Lista 5 - Geometria Analítica - Letícia Santos Alves - 2025.1.08.016

01- a) $\vec{BF} \to -\vec{b} + \vec{y}$

b) $\vec{AG} \to \vec{AG} = \vec{AB} + \vec{BC} + \vec{CG} \to \vec{BC} = -b + c \to \vec{AG} = \vec{b} - \vec{b} + \vec{c} + \vec{y} \to$
$\vec{AG} = \vec{c} + \vec{y}$

c) $\vec{AE} \to \vec{AE} = \vec{AF} + \vec{FE} \to \vec{AE} = \vec{y} - \vec{b}$

D) $\vec{BG} \to \vec{BG} = \vec{BA} + \vec{AG} \to \vec{BG} = -\vec{b} + \vec{c}$

E) $\vec{HB} \to \vec{HB} = \vec{HG} + \vec{GB} \to \vec{HB} = b - (-b + c + y) \to \vec{HB} = 2b - c - f$

f) $\vec{AB} + \vec{FG} \to \vec{FG} = \vec{BC} = -\vec{b} + \vec{c} \to b + (-b) + c \to \vec{C}$

G) $\vec{AD} + \vec{HG} \to \vec{AD} = \vec{BC} = -\vec{b} + \vec{c} \to \vec{HG} = \vec{AB} = \vec{b} \to -\vec{b} + \vec{c} + \vec{b} = \vec{c}$

H) $\vec{HF} + \vec{AG} - \vec{EF} \to \vec{HF} = \vec{HA} + \vec{AF} = b - c + y \to -\vec{c} + \vec{b} + \vec{f} + \vec{c} - \vec{b} = \vec{f}$

I) $2\vec{AD} - \vec{FG} - \vec{BH} + \vec{GH} \to -2b + 2c - (-b + c) - (2b - c) + (-b) \to -2b + 2c +$
$b - c + 2b + c - b \to 0c + 0b = \vec{0}$

02- a) $\vec{DF} \to \vec{DF} = \vec{DE} + \vec{EF} \to \vec{EF} = \vec{DC} \to \vec{DF} = \vec{DE} + \vec{DC}$

b) $\vec{DA} \to \vec{DA} = -\vec{DC}$

c) $\vec{DB} \to \vec{DB} = \vec{DA} + \vec{AB} \to \vec{DB} = -DC + DE$

d) $\vec{DO} \to \vec{DO} = (\vec{DC} + \vec{DE}) \div 2$

e) $\vec{EC} \to \vec{EC} = \vec{EB} + \vec{DC} \to \vec{EC} = -DE + DC \to \vec{EC} = \vec{DC} - \vec{DE}$

f) $\vec{EB} \to \vec{EB} = \vec{ED} + \vec{DB} \to \vec{EB} = -\vec{DE} + (-\vec{DC} + \vec{DE}) \to \vec{EB} = -\vec{DC}$

g) $\vec{OB} \to \vec{OB} = \vec{OD} + \vec{DB} \to \vec{OB} = -\frac{DC + DE}{2} + (-\vec{DC}) + \vec{DE} \to -\frac{3}{2}DC + \frac{1}{2}\vec{DE}$

h) $\vec{AF} \to \vec{AF} = \vec{AB} + \vec{BF} \to \vec{AF} = \vec{DE} + \vec{DE} \to \vec{AF} = 2\vec{DE}$

03- a) $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} + \vec{OF} \to \vec{0}$

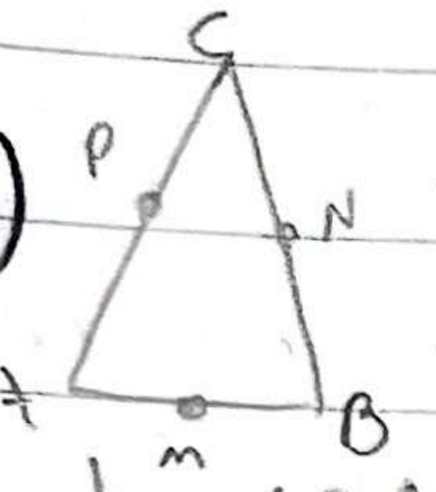
B) $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} + \vec{FA} = \vec{0}$

