Lista 06 - Geometria Analítica - Letícia Santos Alves - 2025.1.08.016

01- a) $||\vec{v}|| = \sqrt{x^2+y^2+z^2} \to \sqrt{1^2+1^2+1^2} = \sqrt{3}$,,

b) $\sqrt{3^2+0+4^2} \to \sqrt{9+16} \to \sqrt{25} = 5$,,

c) $\sqrt{(-1)^2+1^2+0} \to \sqrt{1+1} \to \sqrt{2}$,,

d) $\sqrt{4^2+3^2+(-1)^2} \to \sqrt{16+9+1} \to \sqrt{26}$,,

02- a) $\vec{DH}, \vec{DC}, \vec{DA}$. É ortogonal pois seus vetores são unitários e ortogonais entre si,, (formam ângulos de 90°),,

b) $\vec{u} = \vec{CB} + \vec{CB} \to \vec{CD} = -e_2 \to \vec{CB} = \vec{DA} = e_3 \to e_3 - e_2 \; (0, -1, 1)$,,

$\vec{v} = \vec{DC} + \vec{CB} \to \vec{DC} = e_2; \; CB = e_3 \to e_2 + e_3 \; (0, 1, 1)$,,

$w = \vec{GC} \to \vec{GC} = \vec{HD} = -e_1 \; (-1, 0, 0)$,,

c) $f_1 = \frac{\vec{u}}{||\vec{u}||} \to ||\vec{u}|| = \sqrt{2} \to f_1 = (0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$,, $f_2 = \frac{v}{||v||} \to ||v|| = \sqrt{2} \to f_2 = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

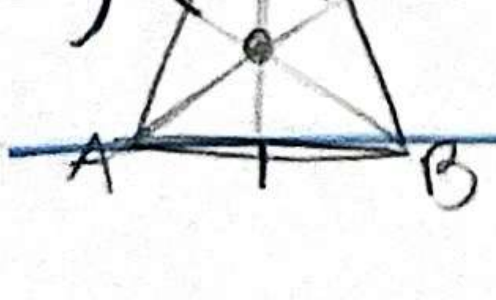
$f_3 = w = (-1, 0, 0) \to f_3 = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$,,

d) $\begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ = Matriz M → Matriz $M^{-1} = M^t$,,

e) Base E: $\vec{HB} = B - H = (1-0, 0-1, 0-1) = (1, -1, -1)$,,; → Base F = $M^t \circ$ base E = $[0, -\sqrt{2}, -\frac{1}{\sqrt{3}})$,,

03- A) $\vec{AB} = B - A = (5-2, 1-4, -3-3) = (3, -3, -6)$,,; $\vec{BC} = C - B = (0-5, -3-1, 1-(-3)) = (-5, -4, 4)$,,; $\vec{CA} = A - C = (2-0, 4-(-3), 3-1) = (2, 7, 2)$,,

b) $||\vec{AB}|| = \sqrt{3^2+(-3)^2+(-6)^2} = \sqrt{9+9+36} = \sqrt{54}$,, $||\vec{BC}|| = \sqrt{(-5)^2+(-4)^2+4^2} = \sqrt{25+16+16} = \sqrt{57}$,,; $||\vec{CA}|| = \sqrt{2^2+7^2+2^2} = \sqrt{4+49+4} = \sqrt{57}$,, → É um triângulo isósceles, pois apresenta dois lados iguais, o lado $\vec{BC}$ e $\vec{CA}$,,

c) 

→ P.M. → $\vec{AB} = (\frac{7}{2}, \frac{5}{2}, 0); \; \vec{BC} = (\frac{5}{2}, -1, -1); \; \vec{CA} = (1, \frac{1}{2}, 2)$. A mediana coincide com a mediatriz, pois é um triângulo isósceles.

d) $B\hat{C}A = \frac{\vec{CB}\cdot\vec{CA}}{||\vec{CB}||\,||\vec{CA}||} \to \frac{(5,4,-4)\cdot(2,7,2)}{\sqrt{57}\cdot\sqrt{57}} \to \frac{10+28+(-8)}{57} = \frac{30}{57} = \frac{10}{19} \to \theta = \arccos\left(\frac{10}{19}\right)$

e) $(3-5+2, -3-4+7, -6+4+2) = (0,0,0) = 0 \to$ O resultado dessa soma deve dar 0, pois é um circuito fechado.

