Lista 07 - Geometria Analítica - Letícia Santos Alves - 2025.1.08.016

01- a) $\vec{v} = (-2-5, 3-4, 2-1) = (-7, -1, 1)$,,

Eq P: $\begin{cases} x = 5 - 7t; \quad z = 2 + t; \\ y = 4 - t; \end{cases}$,, | Sim: $\dfrac{x-5}{-7} = \dfrac{y-4}{-1} = \dfrac{z-2}{1}$,,

b) $\vec{v} = (1-0, 0-(-1), 0-0) = (1, 1, 0)$,,

Eq P: $\begin{cases} x = 0 + t; \quad z = 0; \\ y = -1 + t; \end{cases}$,, | Sim = $\dfrac{x+0}{1} = \dfrac{y+1}{1} = \dfrac{z-0}{0} \Rightarrow \dfrac{x}{1} = \dfrac{y+1}{1}, z = 0$,,
↳ não existe

c) $\vec{v} = (0-0, 0-1, 0-(-1)) = (0, -1, 1)$,,

Eq P: $\begin{cases} x = 0; \quad z = -1 + t; \\ y = 1 - t; \end{cases}$,, | Sim = $\dfrac{x+0}{0} = \dfrac{y+1}{-1} = \dfrac{z-1}{1} \Rightarrow x = 0, \dfrac{y-1}{-1} = \dfrac{z+1}{1}$,,

d) $\vec{v} = (6-3, 1-2, -4-1) = (3, -1, -5)$,,

Eq P: $\begin{cases} x = 3 + 3t; \quad z = 1 - 5t; \\ y = 2 - t; \end{cases}$ | Sim = $\dfrac{x-3}{3} = \dfrac{y-2}{-1} = \dfrac{z-1}{-5}$,,

02- a) $\begin{cases} x = 1 - \lambda \\ y = \lambda \\ z = 4 + 2\lambda \end{cases}$ | Para $\lambda = 0 \Rightarrow x = 1; y = 0; z = 4;$,, $(1, 0, 4)$,,
Para $\lambda = 1 \Rightarrow x = 0; y = 1; z = 6;$,, $(0, 1, 6)$,,
$\vec{v} = (-1, 1, 2)$,, $\vec{v}_2 = 2 \cdot \vec{v} \Rightarrow (-2, 2, 4)$,,

b) $P = (1, 3, -3) \Rightarrow y = \lambda = 3; x = 1 - 3 = -2; x \neq 1$, ou seja, $P \notin r$;
$Q = (-3, 4, 12) \Rightarrow y = \lambda = 4; x = 1 - 4 = -3; z = 4 + 2.4 = 12; Q \in r$;

c) $\vec{v} = (-1, 1, 2); \Rightarrow$ Eq. P: $\begin{cases} x = 1 - t; \\ y = 4 + t; \\ z = -7 + 2t; \end{cases}$,,
$(1, 4, -7)$

03- a) $\vec{AB} = (-5-3, 2-6, 3-(-7)) = (-8, -4, 10)$,,

$\vec{AC} = (4-3, -7-6, -6-(-7)) = (1, -13, 1)$,,

$\frac{-8}{1} \neq \frac{-4}{-13} \neq \frac{10}{1}$,, $\to$ Logo, não são colineares, ou seja, podem ser vértices de um triangulo.

b) $M = \left(\frac{3+(-5)}{2}, \frac{6+2}{2}, \frac{-7+3}{2}\right) \to M_{AB} = (-1, 4, -2)$,,

$\vec{v} = M - C \to \vec{v} = (-1-4, 4-(-7), -2-(-6)) = (-5, 11, 4)$,,

$E_q\ r = (4, -7, -6) + t.(-5, 11, 4) \to$ Equação da reta,,

04- a) $C = (1+\lambda, 2+\lambda, -3\lambda)$,, $\to \vec{CA} \cdot \vec{CB} = 0 \to \vec{CA} = (0-(1+\lambda), 1-(2+\lambda), 8-(3\lambda)) \to \vec{CA} = (-1-\lambda, -1+\lambda, 8+3\lambda)$,, $\to \vec{CB} = (-3-(1+x), 0-(2+\lambda), 9-(-3\lambda)) \to \vec{CB} = (-4-\lambda, -2-\lambda, 9+3\lambda)$,,

