanh(W_{ig}x + b_{ig} + W_{ho}h + b_{ho}) \\ o = \sigma(W_{io}x + b_{io} + W_{ho}h + b_{ho}) \\ c' = f \cdot c + i \cdot g \\ h' = o \cdot \tanh(c') \end{cases}$$

During the process above, σ equation is considered as the activation function, which can conserve the critical data that remains a great significance.

The notations of each equation can be refer to in the table:

Mean square error (MSE) is used as the loss function in LSTM:

$$MSE = \frac{1}{N}\sum^{N}(price'_i - price_i)^2$$

Table 5: Notations in LSTM Model

Symbol	Description
x	The concatenated input vector foe the LSTM
h	Hidden state, containing encoded information for the sequence flow
c	Cell state, tracking dependencies between the elements in the input sequence
i	Input gate, controlling the extent to which a new value remains in the cell
f	Forget gate, controlling the extent to which a value remains in the cell
g	Gather input value from input x and current hidden state
o	Output gate controlling the extent to which the cell is used to compute outputs
W	The weight matrix for transitions
b	The bias for transitions
σ	The sigmoid function
$\tanh$	The hyperbolic tangent function

The loss function will be regarded as an optimization object in LSTM, which will try to reach a minimum value in the training process.

5.3 Support Vector Regression Model (SVR)

Support vector regression (SVR) is a model based on support vector machine (SVM), which purpose is to determine the model closest to the real situation under the condition of effective supervised learning of data features.

In our case, the data of PCA forms a five-dimensional space. We expect to find a hypergeometric surface that maximizes the sum of the distances between the points farthest from the surface. The space could be expressed as the following equation:

$$\mathbb{R}^5 = [x_1, x_2, x_3, x_4, x_5]$$

In order to determine the hypergeometric surface to be closer to the actual position, meaning that there are more points around the surface, we need to set certain limits ε on the width of the interval of the surface. Otherwise, when all the data contained, the surface will tend to be extremely distant from the data. According to the discussion above, we can use the equation to solve the problem:

$$\min_{\mathbf{w},b} \frac{1}{2}\|\mathbf{w}\|_2^2$$
$$s.t. \quad |y_i - (\mathbf{w}^T\mathbf{x}_i + b)| \le \varepsilon, \quad i = 1, 2, \cdots, N$$

When we train with the data by SVR, this hypergeometrc surface can tell us the prediction of any point in space. This is the result of a projection into space.

5.4 Results

We can see that the SVR performs better than the LSTM on earlier trading days, and is more reliable during periods of high price volatility. However, LSTM is more reliable in other cases because of the fluctuation caused by the lack of information in the projection of SVR.