C) $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} = \vec{AF} \to \vec{AF} = 2\vec{OE} \,//\, \vec{AF} = 2\vec{OE} \to \vec{AF} = 2\vec{e}$

d) $\vec{OA} + \vec{OB} + \vec{OD} + \vec{OE} \to \vec{OA} + \vec{OB} + \vec{d} + \vec{e} \to OA = -\vec{OD} \to \vec{OB} = -\vec{OE} \to$
$-d - e + d + e = \vec{0}$

e) $\vec{OC} + \vec{AF} + \vec{EF} \to -\vec{OF} + 2\vec{e} + \vec{EF} \to \vec{EF} = d - (-d - e) = 2d + e \to (-d - e) + 2e +$
$(2d + e) \to \vec{d} + 2\vec{e}$

1) $\vec{AF} + \vec{DE} \to 2\vec{e} + \vec{DE} \to \vec{DE} = \vec{OE} - \vec{OD} = \vec{e} - \vec{d} \to 2\vec{e} + \vec{e} - \vec{d} \to 3\vec{e} - \vec{d}$,,

04) 

$\to \vec{AB} = \vec{u}$; $\vec{AP} = \frac{1}{2}\vec{AC} \to \vec{P} = \vec{A} + \frac{1}{2}\vec{AC}$,, $\vec{BP} = (A + \frac{1}{2}AC) - (A + AB) \to$

$\vec{BC} = \vec{v}$; $\vec{BP} = \frac{1}{2}\vec{AC} - \vec{AB}$,, $\|$ $\vec{N} = \frac{1}{2}(A + AB) + (A + AC) \to$

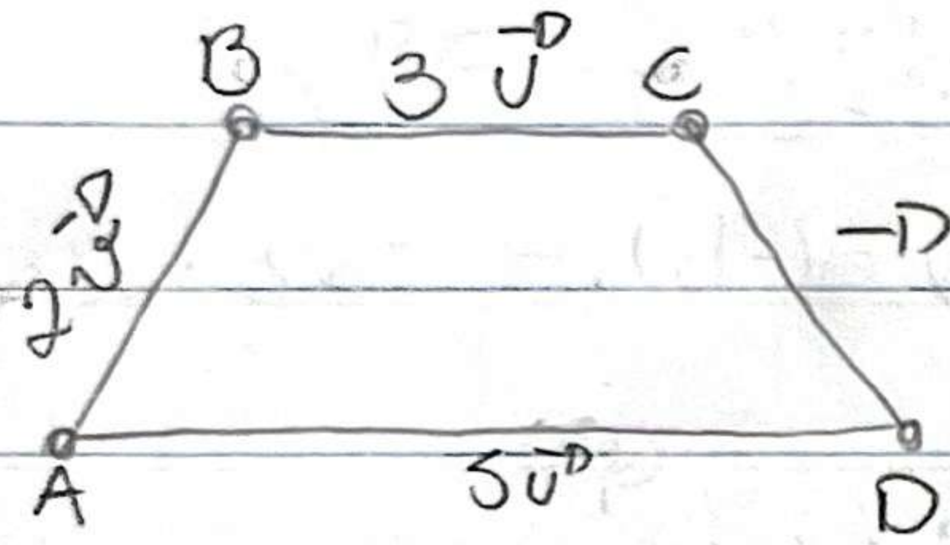
$\vec{N} = \frac{1}{2}(2A + AB + AC) \to \vec{N} = A + \frac{1}{2}(AB + AC) \to \vec{AN} = \frac{1}{2}(\vec{AB} + \vec{AC})$,,

$\vec{CM} = \vec{M} - \vec{C} = (A + \frac{1}{2}AB) - (\vec{A} + \vec{AC}) = \frac{1}{2}AB - AC$,,

05- a) $\vec{CB} = -BC = -3\vec{u} \to \vec{BA} = -\vec{AB} = -2\vec{v} \to \vec{CA} = \vec{CB} + \vec{BA} \to$

$\vec{CA} = -3\vec{u} - 2\vec{v}$,, $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = \vec{0} \to 2\vec{v} + 3\vec{u} + CD - 5\vec{u} = \vec{0} \to$

$CD = -2v - 3u + 5u \to \vec{CD} = 2\vec{u} - 2\vec{v}$,, $\vec{BD} = \vec{BA} + \vec{AD} \to \vec{BD} = -2\vec{v} + 5\vec{u}$,,

b) 

$\to$ AD e BC são lados opostos, sendo o AD a base maior.