$((-1-\lambda)(-4-\lambda) + (-1-\lambda)(-2-\lambda) + (8+3\lambda)(9+3\lambda)) = 0 \to ((4+\lambda+4\lambda+\lambda^2) + (2+\lambda+2\lambda+\lambda^2) + (72+24\lambda+27\lambda+9\lambda^2)) \to (4+5\lambda+\lambda^2 + 2+3\lambda+\lambda^2 + (72+51\lambda+9\lambda^2)) \to \underline{78+59\lambda+11\lambda^2 = 0}$,,

$\Delta = 59^2 - 4.78.11 \to \Delta = 3481 - 3492 \to \Delta = 49$,,

$\lambda = \frac{-59 \pm \sqrt{49}}{2.11} \to \frac{-59 \pm 7}{22} \to \lambda_1 = \frac{-26}{11}$,, $\lambda_2 = \frac{-66}{22} = -3$,,

$C = \left(1 - \frac{26}{11}, 2 - \frac{26}{11}, -3.\left(\frac{-26}{11}\right)\right) = \left(\frac{-15}{11}, \frac{-4}{11}, \frac{78}{11}\right)$,, ou

$C = (1-3, 2-3, -3.(-3)) = (-2, -1, 9)$,,

b) Ponto $= (1+\lambda, \lambda, \lambda) \to \sqrt{(1+\lambda-1)^2 + (\lambda-1)^2 + (\lambda-1)^2} = \sqrt{(1+\lambda-0)^2 + (\lambda-0)^2 + (\lambda-1)^2} \to \lambda^2 + (\lambda-1)^2 + (\lambda-1)^2 = (1+\lambda)^2 + \lambda^2 + (\lambda-1)^2 \to$

$\lambda = \frac{1}{2}$,, Ponto $= \left(\frac{3}{2}, \frac{1}{2}, \frac{1}{2}\right)$,, 05- a) $\vec{r}(s,t) = A + s\cdot\vec{v} + t\cdot\vec{u} \rightarrow \vec{r} = (1,2,0) + s(1,1,0) + t(2,3,-1)$

Eq. P: $\begin{cases} x = 1+s+2t; \quad z = 0-t; \\ y = 2+s+3t; \end{cases}$

b) $\vec{v} = B - A \rightarrow (1-1, -1-1, -1-0) = (0,-2,-1) \rightarrow \vec{r} = (1,1,0) + s(0,-2,-1) + t(2,1,0)$

$\begin{cases} x = 1+2t; \quad z = -s; \\ y = 1-2s+t; \end{cases}$

c) $\vec{u} = \vec{AB} \rightarrow (2-1, 1-0, -2-1) = (1,1,-3) \rightarrow \vec{v} = \vec{AC} \rightarrow (1-1, -1-0, 0-1) = (0,-1,-1) \rightarrow \vec{r} = (1,0,1) + s(1,1,-3) + t(0,-1,-1)$

Eq. P.: $\begin{cases} x = 1+s; \quad z = 1-3s-t; \\ y = s-t; \end{cases}$

06- a) $\vec{n} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (1,0,1)$ $a(x-x_0)+b(y-y_0)+c(z-z_0) = 0 \rightarrow$ $1(x-9)+0(y+1)-1(z-0)=0 \rightarrow x-9-z-0=0$

$x - z = 9$

b) $\vec{u} = \vec{AB} \rightarrow (-1-1, 0-0, 1-1) = (-2,0,0) \rightarrow v = (2-1, 1-0, 2-1) = (1,1,1)$

