Lista 08 - Geometria Analítica - Letícia Santos Alves - 2025.1.08.016

01- a) $\vec{V_r} = (\frac{1}{2}, 1, 1)$; $\vec{V_s} = (1, \frac{3}{2}, 3)$; $\text{sen } a = \dfrac{\frac{\sqrt{37}}{4}}{\frac{3}{2} \cdot \frac{7}{2}} = \boxed{\dfrac{\sqrt{37}}{21}}$

$\vec{V_r} \times \vec{V_s} = (1.3 - 1.\frac{3}{2}, -(\frac{1}{2}.3 - 1.1), \frac{1}{2}.\frac{3}{2} - 1.1) = (\frac{3}{2}, \frac{1}{2}, -\frac{1}{4})$

$|\vec{V_r} \times \vec{V_s}| = \sqrt{(\frac{3}{2})^2 + (\frac{1}{2})^2 + (-\frac{1}{4})^2} \to \sqrt{\frac{9}{4} + \frac{1}{4} + \frac{1}{16}} = \sqrt{\frac{37}{16}} = \frac{\sqrt{37}}{4}$

$|\vec{V_r}| = \sqrt{(\frac{1}{2})^2 + 1^2 + 1^2} \to \sqrt{\frac{1}{4} + 1 + 1} \to \sqrt{\frac{9}{4}} = \frac{3}{2}$

$|\vec{V_s}| = \sqrt{1^2 + (\frac{3}{2})^2 + 3^2} \to \sqrt{1 + \frac{9}{4} + 9} \to \sqrt{\frac{49}{4}} = \frac{7}{2}$

b) $\vec{V_r} \times \vec{V_s} = (-1.0 - 1.1, -(0.0 - 1.1), 0.1 - (-1).1) = (-1, -1, 1)$

$|\vec{V_r} \times \vec{V_s}| = \sqrt{(-1)^2 + (-1)^2 + 1^2} = \sqrt{1+1+1} = \sqrt{3}$

$|\vec{V_r}| = \sqrt{0^2 + (-1)^2 + 1^2} \to \sqrt{0+1+1} = \sqrt{2}$ $\qquad \text{sen } a = \dfrac{\sqrt{3}}{\sqrt{2}.\sqrt{2}} = \boxed{\dfrac{\sqrt{3}}{2}}$

$|\vec{V_s}| = \sqrt{1^2 + 1^2 + 0^2} \to \sqrt{1+1+0} = \sqrt{2}$

c) $\cos a = \dfrac{|(-3).2 + 0.0 + 1.1)|}{\sqrt{(-3)^2 + 0^2 + 1^2} \cdot \sqrt{2^2 + 0^2 + 1^2}} \to \dfrac{5}{\sqrt{10}.\sqrt{5}} = \dfrac{5}{\sqrt{50}} \to \dfrac{1}{\sqrt{2}} = \boxed{\dfrac{\sqrt{2}}{2}}$

d) $\cos a = \dfrac{|1.1 + (-2).2 + 3.1|}{\sqrt{1^2 + (-2)^2 + 3^2} \cdot \sqrt{1^2 + 2^2 + 1^2}} \to \dfrac{0}{\sqrt{14}.\sqrt{6}} \to$ (perpendicular)

02- a) 45° com r: $\cos 45 = \dfrac{|\vec{PQ} \cdot \vec{V_r}|}{|\vec{PQ}| \cdot |\vec{V_r}|} \to \dfrac{\sqrt{2}}{2} = \dfrac{|-\lambda|}{\sqrt{1+\lambda^2+\mu^2} \cdot 1} \to \sqrt{2}\sqrt{1+\lambda^2+\mu^2} = 2|\lambda| \to$

$\sqrt{2} \cdot 2|\mu| = 2|\lambda| \to \lambda = \sqrt{2}\mu$

60° com s: $\dfrac{1}{2} = \dfrac{|\mu|}{\sqrt{1+\lambda^2+\mu^2} \cdot 1} \to \sqrt{1+\lambda^2+\mu^2} = 2|\mu| \to 1 + 3\mu^2 = 4\mu^2 \to \mu^2 = 1 \to \mu = \pm 1$

se $\mu = 1$; $\lambda = \sqrt{2}$ / se $\mu = -1$; $\lambda = -\sqrt{2}$

$P = (0, 2 \pm \sqrt{2}, 0)$ e $Q = (1, 2, \pm 1)$

03- a) $\vec{V_r} = (0,1,1)$; $\to \text{sen } \theta = \dfrac{1}{\sqrt{2} \cdot 1} \to \text{sen } \theta = \dfrac{1}{\sqrt{2}} \to \theta = \dfrac{\pi}{4}$ rad