5.4.1 Results of Model Selector

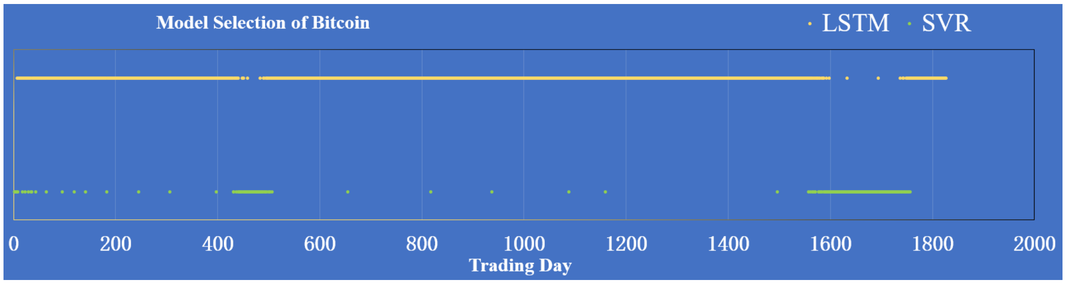

Figure 3: Model Selection of Bitcoin

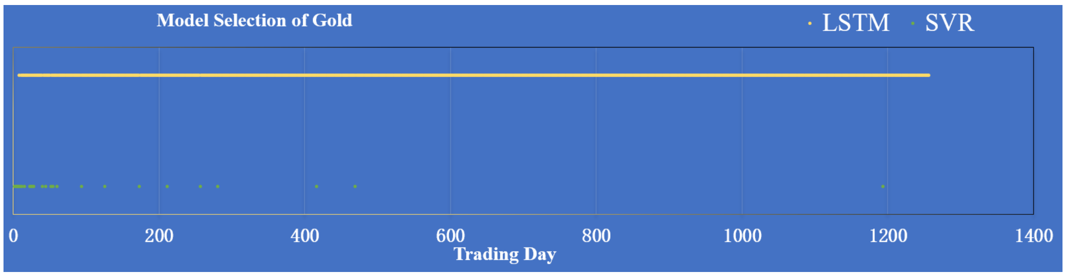

Figure 4: Model Selection of Gold

5.4.2 Results of Prediction

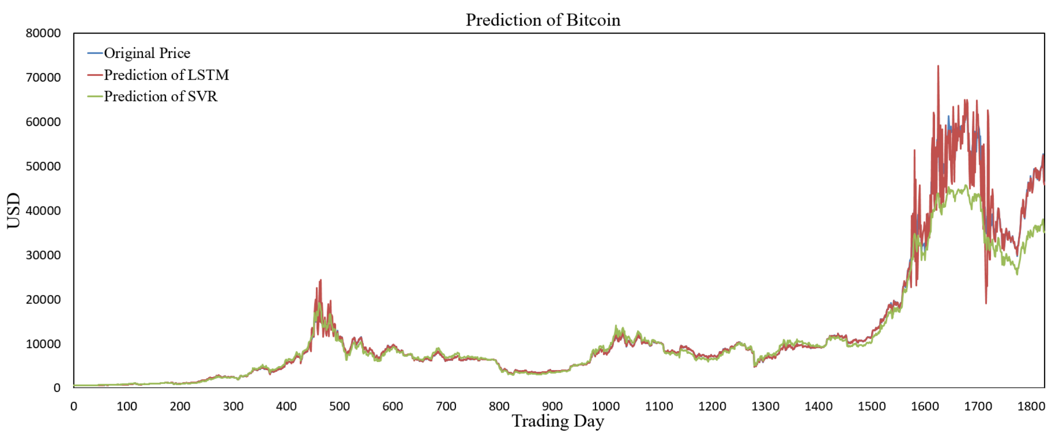

Figure 5: Prediction of Bitcoin

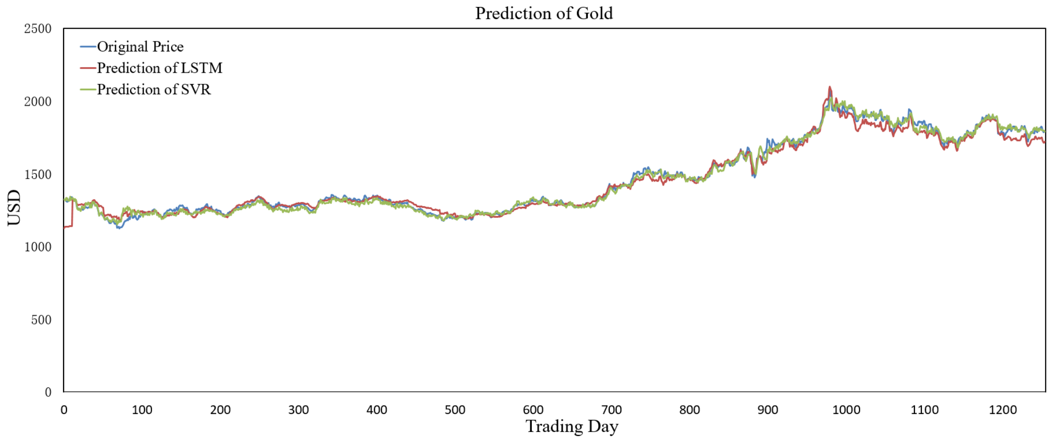

Figure 6: Prediction of Gold

6 Sub-model III: Risk Control Model

Investment risk is the possibility that an investor will experience a loss from a decline in the price after purchasing the bitcoin or gold, which is generally perceived as the downward deviation between selling price and anticipated price. Since instantaneous changes happening in the market at every moment, opportunities and risks are always born together, and investors are bound to take huge risks while expecting to obtain high returns. From this perspective, it is necessary to build a risk control model. In the implementation of actual trading strategies, the frequently used methods include but are not limited to position management, investment diversification, stop-loss and take-profit, etc. Here we expand on the strategy of position management in detail.

6.1 Position Management

Position is the ratio of the market value of an investor's risky assets to the total assets (risk assets & non-risk assets). The aim of position management is to maximize the long-term growth rate of the trader's assets, which is a multi-period investment decision, the key point falling on how to deal with the asymmetrical return volatility of risk assets.

6.2 The Establishment of Position Management Model

6.2.1 The Kelly Criterion and its Extension

One of the theoretical foundations of position management is the Kelly Criterion, which was developed by John Larry Kelly when studying how bettors choose to bet in pursuit of the maximum long-term wealth growth. He argued that in a game with infinite rounds, a bettor can obtain the supreme return, if each time betting a certain percentage (called the optimal betting ratio) of his wealth.