$\vec{n} = \begin{vmatrix} i & j & k \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,2,-2) \rightarrow 0(x-1)+2(y-0)-2(z-1) = 0 \rightarrow 2y - 2z + 2 = 0 \rightarrow y - z + 1 = 0$

c) $\vec{n} = (0,-2,-1) \times (2,1,0) = (1,-2,4) \rightarrow 1(x-1) - 2(y-1) + 4(z-0) \rightarrow$ $x - 1 - 2y + 2 + 4z = 0 \rightarrow x - 2y + 4z + 1 = 0 \rightarrow x - 2y + 4z = -1$

d) $\vec{v} = (1-0, -1-2, 1-2) = (1,-3,-1) \rightarrow \vec{n} = (1,1,-1) \times (1,-3,-1) = (-4,0,-4) \rightarrow$ $-4(x-1) + 0(y+1) - 4(z-1) = 0 \rightarrow -4x + 4 - 4z + 4 = 0 \rightarrow -4x - 4z + 8 = 0 \rightarrow$ $x + z - 2 = 0 \rightarrow x - z = +2$

07- a) $4x+2y-z=-5 \to z=4x+2y+5$; $x=\lambda$; $y=\mu$;

b) $5x-y+0z=1 \to y=5x-1$; $x=\lambda$; $z=\mu$;

c) $z=3$; $x=\lambda$; $y=\mu$

d) $y-z=2 \to y=z+2$; $x=\lambda$; $z=\mu$; $y=\mu+2$

08- a) $\vec{u}=(1,2,0)$; $\vec{v}=(-1,1,-1)$; $\vec{n}=\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ -1 & 1 & -1 \end{vmatrix}=(-2,1,3)$

$-2(x-1)+1(y-0)+3(z-3)=0 \to$

$-2x+2+y+3z-9=0 \to -2x+y+3x-7=0$

b) $P=(1,2,3)$; $\vec{u}=(1,0,-1)$; $\vec{v}=(0,0,1)$; $\vec{n}=\begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix}=(0,-1,0)$

$0(x-1)-1(y-2)+0(z-3)=0 \to -y+2=0$; $y=2$

c) $P=(-2,0,0)$; $\vec{u}=(1,2,1)$; $\vec{v}=(-1,2,1)$; $\vec{n}=\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 2 & 1 \end{vmatrix}=(0,-2,4)$

$0(x+2)-2(y-0)+4(z-0)=0 \to -2y+4z=0$; $y=2z$

09- a) $\begin{cases} 1+2\lambda=-1+4\mu; \to 1+2(-1+2\mu)=-1+4\mu \to 1-2+4\mu=-1+4\mu \\ \lambda=-1+2\mu; \to 1+3(-1+2\mu)=-2+6\mu \to 1-3+6\mu=-2+6\mu \to -2=-2 \\ 1+3\lambda=-2+6\mu \end{cases}$ As retas são coincidentes, não concorrentes.

b) $\begin{cases} 1+\lambda=2+3\mu; \to 1+\lambda=2+3(3\lambda-3) \to 1+\lambda=2+9\lambda-9 \to 8=8\lambda \to \lambda=1 \\ 1+2\lambda=3+2\mu; \quad 1+2(1)=3+2(0) \to 3=3 \to P=(1+1,1+2,0+3)= \\ 0+3\lambda=3+\mu; \quad (2,3,3) = \text{Ponto de intersecsão} \end{cases}$

$1(x-2)-2(y-3)+1(z-3)=0 \to x-2-2y+6+z-3=0 \to x-2y+z+1=0$

c) $\frac{x}{2}=\frac{y-1}{-2}=z=t \to \begin{cases} x=2t; \\ y=-2t+1; \\ z=t \end{cases}$ $\to x=2-4\lambda=2t$; $y=4+5\lambda=-2t+1$; $z=11=t$;