05- a) $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{||\vec{u}||\cdot||\vec{v}||} \to ||u|| = \sqrt{2};\ ||v|| = \sqrt{4+100+4} = \sqrt{108} = 6\sqrt{3};\ \to \frac{(-2+0+2)}{\sqrt{2}\cdot 6\sqrt{3}} \to \frac{0}{2\cdot 6\sqrt{3}} = 0 \to \theta = \frac{\pi}{2}$

b) $||\vec{u}|| = \sqrt{3},\ ||\vec{v}|| = \sqrt{3};\ \to \frac{(-1+1+1)}{\sqrt{3}\cdot\sqrt{3}} = \frac{1}{3} \to \theta = \arccos\left(\frac{1}{3}\right)$

c) $||\vec{u}|| = \sqrt{3^2+3^2+0^2} = \sqrt{9+9} = \sqrt{18};\ ||v|| = \sqrt{2^2+1^2+(-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3;\ \to \frac{(6+3+0)}{3\sqrt{2}\cdot 3} \to \frac{9}{9\sqrt{2}} = \frac{1}{\sqrt{2}} \to \theta = \frac{\pi}{4}$

d) $||u|| = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2;\ ||v|| = \sqrt{(\sqrt{3})^2+1^2+(2\sqrt{3})^2} = \sqrt{3+1+4\cdot3} = \sqrt{16} = 4;\ \to \frac{(3+1+0)}{2\cdot 4} \to \frac{4}{8} = \frac{1}{2} \to \theta = \frac{\pi}{3}$

06- a) $(x+1)\cdot(x-1), -1, -4 = 0 \to x^2 - x + x - 1, -1, -4 = 0 \to x^2 = 6 \to x = \pm\sqrt{6}$
$x' = \sqrt{6}$ ou $x'' = -\sqrt{6}$

b) $4x, x^2+4 = 0 \to \Delta = 4^2 - 4\cdot1\cdot4 \to \Delta = 16-16 = 0,\ x = \frac{-4 \pm \sqrt{0}}{2} \to x = -2$

07- a) $\begin{vmatrix} i & j & k \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{vmatrix} \to i(-1\cdot3 - 5\cdot(-2)) - j(4\cdot3 - 5\cdot1) + k(-4\cdot(-2) - (-1)\cdot1) = (7,-7,-7) \to k(7\cdot1 + (-7)\cdot1 + (-7)\cdot1) = -1 \to k(-7) = -1 \to k = 1/7 \to \vec{u} = 1/7(7,-7,-7) \to \vec{u} = (1,-1,-1)$

b) $\begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{vmatrix} \to i(3\cdot6 - (-1)\cdot(-4)) - j(2\cdot6 - (-1)\cdot2) + k(2\cdot(-4) - 3\cdot2) \to (14,-14,-14) \to ||(1,-1,-1)|| = \sqrt{3} \to \vec{u} = \pm 3\sqrt{3}\,\frac{(1,-1,-1)}{\sqrt{3}} = \pm(1,-1,-1) \to \vec{u} = (3,-3,-3)$

c) $\cos\theta = \frac{(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})}{||\vec{u}+\vec{v}||\ ||\vec{u}-\vec{v}||} \to \frac{||\vec{u}||^2 - ||\vec{v}||^2}{\sqrt{||u||^2+||\vec{v}||^2+2\vec{u}\cdot\vec{v}}\cdot\sqrt{||\vec{u}||^2+||\vec{v}||^2-2\vec{u}\cdot\vec{v}}} \to$

$\cos\theta = \frac{5-1}{\sqrt{5+1+\sqrt{10}}\cdot\sqrt{5+1-\sqrt{10}}} \to \frac{4}{\sqrt{36-10}} = \frac{4}{\sqrt{26}} \to \theta = \arccos\left(\frac{4}{\sqrt{26}}\right)$