Suppose a bettor has an initial wealth of W_0. The bettor can put a fraction of his wealth into a risky game over and over again. In each round of the game, the bettor has a winning probability of p and inversely, $1-p$. If winning, the bettor will get twice his bet, otherwise nothing. Let the rate of return in each round is R_t, then the bettor wins with $R_t = 1$ and loses with $R_t = -1$ At the same time, the optimal betting ratio (position) is f for each round, then the investor's wealth W_T at the end of period T,

$$W_T = W_0 \prod_{t=1}^{T} (1 + fR_t)$$

Since the rate of return R_t for bettors in each round of game is independently identically distributed over a two-point distribution $(1, -1; p, 1-p)$, the long-term exponential growth rate of bettors' assets is thus,

$$g = \lim_{T\to\infty} (1/T) ln(W_T / W_0) = pln(1+f) + (1-p)ln(1-f)$$

By maximizing the equation (2), we can find the optimal position,

$$f^* = (2p-1) \times 100\%$$

Applying the Kelly Criterion to the stock market, we can find the optimal position,

$$f^* = [pr_w - (1-p)r_1]/r_w r_1 \times 100\% = E(R)/r_w r_1 \times 100\%$$

Equation above shows that, if the expected return is positive, the investor should go long; otherwise go short. The optimal position is positively related to the expected return and negatively related to $r_w r_l$. In fact, since the variance of this two-point distribution is $Var(R) = p(1-p)(r_w + r_1)^2$, $r_w r_l$ increases with the variance $Var(R)$ when p and $E(R)$ are fixed. Therefore, it can be derived that the larger the variance of this two-point distribution, the smaller the optimal position 1 is. In other words, the optimal position f^* is negatively related to the variance of this two-point distribution.

Considering that the rate of returns following the assumption of a two-point distribution is special comparatively, we alter the assumption to this one, the variation of price S follows the Geometric Brownian motion, $dS = \mu S dt + \sigma S dz$. Under this assumption, the price obeys a log-normal distribution and the short-term rate of return approximately follows a normal distribution $N(\mu - \sigma^2/2, \sigma)$. Consider a portfolio consisting of stocks and non-risk assets, let the positions (investment ratios) are f and $1-f$, respectively, meanwhile the non-risk rate of return is r, then the long-term exponential growth rate of the investor's assets is,

$$g = f\mu - 1/2(f\sigma)^2 + (1-f)r$$

By maximizing the equation (6), we can find the optimal position,

$$f^* = (\mu - r)/\sigma^2 \times 100\%$$

Based on the equation above, we prove theoretically that the optimal position is positively related to the expectation of the rate of returns and negatively related to its variance. Investors should make expectations of the future rate of returns and its volatility, thus deciding whether to go long or short, add or subtract positions, increase or decrease leverage (leverage can adjust for the situation where optimal position is greater than 1).

However, in the actual trading process, using a fixed position in the whole sample interval may cause forward Look-Ahead Bias. Consequently, we adopt the management strategy of dynamically adjusting the position to optimize our model.

6.2.2 The Dynamic Position Management

The dynamic position management strategy is adjusted by volatility and takes the signal of purchase or sale into account.

Referring to theories of Barroso and Santa-Clara on Momentum Crashes, we put forward the idea of volatility adjustment as follows. Since the volatility has fairly great predictability, we adopt the auto-regressive model to predict the future volatility, and then use the predicted value to amend or adjust the optimal position. Concretely speaking, when the predicted value of volatility is relatively high, decrease the position; otherwise increase the position. The dynamic position adjusted by volatility is figured by the following formula,

$$f_{m} = 1 + (D_{max} - D)/(D_{max} - D_{min})$$

$$D = w^{T} \sum_{i=1}^{2} \sum_{j=1}^{2} Cov\left(\tilde{r}_{i}, \tilde{r}_{j}\right) w$$

D, D_{max} and D_{min} respectively represent the estimated value of current variance, maximum and minimum variance in the past n periods.

Equation above shows that when the estimated value of current variance is relatively high, traders are recommended to increase the position in small degree and conversely by a large scale. We should make adjustments to the optimal position according to the variation and predicted value of volatility. We replace the optimal position with the position at the beginning of the period and simultaneously adjust the second item on the right of the equal sign.

$$f_{t} = f_{t-1}\left[1 + sign(t)\frac{D_{t-1} - D_{t}}{D_{max} - D_{min}}\right]\left(1 + \Delta\frac{SSE}{SST}\right)$$

$sign(t)$ means the signal of purchasing or selling. When the price index is greater than its value of average line, $sign = 1$; otherwise $sign = -1$. The value of $sign$ is determined by the prediction of future trend, based on the Trend Prediction Model above.

Equation above means that increase the position when the price index is above the average line and decrease the position when the index is located below. Exactly how much to add or subtract depends on the relative value of volatility.