tilibra

$2-4\lambda = 2.11 = 22 \rightarrow 4\lambda = 20 \rightarrow \lambda = 5$; $y = 4 + 5(-5) = 4 - 25 = -21$;
$(22, -21, 11)$ $\rightarrow \vec{v}_r = (-4, 5, 0)$; $\vec{v}_s = (2, -2, 1)$; $\vec{n} = \begin{vmatrix} i & j & k \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix} = (5, 4, -2)$
$5(x-22) + 4(y+21) - 2(z-11) = 0 \rightarrow 5x + 4y$
$-2z = 110 - 84 + 22 \rightarrow \underline{5x + 4y - 2z = 48}$

d) $\begin{cases} 3t + 2 = \frac{v}{4} \\ 4t - 2 = \frac{v}{2} \\ t = \frac{v-3}{2} \end{cases}$ $\rightarrow 3\left(\frac{v-3}{2}\right) + 2 = \frac{v}{4} \rightarrow \frac{3v-9}{2} + 2 = \frac{v}{4} \rightarrow \frac{3v-9+4}{2} = \frac{v}{4} \rightarrow$
$\frac{3v-5}{2} = \frac{v}{4} \rightarrow 2(3v-5) = v \rightarrow 6v - 10 = v \rightarrow \underline{v = 2}$ $\rightarrow t = \frac{2-3}{2} =$
$-\frac{1}{2}$; $\rightarrow 4t - 2 = 4\left(-\frac{1}{2}\right) - 2 \rightarrow -2 - 2 = -4$;
As equações não são compatíveis, logo as retas não se cruzam

10 - a) $x + 2y + 3\lambda = 1 \rightarrow x + 2y = 1 - 3\lambda$; $-x + y + 2\lambda = 0 \rightarrow -x + y = -2\lambda$;
$(x + 2y) + (-x + y) = (1 - 3\lambda) + (-2\lambda) \rightarrow x - x + 2y + y = 1 - 5\lambda \rightarrow 3y = 1 - 5\lambda$
$-x + \frac{1-5\lambda}{3} = 2\lambda \rightarrow -x = -2\lambda - \frac{1-5\lambda}{3} \rightarrow -x = \frac{-6\lambda - (1 - 5\lambda)}{3} = \frac{-\lambda - 1}{3} \rightarrow x = \frac{\lambda + 1}{3}$
Equação $= \left(\frac{1}{3}, \frac{1}{3}, 0\right) + \lambda\left(\frac{1}{3}, -\frac{5}{3}, 1\right)$

b) $2x + 2y = 1 \rightarrow x + 1 = \frac{1}{2}$; $2z = 1 \rightarrow z = \frac{1}{2}$; $\rightarrow y = \frac{1}{2} - \lambda \rightarrow z = \frac{1}{2}$
$\vec{X} = \left(0, \frac{1}{2}, \frac{1}{2}\right) + \lambda(1, -1, 0)$

c) $2(3) - z + 1 = 0 \rightarrow \underline{z = 7}$ $\rightarrow \underline{\vec{X} = (3, 0, 7) + \lambda(0, 1, 0)}$

d) $\underline{\vec{X} = (0, 2, 0) + \lambda(1, 0, 0)}$

11 - a) $y + z = 3 \rightarrow z = 3 - y$; $x + y - (3 - y) = 6 \rightarrow x + y - 3 + y = 6 \rightarrow x + 2y = 9 \rightarrow$
$x = 9 - 2y$; $s: X = (9 - 2t, t, 3 - t)$; $\begin{cases} x = 1 - 2\lambda = 9 - 2t ; \quad z = 1 - \lambda = 3 - t ; \\ y = -1 + \lambda = t ; \end{cases}$
Retas paralelas distintas

b) $t = \frac{x+1}{2} \rightarrow x = 2t - 1$; $y = 3t$; $z = 2t - 1$; $\rightarrow r: X = (-1, 0, -1) + t(2, 3, 2)$;
$s: X = (0, 0, 0) + \lambda(1, 2, 0)$; Vetores não multiplos ; $\begin{cases} x = 2t - 1 = \lambda ; \\ y = 3t = 2\lambda ; \\ z = 2t - 1 = 0 ; \end{cases}$
Sistema incompatível!
Retas reversas

c) $r:(2,-1,3)$; $s:(1,-2,-2)$; $\rightarrow$ Não são paralelas;

$$\begin{cases} 8+2\lambda=3+\mu \\ 1-\lambda=-4-2\mu \\ 9+3\lambda=4-2\mu \end{cases} \quad \begin{vmatrix} 2\lambda-\mu=-5; \\ -\lambda+2\mu=-5; \\ 3\lambda+2\mu=-5; \end{vmatrix}$$