$\vec{n_\pi} = (0,0,1)$;

b) $\vec{V_r} = (-1,1,2)$; $\to \text{sen } \theta = \dfrac{|(-1).2 + 1.(-1) + 2.0)|}{\sqrt{6} \cdot \sqrt{5}} \to \text{sen} = \dfrac{3}{\sqrt{3}} = \dfrac{\sqrt{30}}{10} \to \theta = \text{arcsen}\left(\dfrac{\sqrt{30}}{10}\right)$ rad

$\vec{n_\pi} = (2,-1,0)$;

c) $\vec{v_r} = (1,1,-2)$; $\text{sen}\,\theta = \dfrac{|1+1+2|}{\sqrt{6}\cdot\sqrt{3}} \rightarrow \dfrac{4}{\sqrt{18}} \rightarrow \dfrac{2\sqrt{2}}{3} \rightarrow \theta = \text{arcsen}\left(\dfrac{2\sqrt{2}}{3}\right)\text{rad}$,,
$\vec{n_\pi} = (1,1,-1)$;

04- $\vec{n_1} = (1,1,1)$; $\text{sen}\,45 = \dfrac{|a-b|}{\sqrt{2}\cdot\sqrt{a^2+b^2+c^2}} \rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{|a-b|}{\sqrt{2}\cdot\sqrt{a^2+b^2+c^2}} \rightarrow |a-b| = \sqrt{a^2+b^2+c^2}$
$\vec{n_2} = (1,-1,0)$;
$(a-b)^2 = a^2+b^2+c^2 \rightarrow 2ab = c^2 \rightarrow -2ab = (a+b)^2 \rightarrow -2ab = a^2+2ab+b^2 \rightarrow$
$a^2+4ab+b^2 = 0 \rightarrow b = (-4a \pm \sqrt{16a^2-4a^2})/2 \rightarrow (-4a \pm 2\sqrt{3}a)/2 \rightarrow$
$(-2\pm\sqrt{3})a$; $\rightarrow a = 1$; $b = -2+\sqrt{3}$; $c = -1-(-2+\sqrt{3}) = 1-\sqrt{3}$,, $\rightarrow$
$|\vec{v}| = \sqrt{1^2+(-2+\sqrt{3})^2+(1-\sqrt{3})^2} = \sqrt{1+4-4\sqrt{3}+3+1-2\sqrt{3}+3} = \sqrt{12-6\sqrt{3}}$
$\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{(1, -2+\sqrt{3}, 1-\sqrt{3})}{\sqrt{12-6\sqrt{3}}}$,,

05- a) $\vec{n_1} = (2,1,-1) \rightarrow \cos\theta = \dfrac{|2.1+1.(-1)+(-1).3|}{\sqrt{6}\cdot\sqrt{11}} = \dfrac{2}{\sqrt{66}} \rightarrow \theta = \arccos\left(\dfrac{\sqrt{66}}{33}\right)\text{rad}$,,
$\vec{n_2} = (1,-1,3)$

b) $\vec{n_1} = (0,-1,0)$; $\rightarrow \cos\theta = \dfrac{|0.1+(-1).1+0.1|}{\sqrt{1}\cdot\sqrt{3}} = \dfrac{1}{\sqrt{3}} \rightarrow \theta = \arccos\left(\dfrac{\sqrt{3}}{3}\right)\text{rad}$,,
$\vec{n_2} = (1,1,1)$;

c) $\vec{n_1} = (0,-1,1)$; $\rightarrow \cos\theta = \dfrac{|0.0+(-1).0+1.(-1)|}{\sqrt{2}\cdot\sqrt{1}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \rightarrow \theta = \dfrac{\pi}{4}\text{ rad}$,,
$\vec{n_2} = (0,0,-1)$;

06- $\vec{n_1} = (2,-1,1)$; $\rightarrow \vec{n_1}\cdot\vec{n_2} = (2.1+(-1).(-2)+1.1) = (2+2+1) = 5$,,
$\vec{n_2} = (1,-2,1)$; $|\vec{n_1}| = \sqrt{6}$,, ; $|\vec{n_2}| = \sqrt{6}$,,
$\cos\theta = \dfrac{|5|}{\sqrt{6}\cdot\sqrt{6}} \rightarrow \dfrac{5}{6} \rightarrow \theta = \arccos\left(\dfrac{5}{6}\right)$,,