$\frac{SSE}{SST}$ measures the rate of volatility explained by our Trend Prediction Model. The predicted value derived from the model above is not precise, thus we need to take the risk from inaccuracy of our Trend Prediction Model into account. SSR for sum squared residual. SSR for explained sum of squares. SST for total sum of squares, SST = SSR + SSE. The increase of $\frac{SSE}{SST}$ implies that the risk from model decreases.

6.3 Results

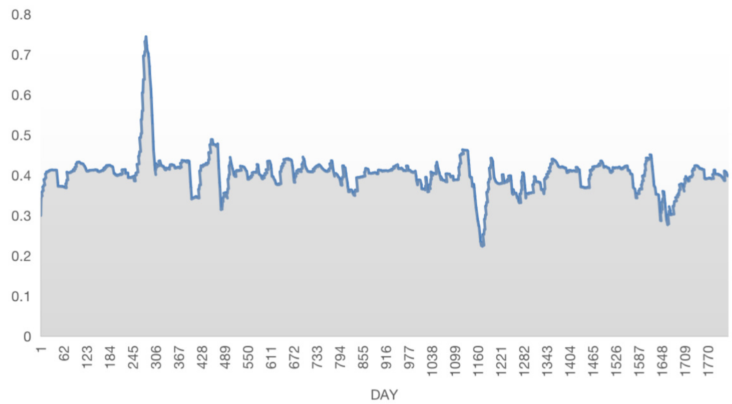

Figure 7: The Results of f

The initial value of f is 0.3. f is calculated by the model above. Figure.3 shows the more specific results.

7 Sub-model IV: Subject Determination Model

7.1 Notations

- v_r=f·v, value of risky assets (gold and bitcoin)
- $w = [w_1, w_2]^T$, vector form of w_1 and w_2
- $e = [r_1, r_2]^T$, vector form of r_1 and r_2

  Based on the Markowitz Mean-Variance Model (MM) in his Modern Portfolio Theory (MPT), we need to calculate the following values, to measure the risk and return of portfolio investments and balance these two indicators for assets allocation.

- $\mu_0 = w^T e = w_1 r_1 + w_2 r_2$, the return rate expectation of [C, G, B] with no commission
- $\mu = w^T e - com/v_p = w_1 r_1 + w_2 r_2 - com/v_p$, the return rate expectation of [G, B]
- $\Omega = \sum_{i=1}^{2} \sum_{j=1}^{2} Cov\left(\widetilde{r}_i, \widetilde{r}_j\right)$
- $\sigma^2 = w^T \Omega w$, the return rate variance of [G, B], σ, the standard deviation.

Table 6: Notations in Subject Determination Model

Symbol	Description
f	Proportion of risky assets in total assets
r_0	Expectation return rate of cash, 0
r_1	Expectation return rate of gold
r_2	Expectation return rate of bitcoin
v_0	Value of cash (C)
v_1	Value of gold (G)
v_2	Value of bitcoin (B)
v	Value of portfolio (before the Optimization Model) , [C, G, B]
v_p	Value of the new portfolio (within the Optimization Model), [G, B]
w_1	Weight of gold in the new portfolio, [G, B]
w_2	Weight of bitcoin in the new portfolio, [G, B]
x_1	Adjustment amount of gold ($x_1 > 0$ means buying-in, otherwise selling-out)
x_2	Adjustment amount of bitcoin ($x_2 > 0$ means buying-in, otherwise selling-out)
α	Upper limit to $\frac{com}{v_0}$
α_1	Commission for each transaction (purchase or sale) of the amount of gold traded
α_2	Commission for each transaction (purchase or sale) of the amount of bitcoin traded
com	Total commission of gold and bitcoin in the trading behavior

7.2 Dual-subject Optimization

7.2.1 The Establishment of non-linear Optimization Model

There is a customary practice in investment that, traders can endure a higher volatility risk when the expected return increase. Therefore, the main objective of a rational investor in portfolio determination is to seek the maximum return for a fixed acceptable risk, or the minimum risk for a fixed expected return. Here we adopt the Sharpe Ratio (Sharpe Index), a standardized indicator of performance evaluation that takes into account both return and risk. The Sharpe Ratio has been studied in modern investment theory, showing that risk plays a fundamental role in determining the portfolio performance, and that in the long term it is possible to exclude the negative impact of risk factors. According to the concept of Capital Allocation Line (CAL), the Sharpe Ratio applies to the risky assets, exactly [G, B] in this problem. In terms of specific figure, the higher the Sharpe Ratio (ratio of investment return to risk), the better the trading strategy of portfolio.

Particularly, we should pay attention to the fact that that bitcoin can be traded every day, but gold is only traded on days the market is open. Hence, To maximize the

$$SharpeRatio = \frac{\mu}{\sigma} = \frac{w_1 r_1 + w_2 r_2 - com/v_p}{\left(w^T \sum_{i=1}^{2} \sum_{j=1}^{2} Cov\left(\tilde{r}_i, \tilde{r}_j\right) w\right)^{1/2}}$$

we need to discuss in 2 cases where gold market closed or not.