Retas concorrentes.

d) $r: X=(-1,0,0)+t(2,1,1)$; $\rightarrow 2x+2y-6z=2 \rightarrow 4x-4z=2 \rightarrow$
$x=\frac{1}{2}+z$, $\rightarrow y=3z-x+1 \rightarrow y=3z-\frac{1}{2}-z+1=2z+\frac{1}{2}$, $\rightarrow s: X=(\frac{1}{2}+t,$
$2t+\frac{1}{2}, t)$, $s:(1,2,1)$ $r(2,1,1)$, $\rightarrow$ não paralelas

$$\begin{cases} x=2\lambda-1=\frac{1}{2}+t; \; z=\lambda=t; \\ y=\lambda=2t+\frac{1}{2}; \end{cases}$$

Retas reversas.

12- a) $x=1$; $y=1+\lambda$; $z=\lambda$; $\rightarrow 1-(1+\lambda)-\lambda=2 \rightarrow 1-1-\lambda-\lambda=2 \rightarrow -2\lambda=$
$2 \rightarrow \lambda=-1$, ; $x=1$; $y=0$; $z=-1$; Transversais. $P=(1,0,-1)$,

b) $(2\lambda+1,\lambda,\lambda)=(3,0,1)+a(1,0,1)+b(2,2,0) \rightarrow \begin{cases} 2\lambda+1=3+a+2b \\ \lambda=2b; \; \lambda=1+a; \end{cases} \rightarrow$
$\lambda=2b \rightarrow 2b=1+a \rightarrow a=2b-1$; $\rightarrow 2(2b)+1=3+(2b-1)+2b$
$\rightarrow 4b+1=3+2b-1+2b=4b+2 \rightarrow 1\neq 2$; Retas reversas;

c) $r: \begin{cases} x-y=1 \\ x-2y=0 \end{cases}$ $\rightarrow (x-y)-(x-2y)=1-0 \rightarrow y=-1$; $x=2$; $\vec{v}=(0,0,1)$;
$\pi=x+y=2 \rightarrow x+y=3 \rightarrow 3\neq 2$,
$\vec{n}=(1,1,0) \rightarrow 0.1+0.1+1.0=0$, $\rightarrow$ Paralela distinta.

d) $\begin{cases} x-2y=3-2z+y \\ 3-2z+y=2x-z \end{cases}$ $\rightarrow$ $\begin{matrix} x-3y+2=3 \\ \rightarrow -2x+y-z=-3 \end{matrix}$ $\vec{v}=\begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ -2 & 1 & -1 \end{vmatrix}=(1,-3,-5)$;

$1.(-1)+(-3).2+(-5).(-1)=-1-6+5=-2$, $\rightarrow$ Transversais.

$$\begin{cases} x-3y=3-2\lambda \\ -2x+y=-3+\lambda \end{cases} \quad x=\frac{6}{5}-\frac{1}{5}\lambda; \; y=-\frac{3}{5}+\frac{7}{5}\lambda;$$

13- a) $\vec{v}_1 = (1,2,0)$; $\vec{v}_2 = (1,0,1)$; $\rightarrow \begin{cases} a + b = 2; \quad b = 1; \rightarrow a + 1 = 2 \rightarrow \underline{a = 1} \\ 2a = m; \quad 2.(1) = m \rightarrow \underline{m = 2} \end{cases}$

b) $n - 3(2) + 0 = 1 \rightarrow \underline{n = 7} \rightarrow (2, m, m) \cdot (1, -3, 1) = 2 - 3m + m \rightarrow 2 - 2m = 0$
$\rightarrow -2m = -2 \rightarrow \underline{m = 1}$

c) $\begin{cases} x = m\lambda + 1; \\ y = 2\lambda; \ z = m\lambda \end{cases}$ $\vec{X} = (1,0,0) + \lambda(m,2,m)$; Plano: $x + my + z = 0 \ (1, m, 1)$
$(m,2,m) \cdot (1,m,1) = m + 2m + m = 0 \rightarrow 4m \neq 0 \rightarrow \underline{m \neq 0}$