07- a) Ponto genérico em r: $X = (1+t, t/2, t)$; $d(X,A)^2 = t^2+(t/2-1)^2+t^2 \rightarrow 2t^2 + \frac{t^2}{4} - t + 1$,, ; $d(X,B)^2 = (1+t)^2+(\frac{t}{2}-1)^2+(t-1)^2 \rightarrow (1+2t+t^2)+(\frac{t^2}{4}-t+1)+(t^2-2t+1) =$
$2t^2+\frac{t^2}{4}-t+3$,, ; Equidistância: $2t^2+\frac{t^2}{4}-t+1 = 2t^2+\frac{t^2}{4}-t+3 \rightarrow 1 \neq 3$,,
Logo, não existem pontos na reta que equidistam de A e B,,

b) $X=(4\lambda, 2\lambda, 4-3\lambda) \to d(X,A)^2 = (4\lambda-2)^2 + (2\lambda-2)^2 + (4-3\lambda-5)^2 = 29\lambda^2 - 18\lambda + 9$,, $d(X,B)^2 = (4\lambda)^2 + (2\lambda)^2 + (4-3\lambda-1)^2 = 29\lambda^2 - 18\lambda + 9$,,; Equidistância: $29\lambda^2 - 18\lambda + 9 = 29\lambda^2 - 18\lambda + 9$,,; Logo, todos os pontos da reta são equidistantes de A e B

c) $X=(2+\lambda, 3+\lambda, -3+\lambda) \to d(X,A)^2 = (1+\lambda)^2 + (2+\lambda)^2 + (-3+\lambda)^2 = 3\lambda^2 + 14$,,; $d(X,B)^2 = (\lambda)^2 + (1+\lambda)^2 + (-7+\lambda)^2 = 3\lambda^2 - 12\lambda + 50$;,, Equidistância $= 3\lambda^2 + 14 = 3\lambda^2 - 12\lambda + 50 \to 12\lambda = 36 \to \lambda = 3$,,; Logo, para $\lambda = 3$, $X = (5,6,0)$,,

08- a) $d(P,r) = \|\vec{P_0P} \times \vec{v}\| / \|\vec{v}\| \to \vec{v} = (3,2,1)$; $A=(1,-2,0)$; $\vec{AP} = (-3,2,1)$,, $\to$

$\begin{vmatrix} i & j & k \\ -3 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0, -(-6), -12 = (0, 6, -12)$,, $\to \|\vec{AP} \times \vec{v}\| = \sqrt{0^2 + 6^2 + (-12)^2} = \sqrt{36+144} = \sqrt{180}$,, $\|\vec{v}\| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$,,; $\to d = \frac{6\sqrt{5} \cdot \sqrt{14}}{14} = \frac{6\sqrt{70}}{14} = \boxed{\frac{3\sqrt{70}}{7}}$,,

b) $A=(2,0,1)$; $\vec{v} = (4,-3,-2)$,, $\to \vec{AP} = (1-2, -1-0, 4-1) = (-1,-1,3)$, $\to$

$\begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ 4 & -3 & -2 \end{vmatrix} = (11, 10, 7)$,, $\to \|\vec{AP} \times \vec{v}\| = \sqrt{11^2+10^2+7^2} = \sqrt{121+100+49} = \sqrt{270}$,, $\to \|\vec{v}\| = \sqrt{4^2+(-3)^2+(-2)^2} = \sqrt{29}$,, $\to d = \frac{3\sqrt{30}}{\sqrt{29}} = \frac{3\sqrt{30} \cdot \sqrt{29}}{29} = \boxed{\frac{3\sqrt{870}}{29}}$,,

c) $X = (0, 3/2, 1/2) + \lambda(1, 1/2, 1/2) \to A = (0, \frac{3}{2}, \frac{1}{2})$; $\vec{v} = (1, \frac{1}{2}, \frac{1}{2})$; $\vec{AP} = (0, -5/2, -1/2)$,,