There are qualifications from 4 aspects.

- Weight Restriction

  In the new portfolio, the sum weight of gold and bitcoin needs to be 1, namely w_1+$w+2$=1.

- Proportion Variation

  It is understandable that $v_1 + x_1 = w_1 v_r$ and $v_2 + x_2 = w_2 v_r$.

- Selling-out Restriction

  Here we consider the situation of $x_1 < 0$, because the most traders can sell is all the available gold. Hence, $v_1 + x_1 \geq 0$. Bitcoin is of the same argument, $v_2 + x_2 \geq 0$.

- Commission Restriction

  As for the commission, $com = |x_1|\alpha_1 + |x_2|\alpha_2$, we need to take a special situation into consideration. Possibly, $x_1 + x_2 = 1$ but the absolute values of x_1 and x_2 are too large. Thus, the trading volume and com jumps to a high level. $v_0 < 0$ through worse case scenarios. Setting an upper limit α to com/v_0 is of great necessity.

$$\max_{x_1,x_2,w_1,w_2} \quad SharpeRatio$$
$$s.t. \quad \begin{cases} w_1 + w_2 = 1 \\ v_1 + x_1 = w_1 v_r \\ v_2 + x_2 = w_2 v_r \\ v_1 + x_1 \geq 0 \\ v_2 + x_2 \geq 0 \\ com = |x_1|\alpha_1 + |x_2|\alpha_2 \\ com/v_0 \leq \alpha \end{cases}$$

7.3 Single-subject Optimization

When the gold market closed, there is no question of optimal portfolio, since there is only one subject, bitcoin. As Figure 3, traders are recommended not to trade if one of the following 3 conditions is met. First, $|r_2| \leq |x_1|\alpha_1$. Second, $|r_2| > |x_1|\alpha_1$ and $v_p - v_1 - v_2 = 0$. Third $|r_2| > |x_1|\alpha_1$ and $r_2(v_p - v_1 - v_2) < 0$. If $|r_2| > |x_1|\alpha_1$ and $|r_2| > 0$ and $v_p > v_1 + v_2$, purchasing the bitcoin is better. If $|r_2| > |x_1|\alpha_1$ and $|r_2| < 0$ and $v_p < v_1 + v_2$, we will advise investors to sell out the bitcoin.

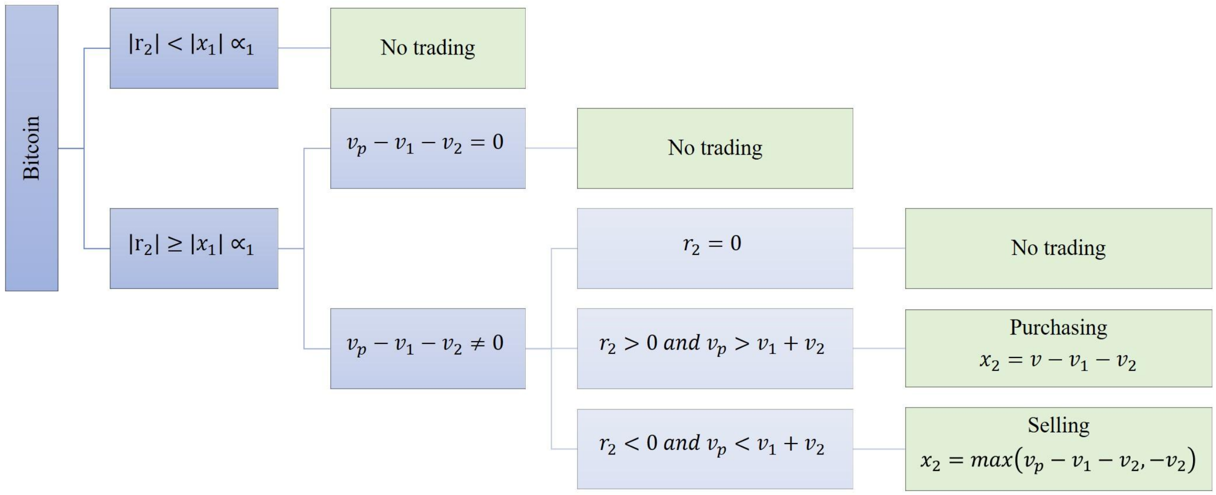

Figure 8: Single-subject Optimization Procedure

7.4 Results

According to the Optimization Model established above, the results of assets allocation and specific trading behavior can be computed by the python code.

If μ<0, there is no trading. If μ>0, summing the original set, $[v_0, v_1, v_2]$, and the adjustment set, $[-x_1 - x_2 - com, x_1, x_2]$, we can derive the post-transaction set of same day.