06- $\vec{OA} = \vec{a}$; $\vec{OB} = \vec{b}$; $\vec{OC} = \vec{c}$; $\vec{D} = \vec{A} + \vec{AD} \to \vec{D} = \vec{a} + \frac{1}{4}\vec{c}$; $\vec{E} = \vec{B} + \vec{BE} \to$

$\vec{E} = \vec{b} + \frac{5}{6}\vec{a}$; $\vec{DE} = \vec{E} - \vec{D} = b + \frac{5}{6}a - a - \frac{1}{4}c \to \vec{DE} = \vec{b} - \frac{1}{6}\vec{a} - \frac{1}{4}\vec{c}$,,

07- LI = vetores independentes; $\vec{AC} = \vec{OC} - \vec{OA} = (5a + xb) - (a + 2b) \to$

$4a + (x-2)b$. $\vec{BC} = \vec{OC} - \vec{OB} = (5a + xb) - (3a + 2b) \to 2a + (x-2)b$; $\to$

$\vec{AC} = K.\vec{BC} \to 4a + (x-2)b = K.(2a + (x-2)b) \to \vec{K} = 2$; $b = (x-2) = K.(x-2) \to$

$(x-2) = 2.(x-2) \to x - 2 = 2x - 4 \to x - 2 + 4 = 2x \to \boxed{2 = x}$,,

08- $\vec{AB} = \vec{OB} - \vec{OA} = \frac{1}{n}\vec{ON} - \vec{OA}$; $\vec{AC} = \vec{OC} - \vec{OA} = \frac{1}{1+n}\vec{OM} - \vec{OA}$; $\to \vec{OM} = \vec{OA} + \vec{ON} \to$

$\vec{AC} = \frac{1}{1+n}(\vec{OA} + \vec{ON}) - \vec{OA} \to \vec{AC} = \frac{1}{1+n}\vec{OA} + \frac{1}{1+n}\vec{ON}$; $\vec{AB} = \frac{1}{n}ON - \vec{OA}$. São colineares, visto que dividem lados nesse paralelogramo, para que a reta A exista...

09- $c_1(2\vec{u} + \vec{v}) + c_2(\vec{u} - 2\vec{v}) = \vec{0}$. $(2c_1 + c_2)\vec{u} + (c_1 - 2c_2)\vec{v} = \vec{0}$; $\to$

$2c_1 + c_2 = 0$ e $c_1 - 2c_2 = \vec{0} \to 2(2c_2) + c_2 = 0 \to 5c_2 = 0 \to$

$c_2 = 0$; $c_1 = 0$,,

Eles são uma base para o plano, pois seus vetores são linearmente independentes.

10 - a) $x(u+v) + y(u-v+w) + z(u+v+w) = 0 \to (x+y+z)u + (x-y+z)v + (y+z)w = 0 \to x+y+z=0;\ x-y+z=0;\ y+z=0;$ $\to$ Logo, $x=0;$ $y=0;$ $z=0$,, os vetores são LI,,

b) $t = au + bv + cw; \to x(u+t) + y(v+t) + z(w+t) = 0 \to (x + xa + ya + za)u + (y + xb + yb + zb)v + (z + xc + yc + zc)w = 0$,, $\to$

$$\begin{vmatrix} 1+a & a & a \\ b & 1+b & b \\ c & c & 1+c \end{vmatrix} = 0 \to$$

$\det = 1 + a + b + c + ab + ac + bc + abc = 0 \to$ Para serem LD: $a+b+c = -1$,,