$\begin{vmatrix} i & j & k \\ 0 & -\frac{5}{2} & -\frac{1}{2} \\ 1 & 1/2 & 1/2 \end{vmatrix} = (-1, -\frac{1}{2}, 5/2)$,, $\to \sqrt{1 + \frac{1}{4} + \frac{25}{4}} \to \sqrt{30/4} = \frac{\sqrt{30}}{2}$,, $\to \|\vec{v}\| = \sqrt{1 + \frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{6}}{2}$,, $\to$ $d = \frac{\sqrt{30}/2}{\sqrt{6}/2} = \frac{\sqrt{30}}{\sqrt{6}} = \underline{\sqrt{5}}$,,

09- $r: (t, 2-t, 2t-2)$; $s: (k, k, k-1) \to P_r = (t, 2-t, 2t-2) \to P_s = (0,0,-1)$; $\vec{v_s} = (1,1,1)$, $\vec{P_sP_r} = (t, 2-t, 2t-1)$; $\vec{P_sP_r} \times \vec{v_s} = (3-3t, t-1, 2t-2)$, $\|\vec{v_s}\|^2 = 1^2+1^2+1^2 = 3$; $\|\vec{P_sP_r}\|^2 = (3-3t)^2 + (t-1)^2 + (2t-2)^2 = 9(t-1)^2 + (t-1)^2 + 4(t-1)^2 \to 14(t-1)^2$,, $\to d = \frac{14(t-1)^2}{3} = \frac{14}{3} \to$ $(t-1)^2 = 1 \to t-1 \pm 1$,, Logo, para $t=2$, $P=(2,0,2)$; para $t=0$, $P=(0,2,-2)$,,

10- a) $P_0 = (1,0,0)$; $\vec{v_1} = (1,0,0)$, $\vec{v_2} = (-1,0,3)$; $\vec{n} = (0.3-0.0), -(1.3-0.(-1)), (1.0-0.(-1)) = (0,-3,0)$,,; $d = |0.1 - 3.3 + 0.4 + 0| / \sqrt{0^2+(-3)^2+0^2} = 9/3 = \boxed{3}$,,

b) $d(P,\pi) = |1.0 - 2.0 - 2.(-6) - 6| / \sqrt{1^2+(-2)^2+(-2)^2} \to |12-6| / \sqrt{9} = \frac{6}{3} = \boxed{2}$,,

c) $d = (|2.1 - 1.1 + 2.1 - 3|) / \sqrt{2^2 + (-1)^2 + 2^2} = (|2 - 1 + 2 - 3|) / \sqrt{9} \to \frac{0}{3}$,

$d(P, \pi) = 0$, Logo, P está no plano,,

11 - $r: x = 2 - y \to x = 2 - t;\ y + z = y \to z = 0;\ P_1 = (2 - t, t, 0); \to x - 2y - z - 1 = 0 \to$
$d = (|1(2-t) - 2(t) - 1.0) - 1|) / \sqrt{1^2 + (-2)^2 + (-1)^2} \to (|2 - t - 2t - 1|) / \sqrt{6} = \frac{|1 - 3t|}{\sqrt{6}}$
$\frac{|1 - 3t|}{\sqrt{6}} = \sqrt{6} \to |1 - 3t| = 6;\ t = 1 - 3t = 6 \to t = \frac{-5}{3}$ ou $t = 1 - 3t = -6 = \frac{7}{3}$,,
Logo, para $t = \frac{-5}{3}$, $P_1 = (\frac{11}{3}, -\frac{5}{3}, 0)$; para $t = \frac{7}{3}$, $P_2 = (-\frac{1}{3}, \frac{7}{3}, 0)$,,

12 - a) $r: P_1 = (2, 1, 0),\ \vec{v_1} = (1, -1, 1);\ s: P_2 = (0, -1, 1),\ \vec{v_2} = (1, 2, -3); \to \overrightarrow{P_1P_2} =$
$(-2, 2, 1);\ \vec{v_1} \times \vec{v_2} = (1, 4, 3); \to -2.1 + (-2).4 + 1.3 = -2 - 8 + 3 = -7$,, $\to$
$\|\vec{v_1} \times \vec{v_2}\| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$,, $\to d = \frac{|7|}{\sqrt{26}} = \boxed{\frac{7.\sqrt{26}}{26}}$,,

b) $P_1 = (-4, 0, -5),\ \vec{v_1} = (3, 4, -2);\ P_2 = (21, -5, 2),\ \vec{v_2} = (6, -4, -1); \to \overrightarrow{P_1P_2} = (25,$
$-5, 7);\ \vec{v_1} \times \vec{v_2} = (-12, -9, -36)\ \vec{n} = (4, 3, 12);\ (\overrightarrow{P_1P_2}) \times \vec{n} = 25.4 + (-5).3 +$
$7.12 = 100 - 15 + 84 = 169$,, $\to \|n\| = \sqrt{16 + 9 + 144} = 13 \to d = \frac{|169|}{13} = \boxed{13}$,,