8 Total Results & Optimality Verification

We assumed the following three strategies and compared our own model ($f = 0.3$, $\alpha = 0.1$) with them from five perspectives.

Three Other Strategies

- Strategy 1: $w_1 = 0.5, w_2 = 0.5$
- Strategy 2: $w_1 = 1, w_2 = 0$
- Strategy 3: $w_1 = 0, w_2 = 1$

Five Perspectives (Table 6 & Figure 4 for specific data)

- Cumulative Rate of Return

  The cumulative rate of return of our model is significantly higher than others. It shows that with the initial state [1000, 0, 0], we can get more return.

- Annualized Rate of Return

  The Annualized rate of return provides more visual data to demonstrate the superiority of our model. The annualized rate of return of our model is highest.

- Standard Deviation of Daily Returns

  It measures the volatility in returns and the risk of the investment strategy. The standard deviation of daily returns of our model is lowest.

- Sharpe Ratio

  The objective of using Sharpe Ratio is to calculate how much excess reward the portfolio will generate for each unit of risk taken, and the higher the ratio, the better the portfolio. The Sharpe Ratio of our model is highest.

- Max Drawdown

  Max Drawdown calculates the maximum value of the retracement of the return when the net value of the asset goes to its lowest point at any historical point backward in the selected period. It is used to describe the worst-case scenario that can occur after buying gold and bitcoin, acting as an important risk metric for quantitative strategy trading. The Max Drawdown of our model is lowest.

	Cumulative rate of return	Annualized rate of return	Std of daily returns	Sharpe ratio	Max drawdown
Model	40336%	232%	1.91%	121.750	28.41%
Strategy1	4319%	112%	3.19%	35.186	68.95%
Strategy2	1483%	71%	4.08%	17.539	83.37%
Strategy3	7213%	135%	4.15%	32.612	83.37%