08- a) $\left(\frac{u\cdot v}{||u||^2}\right)\vec{u} \to ||u||^2 = \sqrt{9+1+1} = \sqrt{11} = 11;\ \to (3+1+2) = 6 \to \frac{6}{11}(3,-1,1) \to$
projeção $= \left(\frac{18}{11}, -\frac{6}{11}, \frac{6}{11}\right)$

b) $||u||^2 = (\sqrt{9+1})^2 \rightarrow (\sqrt{10})^2 = 10$; $(-3+3+0) = 0 \rightarrow$ projeção: $(0,0,0)$

c) $||u||^2 = (\sqrt{4+1+4})^2 \rightarrow (\sqrt{9})^2 = 9$; $(2+1+2) = 5 \rightarrow \frac{5}{9} \cdot (-2,1,2) \rightarrow$ projeção $= (-\frac{10}{9}, \frac{5}{9}, \frac{10}{9})$

d) $||u||^2 = (\sqrt{4+16+64})^2 \rightarrow (\sqrt{84})^2 = 84$; $(-2+(-8)+(-32)) = -42 \rightarrow -\frac{42}{84} = -\frac{21}{42} = -\frac{1}{2} \cdot (-2,-4,-8) = (1,2,4) =$ projeção

09- a) $||u||^2 = (\sqrt{4+4+1})^2 \rightarrow (\sqrt{9})^2 = 9$; $(6+12+0) = 18 \rightarrow \frac{18}{9} = \frac{2}{1} \cdot (2,-2,1) \rightarrow$ projeção $= (4,-4,2) \rightarrow$ projeção de v sobre u

$||v||^2 = (\sqrt{9+36})^2 \rightarrow (\sqrt{45})^2 = 45 \rightarrow \frac{18}{45} = \frac{2}{5} \cdot (3,-6,0) \rightarrow$ projeção $= (\frac{6}{5}, -\frac{12}{5}, 0) \rightarrow$ projeção de u sobre v

b) $\vec{v} = \vec{p} + \vec{q} \rightarrow \vec{p} =$ projeção de $\vec{v} = (4,-4,2) \rightarrow \vec{q} = \vec{v} - \vec{p} \rightarrow q = (3,-6,0) - (4,-4,2) \rightarrow \vec{q} = (-1,-2,-2)$

c) $\begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{vmatrix} \rightarrow \det = i(-(-6)) - j \cdot (-3) + k(-12-(-6)) \rightarrow i(6) - j(-3) + k(-6) \rightarrow (6,-3,-6) \rightarrow ||u \cdot v|| = \sqrt{36+9+36} \rightarrow \sqrt{81} = 9$

10- a) $\begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{vmatrix} \rightarrow \det = k(12-15) \rightarrow \det = -3k$; $||u \cdot v|| = \sqrt{3^2} = 3$

b) $\begin{vmatrix} i & j & k \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{vmatrix} \rightarrow \det = i(10) - j(2) + k(14) \rightarrow (10,2,14)$; $||u \cdot v|| = \sqrt{10^2+2^2+14^2} = \sqrt{100+4+196} \rightarrow \sqrt{300} = 10\sqrt{3}$

c) $\begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{vmatrix} \rightarrow \det = i(-12-1) - j(4-1) + k(1-(-3)) \rightarrow i(-13) - j(3) + k(4) \rightarrow (-13,3,4) \rightarrow ||\vec{u} \cdot \vec{v}|| = \sqrt{(-13)^2+3^2+4^2} \rightarrow \sqrt{169+9+16} = \sqrt{194}$

d) $\begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{vmatrix} \rightarrow \det = i(4-4) - j(8-8) + k(4-4) \rightarrow i(0) - j(0) + k(0) \rightarrow (0,0,0)$; $\rightarrow ||u \cdot v|| = 0$

11- a) $\|u.v\| = \|u\|\|v\| \operatorname{sen}\theta \rightarrow u.v = \|u\|\|v\|\cos\theta \rightarrow \|u.v\|^2 + (u.v)^2 = \|\vec{u}\|^2\|\vec{v}\|^2(\operatorname{sen}^2\theta + \cos^2\theta) = \|u\|^2\|v\|^2 \rightarrow$ logo: $\|u.v\|^2 = \|u\|^2 \|v\|^2 - (\vec{u}.\vec{v})^2$,,

b) $\|u.v\| = \sqrt{1^2 \cdot 5^2 - 3^2} \rightarrow \sqrt{25-9} \rightarrow \sqrt{16} = \underline{4}$,,

c) $\|\vec{AB}.\vec{AC}\| = l.l.\operatorname{sen}\pi/3 \rightarrow \frac{l^2\sqrt{3}}{2}$,,

12- a) $\begin{cases} \vec{x}.(2\vec{i}+3\vec{j}+4\vec{k}) = 9 \\ \vec{x}\times(-\vec{i}+\vec{j}-\vec{k}) = -2\vec{i}+2\vec{k} \end{cases} \rightarrow \begin{cases} \vec{x}\cdot(2,3,4) = 9 \\ \vec{x}\times(-1,1,-1) = (-2,0,2) \end{cases} \rightarrow$