c) $P_1 = (1, 0, 0),\ \vec{v_1} = (-4, 1, 2);\ P_2 = (0, 0, 2),\ \vec{v_2} = (-4, 1, 2) \to$ Logo, $\vec{v_1} = \vec{v_2}$;
$\overrightarrow{P_2P_1} = (1, 0, -2);\ \overrightarrow{P_2P_1} \times \vec{v_2} = (2, 6, 1) \to \sqrt{4 + 36 + 1} = \sqrt{41}$,, $\to \|\vec{v_2}\| = \sqrt{16 + 1 + 4} =$
$\sqrt{21} \to d = \frac{\sqrt{41}}{\sqrt{21}} = \frac{\sqrt{41} \cdot \sqrt{21}}{\sqrt{21} \cdot \sqrt{21}} = \boxed{\frac{\sqrt{861}}{21}}$,,

13 - a) $\vec{v_r} = (3, 3, 3)$ ou $(1, 1, 1);\ \vec{n_\pi} = (1, 0, 0) \times (0, 1, 0) = (0, 0, 1)$,, $\to \vec{v_r} \times$
$\vec{n_\pi} = 1.0 + 1.0 + 1.1 \to 1 \neq 0$,, Reta e plano não paralelos, distância $= 0$,,

b) $\vec{v_r} = (1, -1, 1) \times (2, 1, -1) = (0, 3, 3)$ ou $(0, 1, 1);\ \vec{n_\pi} = (1, 1, -1); \to \vec{v_r} \times \vec{n_\pi} = 0 + 1 - 1 = 0$,
São paralelas, $P_r = (1, 1, 0)$,,; $d = \frac{|-2|}{\sqrt{3}} \to \boxed{d = \frac{2\sqrt{3}}{3}}$,,

c) $\vec{v_r} = (1, 1, 1),\ \vec{n_\pi} = (2, 1, -3); \to \vec{v_r} \times \vec{n_\pi} = 2 + 1 - 3 = 0 \to$ São paralelos,
$P_r = (0, 1, -3) \to d = |2.0 + 1.1 - 3.(-3) - 10| / \sqrt{2^2 + 1^2 + 3^2} \to d = \frac{|0|}{\sqrt{14}}$,,
Logo, a reta está contida no plano,,

14- a) $\vec{n_1}=(2,-1,2)$; $\vec{n_2}=(4,-2,4)$, ou seja, $\vec{n_2}=2\vec{n_1}$,, $\rightarrow \pi_1 = 4x-2y+4z+0$ $=0$,, $\rightarrow d=|0-(-21)|/\sqrt{16+4+16} \rightarrow d=|21|/6 \rightarrow \boxed{d=7/2}$,,

b) $\vec{n_1}=(2,2,2)$ ou $(1,1,1)$; $\vec{n_2}=(-1,0,3)\times(1,1,0)=(-3,3,-1)$; Não paralelos, $d=0$,,

c) $\vec{n_1}=(1,1,1)$; $\vec{n_2}=(2,1,1) \rightarrow$ Não paralelos; $d=0$,,

15- Reta r: $y=1$, $z=5-x$; Ponto $P_r=(0,1,5)$; $\vec{v_r}=(1,0,-1)$; Reta s: $P_s=$ $(4,1,1)$; $\vec{v_s}=(4,2,-3)$; Para r, $t=4$, as retas se interceptam em $(4,1,1)$; $\vec{n_\pi}=(1,0,-1)\times(4,2,-3)=(2,-1,2)$; Eq. de π: $2(x-4)-1(y-1)+2(z-1)=0 \rightarrow$ $2x-y+2z-9=0$,, $\rightarrow d=|-9-d'|/\sqrt{4+1+4} \rightarrow |-9-d'|/\sqrt{9} \rightarrow$ $|-9-d'|/3=2 \rightarrow |-9-d'|=6$;

Valores de d': 1° $\rightarrow -9-d'=6 \rightarrow d'=-15$,, Plano: $2x-y+2z-15=0$,,

2° $\rightarrow -9-d'=-6 \rightarrow d'=-3$,, Plano: $2x-y+2z-3=0$,,