Table 7: Optimality Evidence

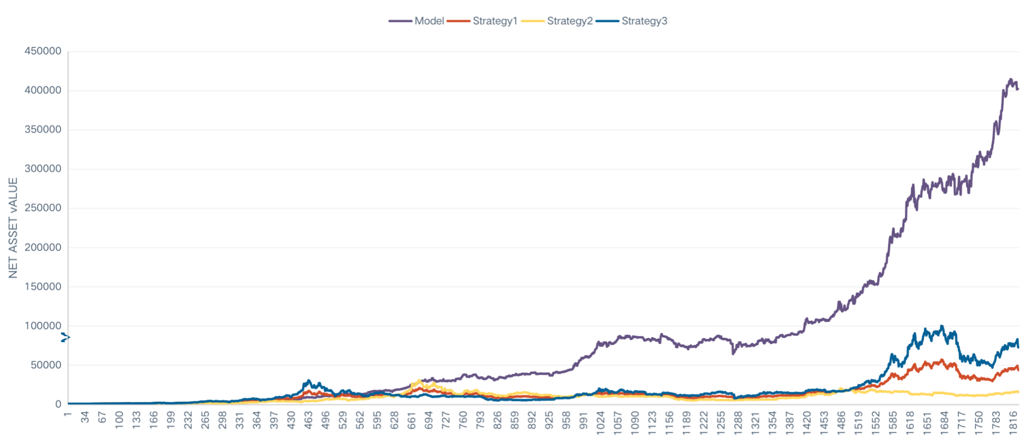

Figure 9: Portfolio values of four Strategies

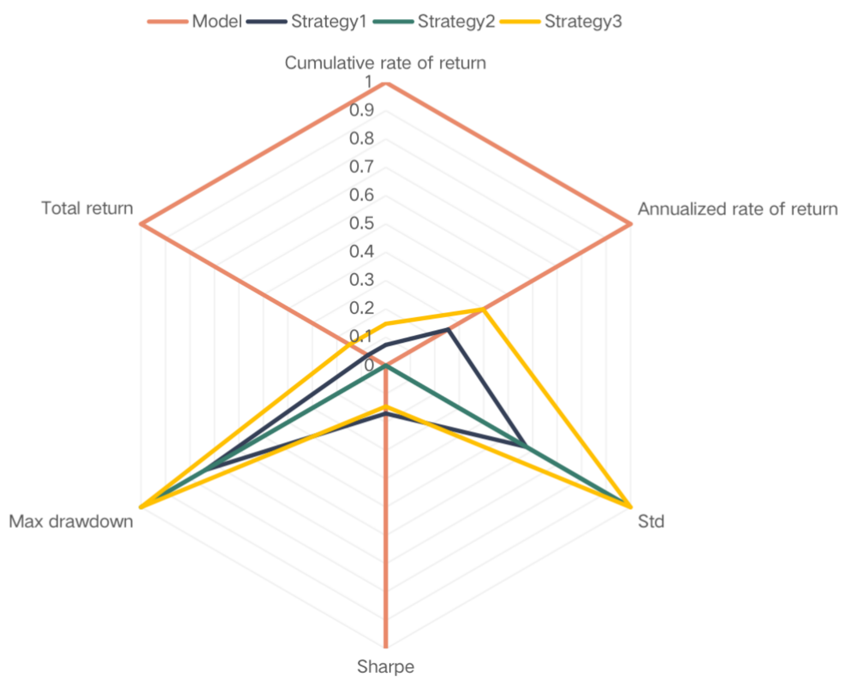

Figure 10: Optimality Evidence (Radar Chart after Standardization of Deviation)

Also, we make a trend chart of the asset value to observe the terminal value of the original investment. The result of our model far exceeds others.

Additionally, Strategy 1 performances better than Strategy 2&3, which echoes to the golden words of wisdom from the investment community, **"Don't put your eggs in one basket"**.

Given all of that, traders with the utilization of our model and strategy will obtain the superb return.

9 Sensitivity Analysis

Comprehensively analyzing the unstable transaction costs, α_{gold} and $\alpha_{bitcoin}$, we also conduct sensitivity analysis.

9.1 Effect on Strategy

- Proportion of Gold in Risk Assets

  According to graphs in the **first line**, proportion of subject has negative correlation with the cost of it. Here we find an interesting phenomenon that, when $\alpha_{bitcoin}$ is infinitely large and α_{gold}=0, **proportion of gold in risk assets** $(v_1 + v_2)$ does not be 1, which exactly echoes the Optimally Verification above. Multi-subject portfolio is better than single-subject investment in most cases. Traders should attach great significance to the investment diversification.

- Num of Trades

  Num of trades has negative correlation with α_{gold} and $\alpha_{bitcoin}$. It has a relatively flat change.

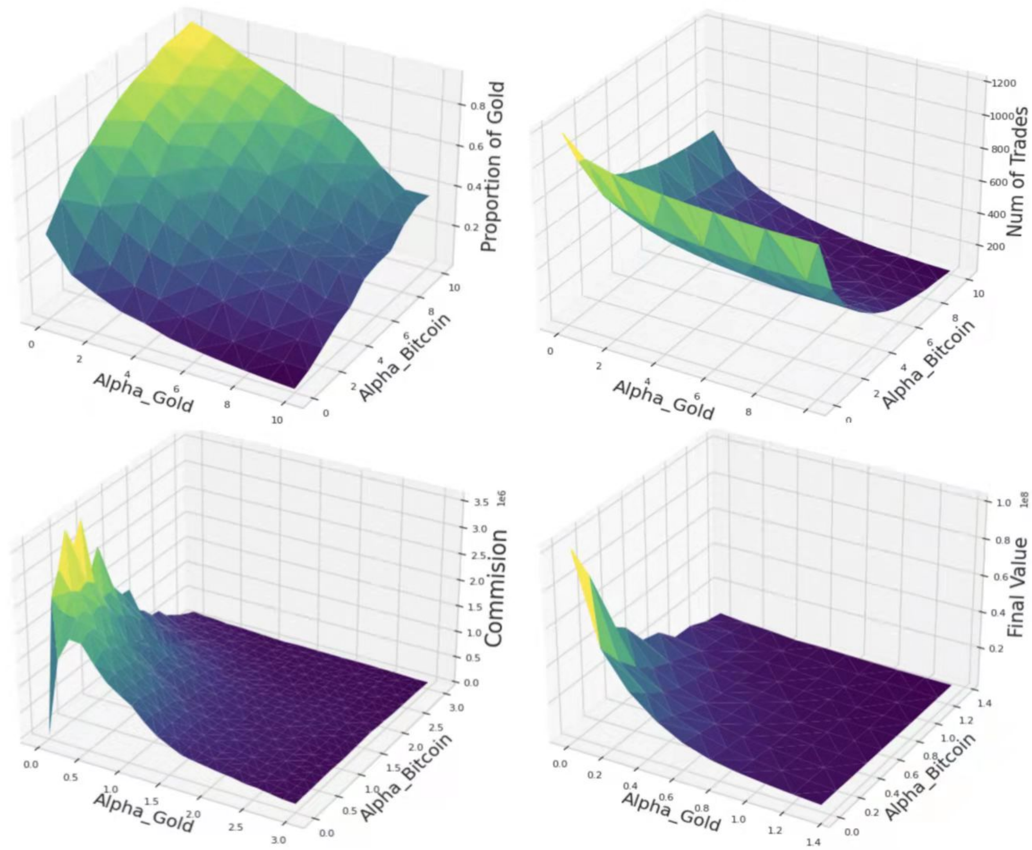

Figure 11: Sensitivity Analysis

9.2 Effect on Results

- Commission

  According to graphs in the **second line**, when α_{gold} and $\alpha_{bitcoin}$ are close to 0, a marginal increase in cost will lead to the surge of commission, since num of trades in this situation is huge. When α_{gold} and $\alpha_{bitcoin}$ are infinitely large, since num of trades is little, it is understandable that commission is limited.