$\vec{x}\times(-1,1,-1) = (x_2\cdot(-1) - x_3\cdot 1,\ x_3\cdot(-1) - x_1\cdot(-1),\ x_1\cdot 1 - x_2\cdot(-1)) \rightarrow$

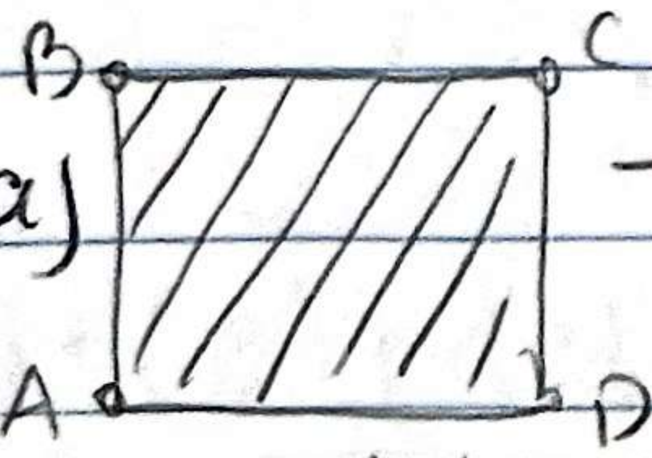
$(-x_2 - x_3,\ -x_3 + x_1,\ x_1 + x_2) = (-2,0,2)$,, $\rightarrow \begin{cases} -x_2 - x_3 = -2 \\ -x_3 + x_1 = 0 \\ x_1 + x_2 = 2 \end{cases} \rightarrow$

$\vec{x} = (1,1,1)$,,

b) $\begin{cases} \vec{x}\times(1,0,1) = 2(1,1,-1) \\ \|\vec{x}\| = \sqrt{6} \end{cases} \rightarrow \vec{x}\times(1,0,1) = (x_2\cdot 1 - x_3\cdot 0,\ x_3\cdot 1 - x_1\cdot 1,\ x_1\cdot 0 - x_2\cdot 1) = (x_2,\ x_3 - x_1,\ -x) = (2,2,-2)$,, $\rightarrow x_2 = 2 \rightarrow \|x\| = \sqrt{x_1^2 + 2^2 + (x_1+2)^2} = \sqrt{6} \rightarrow \sqrt{2x_1^2 + 4x_1 + 8} = 6 \rightarrow$

$\Delta = b^2 - 4ac \rightarrow \Delta = 16 - 4.2.2 \rightarrow \Delta = 0 \rightarrow x = \frac{-4\pm 0}{4} = -1$,, $\underline{(-1,2,1)}$,,

c) $-3x_1 + 3x_3 = 0 \rightarrow x_1 = x_3 \rightarrow 2x_1 - 2x_2 = 0 \rightarrow x_1 = x_2 \rightarrow \|\vec{x}\| = \sqrt{x_1^2 + x_1^2 + x_1^2} \rightarrow \sqrt{3x_1^2} \rightarrow x = \pm 1$,, $\rightarrow \underline{(-1,-1,-1)}$

13- a) 

$\rightarrow \vec{AD} = (D-A) = (5-3,\ 3-2,\ 3-(-1)) \rightarrow (2,1,4) \rightarrow \det = i(4+1) - j(4+2) + k(1-2) \rightarrow \det = i(5) - j(6) + k(-1)$,, $\|\vec{AB}\times\vec{AD}\| = \sqrt{5^2 + 6^2 + 1^2} \rightarrow \sqrt{25+36+1} \rightarrow \sqrt{62}$,,

$$\begin{bmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{bmatrix}$$

b) $\begin{bmatrix} i & j & k \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \rightarrow \det = i(3) - j(-3) + k(-1) \rightarrow (3,3,-1) \rightarrow$ Area $= \frac{1}{2}\cdot(\sqrt{9+9+1}) = \frac{1}{2}\cdot(\sqrt{19}) = \frac{\sqrt{19}}{2}$,, $\vec{BC} = \vec{AC} - \vec{AB} \rightarrow \vec{BC} = (1,0,3) \rightarrow \|BC\| = \sqrt{1+9} = \sqrt{10} \rightarrow \frac{\sqrt{19}}{2} = \frac{1}{2}\cdot\sqrt{10}\cdot h \rightarrow \underline{h = \frac{\sqrt{190}}{10}}$,, 14 - a) Produto misto $= \vec{u}\cdot(\vec{v}\times\vec{w}) = (\vec{u}\times\vec{v})\cdot\vec{w} \to \vec{v}\times\vec{w} = \begin{vmatrix} i & j & k \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$