14- a) $\vec{n}_1 = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = (4-0)-(4-0)(1-0) = (4,-4,1)$; $\vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix} = (-5, 5, 0)$

$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ -5 & 5 & 0 \\ 4 & -4 & 1 \end{vmatrix} = (5, 5, -20) \rightarrow P = (4,2,4) \rightarrow$ Planos transversais
$\vec{X} = (4,2,4) + t(1,1,-4)$

b) $\vec{n}_1 = (1,-1,2)$, $\vec{n}_2 = (1,0,3) \times (-1,1,1) = (-3,-4,1) \rightarrow$ Não paralelas
$\vec{d} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -3 & -4 & 1 \end{vmatrix} = (-1+8)-(1+6)(-4-3) = (7,7,-7) = (1,1,-1) \rightarrow$ São transversais
$\vec{X} = (0,0,1) + t(1,1,-1)$

c) $\frac{4}{2} = 2$, $\frac{-2}{-1} = 2$, $\frac{2}{1} = 2$, $\frac{-9}{-1} \neq 2 \rightarrow$ Paralelas distintas

d) $\vec{AB} = (5-0, 0-1, 1-6) = (5,-1,-5)$; $\vec{AC} = (4-0, 0-1, 0-6) = (4,-1,-6)$
$\vec{n}_1 = \begin{vmatrix} i & j & k \\ 5 & -1 & -5 \\ 4 & -1 & -6 \end{vmatrix} = (1, 10, -1) \rightarrow (x-0) + 10(y-1) - (z-6) = 0 \rightarrow x + 10y - z - 4 = 0 \rightarrow$
$\pi_2 = x + 10y - z - 4 = 0$ Planos coincidentes

15- a) $\vec{n}_1 = \begin{vmatrix} i & j & k \\ -1 & m & 1 \\ 2 & 0 & 1 \end{vmatrix} = (m, 3, -2m)$; $\vec{n}_2 = \begin{vmatrix} i & j & k \\ m & 1 & 0 \\ 1 & 0 & m \end{vmatrix} = (m, -m^2, -1) \rightarrow$
$(m, 3, -2m) = \lambda(m, -m^2, -1) \rightarrow m = \lambda m \rightarrow \lambda = 1$ (se $m \neq 0$) $\rightarrow 3m \neq -m^2$
Os planos são transversais qualquer que seja $m \in \mathbb{R}$

b) $\vec{n_1} = (1.m-1.1,\ 1.1-m.m,\ m.1-1.1) = (m-1,\ 1-m^2,\ m-1)$, $\vec{n_2} = (2,3,2)$,

$\frac{m-1}{2} = \frac{1-m^2}{3} = \frac{m-1}{2} \to 3(m-1) = 2(1-m^2) \to 3m-3 = 2-2m^2 \to 2m^2+3m-5=0 \to$

$\Delta = 3^2 - 4.2.(-5) = 9+40 = 49 \to x = \frac{-3\pm 7}{4} \to x_1 = 1,\ x_2 = \frac{-10}{4} = \frac{-5}{2}$

Para $m=1 \to \vec{n_1} = (0,0,0) \to$ inválido,

Para $m = -5/2 \to \vec{n_1} = \left(-\frac{7}{2}; -\frac{21}{4}, -\frac{7}{2}\right)$; Logo, o ponto $(1,1,0)$ de π_1 não deve pertencer a π_2: $2(1)+3(1)+2(0)+n \neq 0 \to 5+n \neq 0 \to n \neq -5$.

$m = -5/2$; $n \neq -5$