Para serem LI: $a+b+c \neq -1$,,

11 - a) $\vec{AB} = B - A = 1-1, 0-3, -1-2 = (0,-3,-3) = \vec{AB}$,,

$\vec{BC} = C - B = 1-1, 1-0, 0-(-1) = (0,1,1) = \vec{BC}$,,

$\vec{CA} = A - C = 1-1, 3-1, 2-0 = (0,2,2) = \vec{AC}$,,

b) $(0,-3,-3) + \frac{2}{3}(0,1,1) \to (0, -\frac{7}{3}, -\frac{7}{3})$,,

c) $(1,1,0) + \frac{1}{2}(0,-3,-3) \to (1, -\frac{1}{2}, -\frac{3}{2})$,,

D) $(1,3,2) - 2.(0,1,1) \to (1-0, 3-2, 2-2) = (1,1,0)$,,

12 - a) $\{(2,3),(0,2)\} \to$ não são paralelos na reta, LI,,

b) $\{(3,0),(-2,0)\} \to$ Paralelos na reta x, LD,,

c) $\{(2,3,4),(0,3,3)\} \to$ não são paralelos na reta, LI,,

d) $\{(1,-1,2),(1,1,0)(1,-1,1)\} \to \det = -2$,, $\to$ LI,,

e) $\{(1,-1,1),(-1,2,1),(-1,2,2)\} \to \det = 1$,, $\to$ LI,,

f) $\{(1,0,1),(0,0,1),(2,0,5)\} \to$ Segunda coordenada $= 0$, LD,,

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

$\det = 2(-1-1) + (1+1)$
$(-4) + 2 \to \det = -2$,,

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

$\det = (-2+2) - (2-1) + 2(2-1) \to$
$(0) - (1) + 2 \to \det = 1$,,

13 - a) $xu + yv = \vec{w} \rightarrow x(2,-1) + y(1,-1) = (1,1)$, $\rightarrow \begin{cases} 2x + y = 1 \\ -x - y = 1 \end{cases}$

$2x + y - x - y = 2 \rightarrow \underline{x = 2}$, $\rightarrow -2 - y = 1 \rightarrow \underline{y = -3}$,

$\vec{w} = 2\vec{u} - 3\vec{v}$,

b) $l(1,1,1) + m(0,1,1) + n(1,1,0) = (1,2,3) \rightarrow \begin{cases} l + n = 1 \\ l + m + n = 2 \\ l + m = 3 \end{cases} \rightarrow$

$l + m + n - l - n = 1 \rightarrow m = 1$, $\rightarrow l + 1 = 3 \rightarrow l = 2$,

$2 + n = 1 \rightarrow n = -1$, $\rightarrow \vec{z} = 2\vec{a} + \vec{b} - \vec{c}$,

14 - a) $\frac{1}{m} = \frac{m-1}{2n} = \frac{m}{4} \rightarrow$ se $m^2 = 4 \rightarrow m = \pm 2$, $\frac{2-1}{2n} \rightarrow \frac{1}{2n} \rightarrow n = 1$, $\rightarrow \frac{1}{-2} = \frac{-2-1}{2n} = \frac{-3}{2n} \rightarrow n = -3$, $\rightarrow (2,1)$ ou $(-2,-3)$,

b) $\frac{1}{m} = \frac{m}{n+1} = \frac{n+1}{8} \rightarrow m(n+1) = 8;\ 8m = (n+1)^2 \rightarrow 8\left(\frac{8}{n+1}\right) = (n+1)^2 \rightarrow (n+1)^3 = 8^2 \rightarrow$

$(n+1)^3 = 64 \rightarrow \underline{n = 3}$, $\rightarrow \frac{8}{4} = 2$, $\rightarrow (2,3)$,

15 - $\begin{bmatrix} m & -1 & m^2+1 \\ m^2+1 & m & 0 \\ m & 1 & 1 \end{bmatrix} \rightarrow \det = m^2+1(m^2+1-m^2) + (m^2 - (-m^2-1)) \rightarrow m^2+1(1) + m^2 + m^2 + 1 \rightarrow m^2 + 1 + 2m^2 + 1 \rightarrow 3m^2 + 2 \neq 0$,

Como o determinante nunca será 0, sempre será uma L

16 - a) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix} \rightarrow \det = 1(-1) - 1(-1-1) \rightarrow -1 + 2 \rightarrow \underline{\det = 1}$, Já que o determinante não é nulo, os vetores são LI e C é uma base V^3,

b) $2(1,1,0) + 3(1,0,1) + 7(1,1,-1) \rightarrow (2+3+7,\ 2+7,\ 3-7) \rightarrow B = (12, 9, -4)$,

c) $x(1,1,0) + y(1,0,1) + z(1,1,-1) = (2,3,7) \rightarrow \begin{cases} x + y + z = 2 \\ x + z = 3 \\ y - z = 7 \end{cases} \rightarrow$

$x + y + z - x - z = -1 \rightarrow \underline{y = -1}$, $\rightarrow -1 - z = 7 \rightarrow$

$\underline{z = 6}$, $\rightarrow x - 1 + 6 = 2 \rightarrow \underline{x = -3}$, $(-3,-1,6)$,