O produto escalar resulta no determinante da matriz formada por $\vec{u}, \vec{v}, \vec{w}$.

b) $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{bmatrix} \to \det = 1(-(-4)) - 3(-2) + 2(-1) = (4 - 6 + 2) = 0 \to$ Como $[\vec{u}, \vec{v}, \vec{w}] = 0$, qualquer combinação linear resultará em 0.

15 - a) $\vec{AD} = \vec{AF} - \vec{AB} - \vec{BE} = (3-1-2, 5-0-2, 6-1-2) = (0,3,3) = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 3 & 3 \end{vmatrix}$

$\to i(-3) - j(3) + k(3) = \underline{(-3,3,3)} \to \sqrt{9+9+9} = \sqrt{27} = \underline{3\sqrt{3}}$

b) $\vec{AE} = \vec{AB} + \vec{BE} = (1+2, 0+2, 1+2) = (3,2,3) \to \begin{vmatrix} 1 & 0 & 1 \\ 0 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix} \to i(3-6) - 0 + (-9) \to$

$3 - 9 = -6 \to |-6| = \underline{6} = \text{Volume}$

c) $V = a.h \to 6 = 3\sqrt{3}.h \to h = \frac{2\sqrt{3}}{3}$

d) $V = \frac{1}{6}\cdot 6 \to \underline{V = 1}$

e) $\vec{DB} = \vec{AB} - \vec{AD} = (1,-3,-2)$; $\vec{DE} = \vec{AE} - \vec{AD} = (3,-1,0)$; $\begin{vmatrix} i & j & k \\ 1 & -3 & -2 \\ 3 & -1 & 0 \end{vmatrix} \to$

$i(-2) - j(6) + k(-1+9) \to \underline{(-2,-6,8)} \to \text{Área} = \frac{1}{2}\sqrt{4 + 36 + 64} \to \frac{1}{2}\sqrt{104} = \underline{\sqrt{26}} \to V = \frac{1}{3}a_b.h \to 1 = \frac{1}{3}\sqrt{26}.h \to h = \underline{\frac{3\sqrt{26}}{26}}$

04 - a) Se $\vec{w} = \vec{u} - \left(\frac{u\cdot v}{\|v\|^2}\right)\vec{v} \to \|\vec{w}\|^2 = \left(\vec{u} - \left(\frac{\vec{u}\cdot\vec{v}}{\|v\|^2}\right)\vec{v}\right)\cdot\left(\vec{u} - \left(\frac{u\cdot v}{\|v\|^2}\right)\vec{v}\right) \geq 0 \to$

$\|\vec{u}\|^2 - 2\left(\frac{(u\cdot v)^2}{\|v\|^2}\right) + \left(\frac{(u\cdot v)^2}{\|v\|^2}\right) \geq 0 \to \|\vec{u}\|^2 - \frac{(u\cdot v)^2}{\|v\|^2} \geq 0 \to (\vec{u}\cdot\vec{v})^2 \leq \|\vec{u}\|^2.\|v\|^2 \to$

$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|.\|\vec{v}\|$

b) $\|u+v\| \leq \|\vec{u}\| + \|\vec{v}\| \to \|\vec{u}+\vec{v}\|^2 = \|u\|^2 + 2u\cdot v + \|v\|^2 \leq (\|u\| + \|v\|)^2 =$

$\|u\|^2 + 2\|u\|\|v\| + \|v\|^2 \to \|u+v\|^2 \leq \|u\|^2 + 2\|u\|\|v\| + \|v\|^2 \to \|u+v\| \leq \|u\| + \|v\|$

c) $4\vec{u}\cdot v = \|u+v\|^2 - \|u-v\|^2 \to \|u+v\|^2 = \|u\|^2 + 2u\cdot v + \|v\|^2 \to$

$\|u-v\|^2 = \|u\|^2 - 2u\cdot v\|v\|^2 \to \|u+v\|^2 - \|u-v\|^2 = 4\vec{u}\cdot\vec{v}$