- Final Value

  Final value has negative correlation with α_{gold} and $\alpha_{bitcoin}$ sharply.

Our sensitivity analysis shows that the strategy and model have strong adaptability and are easier to popularize.

10 Strength and Weakness

Strength

- First of all, our model fully applied and developed the classical theoretical achievements of investment including but not limited to technical analysis, Kelly Criterion and its extension, Markowitz Portfolio Model and so on.

- The existing market information can be entirely extracted by our three-level index system using more than 30 specific indicators.
- Based on the advanced artificial intelligence models, our Trend Prediction Model is much better than the prediction through the simple technical indicators.
- Taking the volatility , trend of market and risk from our prediction model into account, our Dynamic Position Management Model can minimize the potential risk and maximize the potential returns from the risk asset towards total asset.
- As for our Subject Determination Model, Dual-subject Optimization creatively replaces the singe objective function of maximizing Sharpe ratio with the traditional two objective functions of maximizing returns and minimizing risks. In addition, Single-subject Optimization still does a good job when Dual-subject Optimization cannot work due to closed gold market.

Weakness

- The results of non-linear optimization in Dual-subject Optimization may merely be local optimal rather than global optimal.
- Due to insufficient data, our range of research is limited. Therefore, the model is still not comprehensive enough.

Despite these weaknesses, we hope our model is helpful to the determination of each day's transaction.

11 Memorandum

Are you thirsty to be a millionaire?

Are you confused by the dazzling market indicators?

Are you anxious because you can't predict tomorrow's stock price?

Are you remorseful about the blowout due to the position mismanagement?

Are you racking your brain to pursue the optimal portfolio in the Markowitz Mean-Variance Model?

If Yes, join us!

Our ITRS Model will live up to your expectations.

Firstly, our Index Model includes historical indicators and technical indicators. The existing market information can be entirely extracted by our three-level index system using more than 30 specific indicators.

Secondly, we develop the Trend Prediction Model, consisting of SVR and LSTM, which can accurately forecast the price.

Thirdly, learning form the Kelly Criterionand its extension, we propose the Risk Control Model. Taking the volatility, trend of market and risk from our prediction model into account, our Dynamic Position Management Model can minimize the potential risk and maximize the potential returns from the risk asset towards total asset.

Fourthly, our Subject Determination Model provides you with the optimal protfolio. Dual-subject Optimization offer the results of assets allocation and specific trading behavior. In addition, Single-subject Optimization still does a good job when Dual-subject Optimization cannot work due to closed gold market.

Then, we compare our model to other three strategies to confirm that our model offers the unrivaled strategy. Using our model and strategy, the initial $1,000 investment worth on 9/10/2021 will turn out to be $402,287. Five perspectives for comparison include, Cumulative Rate of Return, Annualized Rate of Return, Standard Deviation of Daily Returns, Sharpe Ratio and Max Drawdown.

As for the results, can you imagin that the initial $1,000 investment worth on 9/10/2021 will turn out to be amazing $402,287?

All of five perspectives, including Cumulative Rate of Return, Annualized Rate of Return, Standard Deviation of Daily Returns, Sharpe Ratio and Max Drawdown, prove dazzling performance.

If you are curious about further information, call us at 12345678.

The only difference between you and millionaire is–ITRS Model!

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Appendices

```
import random
from scipy.optimize import minimize
def dual_pro(M,V,omega,r,cost=(0.01,0.02,0.1)):#omega=(var1,var2,cov)
    '''
     M=f*V

    V=(V0,V1,V2)
    V0: value of cash
    V1:value of gold
    V2: value of bitcoin

    omega=(var1,var2,cov)
    var1: variance of gold
    var2: variance of bitcoin
    cov: covariance of two

    r=(r1,r2)
    r1,r2:return of gold and cash

    cost=(alpha1,aplha2,alpha), default as (0.01,0.02, 0.1)
    '''
    V0,V1,V2=V
    r1,r2=r
    var1,var2,cov=omega
    x1=0
    x2=0
    x3=0
    def obj(w):
        w1,w2=w
        mu0=w1*r1+w2*r2#
        mu=mu0-abs(w1)*cost[0]-abs(w2)*cost[1]#
        var=(w1**2*var1+w2**2*var2+2*w1*w2*cov)
        return -mu/np.sqrt(var)
```